Opportunistic Multiple Relay Selection for Two-Way Relay Networks with Outdated Channel State Information

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Abstract

Outdated Channel State Information (CSI) was proved to have negative effect on performance in two-way relay networks. The diversity order of widely used opportunistic relay selection (ORS) was degraded to unity in networks with outdated CSI. This paper proposed a multiple relay selection scheme for amplify-and-forward (AF) based two-way relay networks (TWRN) with outdated CSI. In this scheme, two sources exchange information through more than one relays. We firstly select N best relays out of all candidate relays with respect to signal-noise ratio (SNR). Then, the ratios of the SNRs on the rest of the candidate relays to that of the Nth highest SNR are tested against a normalized threshold $\mu \in [0, 1]$, and only those relays passing this test are selected in addition to the N best relays. Expressions of outage probability, average bit error rate (BER) and ergodic channel capacity were obtained in closed-form for the proposed scheme. Numerical results and Simulations verified our theoretical analyses, and showed that the proposed scheme had significant gains comparing with conventional ORS.

Keywords: Multiple relay selection, Two-way relay networks, Outdated CSI, Amplify-and-forward, Outage probability, BER, Ergodic channel capacity

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1. Introduction

Cooperative communications has been widely studied recently for its ability to enhance throughput and reliability of wireless networks [1]. Two-way relaying obtains lots of interests for its spectral efficiency [2]. Several network-coding schemes have been presented for two-way relay systems, such as amplify-and-forward (AF), decode-and-forward (DF), etc. Two-way AF relaying is more practically attractive than DF relaying due to the simple processing at the relay node. The received signal-to-noise ratio (SNR) expression of two-way AF relaying had been given in [3] and the achievable sum rate had been derived in [2].

Relay selection (RS) protocol is a widely used method in Cooperative communications to improve system performance. Most of RS schemes select only one best relay to forward the information through a certain criterion [4]–[8]. Sum rate, symbol error rate and outage probability of single relay selection based on maximizing the minimum receiving signal-to-noise ratio were analyzed in [4]-[5]. And some other criteria such as maximizing sum rate, minimizing outage probability and maximizing mutual information were proposed in [6]-[8]. Authors in [9] studied the diversity-multiplexing tradeoff for two-way relaying networks with all and partial CSI knowledge.

The aforementioned literatures are all based on perfect CSI knowledge. However, in a highly dynamic network, the delay between selection process and transmit process cannot be ignored [10]. In this case, the selected best relay may not remain the best at the time of transmission. Therefore, recently, study on the impact of outdated CSI attracted lots of attention. In [11] and [12], the effects of outdated CSI on outage probability and error rate of single relay selection were studied for both AF and DF relaying systems. Two variations of relay selection, named best relay selection and partial relay selection, were considered under the outdated CSI condition in [11] and [13]. Both strategies selected only one relay for information forwarding. Authors in [14] analyzed BER of BPSK, M-PSK and square M-QAM modulations in AF relay networks with outdated CSI. All of investigations mentioned in [11]-[14] were focusing on performance analyses of one-way relaying networks. In [15] and [16] settings were extended to two-way scenarios. Authors in [15] studied performance of bidirectional cooperative networks in the presence of imperfect CSI for outdated channel models with different correlation coefficient. In [16], the Nth best relay was selected to assist two sources to exchange signals, then based on the proposed algorithm an asymptotic expression of outage probability for high SNR was derived. These studies show that only one relay is inadequate for cooperative communications with outdated CSI, and diversity order is degraded to unity.

To combat the severe diversity loss due to outdated CSI, appropriate multiple relay selection (MRS) schemes are required. In [17], the authors propose a novel optimal method to complete the information exchange by selecting multiple best sub-channels out of all channels. Authors in [18] and [19] investigated MRS schemes. More than one relays were allowed to participate in second phase of cooperative communications, which brought a significant performance gain comparing with conventional ORS. However, all participating relays had to adjust the phases of their transmission signals precisely, since the end node received multiple signals from different relays simultaneously, which was difficult to realize in actual communications. Authors in [20] proposed an N+NT-ORS scheme for one-way relaying networks. This scheme selected N or more relays through a preset criterion. A novel approach based on partitioning the probability space by an anchor element was used to analyze system performance, which was proposed in [21]. It obviated nested integrals originated from ordering all of the instantaneous SNR’s, which made formulae simpler.

The MRS schemes are rarely studied in TWRN with outdated CSI. To the best of our knowledge, there has been no open published paper working on analyzing performance for this setup, which has motivated our work. Our main contributions in this paper are as follows. Firstly, the MRS scheme in [20] is extended to bidirectional relay systems with outdated CSI. The extension is not straightforward since there are two communication tasks for two-way relay networks, each task with its own end-to-end SNR. The fairness and performance of overall networks are under consideration. The closed-form expressions of BER and ergodic channel capacity, as well as outage probability, are derived for TWRN with outdated CSI. Numerical results and simulations verified our theoretical analyses, and showed that the proposed scheme had significant gains comparing with conventional ORS.

The rest of this paper is organized as follows. In Section II, we present the system model. In Section III, we propose an N+NT MRS scheme for two-way system with outdated CSI, and analyze performance of the proposed scheme. Numerical simulations are presented in Section IV and Section V concludes the paper.

2. System Model

We consider a two-way cooperative system with two sources and K relays. The two sources tend to exchange information over Rayleigh flat-fading channels with the help of relay nodes. We use S_1, S_2 and $\boldsymbol{R}_i$ to denote the first source, the second source, and the ith relay, respectively. The sources select one or more relays from all candidate relays to assist information exchanging. All nodes are equipped with single antenna, and operated in a half-duplex mode. The fading coefficients between S_1 and R_i, S_2 and R_i are denoted as $h_{1,i}$, $h_{2,i}$ respectively. All channels are assumed to be independent and identically distributed (i.i.d) complex Gaussian fading with distributions $h_{1,i} \sim CN(0,\Omega_{1,i})$ and $h_{2,i} \sim CN(0,\Omega_{2,i})$. Equal power allocation scheme is used for ease of analysis, which means the power budget at each node is the same. Extension to unequal power allocation and independent and non-identically distributed (i.n.i.d) channels can be made following a similar approach presented herein. We assume that there is no direct path between sources due to direct line-of-sight blockage.

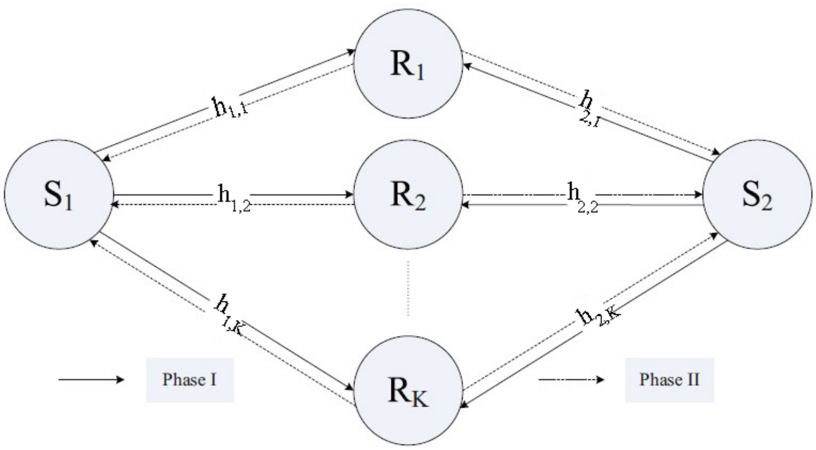

Fig. 1. System Model

The protocol shown in **Fig. 1**, which works under time division duplex (TDD) mode, takes two phases to exchange information. At the first phase, S_1 and S_2 send signals to all candidate relay nodes. The received signal at *i*th relay is expressed as

$$x_i = \sqrt{P}h_{1,i}x_1 + \sqrt{P}h_{2,i}x_2 + n_i, \tag{1}$$

where x_1, x_2 are transmitted symbols from S_1 and S_2, n_i is additive white Gaussian noise at relay *i* with double-sided spectrum density of $N_0/2$.

At the second phase, partial relays are selected from all candidates according to their channel gains. Then these selected relays amplify x_i with coefficient ρ_i and broadcast it to S_1 and S_2. The amplifying coefficient ρ_i can be chosen as

$$\rho_i = \sqrt{\frac{P}{P\left|h_{1,i}\right|^2 + P\left|h_{2,i}\right|^2 + N_0}}. \tag{2}$$

It is approximated in the following derivation as

$$\rho_i = \sqrt{\frac{1}{\left|h_{1,i}\right|^2 + \left|h_{2,i}\right|^2}}, \tag{3}$$

which has been proved very accurate to the exact value for the entire SNR region [22]-[23].

Considering channel reciprocity, the received signals at S_1 and S_2 can be expressed as

$$y_{i1} = \sqrt{P}\rho_i h_{1,i}^2 x_1 + \sqrt{P}\rho_i h_{1,i}h_{2,i}x_2 + \rho_i h_{1,i}n_i + n_1, \tag{4}$$

$$y_{i2} = \sqrt{P}\rho_i h_{2,i}^2 x_2 + \sqrt{P}\rho_i h_{1,i}h_{2,i}x_1 + \rho_i h_{2,i}n_i + n_2, \tag{5}$$

where n_1, n_2 denote additive white Gaussian noise at S_1, S_2 respectively. We can subtract the self-interference from y_{i1} and y_{i2}, then obtain

$$\tilde{y}_{i1} = y_{i1} - \sqrt{P}\rho_i h_{1,i}^2 x_1 = \sqrt{P}\rho_i h_{1,i}h_{2,i}x_2 + \rho_i h_{1,i}n_i + n_1, \tag{6}$$

$$\tilde{y}_{i2} = y_{i2} - \sqrt{P}\rho_i h_{2,i}^2 x_2 = \sqrt{P}\rho_i h_{1,i}h_{2,i}x_1 + \rho_i h_{2,i}n_i + n_2. \tag{7}$$

The sources can obtain signals x_2 and x_1 by decoding the received signals $\tilde{y}_{i1}$ and $\tilde{y}_{i2}$.

Using $\gamma_{A,B} = P\left|h_{A,B}\right|^2 / N_0$ to represent instantaneous SNR between A and B, the instantaneous SNR at S_1 and S_2 can be respectively expressed as

$$\gamma_{i1} = \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i} + \gamma_{2,i}}, \tag{8}$$

$$\gamma_{i2} = \frac{\gamma_{1,i}\gamma_{2,i}}{\gamma_{1,i} + 2\gamma_{2,i}}. \tag{9}$$

Due to the possible dynamic delay between selection process and transmit process, the selection based on instantaneous CSI is outdated. Thus it is possible that the best relays selected before cannot remain the best during transmit process. Therefore, it is necessary to study the impact of this delay in two-way relay networks. We define $\hat{h}_{A,B}$ as delayed version of the instantaneous CSI $h_{A,B}$ at selection instant. $h_{A,B}$ and $\hat{h}_{A,B}$ are called instantaneous CSI and outdated CSI for convenience in the next statement. The outdated CSI obviously follow the same distribution as the instantaneous CSI. So we can obtain $h_{A,B} \sim CN(0,\Omega_{A,B})$, and $h_{A,B}$ conditioned on $\hat{h}_{A,B}$ follows a Gaussian distribution [10]:

$$h_{A,B} \mid \hat{h}_{A,B} \sim CN(\rho\hat{h}_{A,B}, 1-\rho^2), \tag{10}$$

where ρ is the correlation coefficient between $h_{A,B}$ and $\hat{h}_{A,B}$. Jake's model is adopted to represent the outdated CSI in the following research, $\rho = J_0(2\pi \mathrm{f}_{d_k} \mathrm{T}_{D_k})$, where f_{d_k} represents Doppler frequency, T_{D_k} is the delay between selection process and transmission process, and $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind. According to [10], instantaneous SNR conditioned on its delayed version follows a non-central chi-square distribution with two degrees of freedom, whose PDF is expressed as

$$f_{\gamma_{A,B}|\hat{\gamma}_{A,B}}(\gamma_{A,B} \mid \hat{\gamma}_{A,B}) = \frac{1}{\bar{\gamma}_{A,B}(1-\rho^2)} \exp\left(\frac{-(\gamma_{A,B}+\rho^2\hat{\gamma}_{A,B})}{\bar{\gamma}_{A,B}(1-\rho^2)}\right) \times \mathrm{I}_0\left(\frac{2\sqrt{\rho^2\gamma_{A,B}\hat{\gamma}_{A,B}}}{\bar{\gamma}_{A,B}(1-\rho^2)}\right), \tag{11}$$

where the delayed SNR is denoted as $\hat{\gamma}_{A,B} = P\left|\hat{h}_{A,B}\right|^2 / N_0$ with expectation

$\bar{\gamma}_{A,B} = P\Omega_{A,B} / N_0$ and $I_0(\cdot)$ stands for the zero-order modified Bessel function of the first kind.

3. Performance Analyses

Since the scheme presented in this paper considers bidirectional networks consisted of two communication tasks, the smaller SNR of each sub-channel, which denoted as $\gamma_i = \min(\gamma_{i1}, \gamma_{i2})$, dominates the end-to-end performance (similar to max-min relay selection in [2]). Thus we use γ_i as the equivalent SNR of the ith sub-channel during ordering process and selection process. We first obtain probability density function (PDF) and cumulative density function (CDF) of γ_i for a certain two-way sub-channel $S_1 - R_i - S_2$. Then we derive the outage probability, BER and ergodic channel capacity of the whole system on the basis of equivalent SNR.

3.1 PDF and CDF of Equivalent SNR

Since the considered two-way sub-channel is a multi-source system, either source's outage will cause outage of the whole system. Thus we can express the CDF of equivalent SNR γ_i of the ith two-way channel as

$$\begin{aligned} F_{\gamma_i}(\gamma) &= \Pr(\gamma_{i1} < \gamma \, or \, \gamma_{i2} < \gamma) \\ &= \Pr(\min(\gamma_{i1}, \gamma_{i2}) < \gamma) \\ &= 1 - \Pr(\min(\gamma_{i1}, \gamma_{i2}) > \gamma) \end{aligned}$$

$$= 1 - \underbrace{\Pr\left(\frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i}+\gamma_{2,i}} > \gamma, \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i}+\gamma_{2,i}} < \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{2,i}+\gamma_{1,i}}\right)}_{\Pr 1}$$

$$- \underbrace{\Pr\left(\frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{2,i}+\gamma_{1,i}} > \gamma, \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i}+\gamma_{2,i}} > \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{2,i}+\gamma_{1,i}}\right)}_{\Pr 2}. \tag{12}$$

The probability of the first part in (12) can be evaluated in the following way

$$\begin{aligned}
&\Pr\left(\frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i}+\gamma_{2,i}} > \gamma, \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i}+\gamma_{2,i}} < \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{2,i}+\gamma_{1,i}}\right) \\
&\quad < \Pr\left(\min(2\gamma_{1,i}, \gamma_{2,i}) > 2\gamma, \gamma_{1,i} > \gamma_{2,i}\right) \\
&\quad = \Pr(\gamma_{2,i} > 2\gamma, \gamma_{1,i} > \gamma_{2,i}) \\
&\quad = \frac{\bar{\gamma}_{1,i}}{\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i}} \exp\left(-\frac{2(\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i})\gamma}{\bar{\gamma}_{1,i}\bar{\gamma}_{2,i}}\right).
\end{aligned} \tag{13}$$

Similarly, the probability of the second part in (12) can be solved as follows:

$$\begin{aligned}
&\Pr\left(\frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{2,i}+\gamma_{1,i}} > \gamma, \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{1,i}+\gamma_{2,i}} > \frac{\gamma_{1,i}\gamma_{2,i}}{2\gamma_{2,i}+\gamma_{1,i}}\right) \\
&\quad < \frac{\bar{\gamma}_{2,i}}{\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i}} \exp\left(-\frac{2(\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i})\gamma}{\bar{\gamma}_{1,i}\bar{\gamma}_{2,i}}\right).
\end{aligned} \tag{14}$$

By substituting (13) and (14) into (12), we obtain the lower bound of CDF of γ_i as

$$\begin{aligned}
&F_{\gamma_i}(\gamma) > \tilde{F}_{\gamma_i}(\gamma) \\
&\quad = 1 - \exp\left(-\frac{2(\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i})\gamma}{\bar{\gamma}_{1,i}\bar{\gamma}_{2,i}}\right).
\end{aligned} \tag{15}$$

The lower bound is proved to be tight with actual value in [24]. Thus, PDF of γ_i can be obtained by differentiating (15) as

$$f_{\gamma_i}(\gamma) > \tilde{f}_{\gamma_i}(\gamma) = \frac{\bar{\gamma}_{1,i}\bar{\gamma}_{2,i}}{2(\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i})} \exp\left(-\frac{2(\bar{\gamma}_{1,i}+\bar{\gamma}_{2,i})\gamma}{\bar{\gamma}_{1,i}\bar{\gamma}_{2,i}}\right). \tag{16}$$

3.2 N+NT MRS Scheme

Before derivation, the details of N+NT MRS scheme for two-way relay networks are described as follows. With the available CSI is outdated at the *i*th relay, the proposed scheme firstly arranges all of equivalent SNR $\gamma_i = \min(\gamma_{i1}, \gamma_{i2})$ in descending order. Thereafter, it opportunistically selects *N* (*N*<*K*) best relays from this ordered set. Finally, the ratio of the SNR on the remaining relays to that of the Nth highest SNR relay is tested against a normalized threshold, $\mu \in [0,1]$, and only those relays passed the test are selected in addition to the N best relays.

The proposed N+NT MRS scheme is a generalized selection scheme. When $\mu = 1$, this scheme is simplified as MRS scheme with N best relays. If *N*=1, this scheme turns to be widely used ORS scheme. If *N*=*K*, it becomes all relays selection scheme. With the help of μ, this scheme is able to ensure that a proper number of relays are selected [20].

Of all independent individual equivalent SNRs $\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_K$, the *N*th largest SNR $\hat{\gamma}_N$ is chosen to be the anchor element. Due to maximum ratio combining (MRC) at the receiver, the received equivalent SNR is $\gamma_{sum} = \sum_{k=1}^{K} T(\hat{\gamma}_k)$, where $T(\hat{\gamma}_k)$ is given by

$$T(\hat{\gamma}_k) = \begin{cases} 0, & \hat{\gamma}_k < \mu \hat{\gamma}_N \\ \gamma_k, & \hat{\gamma}_k \geq \mu \hat{\gamma}_N \end{cases}. \quad (17)$$

To obtain the MGF of γ_{sum}, we use the probability space partition approach proposed in [21]. We define a set Φ_i (i=1, 2…K), as $\hat{\gamma}_i$ is the *N*th largest element in the collection of $\hat{\gamma}_1, \hat{\gamma}_2 ... \hat{\gamma}_K$, which can denoted as

$$\begin{aligned} &\boldsymbol{\Phi_i : \{\hat{\gamma}_1, \hat{\gamma}_2 ... \hat{\gamma}_K \mid \exists \mid I \mid = N-1, \text{s.t.}} \\ &\boldsymbol{k \in I \Leftrightarrow (\hat{\gamma}_k > \hat{\gamma}_i) or (\hat{\gamma}_k = \hat{\gamma}_i, k > i)\}}, \end{aligned} \quad (18)$$

where $I \subset \{1, 2, ..., \text{K}\} - \{\text{i}\}$ is an index set that consists of indices of relays that equivalent SNR larger than $\hat{\gamma}_i$.

Following the analyses in [20] and [21], the MGF of γ_{sum} can be expressed as

$$\begin{aligned} M_{\gamma_{sum}}(s) &= \sum_{i=1}^{K} \text{E}_{\Phi_i}[\exp(s \sum_{k=1}^{K} T(\gamma_k))] \\ &= \sum_{i=1}^{K} \int_0^{\infty} \sum_{All\ I} \underbrace{\int_0^{\infty} ... \int_0^{\infty}}_{K} \times \underbrace{\int_0^{\hat{\gamma}_i} ... \int_0^{\hat{\gamma}_i}}_{K-N} \\ &\times \underbrace{\int_{\hat{\gamma}_i}^{\infty} ... \int_{\hat{\gamma}_i}^{\infty}}_{N-1} \prod_{k=1}^{K} [\exp(sT(\hat{\gamma}_k) f_{\gamma_k|\hat{\gamma}_k}(\gamma_k \mid \hat{\gamma}_k) f_{\hat{\gamma}_k}(\hat{\gamma}_k)] \\ &\quad d\hat{\gamma}_{k_1} ... d\hat{\gamma}_{k_{N-1}} d\hat{\gamma}_{k'_1} ... d\hat{\gamma}_{k'_{K-N}} d\gamma_1 ... d\gamma_K d\hat{\gamma}_i, \end{aligned} \quad (19)$$

where $k_l \in I$, and $k'_l \in \{1, 2, ..., \text{K}\} - \{\text{i}\} - \text{I}$.

For i.i.d channels, the calculation can be simplified by using [25, eqs. (6.614.3), (9.220.2), and (9.215.1)]. After substituting (16) into (19), we can obtain:

$$\begin{aligned} M_{\gamma_{sum}}^{i.i.d}(s) &= K \binom{K-1}{N-1} \sum_{k=0}^{K-N} \sum_{l=0}^{K-N-k} \sum_{m=0}^{k} \left(\frac{1}{1 - s\bar{\gamma}_{two-way}} \right)^{K-1-k} \\ &\times \frac{(K-N)!(-1)^{l+m}}{(\text{K}-\text{N}-\text{k}-l)!\text{m}!l!(\text{k}-\text{m})!} \end{aligned}$$

$$\times \frac{1}{(1-s\overline{\gamma}_{two-way})[(\mathrm{K}-\mathrm{N}-\mathrm{k}-l)\mu+\mathrm{q}+N]+\mathrm{m}\,\mu(1-(1-\rho^2)s\overline{\gamma}_{two-way})},(20)$$

where $\overline{\gamma}_{two-way} = \overline{\gamma}_{1,i}\overline{\gamma}_{2,i} / (2(\overline{\gamma}_{1,i} + \overline{\gamma}_{2,i}))$.

The number of selected relays J is a discrete random variable with range 1<*J*<*K*. It denotes the actual number of relays used in a certain transmission.

$\overline{J}$, which represents the average of *J*, can be obtained easily by substituting the following formula into (19)

$$T'(\hat{\gamma}_k) = \begin{cases} 0, & \hat{\gamma}_k < \mu\hat{\gamma}_N \\ 1, & \hat{\gamma}_k \geq \mu\hat{\gamma}_N \end{cases}. \tag{21}$$

After some manipulation, we can calculate $\overline{J}$ as

$$\overline{J} = K(K-N)\binom{K-1}{N-1}\sum_{l=0}^{K-N-1}\frac{(-1)^l}{N+l+\mu}\binom{K-N-1}{l}. \tag{22}$$

The outage probability of bidirectional system is defined as the probability that the mutual information of each direction is lower than *R*/2 while the required rate of whole system is set to *R*. The mutual information is given by $I = 1/(J+1)\log_2(1+\gamma_{sum})$. Therefore, the outage probability is calculated by

$$P_{out}(J) = \Pr(I < R/2) = \Pr(\gamma_{sum} < 2^{(1+J)R/2} - 1). \tag{23}$$

Since J is discrete random variable, the outage probability can be calculated as the following long-term average

$$\mathrm{E}[P_{out}(J)] = \sum_{j=1}^{K}\Pr(J=j)P_{out}(J). \tag{24}$$

From (22), we can find that $\overline{J}$ is independent of γ, thus we can obtain the following formula:

$$\begin{aligned} P_{out} &= \mathrm{E}[P_{out}(J)] = \Pr(\gamma_{sum} < 2^{(1+\mathrm{E}[J])R/2} - 1) \\ &= \Pr(\gamma_{sum} < 2^{(1+\overline{J})R/2} - 1). \end{aligned} \tag{25}$$

It is obviously that one can evaluate the outage probability by a given MGF. Here we use a simple algorithm proposed in [26] to approximate the outage probability. This algorithm can obtain lower computational complexity and high accuracy. With the MGF in (20), we can obtain

$$\begin{aligned} P_{out} &= \frac{2^{-Q}\exp(A/2)}{\gamma_{th}}\sum_{q=0}^{Q}\binom{Q}{q}\sum_{n=0}^{M+q}\frac{(-1)^n}{\beta_n} \\ &\times R\left\{\frac{M_\gamma\left(-\frac{A+2\pi jn}{2\gamma_{th}}\right)}{\frac{A+2\pi jn}{2\gamma_{th}}}\right\} + E(A,M,Q), \end{aligned} \tag{26}$$

where $\gamma_{th} = 2^{(1+\overline{J})R/2} - 1$, $E(A,M,Q)$ represents overall error term, which decreases to 10^{-10} while the parameters are set to $A = 10\ln 10$, *M*=21, *Q*=15. And β_n defined as

$$\beta_n = \begin{cases} 2, & n = 0 \\ 1, & n = 1, 2, \ldots, N \end{cases} . \tag{27}$$

Substituting (20) into (26), we can obtain the expression of bidirectional system outage probability of N+NT MRS in closed-form.

In order to further calculation, we obtain PDF expression for TWRN with outdated CSI as follows:

$$f_{\gamma_{sum}}(\gamma) = K\binom{K-1}{N-1}\sum_{k=0}^{K-N}\sum_{l=0}^{K-N-k}\sum_{m=0}^{k}\frac{(K-N)!(-1)^{l+m}}{(\mathrm{K-N-k-}l)!\mathrm{m}!l!(\mathrm{k-m})!} \times\left[\frac{\alpha\tau}{\beta\overline{\gamma}_{two-way}}\exp\left(\frac{-\alpha\gamma}{\beta\overline{\gamma}_{two-way}}\right)+\sum_{j=1}^{K-1-k}\frac{\varphi_j\gamma^{j-1}}{(j-1)!\overline{\gamma}_{two-way}^{j}}\exp\left(\frac{-\gamma}{\overline{\gamma}_{two-way}}\right)\right]. \tag{28}$$

Proof: See Appendix A.

The BER of the two-way MRS scheme can be obtained by

$$P_{ber}(e) = \int_0^\infty P_{ber}(e\,|\,\gamma) f_{\gamma_{sum}}(\gamma)d\gamma . \tag{29}$$

Error probability conditioned on a specific SNR can be denoted as $P_{ber}(e\,|\,\gamma) = Aerfc(\sqrt{B\gamma})$, where the values A and B depend on modulation in practical systems. Therefore, the total error probability can be expressed as

$$P_{ber}(e) = A\int_0^\infty erfc(\sqrt{B\gamma}) f_{\gamma_{sum}}(\gamma)d\gamma . \tag{30}$$

By substituting (28) into (30) and solving the integration, the exact closed-form expression for the error probability of N+NT MRS scheme can be obtained as

$$P_{ber}(e) = K\binom{K-1}{N-1}\sum_{k=0}^{K-N}\sum_{l=0}^{K-N-k}\sum_{m=0}^{k}\frac{(K-N)!(-1)^{l+m}}{(\mathrm{K-N-k-}l)!\mathrm{m}!l!(\mathrm{k-m})!} \times\left[\tau\eta(\beta\overline{\gamma}_{two-way}/\alpha)+\sum_{j=1}^{K-1-k}\varphi_j\delta(j)\right], \tag{31}$$

where $\eta(x) = A\left[1-\sqrt{\frac{Bx}{1+Bx}}\right]$, $\delta(x) = A\left[1-I_0(c)\sum_{q=0}^{x-1}\binom{2q}{q}\left(\frac{1-I_0^2(c)}{4}\right)^q\right]$, and

$$I_0(c) = \sqrt{\frac{Bc}{1+Bc}} .$$

Channel capacity is an important performance metric since it provides the maximum achievable transmitting rate under which the errors are recoverable. The ergodic channel capacity can be expressed as

$$\begin{aligned}\overline{C} &= \mathrm{E}\left(\frac{1}{1+J}\log_2(1+\gamma_{sum})\right) \\ &= \frac{1}{1+\overline{J}}\int_0^\infty \log_2(1+\gamma) f_{\gamma_{sum}}(\gamma)d\gamma .\end{aligned} \tag{32}$$

By substituting (28) into (32) and solving the integration with the help of [27, eq. (78)], the ergodic channel capacity can be obtained in closed-form as

$$\bar{C} = K\begin{pmatrix} K-1 \\ N-1 \end{pmatrix}\sum_{k=0}^{K-N}\sum_{l=0}^{K-N-k}\sum_{m=0}^{k}\frac{(K-N)!(-1)^{l+m}}{(\mathrm{K-N-k-}l)!\mathrm{m}!l!(\mathrm{k-m})!} \times \frac{1}{1+\bar{J}}\left[\tau\exp\left(\frac{1}{\bar{\gamma}_{two-way}}\right)\omega\left(\frac{1}{\bar{\gamma}_{two-way}}\right)+\sum_{j=1}^{K-1-k}\varphi_j\phi(j)\right], \qquad (33)$$

where $\omega(x) = \int_x^\infty \frac{\exp(-t)}{t}dt$, $\phi(j) = \bar{\gamma}_{two-way}^{-j}\exp(1/\bar{\gamma}_{two-way})\sum_{k=1}^{j}\frac{\Gamma(k-j,1/\bar{\gamma}_{two-way})}{(1/\bar{\gamma}_{two-way})^k}$ [25].

4. Numerical Results

In this section we verify previous analyses and evaluate the performance of the proposed relay selection scheme for bidirectional relay networks with outdated CSI by simulations and numerical calculations. We consider the scenario with $K = 8$ relays, a required transmit rate $R = 1$ bit/s/Hz, channel fading coefficients are assumed i.i.d with distributions $\Omega_h = \Omega_f = 1$. Outdated correlation coefficient ρ is set to 0.2, unless otherwise specified. Binary phase-shift keying (BPSK) is used as the modulation scheme in the BER analysis by which $A = 1$, $B = 2$. In the following figures, the blue curves stand for our scheme and red curves stand for ORS or all relay selection schemes as references.

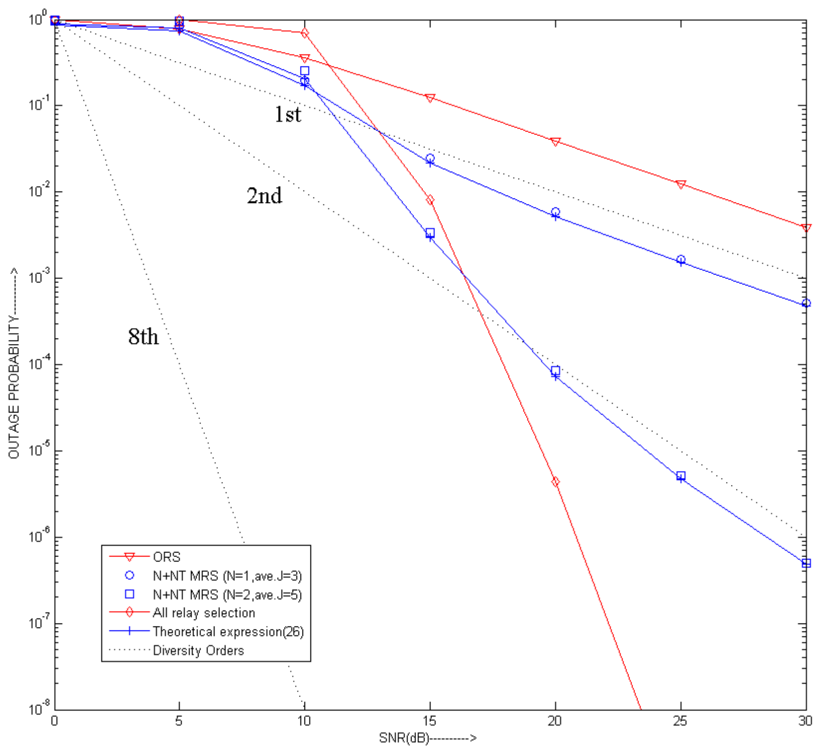

Fig. 2. Comparison of outage probability

with K=8, ρ =0.2 for AF based TWRN

Fig. 2 shows the outage probability of ORS, the proposed N+NT MRS scheme, random select combining MRS and all relay selection in two-way relay networks with outdated CSI. The continuous curves and discrete markers represent the analytical expressions given in former section and the simulation results, respectively. This figure shows the tightness of the analytical expressions and the diversity orders in the sense of the outage probability. As expected, the diversity order of conventional ORS scheme decreases to unity in outdated CSI. We can see that all the MRS schemes have a performance gain comparing with ORS since multiple relays participate in forwarding. Furthermore, the proposed N+NT MRS scheme with N=2, μ =0.308 achieves better outage performance than all relay selection in the low SNR region since a proper number of relays are exploited to conserve system resources. All relay selection scheme, which has more relays participated in forwarding, obtains a better outage performance eventually in high SNR region.

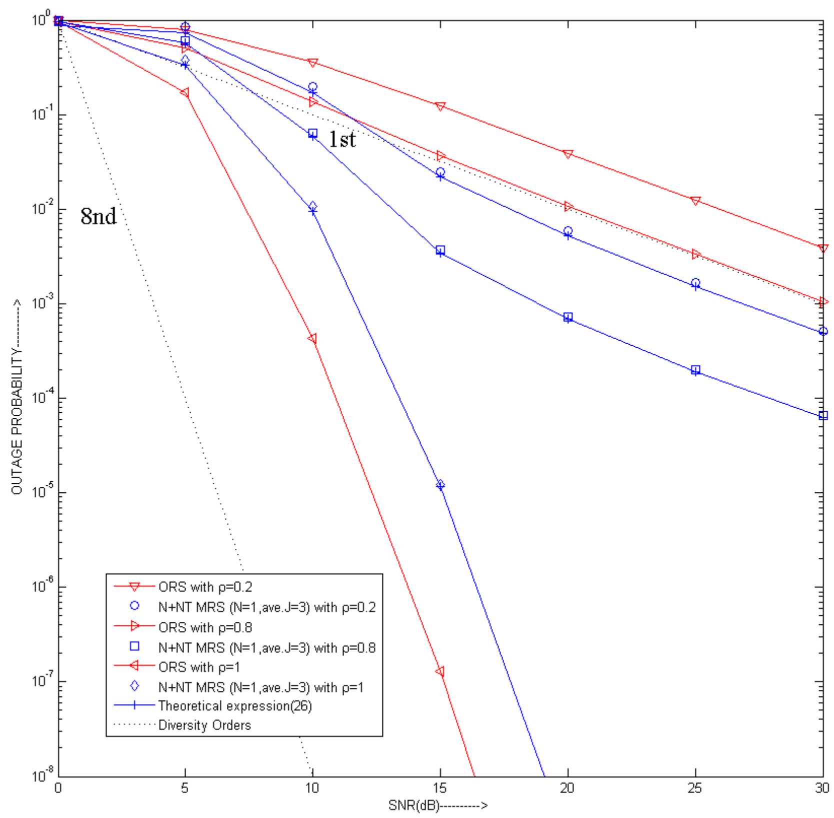

Fig. 3. Comparison of outage probability between ORS and N+NT MRS with different ρ

The comparison of outage probability with different ρ is evaluated in **Fig. 3**. The two-way system has better outage performance as ρ increases. When $\rho \neq 1$, outdate exists, then the diversity order of outage probability decreases to unity as we said before. Meanwhile, the scheme we proposed gains better performance than conventional ORS. However, ORS obtains full diversity order when ρ =1, which means no delay exists, but our scheme experiences performance loss due to low resource efficiency.

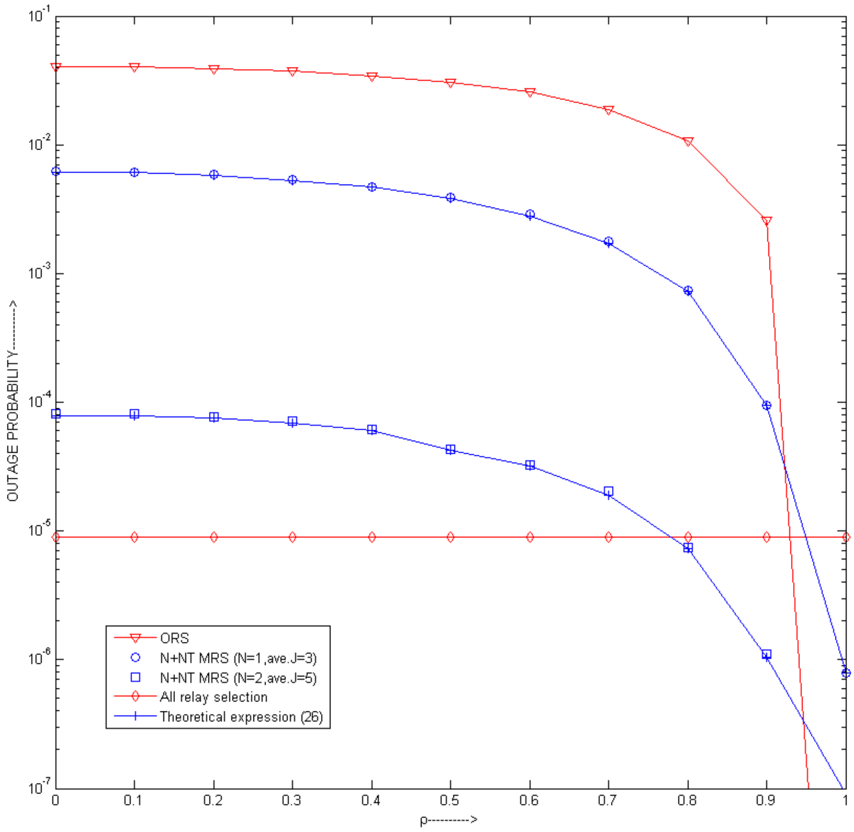

Fig. 4. Comparison of outage probability against ρ under SNR=20dB

In **Fig. 4**, we compare the outage performance between different schemes against outdated correlation coefficient ρ. SNR is set to 20dB in the simulation. From the figure, we can see outage probabilities of ORS and two N+NT MRS schemes decrease as ρ increases. N+NT MRS outperforms conventional ORS unless ρ is close to 1, i.e. the channel estimation tends to be perfect. The performance of N+NT MRS with N=2, $\overline{J}$ =5 exceeds All relay selection in high value of ρ, which means when channel estimation is comparative accurate the proposed scheme can improve system performance.

Fig. 5 shows the BER curves of N+NT MRS schemes. As shown in the figure, two N+NT MRS schemes with different parameters both can provide significant performance gains comparing with conventional ORS scheme. All relay selection, which keeps full diversity order though using all the candidates relaying, still has the best performance. Since more relays participated, the N+NT MRS with N=2, $\overline{J}$ =5 can obtain better BER performance than that with N=1, $\overline{J}$ =3. What's more, the BER curves calculated by formula (31) are very close to that by simulation.

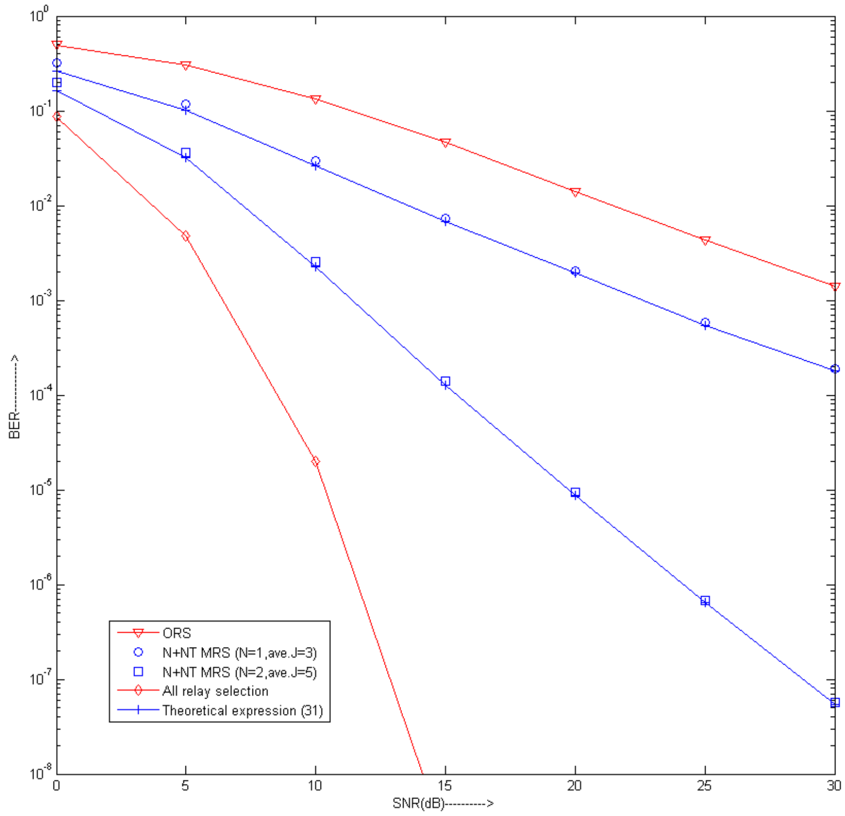

Fig. 5. Comparison of BER
with K=8, ρ =0.2 for AF based TWRN

Fig. 6. shows ergodic channel capacity of each relay selection scheme. We can find out MRS schemes bring capacity loss since more time slots are used during the forwarding process. Though MRS schemes obtain poor ergodic channel capacities than the ORS scheme, outage performances of these MRS schemes are better as shown in **Fig. 2**, which can be considered as MRS schemes enhance the stabilization of the bidirectional transmission and decrease outage probability. Comparing with all relay selection, which has the worst ergodic channel capacity, the N+NT MRS schemes we proposed can reduce capacity loss since an appropriate number of relays are selected.

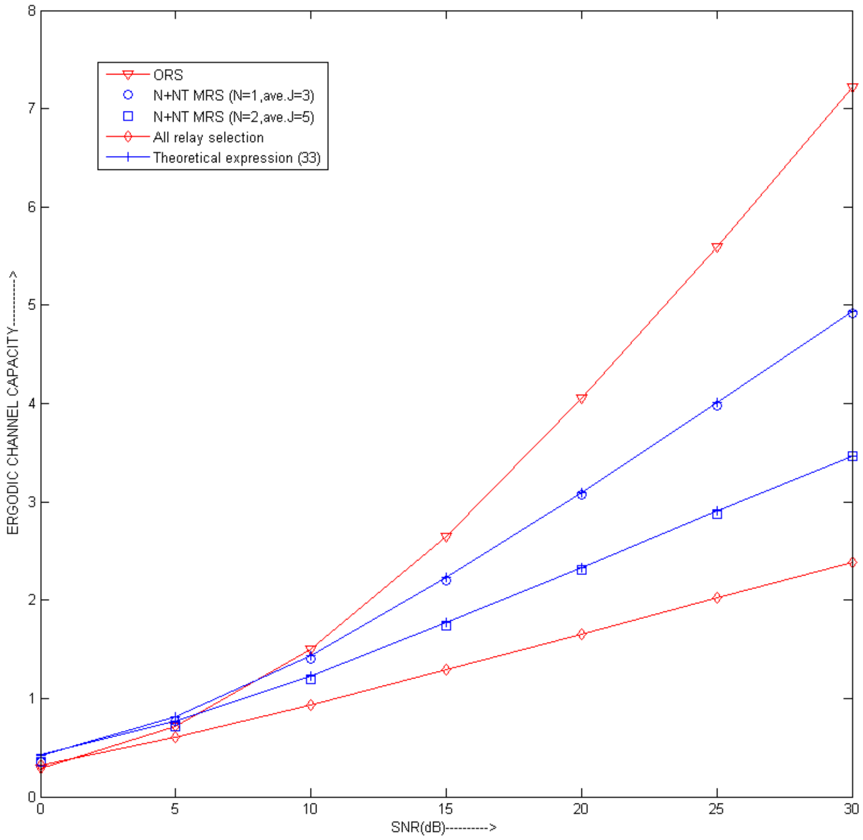

Fig. 6. Comparison of ergodic channel capacity with K=8, ρ =0.2 for AF based TWRN

5. Conclusion

In this paper, we studied relay selection schemes for two-way relay networks with outdated CSI. Diversity order of widely used ORS scheme decreased to unity with outdated CSI. For solving this problem, we propose an N+NT MRS scheme, in which N or more relays were involved to improve system performance. The expressions of outage probability, BER and ergodic channel capacity were formulated for the proposed scheme. Analyses and performance gain for the proposed scheme were verified by numerical calculations and simulations. Numerical results showed the N+NT MRS scheme obtained a significant performance gain comparing with conventional ORS. Meanwhile this scheme achieved a trade-off between power and performance.

Appendix:

Firstly, we rewrite the MGF of γ_{sum} in a more tractable manner.

$$M_{\gamma_{sum}}^{i.i.d}(s) = K\binom{K-1}{N-1}\sum_{k=0}^{K-N}\sum_{l=0}^{K-N-k}\sum_{m=0}^{k}\frac{(K-N)!(-1)^{l+m}}{(K-N-k-l)!m!l!(k-m)!}$$
$$\times\left(\frac{1}{1-s\bar{\gamma}_{two-way}}\right)^{K-1-k}\left(\frac{1}{\alpha-\beta s\bar{\gamma}_{two-way}}\right), \tag{34}$$

where $\alpha = (K-N-k-l)\mu + q + N + m\mu$, $\beta = (K-N-k-l)\mu + q + N + m\mu(1-\rho^2)$.

Then, (34) can be expressed as follows after some manufacture:

$$M_{\gamma_{sum}}^{i.i.d}(s) = K\binom{K-1}{N-1}\sum_{k=0}^{K-N}\sum_{l=0}^{K-N-k}\sum_{m=0}^{k}\frac{(K-N)!(-1)^{l+m}}{(K-N-k-l)!m!l!(k-m)!}$$
$$\times\left[\frac{\tau}{\alpha-\beta s\bar{\gamma}_{two-way}} + \sum_{j=1}^{K-1-k}\frac{\varphi_j}{(1-s\bar{\gamma}_{two-way})^j}\right], \tag{35}$$

where $\tau = \left(1-\frac{\alpha}{\beta}\right)^{-(K-1-k)}$, $\varphi_j = \frac{1}{(\alpha-\beta)(1-\alpha/\beta)^{K-1-k-j}}$.

Using the relation $\hat{p}_\gamma(s) = M_\gamma(-s)$, and inverse Laplace transform of the MGF (36)

$$S\{(1+As)^{-m}\} = \left(\frac{1}{(m-1)!A^m}\right)x^{m-1}\exp(-x/A), \tag{36}$$

we can obtain PDF of proposed N+NT-MRS scheme for two-way relay networks in (28).

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