

λ -domain VVC Rate Control Based on Game Theory

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ABSTRACT

Versatile Video Coding (VVC) has set a new milestone in high-efficiency video coding. In the standard encoder, the λ -domain rate control is incorporated for its high accuracy and good Rate-Distortion (RD) performance. In this paper, we formulate this task as a Nash equilibrium problem that effectively bargains between multiple agents, *i.e.*, Coding Tree Units (CTUs) in the frame. After that, we calculate the optimal λ value with a two-step strategy: a Newton method to iteratively obtain an intermediate variable, and a solution of Nash equilibrium to obtain the optimal λ . Finally, we propose an effective CTU-level rate allocation with the optimal λ value. To the best of our knowledge, we are the first to combine game theory with λ -domain rate control. Experimental results with Common Test Conditions (CTC) demonstrate the efficiency of the proposed method, which outperforms the state-of-the-art CTU-level rate allocation algorithms.

KEYWORDS

VVC, rate control, λ -domain RD model, game theory.

1 INTRODUCTION

To date, different types of videos have been widely used in broadcasting, teaching, entertainment, and so on. With the explosive growth of video consumption, efficient video compression and delivery have become urgent problems. In the past decades, video coding standards (*e.g.*, H.264/AVC [25], High Efficiency Video Coding (H.265/HEVC) [20], and Versatile Video Coding (VVC) [4]) have been developed to solve these problems in different periods. The up-to-date standard VVC, published in 2020 by the Joint Video Experts Team (JVET), has enhanced the coding performance of H.265/HEVC by reducing its bit rate by 50% at the same coding quality. However, as there is limited bandwidth and storage to transmit video data, Rate Control (RC) [14, 16, 17] is still one of the major research areas in the VVC standard.

In video coding, the primary problem of RC is to maximize the encoded video's perceptual quality under a constrained bit rate.

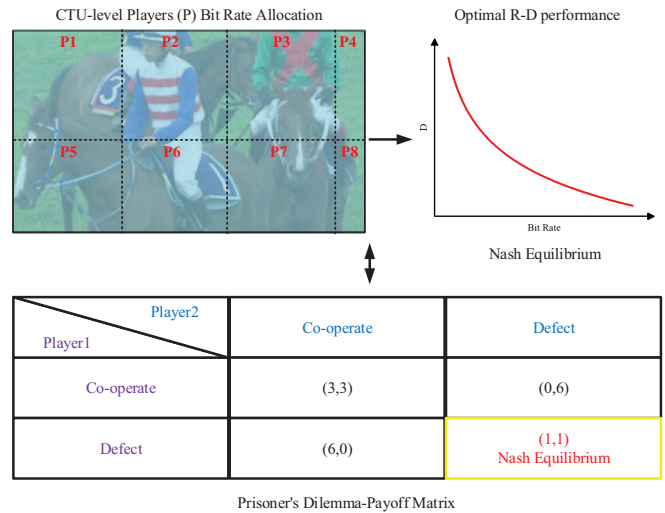


Figure 1: CTU-level Cooperative Bargaining Game.

In the past decades, many approaches have been proposed to enhance the Rate Distortion (RD) performance by adjusting the Quantization Parameter (Qp), bit rate, or Lagrange multiplier parameters. For H.264/AVC and H.265/HEVC, many methods (*e.g.*, the λ -domain RD model [9, 11, 12], deep-learning-based quantization parameters prediction [7, 29], perception-quality-based [15, 27, 28] bit rate allocation) have been proposed to minimize compression distortion under a constrained bit rate. While in the current VVC reference software VTM13.0 [19], the λ -domain RD model with the outstanding RD performance was set as the default RC scheme. To date, the RC methods [17, 22] for VVC are more inclined to precisely characterize the relationship between R and D. However, the bit allocation strategy based on the λ -domain RD model has not been explored for all uncoded CTUs in an updated way. Although there is a sliding window to smooth the bit rate of CTUs in the window, it lacks sufficient theoretical support to balance bit allocation

of all uncoded CTUs to achieve globally optimized coding performance. Therefore, it is still necessary to develop a new method to enhance the RD performance in video coding.

Game theory is an efficient strategy to balance fairness and efficiency in the case of opaque information for the limited resource allocation problem (e.g., the one-pass RC problem in HEVC). Some studies have implied that game theory can perform well in the CTU-based [1, 5], band-based [26], and frame-based [24] bit rate allocations in H.264 or HEVC. In these methods, the bit rate allocation for different levels of coding unit was defined as a Nash equilibrium problem. Although these methods have presented persuasive performance, all of them were designed for the RC in H.264 or HEVC. In addition, the CTU-level bit rate allocation based on the outstanding λ -domain RD model has not been explored and it still is a cooperative bargaining game problem, as shown in Fig. 1. Therefore, the λ -domain RC based on game theory is encouraged and has the potential to enhance the RD performance in VVC.

Based on the above investigation, we formulate the CTU-level RC in VVC as Nash equilibrium problem and attempt to address it by considering the popular λ -domain RD model. In summary, the major contributions of this paper are presented as follows:

- We are the first to address the λ -domain RC problem with game theory. We formulate the one-pass CTU-level RC problem with the popular λ -domain RD model.
- We obtain the optimal λ value with a two-step strategy: a Newton method to iteratively obtain an intermediate variable, and a solution of Nash equilibrium to obtain the optimal λ .
- We propose an overall RC framework with the optimal λ and implement in the newest video coding standard H.266/VVC. Experimental results demonstrate that the proposed method outperforms benchmarks.

2 RELATED WORK

2.1 Rate Control for VVC

Nowadays, to enhance the coding performance of VVC, Mao *et al.* [17] exploited RD characteristics for sequences by deriving new R-Q and D-Q models for frame-level RC. A dependency factor models the inter-frame dependency characteristics. Then, they proposed an adaptive bit allocation method based on the R-Q model, D-Q model, and the dependency factor. In [14], Li *et al.* proposed an RD parameter updating strategy and explored the quality dependency between frames. In [8], a frame-level constant bit-rate (CBR) control method using Recursive Bayesian Estimation (RBE) was proposed. The RBE with alternating prediction and updated steps could estimate bit rates and allocate target bit rates based on the distortions changes of the previously coded frames. Wang *et al.* [22] shifted the traditional bit rate-centered paradigm to a quality-centered paradigm. In addition, they developed the relationship between Structural Similarity Index Metric (SSIM) and Qp for quality control. Liu *et al.* [16] proposed a multi-objective optimization of quality-based CTU level RC method. Their method converts the multi-objective optimization problem into a single-objective problem to minimize average distortion and quality fluctuation for the constrained bit rate. Then, a two-stage method based on the D- λ

model was proposed to obtain the optimal solution. In this method, the obtained solution allocates the bit rate to all CTUs once for all.

2.2 λ -domain RD model

In [9], the λ -domain RC was firstly proposed and integrated into HEVC. It provided a R- λ model that can better characterize the relationship between R and D than traditional models. Then, to achieve the optimal RD performance, Li *et al.* [11] further proposed a D- λ model to characterize the RD behavior of video content. Based on the λ -domain RD model, many approaches were proposed. Zhou *et al.* [30] proposed a High Dynamic Range (HDR)-visual difference predictor-2-based RD model to improve video coding performance by considering the HDR characteristics. In [13], Li *et al.* designed an optimal CTU level weight for the equi-rectangle projection format of the 360-degree video and proposed a weighted CTU level bit allocation method. Recently, Li *et al.* [15] proposed an SSIM-based optimal bit allocation and an SSIM-based RD optimization (SOSR) to solve the inconsistency between optimal bit allocation and RD optimization results in non-optimal SSIM-based coding. In [6], by considering the relationship between weighted Lagrange multiplier and temporal dependency, Guo *et al.* proposed a CTU level RD model. They also developed a formulation by combining inter-block dependency and RD characteristics.

2.3 Game Theory for Video Coding

Game theory can solve the problem of how to achieve both fairness and efficiency in the case of opaque information. Based on this theory, Ahmad *et al.* [1] first defined bit rate allocation between macroblocks in a frame as a cooperative game. In [23], Generalized Nash Bargaining Solution (GNBS) was developed to solve the inner-layer bit allocation of H.264. Wang *et al.* [24] modeled frame-level bit rate allocation in different temporal layers as a game theory problem. They also analyzed the relationship between different temporal layers and employed it to define the utility function for Nash equilibrium. Based on the statistical characteristics of panoramic pictures, Zhao *et al.* [26] introduced the game theory to the RC for 360-degree video coding and allocated the bit rate band by band. Gao *et al.* [5] adopted support vector machine-based multi-classification scheme to determine RD model for all CTUs. They also designed the bit rate allocation scheme based on the mixed RD model-based cooperative bargaining game theory. The performance of these works has verified the efficiency of game theory for the one-pass RC problem.

To date, the game theory has been adopted in the RC at different levels of coding units, such as CTU [1, 5], band [26], and frame [23, 24]. In such a bargaining game of RC, each player is expected to obtain a fair share of total bit rate whilst minimizing the overall distortion. In other words, each player needs to reach a beneficial agreement. It is worthy to notice that if this agreement has been reached, every player has its utility function u_p , where p is the player's index. However, if such an agreement is impossible, a minimum utilities u_p^0 for every player. These minimum utilities are denoted as the minimum utility pair $u^0 = (u_1^0, u_2^0, \dots, u_p^0)$. As there is a feasible utility set U , which is all the possible utility pairs, for the bargaining game, the Pareto optimality solution

$u^* = (u_1^*, u_2^*, \dots, u_p^*)$ can be obtained by the Nash Bargaining Solution (NBS) [18] in these works. u^* can be represented as:

$$u^* = H(U, u^0) = \arg \max_{\langle U, u^0 \rangle} \prod_{p=1}^P (u_p - u_p^0), \quad (1)$$

where P is the number of players in the bargaining game.

3 PROPOSED METHOD

3.1 Problem Formulation

Most of the RC methods for VVC optimize the RD performance by formulating a new model. However, the default λ -domain RD model has not been explored to balance the bit rate for all uncoded CTUs until now. Since some CTUs may consume too many bit rates, the remaining bits cannot obtain the minimum coding quality for uncoded CTUs. Then the overall coding performance cannot reach the optimal performance. Therefore, a bit rate allocation scheme requires to allocate a fair bit rate for all uncoded CTUs. To our best knowledge, the solution of the λ -domain RD model based on game theory for the one-pass RC has not been exploited and analyzed. In this paper, the λ -domain RC scheme based on game theory is modeled and solved.

The RC problem is formulated as to minimize the total distortion, subject to a constraint on the target bit rate of compressed videos. It can be expressed as:

$$\min \sum_{j=1}^{N_i} M_{i,j} d_{i,j}, \text{ s.t. } \sum_{j=1}^{N_i} M_{i,j} r_{i,j} \leq R_i, \quad (2)$$

where $M_{i,j}$ is the number of pixels in the j -th CTU of the i -th frame. $d_{i,j}$ is the mean distortion in the same CTU. N_i is the number of uncoded CTUs in this frame. $r_{i,j}$ is the Bits Per Pixel (bpp) of this CTU and R_i is the target bit rate of the i -th frame.

To date, the default RD model of VVC regulates the relationship between $r_{i,j}$ and $d_{i,j}$ as an exponential function [10], which is expressed as:

$$d_{i,j}(r_{i,j}) = k_{i,j} r_{i,j}^{-c_{i,j}}, \quad (3)$$

where $k_{i,j}$ and $c_{i,j}$ are the model parameters related to the contents of the video. Then, $\lambda_{i,j}$ could be obtained by:

$$\lambda_{i,j} = -\frac{\partial d_{i,j}}{\partial r_{i,j}} = c_{i,j} k_{i,j} r_{i,j}^{-c_{i,j}-1}. \quad (4)$$

Therefore,

$$r_{i,j} = \left(\frac{\lambda_{i,j}}{c_{i,j} k_{i,j}} \right)^{-\frac{1}{c_{i,j}+1}}. \quad (5)$$

By substituting Eq. (5), we can rewrite $d_{i,j}$ as:

$$d_{i,j} = \left(\frac{\lambda_{i,j} k_{i,j}^{\frac{1}{c_{i,j}+1}}}{c_{i,j}} \right)^{\frac{c_{i,j}}{c_{i,j}+1}}. \quad (6)$$

According to this RD relationship, a lower distortion leads to a higher bit rate cost, and vice versa. Due to the limited R_i , the requirements of all CTUs cannot be satisfied to the maximum. Therefore, the bit rate balance is required to get the optimal distortion $\min \sum_{j=1}^{N_i} d_{i,j}$. This balancing procedure is a typical Nash equilibrium problem of game theory. The objective of each player is to

Table 1: Key Symbol Table

Symbol	Explanation
$d_{i,j}, r_{i,j}$	the distortion and bpp of the j -th CTU in the i -th frame
R_i	the target bit rate of the i -th frame
N_i	the number of uncoded CTUs in the i -th frame
$M_{i,j}$	the number of pixels of the j -th CTU in the i -th frame
$c_{i,j}, k_{i,j}$	the RD model parameters in λ -domain
$\lambda_{i,j}$	the λ value of the j -th CTU in the i -th frame
$u_{i,j}, u_{i,j}^0$	the utility and minimal utility of the j -th CTU in the i -th frame
η, η^*	an intermediate variable and its optimal value to derive $\lambda_{i,j}$
$S_{i,j}$	the scale factor of
ζ	the ratio between the guaranteed minimum utility bit rate and the target bit rate

maximize its positive utility. Then, the utility function of the j -th CTU in the i -th frame $u_{i,j}$ could be expressed as:

$$u_{i,j} = \frac{1}{d_{i,j}}, \quad (7)$$

By substituting Eq. (6) into Eq. (7), we update the utility function $u_{i,j}$ as:

$$u_{i,j} = \left(\frac{\lambda_{i,j} k_{i,j}^{\frac{1}{c_{i,j}+1}}}{c_{i,j}} \right)^{-\frac{c_{i,j}}{c_{i,j}+1}}. \quad (8)$$

Furthermore, a minimal utility $u_{i,j}^0$ is defined to avoid severe quality degradation at each CTU. For ease of derivation, we estimate the minimal utility with information of previously coded CTUs. Therefore, it is irrelevant to any encoding parameters and outputs at j , i.e.,

$$\frac{\partial u_{i,j}^0}{\partial \lambda_{i,j}} = \frac{\partial u_{i,j}^0}{\partial d_{i,j}} = 0. \quad (9)$$

With the utility $u_{i,j}$ and minimal utility $u_{i,j}^0$, the objective of CTU bit rate allocation is formulated by game theory:

$$\max \sum_{j=1}^{N_i} \ln(u_{i,j}(\lambda_{i,j}) - u_{i,j}^0), \text{ s.t. } \begin{cases} \sum_{j=1}^{N_i} M_{i,j} r_{i,j} \leq R_i \\ r_{i,j} > r_{i,j}^0, \forall j \end{cases}, \quad (10)$$

where $r_{i,j}^0$ is the required bpp for minimal utility $u_{i,j}^0$. For ease of derivation, the multiplication function Eq. (1) is changed to summation by the $\ln(\cdot)$ function. The key symbols used for bit allocation are summarized in Table 1.

3.2 Solution for the Optimized Lagrange Multiplier

In this paper, the constraint $\sum_{j=1}^{N_i} M_{i,j} \cdot r_{i,j} \leq R_i$ is relaxed to $\prod_{j=1}^{N_i} r_{i,j} \leq (R_i / (M_{i,j} \cdot N_i))^{N_i}$. The reasons to utilize this approximation is three-fold. First, the geometric and arithmetic average values are close for the allocated bit rate. As shown in Fig. 2, we summarize that the geometric mean and the arithmetic mean have similar statistical significance. It can be readily seen that in most cases, the two

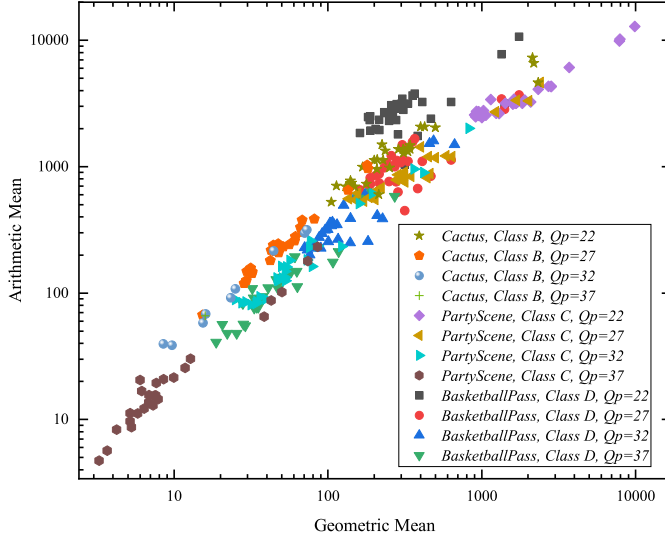


Figure 2: The relationship between arithmetic mean and geometric mean of CTU-level bitrates.

values are very close to each other. Second, this approximation benefits our following derivation to obtain an optimal $\lambda_{i,j}$. Third, this approximation has been widely adopted in similar tasks with promising coding results (e.g., [21]). With the utility $u_{i,j}$ and minimal utility $u_{i,j}^0$, the objective of CTU bit rate allocation is updated as:

$$F = \sum_{j=1}^{N_i} \ln(u_{i,j}(\lambda_{i,j}) - u_{i,j}^0) + \sum_{j=1}^{N_i} \theta_{i,j} (r_{i,j} - r_{i,j}^0) + \eta \left(\sum_{j=1}^{N_i} \ln \left(\left(\frac{\lambda_{i,j}}{c_{i,j} k_{i,j}} \right)^{-\frac{1}{c_{i,j}+1}} \right) - N_i \ln \left(\frac{R_i}{M_{i,j} \cdot N_i} \right) \right). \quad (11)$$

It can be maximized at Karush-Kuhn-Tucker (KKT) conditions,

$$\begin{cases} \frac{\partial F}{\partial \lambda_{i,j}} = 0, \forall j \\ \theta_{i,j} (r_{i,j} - r_{i,j}^0) = 0, \forall j \\ \eta \left(\sum_{j=1}^{N_i} \ln \left(\left(\frac{\lambda_{i,j}}{c_{i,j} k_{i,j}} \right)^{-\frac{1}{c_{i,j}+1}} \right) - N_i \ln \left(\frac{R_i}{M_{i,j} \cdot N_i} \right) \right) = 0 \end{cases}. \quad (12)$$

Because $r_{i,j} - r_{i,j}^0 > 0$, $\theta_{i,j}$ should be 0 to ensure $\theta_{i,j} (r_{i,j} - r_{i,j}^0) = 0$. Meanwhile, $\frac{\partial u_{i,j}}{\partial \lambda_{i,j}} = 0, \forall g < j$. Substituting all these conclusions in $\frac{\partial F}{\partial \lambda_{i,j}}$, we get:

$$\frac{\partial F}{\partial \lambda_{i,j}} = \frac{\partial \ln(u_{i,j}(\lambda_{i,j}) - u_{i,j}^0)}{\partial \lambda_{i,j}} - \frac{\eta}{(c_{i,j} + 1)\lambda_{i,j}} = 0. \quad (13)$$

Therefore,

$$\lambda_{i,j} = c_{i,j} k_{i,j}^{-\frac{1}{c_{i,j}}} \left(\frac{c_{i,j} + \eta}{\eta u_{i,j}^0} \right)^{\frac{c_{i,j}+1}{c_{i,j}}}, \quad (14)$$

Algorithm 1 Solution for Eq. (16)

Formulate Eq. (16) as a function: $Z^x = f(\eta^x) - \xi$, x is the iterator index. τ is the tolerance. τ is set as 0.001.

Input: the start point η^0

Output: final solution η^*

repeat:

Calculate the gradient: $v^x = \nabla f(\eta^x)$;

Update the next step point: $\eta^{x+1} = \eta^x - f(\eta^x)/v^x$;

Calculate the function: Z^{x+1} ;

until: $|Z^x - Z^{x+1}| < \tau$ or $|Z^{x+1}| < \tau$

where $\eta \neq 0$ is known from Eq. (12). Then the condition is updated as:

$$\sum_{j=1}^{N_i} \ln \left(\left(\frac{\lambda_{i,j}}{c_{i,j} k_{i,j}} \right)^{-\frac{1}{c_{i,j}+1}} \right) - N_i \ln \left(\frac{R_{i,j}}{M_{i,j} \cdot N_i} \right) = 0. \quad (15)$$

By substituting Eq. (14) to Eq. (15), we get:

$$\sum_{j=1}^{N_i} \frac{1}{c_{i,j}} \ln \left(\frac{c_{i,j}}{\eta} + 1 \right) - \sum_{j=1}^{N_i} \frac{1}{c_{i,j}} \ln(u_{i,j}^0 k_{i,j}) + N_i \ln \left(\frac{R_i}{M_{i,j} \cdot N_i} \right) = 0. \quad (16)$$

Since the parameters $c_{i,j}$ of uncoded CTUs are not constant, the closed-form solution of Eq. (16) can not be got. To obtain the accurate solution of Eq. (16), the Newton method is applied. The detailed solution is summarized in Algorithm 1. Eq. (16) is divided into two parts: ξ and $f(\eta)$. ξ is represented as:

$$\xi = \sum_{j=1}^{N_i} \frac{1}{c_{i,j}} \ln(u_{i,j}^0 k_{i,j}) - N_i \ln \left(\frac{R_i}{M_{i,j} \cdot N_i} \right), \quad (17)$$

$f(\eta)$ is defined as:

$$f(\eta) = \sum_{j=1}^{N_i} \frac{1}{c_{i,j}} \ln \left(\frac{c_{i,j}}{\eta} + 1 \right). \quad (18)$$

Then, the solution η^* of Eq. (16) can be obtained by Algorithm 1, which can be considered as an intermediate variable. After that, Submitting η^* into Eq. (14), we can obtain the optimal $\lambda_{i,j}$ as:

$$\lambda_{i,j} = c_{i,j} k_{i,j}^{-\frac{1}{c_{i,j}}} \left(\frac{c_{i,j} + \eta^*}{\eta^* u_{i,j}^0} \right)^{\frac{c_{i,j}+1}{c_{i,j}}}. \quad (19)$$

3.3 The Overall Algorithm

To refine the bit rate allocation, several issues need to be addressed. Firstly, as the parameters $k_{i,j}$ and $c_{i,j}$ can not be obtained before coding current CTU, we utilize the parameters of co-located CTU to estimate the bit rate of current CTU. These parameters $c_{i,j}$ and $k_{i,j}$ are updated as:

$$c_{i,j} = \frac{r_{i,j}^{\text{actual}} \lambda_{i,j}^{\text{actual}}}{d_{i,j}^{\text{actual}}}, k_{i,j} = \frac{d_{i,j}^{\text{actual}}}{(r_{i,j}^{\text{actual}})^{c_{i,j}}}, \quad (20)$$

where $r_{i,j}^{\text{actual}}$, $\lambda_{i,j}^{\text{actual}}$, and $d_{i,j}^{\text{actual}}$ are the actual coding results. R_i is also updated by the actual texture bit rate $R_{i,j}^{\text{actual}}$ of the j -th CTU

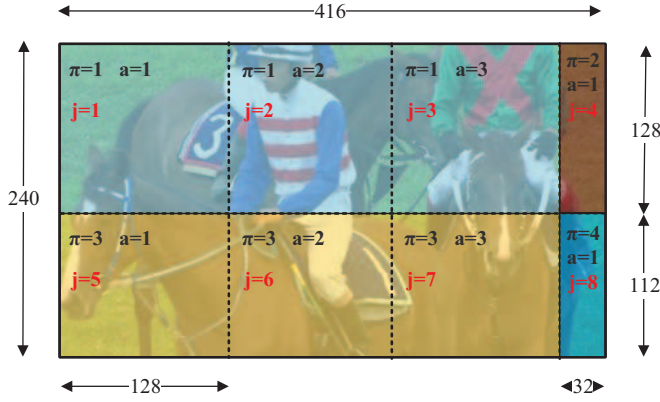


Figure 3: The typical effective CTU sizes in the frame.

as:

$$R_i = R_i - R_{i,j}^{\text{actual}}. \quad (21)$$

Secondly, due to different video resolutions, the Largest CU (LCU) size (128×128) may lead to four different effective CTU sizes (e.g., 128×128, 32×128, 128×112, and 32×112, as illustrated in Fig. 3) of CTUs in the frame. In general, larger effective CTU sizes need more bit rates to guarantee quality smoothness between different types of CTUs. Therefore, the effective CTU sizes are also taken into consideration. In the following paper, the j -th CTU is also denoted by πi and a : it is also numbered as the a -th among all πi -th type of CTUs, as shown in Fig. 3.

To refine the bit rate allocation for different types of CTUs, the target bit rate of the π -th type CTUs. R_i^π in the i -th frame is calculated by:

$$R_i^\pi = \frac{R_i \cdot M_i^\pi}{M_i^l}, \quad (22)$$

where M_i^π and M_i^l are the pixel number of the π -th type of CTUs and the number of remaining pixels in the frame, respectively.

As analyzed above, the minimal utility $u_{i,j}^0$ of the j -th CTU is also represented as $u_{i,j(\pi,a)}^0 \cdot u_{i,j}^0$ has been mentioned in Eq. (10), which regulates the minimum coding quality for each CTU. Therefore, the sum of bits corresponding to the uncoded π type of CTUs should satisfy $\sum_{b=a}^{A_\pi} M_{i,j(\pi,b)} r_{i,j(\pi,b)}^0 \leq R_i^\pi$. A_π is the number of the uncoded π -th type of CTUs in the frame. b is the index number. In this paper, we set a scale factor $S_{i,j}$, which is also represented as $S_{i,j(\pi,a)}$, to limit the minimum coding quality of the j -th uncoded CTU [1, 24]. $S_{i,j(\pi,a)}$ is computed as following:

$$S_{i,j} = S_{i,j(\pi,a)} = \frac{\zeta R_i^\pi}{\sum_{b=a}^{A_\pi} \left(M_{i,j(\pi,b)} \left(\frac{k_{i,j(\pi,b)}}{d_{i,j(\pi,b)}} \right)^{\frac{1}{c_{i,j(\pi,b)}}} \right)}, \quad (23)$$

where ζ is set as 0.7. $\tilde{d}_{i,j(\pi,b)}$ is the average distortion of all co-located CTUs of previous frames at the same hierarchy of coding structure. In this paper, the adjusted minimal utility $u_{i,j(\pi,a)}^0$ is obtained by:

Algorithm 2 CTU Level Bit Rate Allocation Method

```

Initialize the parameters  $k_{i,j}$  and  $c_{i,j}$ 
if the type of the current frame is intra then
    Allocate the bit rate with the default VVC RC
else
    for all uncoded CTUs in a frame do
        Calculate the minimal utility  $u_{i,j}^0$  with Eqs. (23 and 24)
        if the distortion value of corresponding CTU in the
            same frame level is equal to 0 then
            Obtain the estimated  $\lambda_{i,j}$  with Eq. (4)
        else
            Obtain the estimated  $\lambda_{i,j}$  with Eq. (19)
        end if
        Adjust  $\lambda_{i,j}$  with the estimated  $\lambda$  value of the current frame
             $\lambda^{\text{est}}$  and the  $\lambda$  value of neighbor CTU  $\lambda^{\text{nei}}$ 
        if  $\lambda^{\text{nei}} > 0$  then
             $\lambda_{i,j} = \max(\min(\lambda_{i,j}, \lambda^{\text{nei}} \cdot 2^{\frac{1}{3}}), \lambda^{\text{nei}} \cdot 2^{-\frac{1}{3}})$ 
        end if
        if  $\lambda^{\text{est}} > 0$  then
             $\lambda_{i,j} = \max(\min(\lambda_{i,j}, \lambda^{\text{est}} \cdot 2^{\frac{2}{3}}), \lambda^{\text{est}} \cdot 2^{-\frac{2}{3}})$ 
        else
             $\lambda_{i,j} = \max(\min(\lambda_{i,j}, 1000 \cdot 2^4), 10 \cdot 2^4)$ 
        end if
        if  $\lambda_{i,j} < 0.1$  then
             $\lambda_{i,j} = 0.1$ 
        end if
        Estimate the bpp of uncoded CTU  $r_{i,j}$  by Eq. (26)
    end for
    Calculate the refined estimated bit rate  $R_{i,j}$  with Eq. (27)
    Encode the  $j$ -th CTU with the software
    Obtain the actual bit rate  $R_{i,j}^{\text{actual}}$  and update the remaining bit
        rates  $R_i$  with Eq. (21)
    Update the  $k_{i,j}$  and  $c_{i,j}$  parameters Eq. (20)
end if

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$$u_{i,j}^0 = u_{i,j(\pi,a)}^0 = \frac{1}{\delta \tilde{d}_{i,j(\pi,a)}} \min(S_{i,j(\pi,a)}, 1), \quad (24)$$

where the adjustment factor δ [5] is defined as:

$$\delta = \frac{\hat{Q}}{Q}, \quad (25)$$

where \hat{Q} is the estimated Quantization Step (Q_{step}) of the current frame. Q is the Q_{step} of the previous frame in the same frame level.

Finally, by substituting Eq. (19) into Eq. (5), we estimate the target bpp $r_{i,j}$ of the j -th CTU as:

$$r_{i,j} = \left(\frac{k_{i,j} u_{i,j}^0 \eta^*}{c_{i,j} + \eta^*} \right)^{\frac{1}{c_{i,j}}}. \quad (26)$$

To optimize the overall coding quality in the frame, we estimate the bit rates for all uncoded CTUs and adopt sliding windows [9] to refine the estimated bit rate $R_{i,j}$. $R_{i,j}$ is calculated by:

$$R_{i,j} = \lfloor M_{i,j} r_{i,j} - \frac{1}{W} \left(\sum_{j=1}^{N_i} M_{i,j} r_{i,j} - R_i \right) + 0.5 \rfloor, \quad (27)$$

Table 2: Comparison of RC performances in terms of the Y-PSNR and RCError.

Class	Sequence	FixQP		VTM13.0RC		SOSR'TBC21		MORC'TIP21		Proposed	
		Y-PSNR (dB)	Rate (kbps)	Y-PSNR (dB)	RCError (%)	Y-PSNR (dB)	RCError (%)	Y-PSNR (dB)	RCError (%)	Y-PSNR (dB)	RCError (%)
B	<i>MarketPlace</i>	37.2014	6216.883	37.5081	0.12	37.3057	0.12	37.4884	0.20	37.4935	0.12
	<i>RitualDance</i>	39.4402	4652.504	39.4283	0.10	39.0506	0.09	39.4462	0.12	39.4089	0.10
	<i>Cactus</i>	35.7330	5093.700	35.7530	0.10	35.5163	0.11	35.7329	0.26	35.7453	0.10
	<i>BasketballDrive</i>	36.7663	6136.736	36.6474	0.09	36.5220	0.09	36.6865	0.12	36.6792	0.09
	<i>BQTerrace</i>	34.7598	13182.93	34.7343	0.18	34.3929	0.18	34.7524	0.19	34.7213	0.18
C	<i>BasketballDrill</i>	36.5083	1245.662	36.8879	0.29	36.8198	0.30	36.9374	0.32	36.8914	0.29
	<i>BQMall</i>	35.9642	1399.921	36.0326	0.31	35.8437	0.31	36.0213	0.34	36.0579	0.31
	<i>PartyScene</i>	33.1892	2716.644	33.4161	0.17	33.3099	0.18	33.4067	0.33	33.3696	0.17
	<i>RaceHorses</i>	35.0175	1953.890	34.9717	0.14	34.7670	0.14	34.9307	0.16	34.9521	0.14
D	<i>BasketballPass</i>	35.7329	629.642	35.9812	0.53	35.8607	0.53	35.9573	0.56	35.9748	0.53
	<i>BQSquare</i>	33.2506	658.309	33.0898	0.97	32.9413	0.97	33.1068	1.04	33.1708	1.05
	<i>BlowingBubbles</i>	32.9515	657.159	33.0947	0.64	33.0090	0.64	33.1026	0.75	33.1220	0.64
	<i>RaceHorses</i>	34.1670	454.172	34.0853	0.48	33.9449	0.48	34.0435	0.49	34.0688	0.48
E	<i>FourPeople</i>	39.2264	690.026	39.1541	0.60	39.0818	0.61	39.1938	0.68	39.1793	0.59
	<i>Johnny</i>	40.4578	406.579	40.3865	1.29	40.2902	1.31	40.3978	1.53	40.3766	1.30
	<i>KristenAndSara</i>	40.3053	604.222	40.3089	0.77	40.1380	0.77	40.3171	0.82	40.3041	0.77
F	<i>BasketballDrillText</i>	36.3085	1253.154	36.7391	0.29	36.6032	0.30	36.7617	0.32	36.7248	0.30
	<i>ArenaOfValor</i>	37.8757	5244.161	38.0913	0.10	38.1763	0.10	38.2164	0.11	38.1012	0.10
	<i>SlideEditing</i>	41.6256	111.518	36.4628	5.69	37.9999	10.43	36.5385	7.85	36.6463	6.19
	<i>SlideShow</i>	43.9056	220.560	43.7232	14.30	43.8177	3.33	43.1223	8.41	44.1698	0.70
Average		37.0193	2676.419	36.8248	1.36	36.7695	1.05	36.8080	1.23	36.8579	0.71

where W is the size of sliding windows, which is set as the minimum value of 4 and N_i .

In summary, the detailed bit rate allocation method is shown in Algorithm 2.

4 EXPERIMENT

4.1 Simulation Setup

The proposed method is implemented on the VVC reference software VTM13.0 [19]. All experimental results are regulated under the Common Test Condition (CTC) document JVET-T2010 [3] to verify the effectiveness of the proposed method. Four target bit rates are generated with four fixed Qps: 22, 27, 32, and 37, respectively, by VTM13.0 without RC (denoted as FixQP). For comparison, we obtain the performance of the up-to-date CTU-level bit rate allocation methods [15, 16] on the same software platform VTM13.0. They are denoted as SOSR'TBC21 [15] and MORC'TIP21 [16], respectively. Since all compared methods were implemented on Low-Delay B (LD) configuration, we also use this configuration for fair comparison.

4.2 Comparison on Bit Rate Allocation

The luminance component of PSNR (Y-PSNR) and RCError metrics are used to evaluate the method's performance. Y-PSNR is positively correlated with the visual quality of compressed videos. RCError indicates the inaccuracy of bitrate allocation. A higher RCError indicates that the method has a lower bit rate allocation

accuracy, and vice versa. It is calculated by:

$$RCError = \frac{|R_{\text{target}} - R_{\text{actual}}|}{R_{\text{target}}} \times 100\%, \quad (28)$$

where R_{target} is the target bit rate. R_{actual} is the actual coding bit rate of the proposed method.

In Table 2, the default RC scheme of VTM13.0 (VTM13.0RC) achieves a relatively acceptable visual performance with an average Y-PSNR of 36.8248 dB and an average RCError of 1.36%. SOSR'TBC21 and MORC'TIP21 achieve lower coding quality with 36.7695dB and 36.8080dB, respectively. Nevertheless, they perform better than VTM13.0RC in the RCError metric with 0.31% and 0.13%. As for the proposed method, its average Y-PSNR difference from FixQP is only 0.1614 dB. Comparing the performance of VTM13.0RC, SOSR'TBC21, and MORC'TIP21 in Y-PSNR and RCError, we can find that the proposed method achieves an average Y-PSNR of 36.8579dB and an average RCError of 0.71%. It outperforms the performances of benchmarks. Especially for the RCError metric, the proposed method reduces the average bit rate accuracy of VTM13.0RC, SOSR'TBC21, and MORC'TIP21 by about 47.8%, 32.4%, and 42.3%, respectively.

4.3 Comparison on RD Performances

The RD performance of the proposed method is evaluated with Bjontegaard Average Peak-Signal-to-Noise-Ratio (BDPSNR) and Bjontegaard Average Bit Rate (BDBR) [2]. These metrics are calculated by Y-PSNR and bit rate. BDPSNR implies the average coding quality in dB with the same bit rate. The other metric, BDBR, implies the average increment of bit rate in percentage with the same visual quality. In other words, the RD performance of a method

Table 3: Comparison of RD performances in terms of BDPSNR and BDBR.

Class	Sequence	VTM13.0RC		SOSR'TBC21		MORC'TIP21		Proposed	
		BDPSNR (dB)	BDBR (%)	BDPSNR (dB)	BDBR (%)	BDPSNR (dB)	BDBR (%)	BDPSNR (dB)	BDBR (%)
B	<i>MarketPlace</i>	0.2913	-11.32	0.0870	-4.32	0.2702	-10.64	0.2769	-10.92
	<i>RitualDance</i>	-0.0180	0.38	-0.4231	9.45	0.0029	-0.06	-0.0437	0.92
	<i>Cactus</i>	0.0085	0.23	-0.2095	10.75	-0.0147	1.25	-0.0177	1.21
	<i>BasketballDrive</i>	-0.1148	5.31	-0.2405	11.19	-0.0707	3.34	-0.0797	3.74
	<i>BQTerrace</i>	-0.0504	3.66	-0.3892	30.96	-0.0447	3.24	-0.0853	5.74
C	<i>BasketballDrill</i>	0.3269	-7.88	0.2688	-6.51	0.3819	-9.16	0.3360	-8.13
	<i>BQMall</i>	0.0554	-1.46	-0.1329	3.56	0.0442	-1.16	0.0818	-2.15
	<i>PartyScene</i>	0.1798	-4.43	0.0684	-1.69	0.1592	-3.95	0.1339	-3.32
	<i>RaceHorses</i>	-0.0201	0.54	-0.2236	6.22	-0.0647	1.76	-0.0404	1.09
D	<i>BasketballPass</i>	0.2098	-4.16	0.0922	-1.86	0.1834	-3.65	0.2031	-4.06
	<i>BQSquare</i>	-0.1394	3.73	-0.3658	11.91	-0.1868	5.85	-0.1194	3.75
	<i>BlowingBubbles</i>	0.1200	-3.07	0.0325	-0.80	0.1239	-3.15	0.1377	-3.50
	<i>RaceHorses</i>	-0.0932	2.13	-0.2215	4.99	-0.1378	3.10	-0.1101	2.51
E	<i>FourPeople</i>	-0.1276	4.16	-0.1720	5.66	-0.0575	2.24	-0.0869	3.10
	<i>Johnny</i>	-0.0799	4.88	-0.2168	10.68	-0.0936	4.65	-0.0927	5.05
	<i>KristenAndSara</i>	-0.0326	1.60	-0.1817	7.83	-0.0051	0.75	-0.0334	1.65
F	<i>BasketballDrillText</i>	0.3749	-8.60	0.2553	-5.86	0.4041	-9.20	0.3696	-8.47
	<i>ArenaOfValor</i>	0.1669	-4.09	0.2524	-6.22	0.2986	-7.41	0.1761	-4.35
	<i>SlideEditing</i>	-6.0720	50.90	-5.4625	46.26	-6.3164	53.22	-6.1526	49.96
	<i>SlideShow</i>	-0.1326	5.93	-0.0566	0.86	-0.4627	10.29	0.3659	-4.65
Average		-0.2574	1.92	-0.3620	6.65	-0.2793	2.07	-0.2391	1.46

shows a positive correlation with BDPSNR and a negative correlation with BDBR.

This paper evaluates BDPSNR and BDBR by comparing the methods with FixQP. As shown in Table 3, SOSR'TBC21 achieves an average BDPSNR of -0.3620dB and an average BDBR of 6.65%. As the method is designed to enhance the visual quality in HEVC and the bit allocation strategy for all uncoded CTUs is not designed, the performance of SOSR'TBC21 does not achieve the best performance. MORC'TIP21 achieves a competitive performance with an average BDPSNR of -0.2793dB and an average BDBR of 2.07%. Although MORC'TIP21 is a CTU-level bit rate allocation scheme, the allocation scheme allocates bit rates once for all. In contrast, our bit rate allocation scheme of the proposed method is dynamic and adaptive to complex video contents. The proposed method achieves the highest average BDPSNR of -0.2357dB and the lowest average BDBR of 1.44%. In Table 3, the proposed method is superior to VTM13.0RC, SOSR'TBC21, and MORC'TIP21 in the average BDPSNR. In addition, compared with VTM13.0RC, SOSR'TBC21, and MORC'TIP21, the proposed method enhances the average BDBR by 0.48%, 5.21%, and 0.63%, respectively.

In Table 3, we can also find that most sequences (15/20) achieve optimal or sub-optimal BDPSNR and BDBR performances. The performances derive from the fact that the estimated target bit rate of current CTU is obtained with the average previous CTU distortion. It also verifies that game theory can well balance bit rates for uncoded CTUs, which can avoid the large-scale quality fluctuations. In addition, the performance of *Slideshow* sequence indicates that the proposed method has significant advantages for the scene where the space complexity of the screen content fluctuates greatly.

Table 4: Video quality and bit rate fluctuation

Method	VTM13.0RC	SOSR'TBC21	MORC'TIP21	Proposed
Average	Rate			
Var	1.29e+10	1.31e+10	1.66e+10	1.28e+10
Mean	50229	50095	50191	50080
Average	Y-PSNR			
Var	7.7934	7.36424	6.5994	7.1703
Mean	36.8248	36.7695	36.8080	36.8579

This benefits from the advantages of the game theory-based model. The greater difference between the utility functions of the players indicates the better global performance of the uncertain information game.

4.4 Video Quality and Bit Rate Fluctuation

Variance and mean values are also used to evaluate the video quality and bit rate fluctuation of the sequence. A lower variance value indicates a lower fluctuation, and vice versa. A higher mean value implies a better performance of algorithm. The average variance and mean values of all mandatory sequences in Table 4 indicate the proposed method achieves the lowest bit rate fluctuation. This achievement may be attributed to the adaptive CTU-level bit allocation in our method. It also implies that the proposed method is more suitable for complex video transmission environments. For the average mean of Y-PSNR, our proposed method achieves the optimal performance. Although the proposed method is not designed to control the video quality fluctuation, it also outperforms

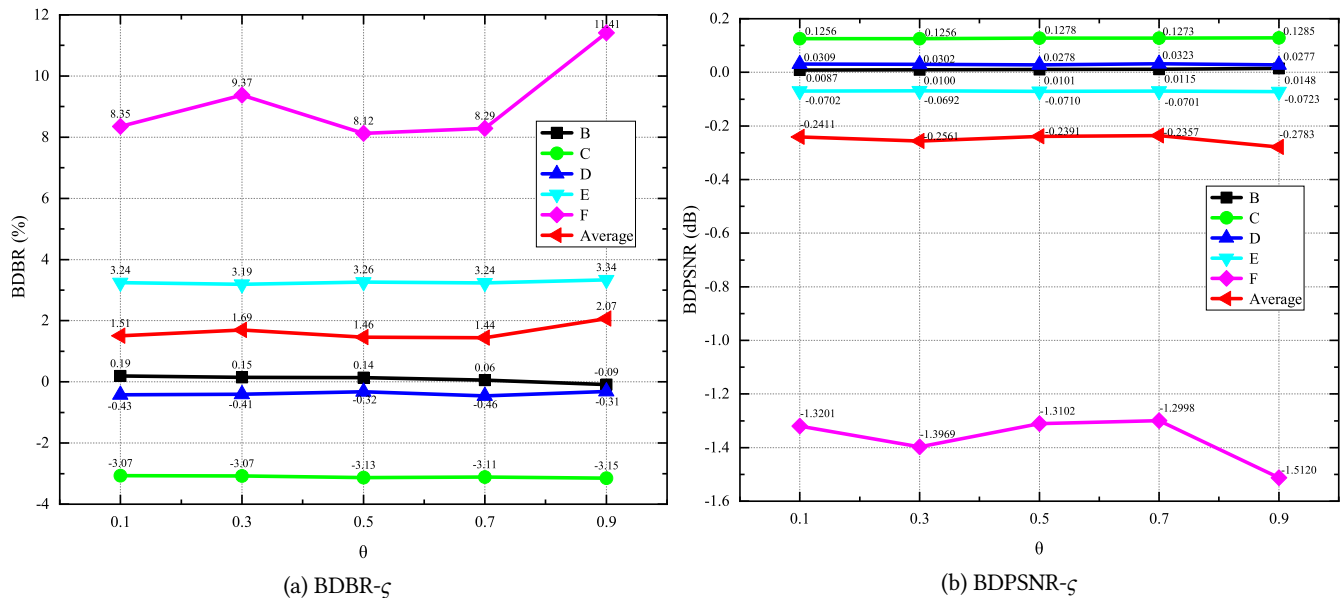
Figure 4: Performance of the proposed method with different ζ values.

Table 5: Computational overhead.

-	Qp	Time cost (s)	Overhead(%)		
		VTM13.0RC	SOSR'TBC21	MORC'TIP21	Proposed
<i>Cactus</i> (Class B)	22	187659	0.35	4.58	0.02
	27	109539	2.04	4.14	0.68
	32	70273	0.34	3.78	1.93
	37	44224	0.60	3.73	0.40
<i>RaceHorses</i> (Class C)	22	35785	-0.25	0.83	-0.08
	27	25723	0.72	0.24	0.07
	32	17559	-0.72	-0.28	-0.70
	37	10932	-2.13	-0.41	0.41
<i>BlowingBubbles</i> (Class D)	22	11793	-3.18	3.01	-1.59
	27	9098	-3.90	-4.24	-3.17
	32	5506	-5.61	-1.19	-1.63
	37	3410	-4.49	-3.17	-2.17
<i>Johnny</i> (Class E)	22	31898	4.98	2.42	1.23
	27	17577	1.43	1.44	2.08
	32	11208	5.86	-1.15	1.43
	37	7323	9.72	-0.77	2.21
Average	-	-	0.36	0.81	0.07

VTM13.0RC and SOSR'TBC21. As MORC'TIP21 is proposed to control the video quality fluctuation, it achieves the best average variance of Y-PSNR. However, it also leads to the highest average variance of bit rate. Compared with MORC'TIP21, the proposed method can better balance the fluctuation between Y-PSNR and bit rate.

4.5 Parameter Sensitivity Analysis

Parameter sensitivity analysis is conducted for different ζ values of Eq. (23) to investigate the performance variations of the proposed method. The parameter ζ is the ratio between the guaranteed minimum utility bit rate and the target bit rate. Here, we examine this parameter from 0.1 to 0.9 with a step size of 0.2. This experiment is conducted on the mandatory test sequences of the CTC. The BDBR and BDPSNR performance of these parameters are shown in Fig. 4,

where the BDBR and BDPSNR values corresponding to Class "B", "C", "D", "E", and "F" represent the average values of all sequences of the corresponding category in the CTC document. "Average" represents the average value of all test sequences. From the figure, the proposed method achieves the optimal performance with an average BDBR of 1.44% and an average BDPSNR of -0.2357dB when the value of ζ parameter is 0.7.

4.6 Computational Overhead

We select four typical sequences (*Cactus*, *RaceHorses*, *BlowingBubbles*, and *Johnny*) to evaluate the overheads of the methods. The experiments are carried out on the computer with @ 3.80GHz, 3.79GHz processor, and 16GB of RAM, running Windows 10. We compare the overhead of SOSR'TBC21, MORC'TIP21, and the proposed method with VTM13.0RC. The overhead ΔT is calculated by:

$$\Delta T = \frac{T_{\text{method}} - T_{\text{org}}}{T_{\text{org}}} \times 100\%, \quad (29)$$

where T_{method} represents the time cost of a compared method. T_{org} is the time cost of VTM13.0RC, which is set as the baseline. From Table 5, the overheads of SOSR'TBC21, MORC'TIP21, and the proposed method are 0.36%, 0.81%, and 0.07%, respectively. It shows that the overhead of the proposed method is lower than the overheads of SOSR'TBC21 and MORC'TIP21. Therefore, our method achieves a high RC performance with a negligible computational overhead.

5 CONCLUSION

This paper proposes a λ -domain RC based on game theory to enhance the RD performance of video coding. We first formulate bit rate allocation with λ -domain RD model and game theory. Then,

we obtain the optimal λ value of uncoded CTU and use it to estimate the bit rates of the next uncoded CTU. In addition, the overall RC framework is also designed. Experimental results show that the proposed method enhances a significant bit rate accuracy and obtains superior BDBR and BDPSNR performances, whilst maintaining a negligible computational overhead.

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