
INCENTIVIZING FEDERATED LEARNING

Shuyu Kong

Department of Electrical and Computer Engineering
Northwestern University
Evanston, IL 60201
shuyukong2020@u.northwestern.edu

You Li

Department of Electrical and Computer Engineering
Northwestern University
Evanston, IL 60201
you.li@u.northwestern.edu

Hai Zhou

Department of Electrical and Computer Engineering
Northwestern University
Evanston, IL 60201
haizhou@northwestern.edu

May 24, 2022

ABSTRACT

Federated Learning is an emerging distributed collaborative learning paradigm used by many of applications nowadays. The effectiveness of federated learning relies on clients' collective efforts and their willingness to contribute local data. However, due to privacy concerns and the costs of data collection and model training, clients may not always contribute all the data they possess, which would negatively affect the performance of the global model.

This paper presents an incentive mechanism that encourages clients to contribute as much data as they can obtain. Unlike previous incentive mechanisms, our approach does not monetize data. Instead, we implicitly use model performance as a reward, i.e., significant contributors are paid off with better models. We theoretically prove that clients will use as much data as they can possibly possess to participate in federated learning under certain conditions with our incentive mechanism.

1 Introduction

Deep learning is data-hungry. Nevertheless, a single party may not have sufficient data for many deep learning training tasks. For example, a hospital that wants to train a disease diagnosis classifier may not have enough cases. Federated learning (FL) is brought up to leverage the strength of massive data distributed among different clients securely and privately. The clients who participate in federated learning collaboratively complete a deep learning task under the coordination of a centralized server or a service provider called the federator. The raw data of each client is stored locally and not transferred. Instead, model parameters or model updates are exchanged to complete the task.

It appears that FL avoids data sharing among clients. However, in the basic FL setting, the centralized server will distribute the same global model to all the clients. As a result, those clients who make only minimal contributions benefit in the same way as those who make significant contributions. Therefore, clients in practice may intentionally contribute fewer or even no data in FL to reduce local training costs and privacy risks. The performance of the global model is only slightly affected if only a few clients "cheat" in this way. Nevertheless, suppose all clients have such "selfish" mindsets that drive them to contribute little while hoping to gain a lot. In that case, there is no way the centralized server could produce a well-performed global model on low-quality training data. Consequently, every client ends up obtaining a worse model, making the whole FL process meaningless.

Recently, many incentive mechanisms have been proposed to incentivize clients to contribute more data [1, 2, 3, 4, 5, 6]. However, almost all those incentive mechanisms use money as a reward. We believe payments could be obstacles for many applications. Furthermore, the value of high-performance models is not always measurable, and many clients

participate in seek of a model instead of money. For those clients, it is hard to compensate for losses on model performance with money.

This paper proposes a non-monetary incentive mechanism that uses model performance as a reward. In our mechanism, the server will evaluate the performance of every client’s uploaded model in each round and distribute different models to clients based on the evaluation results. With the assumption that clients who contribute better quality data will upload a better local model, the objective of our incentive mechanism is to reward significant contributors with better aggregated models. We prove that our approach satisfies the baseline incentive requirements and can drive clients to contribute the maximum amount of data under certain conditions.

2 Related Work

2.1 Incentive Mechanisms in Federated Learning

Recently, various techniques including Stackelberg game, auction, contract, Shapley value, blockchain and reinforcement learning, have been adopted as the incentive mechanisms for federated learning. Stackelberg game [1, 2, 3], auction [4, 5, 6], and contract [7, 8] are mainly employed for node selection and payment allocation, while Shapley value [9, 10] is used for contribution measurement. Both blockchain [11] and reinforcement learning [5, 12] are auxiliary techniques to improve performance and robustness.

2.2 Fairness in Federated Learning

Fairness for federated learning is another close topic to our research. Li [13] proposes q-FFL, a novel optimization objective that addresses fairness issues in federated learning. Inspired by fair resource allocation for wireless networks, q-FFL minimizes an aggregate reweighted loss parameterized by q , such that the devices with higher loss are given higher relative weight. Authors claim that q-FFL maintains the same overall average accuracy while ensuring a more fair and uniform service quality across the network. Agnostic Federated Learning [14] is proposed to resolve the trade-off between performance and fairness [15].

Those approaches allocate appropriate weights in aggregating client models to simultaneously achieve fairness and performance. Hence, they still lie in the vanilla federated learning category and do not necessarily encourage clients to provide more data. Compared with these studies, our work takes one step further in the sense that we achieve fairness in learning and leverage fairness to incentivize all the clients to contribute as much data as they possess.

2.3 Privacy Threat in Federated Learning

Several earlier studies demonstrate that attackers could leverage the model information uploaded by clients, e.g., gradient, to infer private data [16, 17]. This imposes a privacy threat on those clients whose data is confidential. As a result, clients may not fully commit to contributing all their data. This phenomenon has motivated us to develop our incentive mechanism.

2.4 Personalization of Federated Learning

While vanilla federated learning aims to train a single global model across decentralized local datasets, a monolithic model may not fit all clients. Federated personalization allows a client to obtain a more potent model per the client’s specific objective. Rather than averaging with constant weights for the entire federation, optimal weighted model combinations are computed for each client after figuring out how much it can benefit from every other client’s model.

Personalization in FL shares a similar methodology with our incentive mechanism - different clients are distributed with different models - but with distinct objectives. Typical personalization aims to adapt models to the client’s data distribution to obtain a well-performing model on its data of interest. On the other hand, our incentive mechanism attempts to use performance discrepancy to encourage clients to contribute more data. Recent personalization work [18, 19] introduces an attention-inducing function to facilitate collaborations between similar clients. In contrast, our approach uses weighted aggregation to ensure that models with similar performance can access similar data, thus achieving our objective.

3 Background and Problem Formulation

3.1 Background

Federated learning is a clever way of utilizing disjointed data and computational resources to train machine learning models. We refer to the set of all clients as \mathcal{C} . \mathcal{D}_k denotes the local dataset of client $\mathcal{C}_k \in \mathcal{C}$, which consists of $|\mathcal{D}_k|$ samples. $\mathcal{D} = \bigcup_{k \in 1 \dots |\mathcal{C}|} \mathcal{D}_k$ represents the full training set. $\mathcal{L}(w, \mathcal{D}_k) = \frac{1}{|\mathcal{D}_k|} \sum_{z \in \mathcal{D}_k} \mathcal{L}(w, z)$ denotes the empirical loss over model w and \mathcal{D}_k . In the t -th round of federated learning, the optimization problem is formulated as minimizing the empirical loss over all the training examples of the clients \mathcal{C}_t who participate in that round:

$$\min_{w \in \mathbb{R}^p} \mathcal{L}(w, \mathcal{D}) = \sum_{k \in \mathcal{C}_t} \frac{|\mathcal{D}_k|}{N(\mathcal{C}_t)} \mathcal{L}(w, \mathcal{D}_k) \quad (1)$$

where $N(\mathcal{C}_t)$ represents the total number of samples belonging to the clients in \mathcal{C}_t . In other words, the global loss function $\mathcal{L}(w, \mathcal{D})$ is the weighted average of local functions $\mathcal{L}(w, \mathcal{D}_k)$ and the weights are proportional to the sizes of the local datasets. We adopt a widely used standard FL algorithm, federated averaging (FedAvg) [20], to solve the optimization problem. FedAvg is the basic building block of our algorithm and it executes as the following. In the initial stage, the centralized server randomly initializes a global model w_0 . Then the training process orchestrates alternated local and global updates and communications between the server and the clients in every round $t \in 1 \dots T$. Specifically, each client performs local training for a few epochs during each round. Afterward, the local models are uploaded online so that the server can perform federated aggregation to generate a single global model. Subsequently, the server distributes the global model to the clients and proceeds to the next round. The training, uploading and distributing processes are repeated until the global model converges.

Formally speaking, during the i -th training epoch of the t -th round, each client updates its local model through gradient descent:

$$w_{t,i}^k = w_{t,i-1}^k - \eta \nabla \mathcal{L}(w_{t,i-1}^k, \mathcal{D}_k), \quad (2)$$

and at the end of the t -th round, the server performs federated aggregation to compute the weighted or unweighted average of selected client models:

$$w_t \leftarrow \sum_{k \in \mathcal{C}_t} \frac{|\mathcal{D}_k|}{N(\mathcal{C}_t)} w_{t,m}^k. \quad (3)$$

The above weighted aggregation assumes that the information of the local data is known, which might be unrealistic in practice. On the contrary, unweighted aggregation is performed when such information is unknown:

$$w_t \leftarrow \sum_{k \in \mathcal{C}_t} w_{t,m}^k. \quad (4)$$

Please refer to Table 1 for a list of notations used in this paper.

The effectiveness of vanilla federated learning relies on clients' collective efforts and their willingness to contribute local data. However, we believe the unselfish assumption is unrealistic when considering the moral hazard. In reality, every client could contribute very little data or even upload a fraudulent model when participating in federated learning. Once it receives the global model, it fine-tunes that model with all the local data. In this way, the client could get almost all the benefits while devoting the least amount of data. Federated learning will be downgraded to individual learning if most clients take this strategy.

3.2 Problem Formulation

It is paramount to develop an incentive mechanism to avoid such a scenario. To address the incentive problem, we first derive the utility function with respect to a client's local data contribution. In the vanilla FL framework, it is assumed that all clients will always contribute all local data. Therefore, a client's utility is only related to the performance of the final model distributed by the centralized server, i.e., $u_i = \gamma p(d_i)$. As already discussed, this is usually not the case. In contrast, our framework assumes that every client has a goal to maximize its utility by choosing the proper amount of data they would collect or devote when participating in federated learning. Hence, along with the model performance,

we add another term to the client’s utility function. This term evaluates the cost of a client’s participation, including the efforts of data collection and the consumption of computational resources on local training.

$$u_i = \gamma p(d_i, D_{\bar{i}}) - \alpha c(d_i). \quad (5)$$

Here, d_i represents the local data possessed and contributed by \mathcal{C}_i and $D_{\bar{i}}$ represents data possessed by all the remaining clients and is available to \mathcal{C}_i . Accordingly, $p_i = p(d_i, D_{\bar{i}})$ denotes the performance of the model sent to client i while $c_i = c(d_i)$ denotes the client’s participation cost. We can then formulate the data contribution decision problem as a utility maximization problem:

Problem 1 (Utility Maximization for Federated Learning). *Design an incentive mechanism and corresponding utility functions that encourages all clients to contribute as much data they can possibly obtain:*

$$\forall i : \operatorname{argmax}_{d_i} u_i(d_i) = d_i^t. \quad (6)$$

In our setting, rewards are not monetized but reflected by the quality of the models the server sends back to clients. We do not require clients to report their training data sizes honestly. Otherwise, the problem becomes trivial as the server can easily compute the contribution levels, based on which the centralized server rewards the significant contributors.

To facilitate our design, we make the following reasonable *Assumptions*:

- I. We assume a maximum threshold, d_i^t , of the amount of data the client i can obtain.
- II. We assume that model quality is completely determined by data quality for a specific FL training task. For simplicity, we further assume that data quality is solely decided by the amount of data used in training. We let a fixed degeneration factor, γ , on the model quality for federated aggregation, compared with the case that a single learner can utilize all data in \mathcal{D} .
- III. We assume the centralized server has a validation dataset so that it can approximately evaluate a local model’s performance on \mathcal{D} .
- IV. $p(d_i)$ is concave because the marginal performance gain decreases with respect to the amount of data used for training when the dataset is sufficiently large. On the other hand, $c(d_i)$ is convex because the marginal cost to obtain new data increases when the dataset is already sufficiently large.

4 The Incentivized Federated Learning Algorithm

Our main idea is to reward a client who makes a more significant contribution with a higher performance model, aggregated from a broader range of other clients’ models. More specifically, the centralized server ranks models uploaded by all clients based on their performances. Then a client’s model will be aggregated with those models with lower rankings and then sent back to the client. Formally, the aggregation process can be stated as:

$$w_i^k \leftarrow \sum_{j=1}^{r(k)} w_{t,m}^j, \quad (7)$$

where $r(k)$ represents the position of client k after ranking. Note that there is no global model since every client will be distributed a different model based on the performance of its own model.

Our proposed incentivized federated learning framework is illustrated by Algorithm 1. Essentially, we ensure that $p(d_i, D_{\bar{i}})$ is depending on both d_i and $D_{\bar{i}}$, while d_i can also impact $D_{\bar{i}}$. To realize this effect, we assume the centralized server has access to a small validation dataset that follows the global data distribution (Assumption III). The centralized server can evaluate model performance on this dataset and determine the aggregation strategy for each client. Our purpose is to make sure that a high-performance model is likely to be aggregated with models trained from a larger total amount of data. In other words, we ensure that $D_{\bar{i}}$ increases as d_i increases. According to Assumption II, a model’s performance is correlated to the size of its training dataset. We model the relation between $D_{\bar{i}}$ and d_i as a function $D_{others}(d_i)$. As a result, a client who contributes a better model will receive a better resultant model from the server with high probability.

Hence, for vanilla FL, the derivative of the utility function with respect to the amount of training data is:

$$\frac{\partial u_i}{\partial d_i} = \left(\gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} - \alpha \frac{\partial c_i}{\partial d_i} \right). \quad (8)$$

Algorithm 1 *The Incentivized Federated Learning Algorithm.* S is the set of all clients; T is the number of rounds; E is the number of local epochs; w_0 is the common initial model; \mathcal{D}_k is the local dataset of client k ; \mathcal{D}_v is the global validation dataset; η is the learning rate.

```

1: // server executes
2: procedure SeverUpdate
3:   for client  $k \in S$  do
4:      $w_0^k \leftarrow w_0$ 
5:   end for
6:   for round  $t \in 1 \dots T$  do
7:      $Acc \leftarrow \{\}$ 
8:     for each client  $k \in S$  in parallel do
9:        $w_{t,m}^k \leftarrow \text{ClientUpdate}(k, w_{t-1}^k)$ 
10:      evaluate  $w_{t,m}^k$  on  $\mathcal{D}_v$  and obtain  $Acc^k$ 
11:       $Acc \leftarrow Acc \cup \{Acc^k\}$ 
12:    end for
13:     $Acc \leftarrow \text{sorted}(Acc)$ 
14:     $r(k) \leftarrow$  the index function of clients according to  $Acc$ 
15:    for client  $k \in S$  do
16:       $w_t^k \leftarrow \sum_{j=1}^{r(k)} w_{t,m}^j$ 
17:    end for
18:  end for
19:
20: // client  $k$  executes
21: procedure ClientUpdate( $k, w^k$ )
22:   for local epoch  $e \in 1 \dots E$  do
23:     for batch  $b \in \mathcal{D}_k$  do
24:        $w^k = w^k - \eta \nabla \mathcal{L}(w^k, b)$ 
25:     end for
26:   end for
27:   return  $w^k$  to server

```

It can be seen that d_i has no impact on $D_{\bar{i}}$. However, for our proposed mechanism, $D_{\bar{i}}$ is depending on d_i :

$$\frac{\partial u_i}{\partial d_i} = \left(\gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} + \gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} \frac{\partial D_{\bar{i}}}{\partial d_i} - \alpha \frac{\partial c_i}{\partial d_i} \right) \quad (9)$$

With the extra term, as long as $\frac{\partial D_{\bar{i}}}{\partial d_i}$ is positive, the optimal value d_i^{opt} for Equation 8 will be greater than that of Equation 9. Given $D_{\bar{i}}$ a concave function over d_i , a sufficiently large $\frac{\partial D_{\bar{i}}}{\partial d_i}$ guarantees that d_i^{opt} equals or exceeds the maximum threshold d_i^t . In this case, every client will attempt to contribute as much data as it can collect to maximize its utility.

Lemma 1 states that those clients who contribute more data can have their models learn from more data from other clients.

Lemma 1. *If the server has access to global data distribution and can precisely measure models' performances, our mechanism ensures $\frac{\partial D_{\bar{i}}}{\partial d_i} \geq 0$ for all i .*

Proof. By Assumption II, a model's performance is monotonically increasing as the amount of data used for training grows, so $\frac{\partial p_i}{\partial d_i} > 0$. Moreover, our aggregation strategy ensures that a higher performance model will always be aggregated with a larger set of models from other clients. Again, by Assumption II, those models represent a larger amount of training data, which implies $\frac{\partial D_{\bar{i}}}{\partial p_i} \geq 0$. Hence, $\frac{\partial D_{\bar{i}}}{\partial d_i} = \frac{\partial D_{\bar{i}}}{\partial p_i} \cdot \frac{\partial p_i}{\partial d_i} > 0$.

□

With the Lemma 1, we can derive Theorem 2. Intuitively, it states that our incentive mechanism satisfies the baseline incentive requirement that it encourages clients to contribute more data compared with the vanilla federated learning.

Theorem 2. *Let d_i^{opt} and d_i^{opt*} denote the optimal values of d_i that maximizes utility under the proposed incentive mechanism and the vanilla federated learning, respectively. The proposed mechanism always guarantees that $d_i^{opt} \geq d_i^{opt*}$.*

Proof. Consider the utility functions given by Equation 9 and Equation 8. For clarity, let $f(d_i) = \gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} - \alpha \frac{\partial c_i}{\partial d_i}$ and $g(d_i) = \gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} \frac{\partial D_{\bar{i}}}{\partial d_i}$. Thus, the utility function for the proposed incentive mechanism is $f(d_i) + g(d_i)$ and that for vanilla federated learning is $f(d_i)$. Note that based on Assumption IV, both $\gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})}$ and $-\alpha \frac{\partial c_i}{\partial d_i}$ monotonically decreases for $d_i \in [0, \mathcal{D}_i]$, and so is $f(d_i)$. Additionally, $g(d_i) > 0$ always holds, because our incentive mechanism guarantees that $\frac{\partial D_{\bar{i}}}{\partial d_i} \geq 0$, while Assumption II implies that $\frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} > 0$. We prove by 2 cases:

Case 1 ($f(d_i)|_{d_i=\mathcal{D}_i} > 0$).

This condition implies that $f(d_i) > 0$ throughout the range $d_i \in [0, \mathcal{D}_i]$. Hence, a client should devote the whole dataset to maximize its utility in both schemes, i.e., $d_i^{opt} = d_i^{opt*} = \mathcal{D}_i$. So $d_i^{opt} \geq d_i^{opt*}$ holds in this case.

Case 2 ($f(d_i)|_{d_i=\mathcal{D}_i} \leq 0$).

If $f(d_i)|_{d_i=0} \leq 0$, $f(d_i)$ is negative throughout the range. Therefore the optimal value for vanilla federated learning $d_i^{opt*} = 0$. It immediately follows that $d_i^{opt} \geq d_i^{opt*}$.

Otherwise, $f(d_i)|_{d_i=0} > 0$. We further decompose the scenario into 2 cases. When $(f(d_i) + g(d_i))|_{d_i=\mathcal{D}_i} \geq 0$, $d_i^{opt} = \mathcal{D}_i$, and therefore $d_i^{opt} \geq d_i^{opt*}$. When $(f(d_i) + g(d_i))|_{d_i=\mathcal{D}_i} < 0$, there must exist a d_i^{opt*} , such that $f(d_i^{opt*}) = 0$. There must also exist a d_i^{opt} , such that $f(d_i^{opt}) + g(d_i^{opt}) = 0$. Because $g(d_i) > 0$ always holds, $f(d_i^{opt}) < 0$. Additionally, $f(d_i)$ is monotonically decreasing, and this indicates that $d_i^{opt} \geq d_i^{opt*}$.

Hence, we conclude that $d_{opt}' \geq d_{opt}$.

□

The guarantee provided by Theorem 2 may not be sufficiently strong for some use cases. We would like to ensure that all clients contribute data to their maximal capabilities. We present a stronger result in Theorem 3. It states that when the magnitude of the term $\frac{\partial D_{\bar{i}}}{\partial d_i}$ is large enough to dominate the utility function, the client will devote all its data. We would like to point out that the term is also depending on the data distributions among all clients along with our incentive mechanism.

Theorem 3. *If $D_{\bar{i}}$ is a concave function over d_i , and $\frac{\partial D_{\bar{i}}}{\partial d_i}$ is sufficiently large, for example, it satisfies Equation 10, all clients will contribute the maximum amount of data they possess to maximize their utilities.*

$$\frac{\partial D_{\bar{i}}}{\partial d_i} \Big|_{d_i=\mathcal{D}_i} > \frac{\frac{\alpha}{\gamma} \cdot \frac{\partial c_i}{\partial d_i} \Big|_{d_i=\mathcal{D}_i} - \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} \Big|_{d_i=\mathcal{D}_i}}{\frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} \Big|_{d_i=\mathcal{D}_i}}. \quad (10)$$

Proof. Let $u(d_i)$ and $u^*(d_i)$ denote the utility functions under the proposed incentive mechanism and the vanilla federated learning, respectively.

Due to Assumption IV, we have

$$\frac{\partial^2 u_i^*}{\partial^2 d_i} = \left(\frac{\partial^2 p_i}{\partial^2 (d_i + D_{\bar{i}})} - \alpha \frac{\partial^2 c_i}{\partial^2 d_i} \right) \leq 0. \quad (11)$$

Thus $u^*(d_i)$ is a concave function within the range. Moreover, given Assumption II, if $D_{\bar{i}}$ is a concave function over d_i , we have

$$\frac{\partial^2 u_i}{\partial^2 d_i} = \left(\gamma \frac{\partial^2 p_i}{\partial^2 (d_i + D_{\bar{i}})} \left(\frac{\partial D_{\bar{i}}}{\partial d_i} + 1 \right)^2 + \gamma \frac{\partial p_i}{\partial (d_i + D_{\bar{i}})} \frac{\partial^2 D_{\bar{i}}}{\partial^2 d_i} - \alpha \frac{\partial^2 c_i}{\partial^2 d_i} \right) \leq 0. \quad (12)$$

When Equation 10, we can immediately derive that $\frac{\partial u_i}{\partial d_i} \Big|_{d_i=\mathcal{D}_i} > 0$. It follows that both $u(d_i)$ and $u^*(d_i)$ monotonically increase as d_i increases. Thus, all clients are willing to contribute as much data as possible. \square

The proof of Theorem 3 requires $D_{\bar{i}}$ to be a concave function over d_i . Note that this is a sufficient condition, but may not be a necessary condition in practice. Additionally, we observe that this is usually the case in reality, particularly when the amounts of data possessed by clients is sub-exponential, *i.e.*, a small number of clients hold a larger amount of data when the majority of the clients hold fewer data. Even without the concavity assumption, the following statement is still valid.

Corollary 3.1. *It is a Nash Equilibrium state that all clients contribute the maximum amount of data they possess in federated learning.*

Under the proposed incentive mechanism, no clients will change their strategy when they have already devoted all their data in federated learning.

Discussions.

i) The proposed approach trades off the model performance for the incentive to contribute data. Only the client who possesses the largest amount of data can obtain a model that has a similar performance as in vanilla federated learning. Therefore, if model performance is a major concern, the vanilla federated learning could be more desirable.

ii) We made a strong assumption in Assumption II that the training data has even quality. It can be interesting to study how data quality can affect the incentive mechanism.

iii) The proof Theorem 3 requires that the magnitude of $\frac{\partial D_{\bar{i}}}{\partial d_i}$ is sufficiently large, and the concavity of $D_{\bar{i}}$. However, both of them rely on the data distribution among the clients, albeit they are reasonable for general federated learning applications. Furthermore, we note that although these two conditions are required for the soundness of the proof, they are not always necessary for our incentive mechanism. We plan to experimentally study when these two conditions can be violated and how the proposed mechanism works in those situations.

5 Conclusion

In this paper, we present an incentive mechanism for federated learning, such that all clients are willing to contribute their data. We replace monetary rewards with rewards in model performance. We proved that all clients are always more willing to contribute data under the proposed mechanism. Moreover, the proposed mechanism ensures that all clients will devote their maximum amount of data under certain conditions. Future works include assessing its effectiveness in practice and designing dedicated mechanisms for more general and diverse scenarios.

References

- [1] Yunus Sarikaya and Ozgur Ercetin. Motivating workers in federated learning: A stackelberg game perspective, 2019.
- [2] Yufeng Zhan, Peng Li, Zhihao Qu, Deze Zeng, and Song Guo. A learning-based incentive mechanism for federated learning. *IEEE Internet of Things Journal*, 7(7):6360–6368, 2020.
- [3] Shaohan Feng, Dusit Niyato, Ping Wang, Dong In Kim, and Ying-Chang Liang. Joint service pricing and cooperative relay communication for federated learning, 2018.
- [4] Rongfei Zeng, Shixun Zhang, Jiaqi Wang, and Xiaowen Chu. Fmore: An incentive scheme of multi-dimensional auction for federated learning in mec. *2020 IEEE 40th International Conference on Distributed Computing Systems (ICDCS)*, Nov 2020.
- [5] Yutao Jiao, Ping Wang, Dusit Niyato, Bin Lin, and Dong In Kim. Toward an automated auction framework for wireless federated learning services market, 2020.
- [6] Tra Huong Thi Le, Nguyen H. Tran, Yan Kyaw Tun, Zhu Han, and Choong Seon Hong. Auction based incentive design for efficient federated learning in cellular wireless networks. In *2020 IEEE Wireless Communications and Networking Conference (WCNC)*, pages 1–6, 2020.
- [7] Jiawen Kang, Zehui Xiong, Dusit Niyato, Han Yu, Ying-Chang Liang, and Dong In Kim. Incentive design for efficient federated learning in mobile networks: A contract theory approach, 2019.
- [8] Ningning Ding, Zhixuan Fang, and Jianwei Huang. Incentive mechanism design for federated learning with multi-dimensional private information. In *2020 18th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT)*, pages 1–8, 2020.

- [9] Guan Wang, Charlie Xiaoqian Dang, and Ziyue Zhou. Measure contribution of participants in federated learning. In *2019 IEEE International Conference on Big Data (Big Data)*, pages 2597–2604, 2019.
- [10] Tianshu Song, Yongxin Tong, and Shuyue Wei. Profit allocation for federated learning. In *2019 IEEE International Conference on Big Data (Big Data)*, pages 2577–2586, 2019.
- [11] Yuan Liu, Shuai Sun, Zhengpeng Ai, Shuangfeng Zhang, Zelei Liu, and Han Yu. Fedcoin: A peer-to-peer payment system for federated learning, 2020.
- [12] Hao Wang, Zakhary Kaplan, Di Niu, and Baochun Li. Optimizing federated learning on non-iid data with reinforcement learning. In *IEEE INFOCOM 2020 - IEEE Conference on Computer Communications*, pages 1698–1707, 2020.
- [13] Tian Li, Maziar Sanjabi, Ahmad Beirami, and Virginia Smith. Fair resource allocation in federated learning, 2020.
- [14] Mehryar Mohri, Gary Sivek, and Ananda Theertha Suresh. Agnostic federated learning, 2019.
- [15] Wei Huang, Tianrui Li, Dexian Wang, Shengdong Du, and Junbo Zhang. Fairness and accuracy in federated learning, 2020.
- [16] Ligeng Zhu, Zhijian Liu, and Song Han. Deep leakage from gradients, 2019.
- [17] Bo Zhao, Konda Reddy Mopuri, and Hakan Bilen. idlg: Improved deep leakage from gradients, 2020.
- [18] Yutao Huang, Lingyang Chu, Zirui Zhou, Lanjun Wang, Jiangchuan Liu, Jian Pei, and Yong Zhang. Personalized cross-silo federated learning on non-iid data, 2020.
- [19] Michael Zhang, Karan Sapra, Sanja Fidler, Serena Yeung, and Jose M. Alvarez. Personalized federated learning with first order model optimization, 2020.
- [20] Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Aguera y Arcas. Communication-efficient learning of deep networks from decentralized data. In *Artificial intelligence and statistics*, pages 1273–1282. PMLR, 2017.

Appendix A Symbol Table

Table 1: Table of Notation

Notation	Description
FL	federated learning
\mathcal{C}	the set of all clients participate in FL
\mathcal{C}_k	k -th client in \mathcal{C}
\mathcal{D}_k	the dataset possessed by \mathcal{C}_k
\mathcal{D}	the full dataset
$\mathcal{L}(w, \mathcal{D})$	loss of model w on dataset \mathcal{D}
t	t -th round of FL, $t \in 1 \cdots T$
\mathcal{C}_t	the set of all clients who participate in or selected for round t
$N(\mathcal{C}_t)$	the total number of samples belonging to the clients in \mathcal{C}_t
$w_{t,i}^k$	local model of \mathcal{C}_k in round t and training epoch i
w_t^k	server aggregated model for \mathcal{C}_k in round t
$u(\cdot), u_i$	utility function of \mathcal{C}_i
d_i	the portion of data possessed and contributed by \mathcal{C}_i
d_i^{opt}	the value of d_i which maximizes a corresponding utility function
\bar{D}_i	the portion of data possessed by other clients and is available to \mathcal{C}_i
D_i^{total}	data possessed by all clients and is available to \mathcal{C}_i
$p(\cdot), p_i$	model performance function of \mathcal{C}_i
$c(\cdot), c_i$	participation cost function of \mathcal{C}_i
$r(\cdot)$	model performance index function
\mathcal{D}_v	the global validation dataset held by the central server

Appendix B Partial Derivatives of $u(d_i)$

Derivations for the partial derivatives of $u(d_i)$:

$$\begin{aligned}
\frac{\partial u_i}{\partial d_i} &= \gamma \frac{\partial p_i}{\partial d_i} - \alpha \frac{\partial c_i}{\partial d_i} \\
&= \gamma \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \frac{\partial(d_i + D_{\bar{i}})}{\partial d_i} - \alpha \frac{\partial c_i}{\partial d_i} \\
&= \gamma \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \left(1 + \frac{\partial D_{\bar{i}}}{\partial d_i}\right) - \alpha \frac{\partial c_i}{\partial d_i} \\
&= \left(\gamma \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} + \gamma \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \frac{\partial D_{\bar{i}}}{\partial d_i} - \alpha \frac{\partial c_i}{\partial d_i} \right) \geq 0.
\end{aligned} \tag{13}$$

Denote $\frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} = F(d_i + D_{\bar{i}})$, then

$$\begin{aligned}
\frac{\partial^2 u}{\partial^2 d_i} &= \gamma \left(\frac{\partial^2 p_i}{\partial(d_i + D_{\bar{i}}) \partial d_i} + \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \frac{\partial^2 D_{\bar{i}}}{\partial^2 d_i} + \frac{\partial^2 p_i}{\partial(d_i + D_{\bar{i}}) \partial d_i} \frac{\partial D_{\bar{i}}}{\partial d_i} \right) - \beta \frac{\partial^2 c_i}{\partial^2 d_i} \\
&= \gamma \left(\frac{\partial F}{\partial d_i} + \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \frac{\partial^2 D_{\bar{i}}}{\partial^2 d_i} + \frac{\partial F}{\partial d_i} \frac{\partial D_{\bar{i}}}{\partial d_i} \right) - \beta \frac{\partial^2 c_i}{\partial^2 d_i} \\
&= \gamma \left(\frac{\partial F}{\partial(d_i + D_{\bar{i}})} \left(\frac{\partial D_{\bar{i}}}{\partial d_i} + 1 \right) + \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \frac{\partial^2 D_{\bar{i}}}{\partial^2 d_i} + \frac{\partial F}{\partial(d_i + D_{\bar{i}})} \left(\frac{\partial D_{\bar{i}}}{\partial d_i} + 1 \right) \frac{\partial D_{\bar{i}}}{\partial d_i} \right) \\
&\quad - \beta \frac{\partial^2 c_i}{\partial^2 d_i} \\
&= \gamma \frac{\partial^2 p_i}{\partial^2(d_i + D_{\bar{i}})} \left(\frac{\partial D_{\bar{i}}}{\partial d_i} + 1 \right)^2 + \gamma \frac{\partial p_i}{\partial(d_i + D_{\bar{i}})} \frac{\partial^2 D_{\bar{i}}}{\partial^2 d_i} - \alpha \frac{\partial^2 c_i}{\partial^2 d_i} \leq 0.
\end{aligned} \tag{14}$$