
A Minimalist Dataset for Systematic Generalization of Perception, Syntax, and Semantics

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Abstract

Inspired by humans’ remarkable ability to master arithmetic and generalize to unseen problems, we present a new dataset, Handwritten arithmetic with INTegers (HINT), to study machines’ capability of learning generalizable concepts at three levels: *perception*, *syntax*, and *semantics*. Learning agents are tasked to reckon how concepts are perceived from raw signals such as images (*i.e.*, perception), how multiple concepts are structurally combined to form a valid expression (*i.e.*, syntax), and how concepts are realized to afford various reasoning tasks (*i.e.*, semantics), all in a weakly supervised manner. With a focus on systematic generalization, we carefully design a five-fold test set to evaluate both the *interpolation* and the *extrapolation* of learned concepts w.r.t. the three levels. We further design a few-shot learning split to test whether models could quickly learn new concepts and generalize them to more complex scenarios. To understand existing models’ limitations, we conduct extensive experiments with various sequence-to-sequence models, including RNNs, Transformers, and GPT-3 (with the chain of thought prompting). The results suggest that current models still struggle in extrapolation to long-range syntactic dependency and semantics. Models show a significant gap toward human-level generalization when tested with new concepts in a few-shot setting. Moreover, we find that it is infeasible to solve HINT by simply scaling up the dataset and the model size; this strategy barely helps the extrapolation over syntax and semantics. Finally, in zero-shot GPT-3 experiments, the chain of thought prompting shows impressive results and significantly boosts the test accuracy. We believe the proposed dataset together with the experimental findings is of great interest to the community on systematic generalization.¹

1 Introduction

Humans possess a versatile mechanism for learning concepts [29]. Take the arithmetic examples in Fig. 1: When we master concepts like digits and operators, we not only know how to recognize, write, and pronounce them—what these concepts mean at the *perceptual* level, but also know how to compose them into valid expressions—at the *syntactic* level, and how to calculate the results—at the *semantic* level. Learning concepts heavily rely on these three-level interweaving meanings. Such an observation also conforms with the classic view of human cognition, which postulates at least three distinct levels of organizations in computation systems [75, 31].

¹We release our dataset, code, and experiments at our project website <https://liqing-ustc.github.io/HINT/>.

Another appealing property of human concept learning is its *systematic compositionality* [12, 69]: the algebraic capacity to understand and produce a potentially infinite number of novel combinations from a finite set of known components, *i.e.*, “infinite use of finite means” [13]. This type of compositionality is crucial to the human ability to make strong generalizations from limited data [59]. Human demonstrate systematic compositionality in many domains. *E.g.*, once we master the syntax of arithmetic using short expressions, we can parse novel, long expressions; once we master operators’ semantics using small numbers, we can apply them over novel, large numbers.

The emerging community on learning models capturing human-like systematic compositionality has introduced various benchmarks [57, 43, 50, 4, 81, 51] and methods [18, 56, 82, 2, 33, 15, 7, 1]. However, as collecting real data with systematic compositionality is challenging, prior benchmarks are from artificial domains with synthetic data and tasks and only cover a partial spectrum of concept learning; see Tab. 1 for an in-depth comparison. Critically, prior datasets often mix syntax and semantics when evaluating systematic compositionality.² For example, the SCAN dataset [57] is a semantic parsing task from natural language commands to action sequences; when a model fails on a longer command than the ones in the training set, the root cause could come from understanding the complex syntactic relations in a long input sequence (command) or its deficiency of generating a long output sequence (actions) (*e.g.*, due to the EOS decision problem [71]). Furthermore, prior benchmarks often involve simple semantics (*e.g.*, a simple mapping or repeating), leading to an undesirable bias towards syntactic generalization.

To extend the systematic compositionality to a full-spectrum systematic generalization w.r.t. perception, syntax, and semantics, we take inspiration from arithmetic and introduce a new benchmark HINT, Handwritten arithmetic with INTegers. The task of HINT is intuitive: Machines take as input images of handwritten expressions and predict the final results of expressions, restricted in the integers. This design is minimal yet complete on systematic generation with real images. HINT differs from prior datasets as: (i) The images are realistic handwriting with considerable visual variance. (ii) The syntactic relations between the tokens in the expressions are more complex with long-range dependency. (iii) The semantics are the functional meanings of these arithmetic concept, which are more complicated than the simple mappings in previous datasets. In HINT, models are tasked to learn the three-level meanings simultaneously to predict the correct results. Since there is no intermediate supervision, the three-level meanings are presumably intertwined during learning.

The three meanings in HINT have clear definition and boundary, thus affords to design tests focusing on a certain type of generalization. For example, to evaluate if a model can extrapolate over syntax, we can build a test set composed of longer expressions with the same range of results as the training set. To provide a holistic and rigorous test on how models generalize the learned concepts, we introduce a carefully designed evaluation scheme with five subsets, focusing on generalization patterns (*i.e.*, interpolation and extrapolation) at different levels (*i.e.*, perception, syntax, and semantics).

To facilitate future research along this direction, we conduct extensive experiments of various sequence-to-sequence models, including Recurrent Neural Networks [40, 14], Transformers [88], and GPT-3 [8] (with the chain of thought prompting). Our experiments show that there is still large room for improvement on HINT: Even the state-of-the-art model, Universal Transformer [17] with relative positional encoding [85, 16], can only achieve an accuracy of 54% on HINT, whereas the

²The concept of “syntax” and “semantics” have various meanings in different communities. In this work, we adopt the original and specific definition, wherein “syntax” is the underlying compositional structure of input sequences, and “semantics” represents what concepts mean in reasoning tasks.

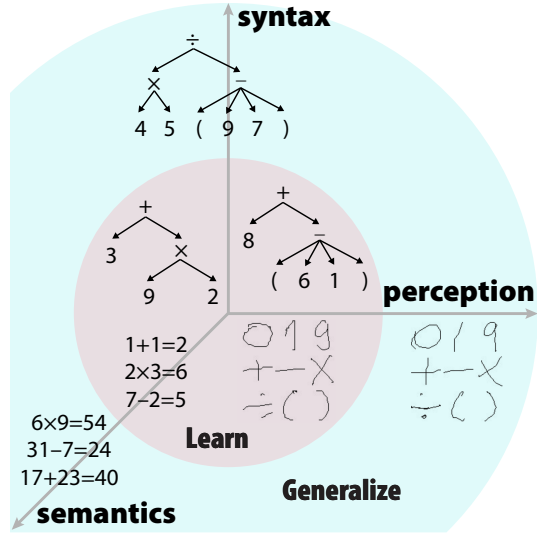


Figure 1: **Concept learning and generalization at three levels.** A learning agent needs to simultaneously master (i) **perception**, how concepts are perceived from raw signals such as images, (ii) **syntax**, how multiple concepts are structurally combined to form a valid expression, and (iii) **semantics**, how concepts are realized to afford various reasoning tasks.

very same model obtains nearly perfect accuracy on previous datasets like SCAN [15]. An in-depth analysis of the results on each test subset reveals that current models still struggle on extrapolation to long-range syntactic dependency and semantics. In the GPT-3 experiments, the chain of thought prompting boost the zero-shot test accuracy by a large margin from 8.6% to 27.6%.

By inspecting the scaling trends of the test accuracy w.r.t. the size of the model and the dataset, we find that it is impractical to solve HINT by scaling up the size of dataset or the model as done in NLP tasks [49, 38]; more data and parameters barely help the extrapolation over syntax and semantics.

Although the top performing models exhibit decent capabilities of learning new concepts, the few-shot learning experiments show that they are still far from the human-level generalization that only requires the learning examples of a new concept in a primitive form and readily generalizes to more complex compositions of the learned concept.

Taking together, we present the HINT dataset for systematic generalization of perception, syntax, and semantics. We benchmark various seq2seq models and identify the key weaknesses. We hope our dataset and the experimental findings inspire new advancements in systematic generalization.

2 Related Work

Three Levels of Concept Learning Although the recent surge of deep learning [60] has significantly advanced the *perception* accuracy given raw signals across multiple modalities, including images [37, 54] and audios [73, 39, 34], learning models fail to disentangle the perceptual generalizations from the other two levels of generalizations. *Syntax* analysis is to understand the compositional and recursive structures in various tasks, such as natural language parsing [9, 52, 46], image and video parsing [87, 99, 98, 35, 77, 76, 44], scene understanding [42, 41, 78, 45, 11, 93, 36], task planning [92, 64, 21, 63, 97], and abstract reasoning [94, 95, 96, 23, 24, 22]. *Semantics* of concepts essentially describe its causal effect. Two primary semantic representations prevail in symbolic reasoning: (i) Logic [66, 68] regards the semantic learning as inductive logic programming [70, 28]—a general framework to induce first-order logic theory from examples. (ii) Program treats the semantic learning as inductive program synthesis [55, 58, 6, 20, 25, 27, 26]. Compared to the aforementioned literature, ours is the first that takes a more holistic stand that simultaneously addresses *all* three levels of concept learning, essentially taking one step closer to realize a versatile mechanism of concept learning under weak supervision.

Table 1: **Dataset categorization and comparison.** SP: semantic parsing, IC: image classification, QA: question answering, NV: navigation, i&t: image & text. Perception/Syntax/Semantics denote whether the task requires models to learning perception/syntax/semantics. Generalization denotes the type of generalization required to handle the test examples. *Images in the dataset are generated with small variance. +Datasets require learning semantics but do not evaluate systematic generalization w.r.t. semantics.

Dataset	Domain	Task	Modality	Perception	Syntax	Semantics	Generalization	Size
SCAN [57]	synthetic	SP	text		✓	✓	systematic	100K
gSCAN [81]	synthetic	SP	i&t	✓*	✓	✓	systematic	300K
PCFG [43]	synthetic	SP	text		✓	✓	systematic	100K
CFQ [50]	realistic	SP	text		✓	✓	systematic	239K
CURI [89]	synthetic	IC	image	✓		✓	systematic	15K
COGS [51]	realistic	SP	text		✓	✓	systematic	30K
SQOOP [5]	synthetic	QA	i&t	✓*		✓	systematic	1M
Mathematics [83]	realistic	QA	text		✓	✓	systematic	2M
HALMA [91]	synthetic	NV	image	✓		✓	systematic	-
CLOSURE [4]	synthetic	QA	i&t	✓	✓	✓	systematic	7K
CLEVR [47]	synthetic	QA	i&t	✓	✓	✓+	i.i.d	865K
HWF [61]	realistic	IC	image	✓			i.i.d	12K
MNIST-Add [68]	realistic	IC	image	✓			i.i.d	-
HINT	realistic	QA	image	✓	✓	✓	systematic	1M

Benchmarks on Systematic Generalization Despite that various benchmarks [57, 43, 50, 4, 81, 51, 50] have boost the progresses on systematic generalization (see Tab. 1), most are built from artificial domains with synthetic tasks and data, involving only one or two aspects of concept learning and often mixing the generalization over syntax and semantics. In contrast, the proposed HINT benchmark originates from a more realistic domain of arithmetic reasoning and requires a joint learning of perception, syntax, and semantics. The clear definitions and boundaries of these

	Prefix	$\times+328$	$--53\times52$	$\div2\times54$	operator semantics
	Infix	$(3+2)\times8$	$5-3-5\times2$	$2\div(5\times4)$	$+(a, b): a + b$
	HW				$-(a, b): \max(0, a - b)$
	Results	40	0	1	$\times(a, b): a \times b$
					$\div(a, b): \text{ceil}(a \div b)$

Figure 2: **The data generation pipeline and operator semantics.** We define $-(a, b)$ as $\max(0, a - b)$ to avoid negative results and $\div(a, b)$ as $\text{ceil}(a \div b)$ to avoid too many zero results in the generated expressions.

meanings in HINT affords building test splits to evaluate the specific generalizations. Of note, HINT possesses more complicated and holistic semantics, which avoids the undesirable bias towards syntactic generalization in previous datasets. The task of the HINT benchmark is inspired by the HWF dataset [61], but requires a full-spectrum learning of perception, syntax, and semantics. By going beyond an i.i.d train/test split in Li *et al.* [61], HINT focuses on evaluating systematic generalization across the different aspects of concepts.

Methods on Systematic Generalization New training regimes [56, 2, 1] and model architectures [18, 82, 15, 33, 7] have been devised to capture systematic generalization. For example, Russin *et al.* [82] extend a seq2seq model by separating semantic and syntactic information. Csordas *et al.* [15] explore various configurations for Transformer to improve its systematic compositionality. Andreas *et al.* [2] and Akyurek *et al.* [1] explore the data augmentation for compositional generalization. Additionally, several neural-symbolic methods with domain-specific designs [10, 72, 65] achieve near perfect accuracy on prior systematic generalization datasets like SCAN [57]. However, these neural-symbolic methods introduce some non-trivial domain-specific symbolic components into the system and thus are not straightforward to transfer to other domains; the flexibility and transferability are questionable [32, 84]. In this work, we benchmark on HINT prevailing seq2seq frameworks, including RNNs, Transformers, and GPT-3, which require minimal domain-specific design and might be of broad interest to the community at present. We leave the exploration of more sophisticated methods like data augmentation and neural-symbolic approaches for future work.

3 The HINT Dataset

In this section, we describe the details about the HINT benchmark, designed to study models’ capability of learning generalizable concepts at three different levels: perception, syntax, and semantics.

The Definitions of Perception, Syntax, and Semantics We first define the perception, syntax, and semantics in the domain of HINT. *Perception* denotes the mapping from image pixels to meaningful patterns, *e.g.*, mapping a handwritten expression to a symbolic sequence. *Syntax* denotes the mechanism of how the concepts in one sample are combined with each other in a structured way, *e.g.*, parsing the symbolic sequence into a tree. The syntax can be represented by a phrase-structure grammar in Tab. A2. *Semantics* denotes the functional meanings of these arithmetic concepts, *e.g.*, what value ‘5’ represents and what value ‘+’ outputs given two arguments 1 and 1; see Fig. 2.

Of note, although these three levels have a clear boundary in their definition, a model does not necessarily represent them by separate and individual modules. For example, an end-to-end neural network trained on this domain will probably have mixed neurons and parameters for the three levels. The notion of perception, syntax, and semantics only requires the models to capture these meanings during evaluation, regardless of how the models finish the tasks, implicitly or explicitly.

Task The task of HINT is intuitive: predict the final results of handwritten arithmetic expressions in a weakly-supervised manner. *I.e.*, only the final results are given as supervision; all the symbolic expressions, parse trees, and intermediate values are latent. Clearly, models must simultaneously master perception, syntax, and semantics to successfully achieve this task.

Data Generation The data generation process follows three steps; see Fig. 2. First, we extract handwritten images for each concept from CROHME [67], including digits $0 \sim 9$, operators $+$, $-$, \times , \div , and parentheses $(,)$. Second, we randomly sample *prefix* expressions and convert them

to *infix* expressions with necessary parentheses based on the operator precedence; only single-digit numbers are allowed. The symbolic expressions are fed into a solver to calculate the final results. Third, we randomly sample handwritten images for symbols in an expression and concatenate them to construct the final handwritten expression. We only keep the handwritten expressions as input and the corresponding final results as supervision; all intermediate results are discarded.

Full-Spectrum Systematic Generalization To rigorously evaluate systematical generalization of the learned concepts, we replace the typical i.i.d. split with a carefully designed evaluation scheme. We randomly split all handwritten images into train (75%), validation (5%), and test (20%).

First, for the training set, we limit the maximum number of operators to 10 and the maximum intermediate values to 100:

$$D_{\text{train}} \subset \mathcal{T}_{\text{train}} = \{(x, y) : |x| \leq 10, \max(v) \leq 100\}, \quad (1)$$

where x is the handwritten expression, $|x|$ its number of operators, y the final result, and v all the intermediate values and the final results. To ensure the diversity in the training set, we sample at most 100,000 unique expressions which have the same number of operators. To avoid the bias in the final results, we limit the maximum percentage of a certain result under 5%.

Next, we carefully curate the test set to evaluate different generalization capabilities (*i.e.*, interpolation and extrapolation) on different levels of meanings (*i.e.*, perception, syntax, and semantics). Specifically, the test set is composed of five subsets, formally defined as:

$$D_{\text{test}} = D^I \cup D^{\text{SS}} \cup D^{\text{LS}} \cup D^{\text{SL}} \cup D^{\text{LL}}, \text{ where} \quad (2)$$

$D^I \subset D_{\text{train}},$	generalization on perception only
$D^{\text{SS}} \subset \mathcal{T}_{\text{train}} \setminus D_{\text{train}},$	interpolation on both syntax and semantics
$D^{\text{LS}} \subset \{(x, y) : x > 10, \max(v) \leq 100\},$	extrapolation on syntax and interpolation on semantics
$D^{\text{SL}} \subset \{(x, y) : x \leq 10, \max(v) > 100\},$	interpolation on syntax and extrapolation on semantics
$D^{\text{LL}} \subset \{(x, y) : x > 10, \max(v) > 100\}.$	extrapolation on both syntax and semantics

All subsets in the test set require generalization on perception since all images in the test set are unseen in training. For the test set, we sample at most 1,000 unique expressions with the same number of operators, and the final results are also balanced. The maximum number of operators is set up to 20, and the maximum intermediate value to 10,000. We also build a small validation set for hyperparameter tuning during model development. See Appx. A for a detailed analysis.

Few-shot Learning and Generalization To further test whether models can learn new concepts as quickly as humans, we build a few-shot learning split to learn six new concepts. This few-shot learning split is used to test if the models pre-trained on the training set can quickly learn a new concept by fine-tuning on only a handful of examples involving the new concept. Here, “few-shot” implies that the examples used to learn a new concept are much fewer than those of the training set, but they are still more than how many humans require to learn a new concept. These six concepts have different means in terms of perception, syntax, and semantics: two new numbers ($x \curvearrowright$ and $y \curvearrowleft$, representing 11 and 12, respectively), two operators of precedence 1 ($a \oslash$ and $b \oslash$, representing max and min), and two operators of precedence 2 ($c \subset$ and $d \oslash$, representing arithmetic mean and harmonic mean). The train, validation, and test splits are built by the same strategy as in the full-spectrum generalization. Expressions are sampled to ensure that the corresponding new concept appears at least once in the expression.

4 Deep Sequence-to-Sequence Baselines

We benchmark deep sequence-to-sequence (seq2seq) frameworks [86] on HINT, since the task of HINT can be naturally formulated as a seq2seq problem: The input is a handwritten expression, segmented into a sequence of images by a sliding window, and the output is an integer, converted into a sequence of digits. Fig. A3 illustrates applying the seq2seq framework to a HINT example.

4.1 Image Tokenizing and Embedding

The current seq2seq frameworks usually take as input a sequence of tokens. To tokenize a handwritten expression, we first resize it by making its height 32 and apply a sliding window of size 32 along

the horizontal axis to render a sequence of images. Next, each image in the sequence is encoded by ResNet-18 [37], which has enough capacity for handling the visual variance in handwriting.

4.2 Encoder-Decoder Architectures

RNNs Recurrent neural networks (RNNs) have long been a dominant choice for sequence modeling tasks, thus we test two popular RNNs in the literature: long short-term memory (LSTM) [40] and gated recurrent units (GRU) [14]. Attention has often been used in RNNs; we evaluate each network both with and without attention [3].

Transformers Since invented [88] for machine translation, Transformers have gradually replaced recurrent or convolutional neural networks as the *de facto* choice for many sequence modeling tasks [19, 79, 8]. However, prior work [17, 43, 51] suggests that the vanilla Transformer [88] fails dramatically in many tasks requiring systematic generalization when the sequence lengths exceed those observed during training. Recently, several simple tricks [15] have been demonstrated to improve the generalization capability of Transformers; two of them work particularly well: (i) using relative positional encoding [85, 16], and (ii) sharing weights across the blocks in the Transformer, *a.k.a.*, Universal Transformer [17]. Therefore, we benchmark three variants of Transformer: the vanilla Transformer, Transformer with relative positional encoding, and Universal Transformer with relative positional encoding.

GPT-3 Since GPT-3 [8] was released, there have been heated discussions and different opinions about the mathematical reasoning capability of pre-trained large language models.³ To systematically and comprehensively evaluate GPT-3’s capability of arithmetic reasoning, we test GPT-3 on the proposed HINT benchmark using symbolic expressions as input. Since all tokens of HINT are in the vocabulary of GPT-3, we directly evaluate GPT-3 via zero-shot prompting using the OpenAI API⁴. We construct the prompt in the following form: “*Q: What is <Expression>? A: The answer is,*” similar to the practice in Brown *et al.* [8] but with more complex expressions.

Very recently, chain of thought prompting (CoT) [90] is extended to the zero-shot setting [53] by adding a simple prompt, “*Let’s think step by step,*” to facilitate step-by-step thinking before answering each question. Zero-shot CoT amazingly outperforms the standard zero-shot prompting by a large margin in a variety of reasoning tasks. Therefore, we also apply zero-shot CoT prompting to evaluate GPT-3 on HINT; see Appx. B.2 for the details of zero-shot CoT.

4.3 Training and Evaluation

Training All models are trained by the Adam optimizer; the gradients larger than 5.0 are clipped. Dropout is applied to each recurrent layer of RNNs and each sub-layer of Transformers: both the multi-head attention layers and the feedforward layers. For zero-shot experiments on GPT-3, no training is required; instead, we randomly select 100 samples for each test subset and feed them to GPT-3 by zero-shot or zero-shot-CoT prompting.

Hyperparameter Tuning To make the results reliable, we conduct a thorough hyperparameter tuning w.r.t. the number of layers in the encoder and the decoder, the dimension of the token embedding, the number of hidden units per layer, the number of attention heads in Transformers, the dropout ratio, and the learning rate. Please refer to Tab. A3 for details.

Evaluation Metric We report the accuracy of final results. A predicted result is considered correct when it *exactly* equals to the ground-truth answer.

Table 2: **The accuracy on the test set using image inputs.** All models are jointly trained with a randomly initialized ResNet-18. Reported accuracy (%) is the median and standard deviation of 5 runs. “rel.” denotes Transformer with relative positional encoding, and “uni.” denotes Universal Transformer.

Model	Variant	I	SS	LS	SL	LL	Avg.
GRU	w/o att	61.3±1.4	53.3±1.7	30.5±1.2	9.2±0.2	11.9±0.5	33.2±0.9
	w/ att	66.7±2.0	58.7±2.2	33.1±2.7	9.4±0.3	12.8±1.0	35.9±1.6
LSTM	w/o att	80.0±5.7	76.2±7.4	55.7±8.2	10.9±0.6	19.8±2.6	48.6±4.9
	w/ att	83.9±0.9	79.7±0.8	62.0±2.5	11.2±0.1	21.0±0.8	51.5±1.0
Transformer	vanilla	20.9±0.4	9.3±0.2	5.7±0.3	1.5±0.3	2.9±0.5	8.3±0.3
	rel.	86.2±0.9	83.1±1.3	60.1±2.3	10.9±0.2	19.4±0.5	51.7±1.0
	rel. uni.	88.4±1.3	86.0±1.3	62.5±4.1	10.9±0.2	19.0±1.0	53.1±1.6

Table 3: **The accuracy on the test set using symbol inputs.**

Model	Variant	I	SS	LS	SL	LL	Avg.
GRU	w/o att	74.9±1.6	68.1±0.5	42.1±1.9	10.5±0.2	14.0±0.8	41.3±0.6
	w/ att	76.2±0.6	69.5±0.6	42.8±1.5	10.5±0.2	15.1±1.2	42.5±0.7
LSTM	w/o att	84.3±5.2	79.6±6.0	63.7±6.1	11.7±0.3	22.1±1.4	52.3±3.8
	w/ att	92.9±1.4	90.9±1.1	74.9±1.5	12.1±0.2	24.3±0.3	58.9±0.7
Transformer	vanilla	93.9±0.3	91.0±0.5	33.2±1.2	11.5±0.1	11.5±0.7	47.4±0.4
	rel.	96.6±0.3	95.1±0.4	72.1±1.5	11.8±0.2	22.3±0.6	59.4±0.5
	rel. uni.	98.0±0.3	96.8±0.6	78.2±2.9	11.7±0.3	22.4±1.1	61.5±0.9
GPT-3	0-shot	19.0	9.0	3.0	10.0	2.0	8.6
	0-shot-CoT	42.0	36.0	5.0	49.0	6.0	27.6

5 Experimental Results

5.1 Joint Learning of Perception, Syntax, and Semantics

Tabs. 2 and 3 summarize the results of all models on HINT using image inputs and symbol inputs, respectively. Among all models, Universal Transformer with relative positional encoding (“Transformer rel. uni.”) achieves the best average accuracy on the test set. Taking a closer look at the results, we draw the following observations and insights:

- **Models achieve high accuracy on the subset I.** Particularly, Transformer rel. uni. using image inputs obtains an accuracy of 88.4%. The test subset I shares the symbolic expressions with training and has different handwritten images for symbols. This indicates that Transformers and RNNs, jointly trained with ResNet-18, have strong generalization over perception. As shown in Fig. A4, the model forms meaningful clusters for each concept and captures syntactic roles to some extent although no direct supervision on perception.
- **Transformers achieve high accuracy on the subset SS.** The expressions in SS come from the same length and value distribution as training. This result indicates that Transformers demonstrate strong interpolation over syntax and semantics.
- **The accuracy of Transformer rel. uni. on LS is significantly lower than its accuracy on SS or I** (see Tab. 3). The very same model produces perfect accuracy on the length cutoff splits of SCAN [15]. This result may be explained by the difference between the syntax of HINT and SCAN, shown in Tab. A2: The expressions in HINT can have longer-range dependency and larger tree depth than the commands in SCAN. This observation indicates that current Transformers, which have finite depth, cannot fully capture the syntax with long dependency and large depth.
- **Transformer with relative positional encoding achieves similar performance on I and SS as the vanilla Transformer with absolute positional encoding, yet relative positional encoding boosts Transformer’s accuracy on LS over twice** (see Tab. 3). This contradiction indicates that relative positional encoding is critical for Transformer to generalize to long expressions. Sharing weights between the layers using the Universal Transformer can further improve the performance.
- **Models behave clumsily on the subsets SL and LL.** The accuracy on SL and LL is much lower than that on I and SS. All models have nearly zero accuracy on samples whose answers are over 100 (the maximum final result in the training set). This result indicates that both RNNs and Transformers have little capability to extrapolate to larger numbers than those in the training set.

³Can GPT-3 do math? <https://www.youtube.com/watch?v=TMxAbNAVrzI>

⁴<https://openai.com/api/>

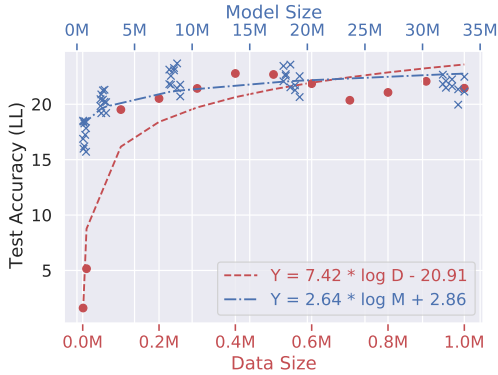


Figure 3: **Scaling trends of the accuracy on the test subset LL w.r.t. model size and dataset size when training Transformer rel. uni. with symbol inputs.**

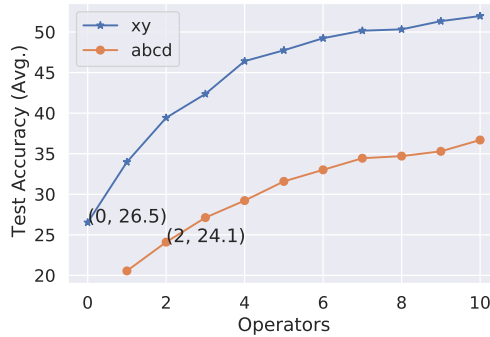


Figure 4: **The performance of Transformer rel. uni. when using varied maximum operators in the few-shot learning.**

- **While GPT-3 with zero-shot prompting has poor performance, chain of thought (CoT) prompting significantly improves the accuracy.** Notably, GPT-3 with zero-shot CoT obtains an accuracy of 49.0% on SL, better than other fine-tuned models. We believe the reason is that GPT-3 is pre-trained on data involving larger numbers, and CoT enhances the reasoning process. However, GPT-3 performs poorly on long expressions in LS and LL even with CoT prompting.

Summary Taking together, we observe a large space for improvement on HINT. Even the best model, Universal Transformer with relative positional encoding, can only achieve an accuracy of 54.3% on HINT, while the same model obtains nearly perfect accuracy on previous datasets of systematic generalization like SCAN [15]. The challenges of HINT lie in that it requires a joint learning and generalization of perception, syntax, and semantics: The perception has large variance from realistic handwriting, the syntax supports long dependency between symbols, and the semantic complexity is well beyond the capability of the state-of-the-art models.

Scaling Laws on HINT Since HINT can produce infinite data for training, one might wonder if simply scaling up the dataset and the model size can solve the problem, as done in NLP tasks [49, 38]. Empirically, Fig. 3 plots the scaling trend of the test accuracy w.r.t. the model size and the number of training samples. Models of various sizes are constructed by varying the hidden dimension, the embedding dimension, and the number of the attention heads. Training sets of various sizes are constructed by randomly sampling from the original training set. Assuming a log-linear scaling trend, we need to train a model of 10^{33} parameters on 10^{15} examples to achieve 90% accuracy on the test subset LL, which is impractical. Therefore, efficient architecture and training algorithm are required to improve the model’s capability of extrapolating over syntax and semantics.

5.2 Few-shot Learning and Generalization

In this section, we fine-tune the top two models on six new concepts; Tab. 4 tabulates the results. Transformer rel. uni. outperforms LSTM w/ attn across all concepts by a large margin, which is over six times their performance gap in Tab. 2. This discrepancy indicates that with limited data, Transformer can learn the new concepts much better than LSTM.

Table 4: **The few-shot learning performance of the top two models: LSTM w/ attn (left) and Transformer rel. uni. (right).** In “Syntax,” “Number,” “Op1,” and “Op2” denote that the concept is appended to the corresponding production rule of the grammar in Tab. A2. In “Semantics,” i and j are the inputs to the operator. Reported results are the median of 5 runs.

Concept	Perception	Syntax	Semantics	I	SS	LS	SL	LL	Avg.
x	x	Number	11	87.8/89.2	47.3/80.2	42.8/58.6	10.8/12.2	16.4/19.3	42.8/52.8
y	y	Number	12	64.5/83.8	39.1/74.8	38.5/54.0	11.6/13.8	18.9/22.4	35.4/50.7
a	a	Op1	$\max(i, j)$	71.8/84.4	44.2/72.0	29.7/48.9	7.9/8.4	11.1/12.3	33.8/46.4
b	b	Op1	$\min(i, j)$	73.4/77.1	29.9/59.1	27.4/39.4	7.4/16.8	12.7/17.1	31.1/42.6
c	c	Op2	$(i + j)/2$	61.5/59.2	19.6/34.0	15.2/24.4	4.5/6.1	6.5/9.4	21.9/27.3
d	d	Op2	$2ij/(i + j)$	59.2/62.8	22.7/39.0	20.2/27.0	7.2/9.2	8.9/10.7	24.7/30.4
Overall	-	-	-	69.7/76.1	33.8/59.9	29.0/42.0	8.2/11.1	12.4/15.2	31.6/41.7

Fig. 4 shows the test accuracy of Transformer rel. uni. while using varied maximum operators for training. Generally, the more data and the longer expressions used for training, the better performance the model can achieve. One test case for learning new numbers (“ xy ”) is (0, 26.5), where the model is exposed to the primitive concept only in the training phrase and required to generalize to complex compositions in test. According to the classic thought experiments [30], this is simple for humans: If you know the meaning of “1,” “ $1 + 1$,” and “ x ,” you should also understand what “ $1 + x$ ” means. A similar test case for learning new operators (“ $abcd$ ”) is (2, 24.1), since expressions containing at least two operators are necessary for capturing the syntax of a new operator. Transformer performs poorly in these two types of tests, indicating that it is still far from the human-level generalization.

6 Discussions: Conclusions and Limitations

In this paper, we take inspiration from arithmetic and present a new challenge for the machine learning community, HINT, which serves as a minimal yet complete benchmark towards studying the full-spectrum systematic generalization of concept learning w.r.t. perception, syntax, and semantics. HINT is inherently more challenging than previous datasets due to its large perceptual variance in realistic handwriting, complex syntax, and sophisticated semantics. We benchmark on HINT the state-of-the-art seq2seq models, including RNNs, Transformers, and GPT-3, and the results point out their ineptitude of extrapolation over syntax and semantics. The scaling trends of test accuracy w.r.t. dataset size and model size indicate that it is impractical to solve HINT by simply scaling up model and dataset. We believe that the HINT dataset together with our experimental findings might inspire new advancement for systematic generalization, especially extrapolation over syntax and semantics.

Limitations and Future Work Despite a large visual variance, the handwritten expressions are relatively simple in terms of spatial locations. It would be more interesting if we can further increase the perceptual complexity w.r.t. spatial relations like natural images [62]. Although syntax and semantics in HINT are already more complicated than previous datasets, they are still context-free. Extending our findings to context-dependent syntax and semantics would be of practical benefits since they are very common in natural languages, *e.g.*, a word might have different syntactic roles or semantic meanings in different contexts.

In terms of model development on HINT, our results indicate that current seq2seq models, including Transformers, still lack the ability to extract the systematic rules for both syntax and semantics from the training data. How to improve the systematic generalization of Transformers, especially extrapolation over semantics, is an important future direction. We also plan to explore more sophisticated methods, such as meta-learning [56], data augmentation [2, 1], Edge Transformer [7], and Neural-Symbolic Stack Machines [10]. Furthermore, it would be helpful to understand the systematic generalization of large language models by evaluating them in few-shot or fine-tuning settings.

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A Dataset Statistics

The handwritten images for each arithmetic concept originate from the handwritten math symbols dataset¹ hosted on Kaggle under the “CC0: Public Domain” license, parsed and extracted from the Competition on Recognition of Online Handwritten Mathematical Expressions (CROHME) [67]². We further clean the dataset by removing duplicate images, resulting in statistics shown in Fig. A1.

To demonstrate the validity of the HINT dataset as a benchmark for systematic generalization, we conduct a detailed analysis of the collected data. Tab. A1 shows the size of each split in HINT, and Tab. A2 shows a comparison between the grammars of HINT and SCAN.

Table A1: **Dataset size.** The first row is the main split of HINT and the rest are the few-shot learning split. As advocated by Csordas *et al.* [15], the validation set also contains five generalization subsets for model selection.

Split	Train	Validation	Test					
			Total	I	SS	LS	SL	LL
main	998000	4698	46620	9980	8000	10000	8640	10000
x	1100	491	4900	1100	900	1000	900	1000
y	1100	493	4900	1100	900	1000	900	1000
a	1000	470	4700	1000	900	1000	800	1000
b	1000	470	4700	1000	900	1000	800	1000
c	1000	470	4700	1000	900	1000	800	1000
d	1000	470	4700	1000	900	1000	800	1000

Table A2: **The phrase-structure grammars for HINT and SCAN.** While the grammars of both HINT and SCAN can generate infinite examples, HINT produces examples with larger depth and longer dependency due to the parentheses; the expression inside parentheses can be arbitrarily long. Specifically, the maximum depth and dependency range in SCAN are 6 and 4, respectively; the maximum length generated by the non-terminal “S” in the grammar of SCAN is 4.

HINT	SCAN
$T = \{\text{Expression, Term, Factor, Number}\}$	$T = \{C, S, V, D, U\}$
Start symbol: Expression	Start symbol: C
$\Sigma = \{+, -, \times, \div, 0, 1, \dots, 9, (,)\}$	$\Sigma = \{\text{walk, look, run, jump, turn, left, right, around, opposite, and after, twice, thrice}\}$
$R = \{$	$R = \{$
Expression \rightarrow Term	C \rightarrow S S and S S after S
Expression \rightarrow Expression Op1 Term	S \rightarrow V V twice V thrice
Op1 \rightarrow + -	V \rightarrow D[1] opposite D[2]
Term \rightarrow Factor	V \rightarrow D[1] around D[2]
Term \rightarrow Term Op2 Factor	V \rightarrow D U
Op2 \rightarrow \times \div	D \rightarrow U left U right
Factor \rightarrow Number	D \rightarrow turn left turn right
Factor \rightarrow (Expression)	U \rightarrow walk look run jump }
Number \rightarrow 0 1 2 3... 9 }	

For each split, we plot the frequency distributions of various aspects, including symbol, number of operators, expression length, tree depth, maximum dependency range, and result, as shown in Fig. A2. The symbol distributions are similar across different splits, and the Kullback–Leibler divergence between train and test is low (0.0055). The digits and operators are approximately equally distributed, except for the test-SL split. The test-SL split has a relatively higher portion of multiplication (“*”) since generating large numbers generally requires more multiplication for short expressions.

The test set’s result distributions differ from the train set. All results in train set are smaller than 100 as desired; about half are in $[0, 10)$. In comparison, 29% of the results in test set are larger than 100.

Several properties of an input expression, including length, number of operators, tree depth, and maximum dependency range, are indicators of the difficulty of calculating the expression. We plot the frequency distributions w.r.t. these input properties in Fig. A2. These distributions demonstrate significant differences between train and test.

¹<https://www.kaggle.com/datasets/xainano/handwrittenmathsymbols>

²<https://www.cs.rit.edu/~crohme2019/>

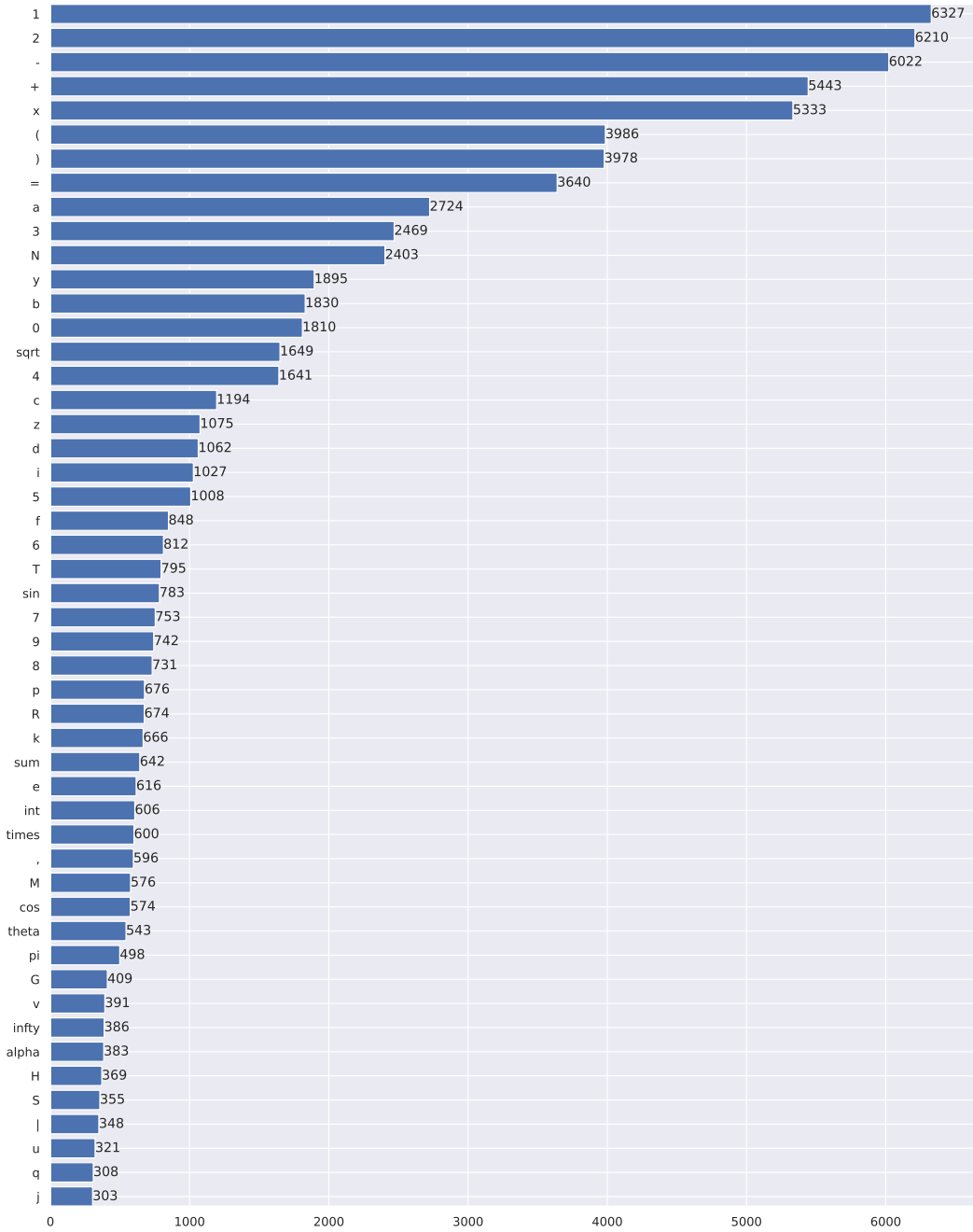


Figure A1: **The number of handwritten images for each symbol.** There are 82 arithmetic symbols (top 50 are shown here) and 83,501 images in total. We use the handwritten images for digits 0 ~ 9, operators +, -, ×, ÷, and parentheses (,) in this work; others are for potential future use.

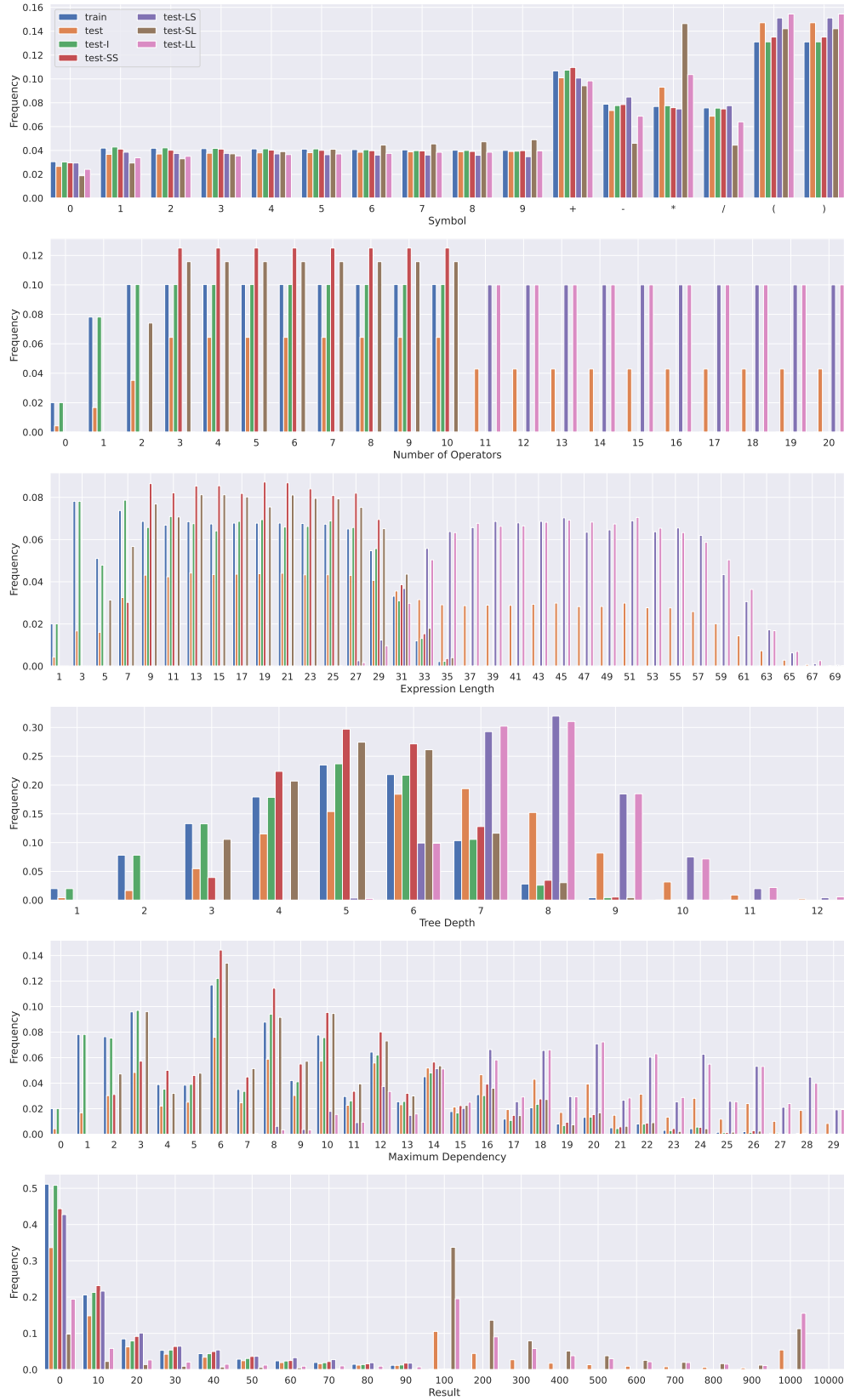


Figure A2: The frequency distributions w.r.t. various aspects, including symbol, number of operators, expression length, tree depth, maximum dependency range, and result.

Dataset License The HINT dataset is licensed under CC BY-NC 4.0³ and hosted on the project website: <https://liqing-ustc.github.io/HINT>. The authors bear all responsibility in case of violation of rights, *etc.*

B Baseline Details

We benchmark deep sequence-to-sequence (seq2seq) frameworks [86] on HINT. Fig. A3 illustrates a HINT example applied by the seq2seq framework. All models are implemented in PyTorch [74].

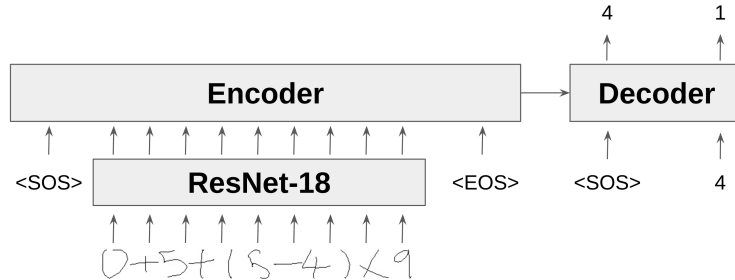


Figure A3: **The seq2seq framework applied to an example in HINT.** The symbols $\langle \text{SOS} \rangle$ and $\langle \text{EOS} \rangle$ represent the start-of-sentence and the end-of-sentence, respectively. The handwritten expression is segmented by a sliding window into a sequence of images, which are then individually encoded by the ResNet-18. $\langle \text{SOS} \rangle$ and $\langle \text{EOS} \rangle$ are appended to the start and the end of the image sequence. The result is converted to a sequence of digits in the reverse order.

B.1 Image Tokenizer and Embedding

To tokenize a handwritten expression, we first resize it by making its height 32 and apply a sliding window of size 32 along the horizontal axis to render a sequence of images. Next, each image in the sequence is encoded by the ResNet-18 [37]. We found in preliminary experiments that pre-training on the ImageNet does not help, likely due to the domain gap between ImageNet and HINT. Therefore, we use a random initialization for ResNet-18 in our experiments.

B.2 Encoder-Decoder Architectures

We consider the following three choices for the encoder-decoder architecture in a seq2seq framework: Recurrent Neural Networks (RNNs), Transformers, and GPT-3.

RNNs We test two popular RNNs: long short-term memory (LSTM) [40] and gated recurrent units (GRU) [14]. Both networks are evaluated with and without attention [3]. Our implementations of RNNs are adapted from a seq2seq tutorial.⁴

Transformers We benchmark three variants of Transformer: the vanilla Transformer, Transformer with relative positional encoding, and Universal Transformer with relative positional encoding. The implementations of these Transformers are adapted from Csordas *et al.* [15].⁵

GPT-3 To test GPT-3’s ability to perform simple arithmetic operations without task-specific training, Brown *et al.* [8] developed a small battery of 10 tests that involve asking GPT-3 a simple arithmetic problem in natural language; see Section 3.9.1 and Table 3.9 in Brown *et al.* [8] for the results. In these tests, GPT-3 displays reasonable proficiency at simple arithmetic in the few-shot setting. However, they do not evaluate the multi-hop reasoning capability required by complex arithmetic expressions, which usually involve more operators and larger numbers.

To systematically and comprehensively evaluate GPT-3’s capability of arithmetic reasoning, we test GPT-3 on the proposed HINT benchmark using symbolic expressions as input. Since all tokens of

³<https://creativecommons.org/licenses/by-nc/4.0/>

⁴<https://github.com/bentrevett/pytorch-seq2seq>

⁵https://github.com/RobertCsordas/transformer_generalization

HINT are in the vocabulary of GPT-3, we directly evaluate GPT-3 via zero-shot prompting using the OpenAI API ⁶. We construct the prompt in the following form: “*Q: What is <Expression>? A: The answer is,*” similar to the practice in Brown *et al.* [8] but with more complex expressions.

Via task-specific zero-shot or few-shot prompting, pre-trained large language models achieve amazing performance in intuitive and single-step *System 1* tasks [48]. However, LLMs struggled on *System 2* tasks that require slow thinking and multi-hop reasoning [80], even at the scale of over 100B parameters like GPT-3. To address this shortcoming, *chain of thought* prompting (CoT) [90], which feeds LLMs with the intermediate step-by-step reasoning to augment the final answer in a few-shot setting, has been proposed to elicit the multi-hop reasoning in LLMs.

Very recently, chain of thought prompting is extended to the zero-shot setting [53] by adding a simple prompt, “*Let’s think step by step*”, to facilitate step-by-step thinking before answering each question. Zero-shot CoT amazingly outperforms the standard zero-shot prompting by a large margin in a variety of reasoning tasks. Therefore, we also apply zero-shot CoT prompting to evaluate GPT-3 on HINT. More concretely, it follows a two-stage prompting strategy similar to [53]:

1st prompt “*Q: What is <Expression>? A: Let’s think step-by-step.*” This prompt extracts the step-by-step reasoning process in the form of natural language from GPT-3, which is denoted by $\langle Z \rangle$.

2st prompt “*Q: What is <Expression>? A: Let’s think step-by-step. <Z> Therefore, the answer (arabic numerals) is*” In the second stage, the response $\langle Z \rangle$ generated in the first step is appended to the initial prompt along with an answer trigger sentence. This second prompt is then fed into GPT-3 to predict the final answer.

In our experiments, we use the ‘text-davinci-002’ engine in the OpenAI API, the most capable GPT-3 model at the time of writing with approximately 175 billion parameters⁷.

B.3 Training

Tab. A3 shows the tuned hyperparameters for the baselines. Our choices for each model are underlined, and the performance is reported under these settings unless explicitly stated otherwise. When generating the output, we use greedy decoding in all models for simplicity.

Table A3: **Hyperparameter tuning.** Our choices are underlined.

Model	Variant	Encoder	Decoder	Embedding	Hidden	Heads	Dropout	Batch	Steps	Learning Rate
RNN	LSTM (+ att)	<u>1,3,6,9</u>	<u>1,3,6,9</u>	<u>128, 256, 512</u>	<u>128, 256, 512</u>	-	0, <u>0.1</u> , 0.5	<u>128</u>	<u>100K</u>	<u>10⁻³</u> , 10 ⁻⁴ , 10 ⁻⁵
	GRU (+ att)	<u>1,3,6,9</u>	<u>1,3,6,9</u>	<u>128, 256, 512</u>	<u>128, 256, 512</u>	-	0, <u>0.1</u> , 0.5	<u>128</u>	<u>100K</u>	<u>10⁻³</u> , 10 ⁻⁴ , 10 ⁻⁵
Transformer	vanilla	<u>1,3,6,9</u>	<u>1,3,6,9</u>	<u>128, 256, 512</u>	<u>128, 256, 512</u>	4,8,12	0, <u>0.1</u> , 0.5	<u>128</u>	<u>100K</u>	10 ⁻³ , <u>10⁻⁴</u> , 10 ⁻⁵
	relative	<u>1,3,6,9</u>	<u>1,3,6,9</u>	<u>128, 256, 512</u>	<u>128, 256, 512</u>	4,8,12	0, <u>0.1</u> , 0.5	<u>128</u>	<u>100K</u>	10 ⁻³ , <u>10⁻⁴</u> , 10 ⁻⁵
	relative universal	<u>1,3,6,9</u>	<u>1,3,6,9</u>	<u>128, 256, 512</u>	<u>128, 256, 512</u>	4,8,12	0, <u>0.1</u> , 0.5	<u>128</u>	<u>100K</u>	10 ⁻³ , <u>10⁻⁴</u> , 10 ⁻⁵

For the few-shot learning experiments, models are first pre-trained on the main training set and then fine-tuned on the training set of each new concept individually. Models are fine-tuned for 1000 iterations using a batch size of 128 with half examples from the main training set to prevent forgetting. The learning rates are 10⁻⁵ and 10⁻³ for Transformers and RNNs, respectively.

All models reported in our paper can be trained on a single NVIDIA TITAN V GPU with 12G memory. It takes at most eight hours to train a model.

C Experimental Results

Fig. A4 shows the t-SNE visualization for the embeddings of handwritten images in the test set using the Transformer rel. univ. model. Fig. A5 shows the test accuracy as a function of several sample properties. Fig. A6 shows the importance of these properties.

⁶<https://openai.com/api/>

⁷OpenAI API GPT-3 model sizes: <https://blog.eleuther.ai/gpt3-model-sizes>



Figure A4: **The t-SNE visualization for the embeddings (the outputs of ResNet-18) of handwritten images in the test set using the Transformer rel. univ. model.** The image embeddings form clear clusters for each concept based on visual appearance. Furthermore, these clusters reflect the concepts' syntactic roles: Most digits are near the bottom part, the operators are near the middle part, and the parentheses appear near the top part.

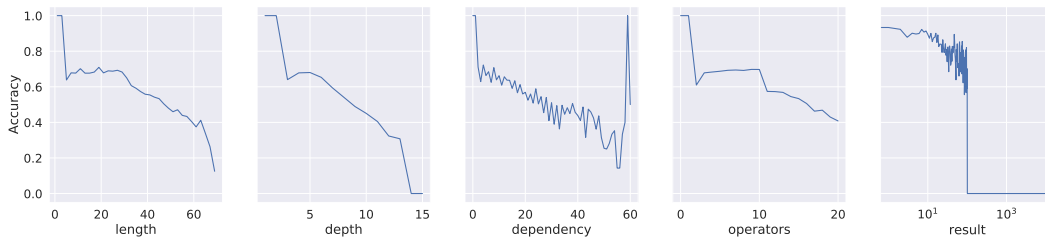


Figure A5: **Test accuracy (avg.) of Transformer rel. uni. using symbol inputs as a function of several properties of samples:** the expression's *length*, the *depth* of the expression's parse tree, the expression's maximum *dependency* range, the number of *operators* in the expression, the final *result*.

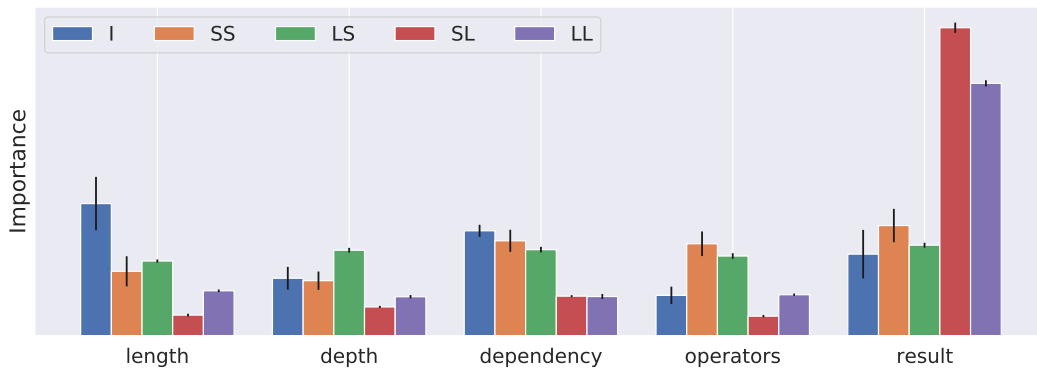


Figure A6: **The importance of sample properties w.r.t. the test accuracy of Transformer rel. uni. using symbol inputs.** Normalized permutation feature importance is reported here using a k-nearest neighbors classifier ($k=3$) to predict if the model can generate correct results.