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# FilFL: Accelerating Federated Learning via Client Filtering

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## Abstract

*Federated learning* is an emerging machine learning paradigm that enables devices to train collaboratively without exchanging their local data. The clients participating in the training process are a random subset selected from the pool of clients. The above procedure is called client selection which is an important area in federated learning as it highly impacts the convergence rate, learning efficiency, and generalization. In this work, we introduce *client filtering* in federated learning (FilFL), a new approach to optimize client selection and training. FilFL first filters the active clients by choosing a subset of them that maximizes a specific objective function; then, a client selection method is applied to that subset. We provide a thorough analysis of its convergence in a heterogeneous setting. Empirical results demonstrate several benefits to our approach, including improved learning efficiency, accelerated convergence, 2-3 $\times$  faster, and higher test accuracy, around 2-10 percentage points higher.

## 1. Introduction

*Federated learning* (FL) is an emerging machine learning paradigm that organizes collaborative training across distributed clients while keeping their data local (Konečný, 2017; Konečný et al., 2015; Li et al., 2020b; Shokri & Shmatikov, 2015). The most popular approach to this setting is the alternation between on-device training (i.e., gradient descent steps on local data) and server aggregation and broadcasting of the latest version of the global model, introduced by McMahan et al. (2017) as FedAvg.

Some of the key challenges in FL include training with a large number of clients and that the data stored on devices are generally non-iid, i.e., different clients have a different number of data points and distributions (Bonawitz et al., 2019; Hosseinalipour et al., 2020; Huba et al., 2022). The

effect of such data heterogeneity on the convergence of local-update SGD is analyzed in a flurry of recent works (Abdelmoniem et al., 2022; Haddadpour & Mahdavi, 2019; Huo et al., 2020; Khaled et al., 2020; Koloskova et al., 2020; Li et al., 2020b; Malinovskiy et al., 2020; Pathak & Wainwright, 2020; Reddi et al., 2020; Stich & Karimireddy, 2019; Woodworth et al., 2020; Zhang et al., 2020). Data heterogeneity generally leads to unstable and slow convergence (Li et al., 2020b), and causes suboptimal or even detrimental model performance (Zhao et al., 2018). This is mainly caused by the fact that data distributions on the devices differ from the global distribution, which leads clients to converge towards their local optima and away from the global optimum.

To overcome the above challenges, first, it is common to consider partial client participation, i.e., in training round  $t$ , only a subset  $\mathcal{A}_t$  of  $K \leq N$  clients are selected to participate in the process (Li et al., 2019). Second, several *client selection* methods were proposed to optimize the process of partial client participation and mitigate the impact of heterogeneous clients, including sampling clients based on the number of local data points (Li et al., 2019), sampling the clients uniformly at random and aggregating the model updates with weights proportional to the local samples (Li et al., 2020b), using power-of-choice, a biased client selection method that selects clients with higher local losses (Cho et al., 2020), and DivFL, which selects a diverse subset of clients, namely those carrying representative gradient information (Balakrishnan et al., 2021).

Although partially capable of optimizing FL in the heterogeneous scenario, all these prior approaches work by selecting participants from the pool of available ones without regard to whether the cohort of clients selected at round  $t$  contains the most suitable ones. Driven by the intuition that synergy among clients leads to better training, in this work, we introduce a yet unexplored approach in which a *client filtering* procedure identifies which clients should be considered at the given stage of the training process. Only the clients that pass this filter are candidates for client selection. In other words, we propose to initially discard some clients that are likely to have only a tiny (if any) improvement to the trained model, which is marginal compared to other, more promising clients. The assessment of this improvement uses a public dataset at the FL server to gauge the representative-

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ness of different local client data towards the global model performance. Thus, our approach, called FilFL, identifies a “good” set of collaborative clients that are filtered based on the joint representativeness of the overall data. FilFL does not simply classify clients as “bad” or “good” clients, as that would eliminate a client throughout the training process. The filtering algorithm discards a client when it is not suitable as an addition to the other available clients based on the server dataset. As such, *client filtering* is conducted periodically, as the filtered clients in the current round might improve the synergy in a later round.

To filter clients, we define a non-monotone combinatorial maximization problem, which aims to find the best subset of available clients with the lowest loss over a server-held public dataset. Inspired by the literature of submodular maximization, we propose a randomized greedy filtering (RGF) algorithm, adapted from the algorithms of Buchbinder et al. (2015) and Fourati et al. (2023), which have the tightest theoretical guarantees for offline and online submodular maximization, respectively.

**Contributions.** The key contributions in this paper are summarized as follows:

- ★ We are the first to propose adding *client filtering* in FL (FilFL) to optimize client selection, accelerate the training, and improve the global model performance (test accuracy).
- ★ We define a non-monotone combinatorial problem, which aims to find the subset of available clients with minimum average losses and propose the randomized greedy filtering (RGF) algorithm, which approximates its solution.
- ★ We theoretically prove that FilFL achieves, under some assumptions, a convergence rate guarantees of  $\mathcal{O}(\frac{1}{t}) + \mathcal{O}(\varphi)$  for  $t$  time steps and a time constant  $\varphi$  that depends on the quality of filtering.
- ★ We empirically show, on different datasets and under various realistic scenarios of time-varying available clients, that FilFL outperforms pure client selection methods, being 2-3× faster, as well as reaching 2-10 higher percentage points test accuracy. The code will be released as open source.

**Notation and Organization.** Please refer to the Table 1 for the notations. In the following, in Sec. 2, we formally define the problem, propose our *client filtering* method, and show its integration with the FL setting, FilFL. In Sec. 3, we provide the theoretical convergence analysis of our method. Then, in Sec. 4, we provide the experimental evaluation for our proposed method. We discuss the limitation and possible future work in Sec. 5.

## 2. Client Filtering

We introduce *client filtering* in FL (or FilFL), which incorporates *client filtering* into the most widely studied FL scheme,

federated averaging (FedAvg). We first present a combinatorial objective for *client filtering*. We then present the randomized greedy algorithm that periodically optimizes the objective by selecting a filtered subset of clients to be used for client selection and training.

$\Omega$	set of all clients, $ \Omega  = N$
$\mathcal{S}_t$	set of active clients in round $t$ , $ \mathcal{S}_t  = n$
$\mathcal{S}_t^*$	set of filtered clients in round $t$ , $\mathcal{S}_t^* \subseteq \mathcal{S}_t$
$\mathcal{A}_t$	set of selected clients in round $t$ , $ \mathcal{A}_t  = K$
$\mathcal{P}$	public dataset, $ \mathcal{P}  = m$
$\mathcal{D}$	union of private datasets
$F_k$	loss of client $k$
$F^{\mathcal{D}}$	average loss of all clients
$F^{\mathcal{P}}$	loss on public dataset
$m_k$	number of data points for client $k$
$E$	number of local steps
$T$	number of communication rounds
$\eta$	learning rate
$\mathbf{w}_t^k$	weight of client $k$ in round $t$

Table 1. Table of notations

### 2.1. Problem Setup

We consider the canonical objective of fitting a single global model to the non-iid data held across clients (McMahan et al., 2017). Thus, we consider the following distributed optimization problem:

$$\min_{\mathbf{w}} \left\{ F^{\mathcal{D}}(\mathbf{w}) \triangleq \sum_{k=1}^N p_k F_k(\mathbf{w}) \right\} \quad (1)$$

where  $N$  is the number of devices, and  $p_k$  is the weight of the  $k$ -th device such that  $p_k \geq 0$  and  $\sum_{k=1}^N p_k = 1$ . Suppose the  $k$ -th device holds the  $m_k$  training data:  $x_{k,1}, x_{k,2}, \dots, x_{k,m_k}$ . The local objective  $F_k(\cdot)$  is defined as:

$$F_k(\mathbf{w}) \triangleq \frac{1}{m_k} \sum_{j=1}^{m_k} \ell(\mathbf{w}; x_{k,j})$$

where  $\ell(\cdot; \cdot)$  is some loss function.

Suppose the server holds a public dataset  $\mathcal{P}$ , which has  $m$  training data:  $x_1, x_2, \dots, x_m$ . We define the loss on the public dataset as follows:

$$F^{\mathcal{P}}(\mathbf{w}) \triangleq \frac{1}{m} \sum_{j=1}^m \ell(\mathbf{w}; x_j) \quad (2)$$

As we consider the partial client participation setting, in each training round  $t$ , only a subset  $\mathcal{A}_t$  of  $K \leq N$  clients are selected to participate in the process. Before selecting the  $K$  clients, we first find the best subset  $\mathcal{S}_t^*$  of available clients out of the active clients  $\mathcal{S}_t$ . Thus, we define the following function (reward), which we maximize:



Figure 1. FiFL phases in round  $t$  (if  $t \bmod h = 0$  or  $S_t \neq S_{t-1}$ ).

$$\mathcal{R}(\mathcal{S}) \triangleq \mathcal{C} - F^{\mathcal{P}} \left( \frac{1}{|\mathcal{S}|} \sum_{k \in \mathcal{S}} \mathbf{w}_t^k \right) \quad (3)$$

where  $\mathcal{C}$  is a sufficiently large constant, such as  $\mathcal{R}(\mathcal{S})$  is positive, and  $\mathbf{w}_t^k$  is the weight of the  $k^{\text{th}}$  client in round  $t$ . Thus, our method finds a subset  $\mathcal{S}_t^*$  that satisfies the following objective function at round  $t$ :

$$\mathcal{S}_t^* = \arg \max_{\mathcal{S} \subseteq \mathcal{S}_t} \mathcal{R}(\mathcal{S}) \quad (4)$$

Then, a client selection method is applied on the chosen subset  $\mathcal{S}_t^*$ , instead of the complete set  $\mathcal{S}_t$ . We note that *client filtering* can be integrated with any FL algorithm; specifically, after applying *client filtering*, any client selection method can be used on the obtained set of clients.

The proposed objective is non-monotone, i.e., adding more clients does not necessarily decrease the loss, especially when dealing with heterogeneous data. Furthermore, intuitively, the objective follows a diminishing return in expectation, i.e., the objective is almost submodular in expectation.

## 2.2. Randomized Greedy Filtering (RGF)

Submodular maximization is an NP-hard problem. Feige et al. (2011) showed that for any constant  $\varepsilon > 0$ , any algorithm achieving an approximation of  $(\frac{1}{2} + \varepsilon)$  requires an exponential number of oracle queries to the objective function. Furthermore, Buchbinder et al. (2015) proposed linear time  $\frac{1}{2}$ -approximation algorithms. More recently, (Fourati et al., 2023) proposed a randomized greedy learning (RGL) multi-armed bandit algorithm under stochastic full-bandit feedback, with proven regret guarantees of  $\tilde{\mathcal{O}}(nT^{\frac{2}{3}})$  for time horizon  $T$  and a number of arms  $n$ . Furthermore, RGL empirically outperforms other methods for submodular and non-submodular problems.

We do not seek the exact solution to the problem in Eq. 4. Instead, we use a greedy method to approximate the solution. Inspired by the greedy algorithms used for non-monotone submodular maximization (Buchbinder et al., 2015; Fourati et al., 2023), we propose a *client filtering* algorithm for FL. Algorithm 1 details the pseudocode of randomized greedy filtering (RGF).

RGF iterates over each available client and decides whether to add it to a set of base clients  $X_i$  (initially empty) or re-

### Algorithm 1 RGF

**Require:** set of clients  $\mathcal{S}$

- 1: **Initialize**  $X_0 \leftarrow \emptyset, Y_0 \leftarrow \mathcal{S}, n \leftarrow |\mathcal{S}|$
- 2: **for** client index  $i \in \{1, \dots, n\}$  **do**
- 3:    $u_i \leftarrow$  client of index  $i$
- 4:    $a_i \leftarrow \mathcal{R}(X_{i-1} \cup \{u_i\}) - \mathcal{R}(X_{i-1})$
- 5:    $b_i \leftarrow \mathcal{R}(Y_{i-1} \setminus \{u_i\}) - \mathcal{R}(Y_{i-1})$
- 6:    $a'_i \leftarrow \max(a_i, 0)$  and  $b'_i \leftarrow \max(b_i, 0)$
- 7:   **with probability**  $(\frac{a'_i}{a'_i + b'_i})$  **do**
- 8:      $X_i \leftarrow X_{i-1} \cup \{u_i\}$  and  $Y_i \leftarrow Y_{i-1}$
- 9:   **else**
- 10:     $Y_i \leftarrow Y_{i-1} \setminus \{u_i\}$  and  $X_i \leftarrow X_{i-1}$
- 11: **end for**
- 12: **Return**  $X_n$

move it from the set of base clients  $Y_i$  (initially containing all the clients). The decisions of adding or removing any client are made in a randomized greedy fashion using empirical estimates of marginal gains until a decision is made for all the individual clients and then exploits the decided best set of clients.

Let  $X_i$  and  $Y_i$  be two sets of clients. Initially,  $X_0 = \emptyset$  and  $Y_0 = \mathcal{S}$ . The algorithm has  $n$  phases, where  $n$  is the number of clients. In phase  $i$  out of  $n$ , RGF computes two variables,  $a_i$  and  $b_i$ , defined as follows:

$$\begin{aligned} a_i &= \mathcal{R}(X_{i-1} \cup \{u_i\}) - \mathcal{R}(X_{i-1}) \\ b_i &= \mathcal{R}(Y_{i-1} \setminus \{u_i\}) - \mathcal{R}(Y_{i-1}). \end{aligned} \quad (5)$$

These two variables are important for the decision-making process.  $a_i$  measures the expected impact of adding client  $u_i$  to  $X_{i-1}$ , while  $b_i$  measures the expected impact of removing client  $u_i$  from  $Y_{i-1}$ . A decision is made greedily and probabilistically by computing a certain probability that depends on these two variables  $a_i$  and  $b_i$ , defined as follows:

$$p = \frac{a'_i}{a'_i + b'_i} \quad (6)$$

where  $a'_i = \max(a_i, 0)$  and  $b'_i = \max(b_i, 0)$ , which explains the randomized greedy name of the algorithm. In the special case when  $a'_i = b'_i = 0$ , we set  $p = 1$ .

With that probability  $p$ , RGF adds the individual client  $i$  to the set of clients  $X_i$  and keeps it in the set of clients  $Y_i$ , and with probability  $1 - p$ , RGF removes the client  $i$  from the set of clients  $Y_i$  and keeps the same clients in  $X_{i-1}$ . Thus,  $X_i \subseteq Y_i$  for all  $i = 1, \dots, n$ . After checking all the  $n$  individual clients, it can be easily seen that by the algorithm's construction, both sets  $X_n$  and  $Y_n$  contain the same clients, i.e.,  $X_n \equiv Y_n$ .

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**Algorithm 2** FilFL

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**Require:**  $T, E, \eta, \omega_0, K, \Omega, \mathcal{S}_0, h$

- 1: **Initialize**  $\mathcal{S}_0^* \leftarrow \mathcal{S}_0$
- 2: **for**  $t = 1, \dots, T - 1$  **do**
- 3:   **if**  $t \bmod h = 0$  or  $\mathcal{S}_t \neq \mathcal{S}_{t-1}$  **then**
- 4:      $\mathcal{S}_t^* = \text{RGF}(\mathcal{S}_t)$      {see Algorithm 1}
- 5:   **else**
- 6:      $\mathcal{S}_t^* \leftarrow \mathcal{S}_{t-1}^*$
- 7:   **end if**
- 8:   Server selects a subset  $\mathcal{A}_t$  of  $K$  clients from  $\mathcal{S}_t^*$ .
- 9:   **for** device  $k \in \mathcal{A}_t$  in parallel **do**
- 10:      $\mathbf{w}^k \leftarrow \mathbf{w}_t$
- 11:     Solve the local sub-problem of client- $k$  inexactly by updating  $\mathbf{w}^k$  for  $E$  local mini-batch SGD steps:
- 12:        $\mathbf{w}^k = \mathbf{w}^k - \eta \nabla F_k(\mathbf{w}^k)$
- 13:      $\mathbf{w}^k$  back to server
- 14:   **end for**
- 15:   Server aggregates  $\mathbf{w}^k$ :
- 16:      $\mathbf{w}_{t+1} \leftarrow \frac{1}{|\mathcal{A}_t|} \sum_{k \in \mathcal{A}_t} \mathbf{w}^k$
- 17: **end for**

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**2.3. Client Filtering in Federated Learning (FilFL)**

We introduce FilFL, a method incorporating *client filtering* using RGF into FedAvg. Algorithm 2 lists the pseudocode of FilFL. For computational efficiency, RGF is employed periodically. For this purpose, we introduce  $h$ , which defines the periodicity for which RGF is employed. Furthermore, suppose we are in a setting where the available clients change over the rounds (behaviour heterogeneity). In that case, we need to run RGF every time we have a new set of available clients. Therefore, in every communication round  $t$ , if  $t \bmod h = 0$ , or if the active clients in that round  $\mathcal{S}_t$  are different from the active clients from the previous round, i.e.,  $\mathcal{S}_t \neq \mathcal{S}_{t-1}$ , we apply RGF to filter clients, before proceeding to the client selection and the training (lines 2-4). At each round, FilFL determines  $\mathcal{S}_t^*$ , which approximates a solution for the objective defined in Eq. 4 (line 4); see Fig. 1 for an explanatory diagram of the case  $\mathcal{S}_t \neq \mathcal{S}_{t-1}$ . In the case when  $\mathcal{S}_t \equiv \mathcal{S}_{t-1}$ , it reuses the previous  $\mathcal{S}_t^*$  (lines 5-6). From the set of filtered clients  $\mathcal{S}_t^*$ , we apply a selection method in order to select  $K$  clients from  $\mathcal{S}_t^*$  (line 8). Note that there is no restriction on the choice of the selection method as long as we obtain  $K$  clients. We proceed with the algorithm by running SGD local steps for each client in  $\mathcal{A}_t$  (lines 9-12). Finally, the server aggregates the weights given back from the clients  $\mathcal{A}_t$  and moves to the next round of training.

**Remark 1.** *FilFL adds an extra layer in FL, which is client filtering. Using a trivial filtering algorithm that accepts all the available clients, i.e.,  $\text{RGF}(\mathcal{S}_t) = \mathcal{S}_t$ , FilFL reduces to FedAvg. Thus, FilFL can be considered as a generalization of FedAvg. In this paper, we propose using RGF as a filter-*

*ing algorithm. However, future work might consider and compare different filtering methods.*

**3. Convergence Analysis**

We provide the theoretical convergence analysis of our proposed FilFL algorithm (c.f. Algorithm 2) for  $L$ -smooth and strongly convex problems under practical assumptions of non-iid data, partial client participation, and local updates. Namely, we provide theoretical guarantees of the convergence of the averaged weights  $\bar{\mathbf{w}}_t$  at round  $t$  to  $\mathbf{w}^*$ . This analysis focuses on the impact of adding *client filtering* at every communication round to the FedAvg setting, assuming random sampling as a client selection method. Although the results of this analysis mainly hold when applied to FedAvg using random sampling, it can be easily extended to other methods.

In the following, we provide the assumptions and definitions required for the analysis and the statement of the Theorem of convergence, as well as a sketch of its proof. We defer the main Lemmas and their proofs to Appendix A.

**3.1. Assumptions and Definitions**

The following assumptions are standard assumptions for the convergence analysis in the literature of FL, such as (Balakrishnan et al., 2021; Li et al., 2019).

**Assumption 1.**  $F_1, \dots, F_N$  are all  $L$ -smooth: for all  $\mathbf{v}$  and  $\mathbf{w}$ ,  $F_k(\mathbf{v}) \leq F_k(\mathbf{w}) + (\mathbf{v} - \mathbf{w})^T \nabla F_k(\mathbf{w}) + \frac{L}{2} \|\mathbf{v} - \mathbf{w}\|_2^2$

**Assumption 2.**  $F_1, \dots, F_N$  are all  $\mu$ -strongly convex: for all  $\mathbf{v}$  and  $\mathbf{w}$ ,  $F_k(\mathbf{v}) \geq F_k(\mathbf{w}) + (\mathbf{v} - \mathbf{w})^T \nabla F_k(\mathbf{w}) + \frac{\mu}{2} \|\mathbf{v} - \mathbf{w}\|_2^2$ .

**Assumption 3.** Let  $\psi_t^k$  be sampled from the  $k$ -th device's local data uniformly at random. The variance of stochastic gradients in each device is bounded:  $\mathbb{E} \left[ \|\nabla F_k(\mathbf{w}_t^k, \psi_t^k) - \nabla F_k(\mathbf{w}_t^k)\|^2 \right] \leq \sigma_k^2$  for  $k = 1, \dots, N$ .

**Assumption 4.** The stochastic gradients are uniformly bounded, i.e.,  $\|\nabla F_k(\mathbf{w}_t^k, \psi_t^k)\|^2 \leq G^2$  for all  $k = 1, \dots, N$  and  $t = 1, \dots, T - 1$

**Assumption 5.** Statistical heterogeneity defined as  $F^* - \sum_{k \in [N]} p_k F_k^*$  is bounded, where  $F^* := \min_{\mathbf{w}} F(\mathbf{w})$  and  $F_k^* := \min_{\mathbf{v}} F_k(\mathbf{v})$

Assumption 6 assumes the  $K$  indices are selected from the distribution  $p_k$  independently and with replacement. The aggregation step is simple averaging. A theoretical analysis of this sampling scheme was provided in (Li et al., 2019).

**Assumption 6.** Assume  $\mathcal{A}_t$  contains a subset of  $K$  indices randomly selected with replacement according to the sampling probabilities  $p_1, \dots, p_N$ . The aggregation step of FilFL performs  $\mathbf{w}_t \leftarrow \frac{1}{K} \sum_{k \in \mathcal{A}_t} \mathbf{w}_t^k$ .



Limited to realistic scenarios (for communication efficiency and low straggler effect), FilFL samples a subset  $\mathcal{A}_t$  of devices and then only perform updates on them. This makes the analysis intricate since  $\mathcal{A}_t$  varies each  $E$  steps. However, we can use an approach similar to the one used in (Li et al., 2019) to circumvent this difficulty. We assume that FilFL activates all devices at the beginning of each round and then uses the parameters maintained in only a few sampled devices to produce the next-round parameter. It is clear that this updating scheme is equivalent to the original.

Let  $\mathbf{w}_t^k$  be the model parameter maintained in the  $k$ -th device at the  $t$ -th step. Let  $\mathcal{I}_E$  be the set of global synchronization steps, i.e.,  $\mathcal{I}_E = \{iE \mid i = 1, 2, \dots\}$ . If  $t + 1 \in \mathcal{I}_E$ , i.e., the time step to communication, FilFL activates all devices. Then the update of our algorithm can be described as: for all  $k \in [N]$ ,

$$\mathbf{v}_{t+1}^k = \mathbf{w}_t^k - \eta_t \nabla F_k(\mathbf{w}_t^k, \psi_t^k),$$

$$\mathbf{w}_{t+1}^k = \begin{cases} \mathbf{v}_{t+1}^k & \text{if } t+1 \notin \mathcal{I}_E, \\ \text{sample } \mathcal{A}_{t+1} \text{ from } \mathcal{S}_{t+1}^* \\ \text{and average } \{\mathbf{v}_{t+1}^k\}_{k \in \mathcal{A}_{t+1}} & \text{if } t+1 \in \mathcal{I}_E. \end{cases}$$

Let  $\mathbf{w}^* \in \arg \min_{\mathbf{w}} F^{\mathcal{D}}(\mathbf{w})$  and  $\mathbf{v}_k^* \in \arg \min_{\mathbf{v}} F_k(\mathbf{v})$  for  $k \in [N]$ .

Let

$$\bar{\mathbf{v}}_t = \sum_{k \in [N]} p_k \mathbf{v}_t^k,$$

$$\bar{\mathbf{w}}_t = \sum_{k \in [N]} p_k \mathbf{w}_t^k.$$

where  $p_k \geq 0$  is the given weight of the  $k^{\text{th}}$  client and w.l.o.g., we assume  $\sum_k p_k = 1$ .

We further note that as RGF is a randomized greedy method, it is biased. Filtering the clients before selection, using biased filtering algorithms, such as RGF, made the theoretical analysis more challenging. Compared to previous theoretical federated convergence analysis, such as (Li et al., 2019) and (Balakrishnan et al., 2021), that introduce  $\bar{\mathbf{v}}_t$  and  $\bar{\mathbf{w}}_t$ , to proceed with our analysis we introduce an extra variable  $\bar{\mathbf{z}}_t$ , defined as follows

$$\bar{\mathbf{z}}_t = \frac{1}{|\mathcal{S}_t^*|} \sum_{k \in \mathcal{S}_t^*} \mathbf{v}_t^k.$$

The filtering algorithm, at every round  $t$ , by optimizing the reward function  $\mathcal{R}$ , is finding a subset  $\mathcal{S}_t^*$  that minimizes  $F^{\mathcal{P}}(\bar{\mathbf{z}}_t)$ , thus decreasing  $F^{\mathcal{P}}(\bar{\mathbf{z}}_t)$ , and thus increasing the gap  $F^{\mathcal{P}}(\bar{\mathbf{v}}_t) - F^{\mathcal{P}}(\bar{\mathbf{z}}_t)$ . Thus, when  $\mathcal{P}$  and  $\mathcal{D}$  are following

a similar distribution, the algorithm is increasing the gap  $F^{\mathcal{D}}(\bar{\mathbf{v}}_t) - F^{\mathcal{D}}(\bar{\mathbf{z}}_t)$ , which we formally define as follows.

$$\delta_t = F^{\mathcal{D}}(\bar{\mathbf{v}}_t) - F^{\mathcal{D}}(\bar{\mathbf{z}}_t) \quad (7)$$

An optimal filtering method leads to the highest  $\delta_t$  possible at every round  $t$ . In FilFL, using RGF as a filtering method, we expect the  $\delta_t$  to be optimized over the rounds. In Lemma 1, we show that  $\mathbb{E}[\delta_t]$  is lower bounded by a constant  $\delta$ .

### 3.2. Convergence Results

We present our main convergence result as follows.

**Theorem 1.** *Let assumptions 1, 2, 3, 4, 5, and 6, hold we have*

$$\mathbb{E}[\|\bar{\mathbf{w}}_{t+1} - \mathbf{w}^*\|^2] \leq \mathcal{O}\left(\frac{1}{t}\right) + \mathcal{O}(\varphi) \quad (8)$$

for some time constant  $\varphi$  that depends on the filtering quality.

*Proof.* Note that

$$\begin{aligned} \mathbb{E}[\|\bar{\mathbf{w}}_{t+1} - \mathbf{w}^*\|^2] &= \mathbb{E}[\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1} + \bar{\mathbf{v}}_{t+1} - \mathbf{w}^*\|^2] \\ &= \mathbb{E}[\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2] \\ &\quad + \mathbb{E}[\|\bar{\mathbf{v}}_{t+1} - \mathbf{w}^*\|^2] \\ &\quad + 2\mathbb{E}[\langle \bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}, \bar{\mathbf{v}}_{t+1} - \mathbf{w}^* \rangle] \end{aligned} \quad (9)$$

Using the results from Lemmas in Appendix A, we bound the three terms in Eq. (9), i.e.,  $\mathcal{T}_1 = \mathbb{E}[\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2]$ ,  $\mathcal{T}_2 = \mathbb{E}[\|\bar{\mathbf{v}}_{t+1} - \mathbf{w}^*\|^2]$ , and  $\mathcal{T}_3 = \mathbb{E}[\langle \bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}, \bar{\mathbf{v}}_{t+1} - \mathbf{w}^* \rangle]$ .

Using Lemma 4 result, shown in Appendix A, we have

$$\mathcal{T}_1 = \mathbb{E}[\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2] \leq \xi. \quad (10)$$

for some constant  $\xi$ .

Using Lemma 1, 2, and 3 in (Li et al., 2019), we have

$$\mathcal{T}_2 \leq (1 - \eta_t \mu) \mathbb{E}[\|\bar{\mathbf{w}}_t - \mathbf{w}^*\|^2] + \eta_t^2 B \quad (11)$$

holds for some constant  $B$ .

Furthermore, using Corollary 1, shown in Appendix A, we have

$$\mathcal{T}_3 = \mathbb{E}[\langle \bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t, \bar{\mathbf{v}}_t - \mathbf{w}^* \rangle] \leq \rho \sqrt{\xi} \quad (12)$$

Define  $\Delta_t = \mathbb{E}[\|\bar{\mathbf{w}}_t - \mathbf{w}^*\|^2]$ , and  $\varphi = \xi + 2\rho\sqrt{\xi}$ , thus

$$\Delta_{t+1} \leq (1 - \eta_t \mu) \Delta_t + \eta_t^2 B + \varphi \quad (13)$$

With a decaying stepsize,  $\eta_t = \frac{\beta}{t}$ ,  $\beta \geq \frac{1}{\mu}$ , the final convergence result follows from Lemma 3 in (Mirzasoileiman et al., 2020).  $\square$

The above result guarantees the convergence rate of  $\mathcal{O}(\frac{1}{t})$  of FilFL up to a certain neighbourhood  $\mathcal{O}(\varphi)$ , which depends on the quality of filtering. The  $\varphi$  term encodes the approximation error of the filtering algorithm. Although we have a different approach than DivFL we end up with similar theoretical guarantees (with different constants) and better empirical results. In experiments, we show that FilFL allows us to achieve faster convergence than DivFL and other FL algorithms.

Following the constant  $\varphi$  dependence, we note that  $\varphi$  depends on  $\xi$ , which in turn depends on the  $\delta$ . Note that a good filtering algorithm implies larger  $\delta_t$  for all  $t$ , defined in Eq. (7). Thus, it implies a larger  $\delta$ . Therefore, a good filtering algorithm implies a smaller  $\xi$ , which implies a smaller  $\varphi$ . Our proposed RGF algorithm is designed to greedily increase  $\delta_t$  as much as possible, hence increase as much as possible  $\delta$ , thus decrease as much as possible  $\xi$ , hence decrease as much as possible  $\varphi$ . It is empirically shown that RGF accelerates the training and provides large  $\delta_t$  for every round  $t$ , thus small  $\varphi$ . We refer the reader to Fig. 8 that shows that the reward  $\mathcal{R}(S_t^*)$  is much larger than  $\mathcal{R}(S_t)$ , i.e., RGF finds a better combination of clients that provides a minor loss in round  $t$ .

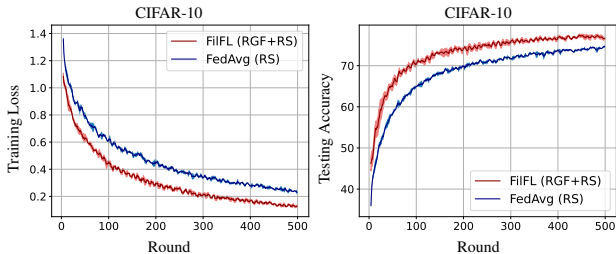


Figure 2. FilFL vs FedAvg using random sampling as a client selection method on the CIFAR-10 dataset.

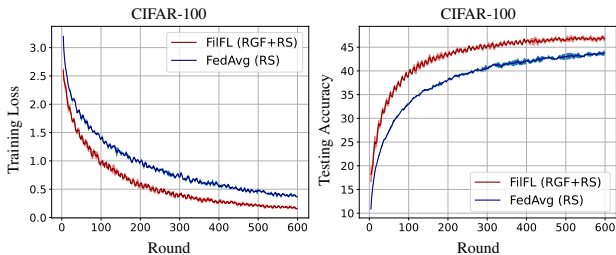


Figure 3. FilFL vs FedAvg using random sampling as a client selection method on the CIFAR-100 dataset.

## 4. Experiments

To evaluate our method, we conduct experiments using different federated datasets. First, we test the impact of *client filtering* on top of client selection in FL. As we are the first

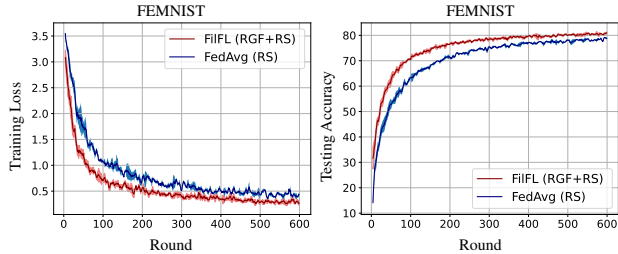


Figure 4. FilFL vs FedAvg using random sampling as a client selection method on the FEMNIST dataset.

to propose *client filtering* in FL, we compare the convergence behaviour of FL when using and not using *client filtering* for different client selection schemes. More specifically, we compare (a) random sampling without filtering (Li et al., 2020b), (b) random sampling with filtering, (c) power-of-choice without filtering (Cho et al., 2020), and (d) power-of-choice with filtering. This will demonstrate the impact of adding filtering using the proposed RGF algorithm. Furthermore, we compare FilFL using the simplest method of client selection, namely random sampling, with a computationally demanding client selection method, namely DivFL, without *client filtering* (Balakrishnan et al., 2021).

### 4.1. Setup

In our experiments, we use CIFAR-10, and CIFAR-100 (Krizhevsky et al., 2009), with a simulated federated setting. Furthermore, we use Federated Extended MNIST (FEMNIST) (Caldas et al., 2018), which is built by partitioning the data in Extended MNIST (Cohen et al., 2017; LeCun, 1998) based on the writer of the digit/character. We experiment with three different seeds for the randomness in the dataset partition across clients as well as the clients selection and present the averaged results together with the error (standard deviation).

#### 4.1.1. CIFAR-BASED BENCHMARKS

*Distribution.* We first split both CIFAR-10 and CIFAR-100 train datasets into private and public datasets, where the public partition fraction is 0.1, and it is utilized by the filtering algorithm. The private dataset is distributed non-iid among all the clients and split into a train (0.9) and validation (0.1) datasets. Similar to existing works (Acar et al., 2021; He et al., 2020; Yurochkin et al., 2019), to simulate the non-iid data distribution among clients, we use the Dirichlet distribution  $\text{Dir}(\alpha)$  where a smaller  $\alpha$  indicates higher data heterogeneity. We report results with  $\alpha = 0.5$ . Finally, we use the existing CIFAR test sets as global test sets.

*Model.* For both CIFAR-10 and CIFAR-100 datasets, we employ ResNet18 (He et al., 2016) as the basic backbone.

*Hyperparameters.* For both datasets, if not otherwise specified, we set the number of local training epoch  $E = 5$ , communication rounds  $T = 500$ , and the number of clients  $N = 200$ . To make the simulation more realistic, we also simulate behaviour heterogeneity by considering a time-varying set of available clients  $\mathcal{S}_t$  of size  $n = 100$ , randomly selected without replacement from the entire pool of clients every 10 rounds. Moreover, we choose  $h = 10$ . Then, we conduct client selection with the fraction  $C = 0.1$  (i.e.,  $|\mathcal{A}_t| = 10$ ). For local training, the batch size is 16, and the weight decay is  $1e - 3$ . The learning rate is 0.1, with a decaying factor of 0.998 every 10 rounds.

#### 4.1.2. FEMNIST-BASED BENCHMARKS

*Distribution.* We use the FEMNIST dataset from the LEAF framework (Caldas et al., 2018). The dataset comprises train and test datasets containing a client-data mapping file that splits the data in a non-iid manner among the clients. It has natural heterogeneity stemming from the writing style of each person. Following (Caldas et al., 2018), we use only 5% of the FEMNIST available dataset with 190 clients. We split the training data of each client into three parts; validation data (0.1), public data (0.1) and training data (0.8). We concatenate all the public datasets from all the clients to obtain a global public dataset representative of all clients. Finally, we use the test set as a global test set.

*Model.* Similar to (Caldas et al., 2018), we use a model with two convolutional layers followed by pooling and ReLU and a final dense layer with 2048 units.

*Hyperparameters.* We set the number of local training epoch  $E = 2$ , communication rounds  $T = 500$ , and the number of clients  $N = 190$ . To make the simulation more realistic, we simulate behaviour heterogeneity by considering a time-varying set of available clients  $\mathcal{S}_t$  of size  $n = 50$ , randomly selected without replacement from the full pool of clients every 10 rounds. Moreover, we choose  $h = 10$ . Then we conduct client selection with the fraction  $C = 0.1$  (i.e.,  $|\mathcal{A}_t| = 5$ ). For local training, the batch size is 50. The learning rate is 0.005.

#### 4.2. FilFL Convergence Behavior

To assess the performance of FilFL, we conduct several experiments on different datasets. We provide training loss and testing accuracy, using random sampling as a client selection method in Fig. 2, Fig. 3, and Fig. 4, for CIFAR-10, CIFAR-100, and FEMNIST, respectively. We provide similar plots, using power-of-choice as a client selection technique in the Appendix B.1. Furthermore, we provide in Fig. 5 an experiment comparing our method against DivFL without filtering.

For the three datasets, FilFL remarkably accelerates the

training. As shown in Table 2, FilFL is around 2 – 3x faster than FedAvg, for all datasets. This acceleration is consistent over all the rounds. As shown in Fig. 2, Fig. 3, and Fig. 4, FilFL has a clear advantage over all the rounds on the training and testing set. Not only it accelerates the training, but it also converges to better weights, i.e., it achieves a better final test accuracy. FilFL achieves an increase of 3%, 3%, and 4% in the final test accuracy for CIFAR-10, CIFAR-100, and FEMNIST, respectively.

The above observations are valid for other selection methods, namely power-of-choice, for which we refer to Appendix B.1. Furthermore, we show that FilFL using RGF as a *client filtering* scheme, combined with the simplest client selection method, i.e., random sampling, can outperform a sophisticated and computationally expensive client selection scheme, namely DivFL. Although DivFL is shown in their work to outperform random sampling and power-of-choice (Balakrishnan et al., 2021), it remains a costly client selection approach. On the CIFAR-10 dataset with  $N = 100$ ,  $n = 25$ , and  $K = 5$ , when both applied periodically with a period  $h = 10$ , as shown in Fig. 5, although DivFL reached a lower training loss (overfitted), FilFL remarkably outperforms this client selection method, with a higher testing accuracy over the rounds.

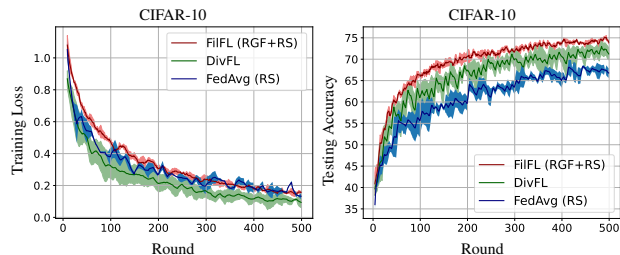


Figure 5. A comparison of FilFL using RGF for filtering and random sampling for selection, DivFL, and FedAvg using random sampling, on CIFAR-10.

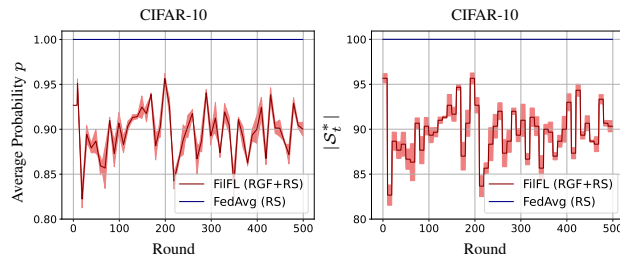


Figure 6. RGF on CIFAR-10 dataset. The left and right plots show the average acceptance probability  $p$  of every available client and the number of accepted (filtered) clients in every round.

	# Rounds Needed		
	CIFAR-10	CIFAR-100	FEMNIST
	70%	60%	70%
FedAvg	207	268	180
FilFL	<b>82</b>	<b>103</b>	<b>90</b>

Table 2. FedAvg vs FilFL using random sampling for client selection on different datasets. It shows the number of communication rounds needed to achieve a certain test accuracy percentage.

	Best Test Accuracy over Rounds		
	CIFAR-10	CIFAR-100	FEMNIST
FedAvg	74.5%	43%	78.7%
FilFL	<b>77.5%</b>	<b>47%</b>	<b>81%</b>

Table 3. FedAvg vs FilFL using random sampling for client selection on different datasets, showing best achieved test accuracy.

### 4.3. RGF Behavior

We test RGF on a stochastic reward function  $\mathcal{R}$ . We simulate  $\mathcal{R}$  to be in expectation an increasing function of time  $t$  and to be a non-monotone function of the chosen set of clients  $X$ , defined as follows,  $\mathcal{R}(x, t) = \mathcal{U}(\alpha t, \beta t)^{\frac{1}{2}}$ , where  $\alpha < \beta$ . ( $\alpha = 0.0025$  and  $\beta = 0.005$ ). Along with RGF, we run a search algorithm to find the exact optimal subset of clients (OPT) for every round  $t$ . We then check the reward value  $\mathcal{R}$  for the initial set of available clients  $\mathcal{S}_t$ , i.e., without any client rejection, the reward value associated with the output of RGF  $\mathcal{S}_t^*$ , and the reward value of the optimal set OPT. From Fig. 8, it can be seen that our proposed algorithm RGF reaches near-optimal rewards over the rounds.

Furthermore, we trace several parameters related to RGF (Algorithm 1) on CIFAR-10; see Fig. 6. The figure shows the results of CIFAR-10 experiments where we simulate a random subset of 100 active clients out of 200 clients every 10 rounds. We track the acceptance probability of each client  $p$  defined in Eq. (6), i.e., the probability of keeping an active client for client selection, and compute its average per round, see the left plot in Fig. 6. While FedAvg accepts all active clients with probability  $p = 1$  (no filtering), RGF, when used with random sampling (RGF+RS), shows a time-varying acceptance probability that oscillates around 0.9 for the CIFAR-10 benchmark. Moreover, we track the actual number of filtered clients every round out of the 100 active clients, i.e., the cardinality of the filtered subset of clients  $\mathcal{S}_t^*$ . As shown on the right plot of Fig. 6, the actual number of filtered clients is variable but mostly oscillates around 90. By construction, RGF is only used if  $\mathcal{S}_t \neq \mathcal{S}_t^*$ , i.e., if a new subset of active clients is generated, thus the number of filtered clients remains the same over 10 rounds.

Furthermore, Fig. 7 shows the percentage of unique clients selected since the beginning of the training. We can observe that FilFL can see all the clients from the pool of all the

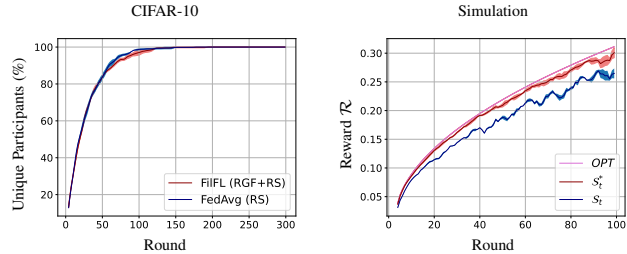


Figure 7. The percentage of unique participants until round  $t$  on CIFAR-10 experiment.

Figure 8. The reward  $\mathcal{R}$ , at round  $t$ , for the initial set  $\mathcal{S}_t$ , the RGF output  $\mathcal{S}_t^*$ , and the optimal set OPT.

clients in around 200 rounds, and FedAvg needs around 150 rounds. This shows that FilFL does not discard clients forever; it just picks the right clients at the right time.

## 5. Discussion

We note that RGF is run on the server, which generally has fewer computational constraints. Moreover, given the oracle values of  $\mathcal{R}$ , RGF has a low complexity of  $\mathcal{O}(n)$  for  $n$  number of available clients. Furthermore, we recall that FilFL only calls RGF periodically, i.e., when  $t \bmod h = 0$ , or when  $\mathcal{S}_t \neq \mathcal{S}_{t-1}$ . Therefore, given its advantage in terms of accelerating the training and increasing the final test accuracy results, the proposed method is indeed efficient.

Among the FilFL limitations is the requirement of a public dataset. Although this assumption is generally reasonable and is common in the literature of FL (Li et al., 2020a; Sattler et al., 2020; Seo et al., 2022; Zhang et al., 2021), it might be limiting in some scenarios. If a public dataset is unavailable, FilFL can be generalized to alleviate this limitation by proposing different filtering methods that do not require a public dataset. A possible direction is the use of knowledge distillation in *client filtering*.

Besides accelerating the training and achieving higher testing accuracy over the rounds, *client filtering* can help detect and reject adversarial and malicious clients. By construction, RGF looks for the best subset of clients; thus, it could eliminate the worst clients. Future work might consider FilFL for more robust federated training.

## 6. Conclusion

We proposed *client filtering* as a promising technique to optimize FL training. We suggested a randomized greedy filtering algorithm called RGF, which is efficient and shows outstanding performance when integrated with FL. Our proposed FL algorithm, FilFL, has proven theoretical conver-



gence guarantees. Moreover, it is empirically shown to provide better learning efficiency, accelerated convergence, and higher final test accuracy. Future works might investigate more algorithms for *client filtering* in FL and the potential of filtering for robust federated training.

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## A. Main Lemmas with Proofs

**Lemma 1.** Under assumptions 1, 2, 3, 4, and 5, for the gap  $\delta_t$  defined in 7, we have

$$\mathbb{E}[\delta_t] \geq \delta \quad (14)$$

for some constant  $\delta$ .

*Proof.* By  $\mu$ -strong convexity, Assumption 1, and L-smoothness, Assumption 2, we have

$$F^{\mathcal{D}}(\bar{\mathbf{z}}_t) - F^{\mathcal{D}}(\bar{\mathbf{v}}_t) \leq \frac{1}{2\mu} \|\nabla F^{\mathcal{D}}(\bar{\mathbf{z}}_t) - \nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2 + \frac{1}{2} \langle \nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t), \bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t \rangle \quad (15)$$

By the Cauchy–Schwarz inequality, we have

$$\begin{aligned} \mathbb{E}[F^{\mathcal{D}}(\bar{\mathbf{z}}_t) - F^{\mathcal{D}}(\bar{\mathbf{v}}_t)] &\leq \frac{1}{2\mu} \mathbb{E}[\|\nabla F^{\mathcal{D}}(\bar{\mathbf{z}}_t) - \nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2] + \frac{1}{2} \mathbb{E}[\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\| \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|] \\ &\leq \frac{1}{2\mu} \sum_k \mathbb{E}[\|\nabla F_k^{\mathcal{D}}(\bar{\mathbf{z}}_t) - \nabla F_k^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2] + \frac{1}{2} \mathbb{E}[\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\| \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|] \\ &\leq \frac{1}{2\mu} \sum_k \sigma_k^2 + \frac{1}{2} \mathbb{E}[G \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|], \end{aligned} \quad (16)$$

where the last inequality follows from Assumption 3 and Assumption 4.

Moreover,

$$\begin{aligned} \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\| &= \left\| \sum_{k \in [N]} p_k \mathbf{v}_t^k - \frac{1}{|\mathcal{S}_t^*|} \sum_{k \in \mathcal{S}_t^*} \mathbf{v}_t^k \right\| \\ &\leq \left\| \sum_{k \in [N]} p_k \mathbf{v}_t^k \right\| + \left\| \frac{1}{|\mathcal{S}_t^*|} \sum_{k \in \mathcal{S}_t^*} \mathbf{v}_t^k \right\| \\ &\leq \sum_{k \in [N]} p_k \|\mathbf{v}_t^k\| + \frac{1}{|\mathcal{S}_t^*|} \sum_{k \in \mathcal{S}_t^*} \|\mathbf{v}_t^k\| \\ &\leq 2 \sum_{k \in [N]} \|\mathbf{v}_t^k\| \\ &\leq 2 \sum_{k \in [N]} [\|\mathbf{v}_t^k - \mathbf{v}_k^*\| + \|\mathbf{v}_k^*\|] \end{aligned} \quad (17)$$

Furthermore, by  $\mu$ -strong convexity, Assumption 2, and Assumption 4, we have

$$\|\bar{\mathbf{v}}_t^k - \mathbf{v}_k^*\| \leq \frac{1}{\mu} \|\nabla F_k(\mathbf{v}_t^k)\| \leq \frac{G}{\mu} \quad (18)$$

Thus, by Eq. (17) and Eq. (18), we have

$$\|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\| \leq \sum_{k \in [N]} 2 \left[ \frac{G}{\mu} + \|\mathbf{v}_k^*\| \right] \quad (19)$$

Using Eq. (16) and Eq. (19), we have

$$\mathbb{E}[F^{\mathcal{D}}(\bar{\mathbf{z}}_t) - F^{\mathcal{D}}(\bar{\mathbf{v}}_t)] \leq \frac{1}{2\mu} \sum_k \sigma_k^2 + G \sum_{k \in [N]} \left[ \frac{G}{\mu} + \|\mathbf{v}_k^*\| \right] \leq -\delta \quad (20)$$

for  $\delta = -\frac{1}{2\mu} \sum_k \sigma_k^2 - G \sum_{k \in [N]} \left[ \frac{G}{\mu} + \|\mathbf{v}_k^*\| \right]$ , which does not depend on  $T$  and only on the problem parameters.

Therefore, we obtain

$$\mathbb{E}[\delta_t] = \mathbb{E}[F^{\mathcal{D}}(\bar{\mathbf{v}}_t) - F^{\mathcal{D}}(\bar{\mathbf{z}}_t)] \geq \delta \quad (21)$$

□

**Lemma 2.** Under assumptions 1, 2, and 4 for the sequences,  $\bar{\mathbf{z}}_t$  and  $\bar{\mathbf{v}}_t$ , we have

$$\mathbb{E} [\|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2] \leq \frac{G^2}{\mu^2} - \frac{2\delta}{\mu} \quad (22)$$

*Proof.* By  $\mu$ -strong convexity, Assumption 1, and L-smoothness, Assumption 2, we have

$$\begin{aligned} \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2 &\leq \frac{2}{\mu} (F^{\mathcal{D}}(\bar{\mathbf{z}}_t) - F^{\mathcal{D}}(\bar{\mathbf{v}}_t) - \langle \nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t), \bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t \rangle) \\ &\stackrel{(7)}{\leq} \frac{2}{\mu} (-\delta_t + \langle \nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t), \bar{\mathbf{v}}_t - \bar{\mathbf{z}}_t \rangle) \\ &\leq \frac{2}{\mu} (\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\| \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\| - \delta_t) \end{aligned} \quad (23)$$

where the last inequality follows from the Cauchy–Schwarz inequality.

Therefore,

$$\|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2 - 2 \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|}{\mu} \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\| \leq -\frac{2}{\mu} \delta_t \quad (24)$$

Thus,

$$\|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2 - 2 \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|}{\mu} \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\| + \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2}{\mu^2} \leq -\frac{2}{\mu} \delta_t + \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2}{\mu^2} \quad (25)$$

Hence,

$$\left( \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\| + \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|}{\mu} \right)^2 \leq -\frac{2}{\mu} \delta_t + \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2}{\mu^2} \quad (26)$$

Hence,

$$\begin{aligned} \|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2 &\leq \frac{\|\nabla F^{\mathcal{D}}(\bar{\mathbf{v}}_t)\|^2}{\mu^2} - \frac{2}{\mu} \delta_t \\ &\stackrel{(1)}{\leq} \frac{\|\sum_{k=1}^N p_k \nabla F_k(\bar{\mathbf{v}}_t)\|^2}{\mu^2} - \frac{2}{\mu} \delta_t \\ &\leq \frac{(\sum_{k=1}^N p_k \|\nabla F_k(\bar{\mathbf{v}}_t)\|)^2}{\mu^2} - \frac{2}{\mu} \delta_t \\ &\stackrel{(4)}{\leq} \frac{(\sum_{k=1}^N p_k G)^2}{\mu^2} - \frac{2}{\mu} \delta_t \end{aligned} \quad (27)$$

Therefore,

$$\mathbb{E} [\|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2] \leq \frac{G^2}{\mu^2} - \frac{2\mathbb{E}[\delta_t]}{\mu} \quad (28)$$

Therefore, by Lemma 1, we have

$$\mathbb{E} [\|\bar{\mathbf{z}}_t - \bar{\mathbf{v}}_t\|^2] \leq \frac{G^2}{\mu^2} - \frac{2\delta}{\mu} \quad (29)$$

□

**Lemma 3.** Under assumptions 1, 2, 3, 4, 5, and 6, we have

$$\|\bar{\mathbf{v}}_t - \mathbf{w}^*\| \leq \rho. \quad (30)$$

for some constant  $\rho$ .



*Proof.* Note that under Assumption 1 and Assumption 5, we have  $\left\| \sum_{k \in [N]} p_k \mathbf{v}_k^* - \mathbf{w}^* \right\|$  is also bounded by a constant  $M$ .

$$\begin{aligned}
 \|\bar{\mathbf{v}}_t - \mathbf{w}^*\| &\leq \left\| \bar{\mathbf{v}}_t - \sum_{k \in [N]} p_k \mathbf{v}_k^* \right\| + \left\| \sum_{k \in [N]} p_k \mathbf{v}_k^* - \mathbf{w}^* \right\| \\
 &\leq \left\| \bar{\mathbf{v}}_t - \sum_{k \in [N]} p_k \mathbf{v}_k^* \right\| + M \\
 &\leq \sum_{k \in [N]} \|p_k (\bar{\mathbf{v}}_t^k - \mathbf{v}_k^*)\| + M \\
 &\leq \sum_{k \in [N]} p_k \|\bar{\mathbf{v}}_t^k - \mathbf{v}_k^*\| + M
 \end{aligned} \tag{31}$$

By  $\mu$ -strong convexity, Assumption 2, we have

$$\|\bar{\mathbf{v}}_t^k - \mathbf{v}_k^*\| \leq \frac{1}{\mu} \|\nabla F_k(\bar{\mathbf{v}}_t^k)\| \tag{32}$$

Therefore,

$$\begin{aligned}
 \|\bar{\mathbf{v}}_t - \mathbf{w}^*\| &\leq \sum_{k \in [N]} \frac{p_k}{\mu} \|\nabla F_k(\bar{\mathbf{v}}_t^k)\| + M \\
 &\stackrel{(4)}{\leq} \frac{G}{\mu} + M \\
 &\leq \rho.
 \end{aligned} \tag{33}$$

where  $\rho = \frac{G}{\mu} + M$ .  $\square$

**Lemma 4.** Under assumptions 1, 2, 3, 4, 5, and 6, for any virtual iteration  $t$ , for the above defined sequences,  $\bar{\mathbf{z}}_t$  and  $\bar{\mathbf{v}}_t$ , we have

$$\mathbb{E} [\|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\|^2] \leq \xi \tag{34}$$

for some constant  $\xi$ .

*Proof.* If not aggregating,

$$\bar{\mathbf{w}}_{t+1} = \bar{\mathbf{v}}_{t+1}.$$

Hence,

$$\mathbb{E} [\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2] = 0 \tag{35}$$

If aggregating, using Lemma 4 in (Li et al., 2019), we know that if  $t + 1 \in \mathcal{I}_E$ , for sampling scheme in Assumption 5, we have

$$\mathbb{E} (\bar{\mathbf{w}}_{t+1}) = \bar{\mathbf{z}}_{t+1} \tag{36}$$

$$\begin{aligned}
 \|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2 &= \|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{z}}_{t+1} + \bar{\mathbf{z}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2 \\
 &= \|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{z}}_{t+1}\|^2 + \|\bar{\mathbf{z}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2 + 2 \langle \bar{\mathbf{w}}_{t+1} - \bar{\mathbf{z}}_{t+1}, \bar{\mathbf{z}}_{t+1} - \bar{\mathbf{v}}_{t+1} \rangle
 \end{aligned}$$

When expectation is taken over  $\mathcal{S}_{t+1}$ , the last term vanishes due to the unbiasedness of  $\bar{\mathbf{w}}_{t+1}$ .

Therefore,

$$\mathbb{E} [\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2] = \mathbb{E} [\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{z}}_{t+1}\|^2] + \mathbb{E} [\|\bar{\mathbf{z}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2]$$

Moreover, using Lemma 5 in (Li et al., 2019), we know that if  $t + 1 \in \mathcal{I}_E$ , for sampling scheme in assumption 5, the expected difference between  $\bar{\mathbf{v}}_{t+1}$  and  $\bar{\mathbf{w}}_{t+1}$  is bounded by

$$\mathbb{E} [\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{z}}_{t+1}\|^2] \leq C. \tag{37}$$

where  $C$  is a constant.

Therefore, using Lemma 2, we have

$$\begin{aligned} \mathbb{E} [\|\bar{\mathbf{w}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2] &\leq C + \mathbb{E} [\|\bar{\mathbf{z}}_{t+1} - \bar{\mathbf{v}}_{t+1}\|^2] \\ &\stackrel{(22)}{\leq} C + \frac{G^2}{\mu^2} - \frac{2\delta}{\mu} \\ &\leq \xi \end{aligned} \tag{38}$$

for  $\xi = C + \frac{G^2}{\mu^2} - \frac{2\delta}{\mu}$ .

□

**Corollary 1.** *Under assumptions 1, 2, 3, 4, 5, and 6, for any virtual iteration  $t$ , for the above defined sequences,  $\bar{\mathbf{z}}_t$  and  $\bar{\mathbf{v}}_t$ , we have*

$$\mathbb{E} [\langle \bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t, \bar{\mathbf{v}}_t - \mathbf{w}^* \rangle] \leq \rho \sqrt{\xi} \tag{39}$$

*Proof.* By Cauchy-Schwarz inequality we have

$$\langle \bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t, \bar{\mathbf{v}}_t - \mathbf{w}^* \rangle \leq \|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\| \|\bar{\mathbf{v}}_t - \mathbf{w}^*\| \tag{40}$$

Moreover, by Lemma 3, we have

$$\|\bar{\mathbf{v}}^t - \mathbf{w}^*\| \leq \rho. \tag{41}$$

Therefore,

$$\langle \bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t, \bar{\mathbf{v}}_t - \mathbf{w}^* \rangle \leq \rho \|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\| \tag{42}$$

Using Jensen inequality (Peajcariac & Tong, 1992) and Lemma 4, it follows that

$$\mathbb{E} [\|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\|] \leq \mathbb{E} [\|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\|^2]^{\frac{1}{2}} \stackrel{(34)}{\leq} \sqrt{\xi} \tag{43}$$

Combine equations (42) and (43), we have

$$\begin{aligned} \mathbb{E} [\langle \bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t, \bar{\mathbf{v}}_t - \mathbf{w}^* \rangle] &\stackrel{(42)}{\leq} \mathbb{E} [\rho \|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\|] \\ &\leq \rho \mathbb{E} [\|\bar{\mathbf{w}}_t - \bar{\mathbf{v}}_t\|] \\ &\stackrel{(43)}{\leq} \rho \sqrt{\xi} \end{aligned} \tag{44}$$

□

## B. Additional Experiments

### B.1. Using Power-of-Choice as a client selection method

In the experiments Sec. 4.2, we mainly use random sampling as a client selection method. In this section, we assess the performance of FiFL vs FedAvg when using another client selection method, called power-of-choice (Cho et al., 2020). We provide training loss, training accuracy, and testing accuracy, in Fig. 9, Fig. 10, and Fig. 11, for CIFAR-10, CIFAR-100, and FEMNIST, respectively.

As shown in Fig. 9, Fig. 10, and Fig. 11, FiFL has a clear advantage over all the rounds on the training and testing set. Not only it accelerates the training, it converges to better weights, i.e., it achieves a better final test accuracy. FiFL, achieves an increase of 10%, 13%, 8%, in the final test accuracy CIFAR-10, CIFAR-100, and FEMNIST, respectively.

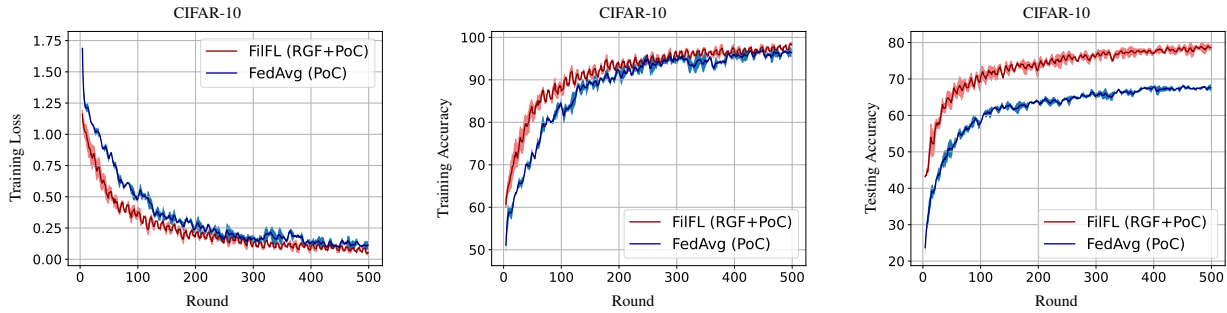


Figure 9. FiFL vs FedAvg using power-of-choice as a client selection method on CIFAR-10 dataset.

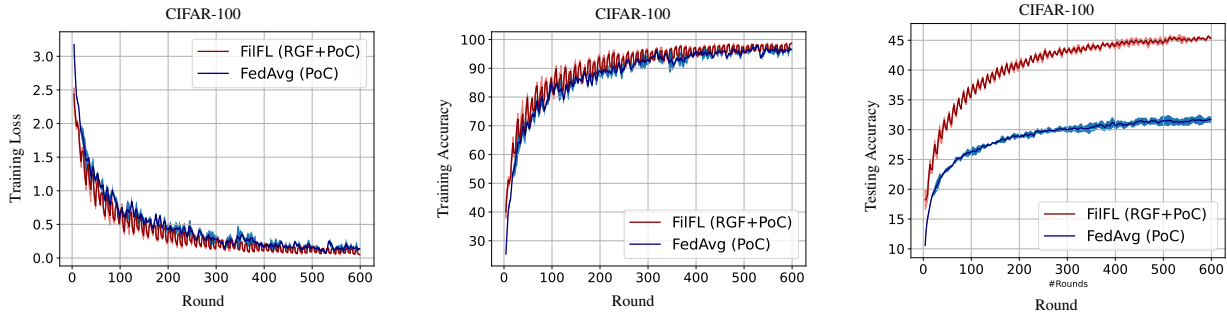


Figure 10. FiFL vs FedAvg using power-of-choice as a client selection method on CIFAR-100 dataset.

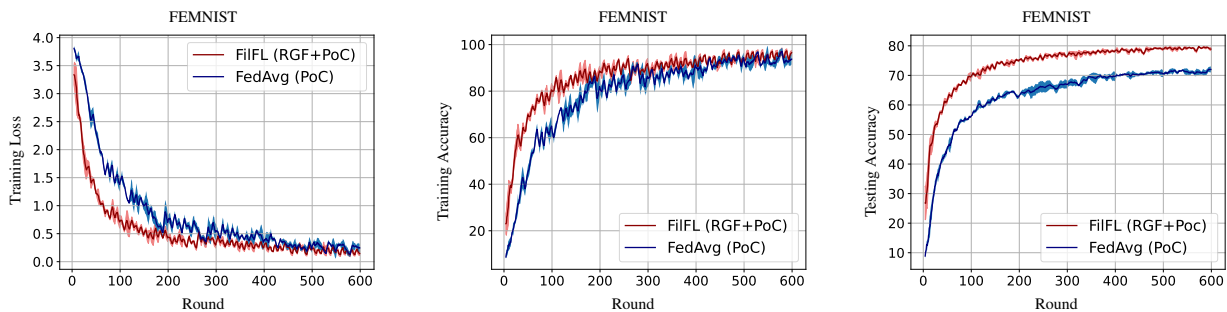


Figure 11. FiFL vs FedAvg using power-of-choice as a client selection method on FEMNIST dataset.