

Topology-Based Reconstruction Prevention for Decentralised Learning

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ABSTRACT

Decentralised learning has recently gained traction as an alternative to federated learning in which both data and coordination are distributed over its users. To preserve the confidentiality of users' data, decentralised learning relies on differential privacy, multi-party computation, or a combination thereof. However, running multiple privacy-preserving summations in sequence may allow adversaries to perform reconstruction attacks. Unfortunately, current reconstruction countermeasures either cannot trivially be adapted to the distributed setting, or add excessive amounts of noise.

In this work, we first show that passive honest-but-curious adversaries can infer other users' private data after several privacy-preserving summations. For example, in subgraphs with 18 users, we show that only three passive honest-but-curious adversaries succeed at reconstructing private data 11.0% of the time, requiring an average of 8.8 summations per adversary. The success rate depends only on the adversaries' direct neighbourhood, and is independent of the size of the full network. We consider weak adversaries, who do not control the graph topology and can exploit neither the inner workings of the summation protocol nor the specifics of users' data, and show that they can infer private data regardless.

We develop a mathematical understanding of how reconstruction relates to topology and propose the first topology-based decentralised defence against reconstruction attacks. Specifically, we show that reconstruction requires a number of adversaries linear in the length of the network's shortest cycle. Consequently, reconstructing private data from privacy-preserving summations is impossible in acyclic networks.

Our work is a stepping stone for a formal theory of decentralised reconstruction defences based on topology. Such a theory would generalise our countermeasure beyond summation, define confidentiality in terms of entropy, and describe the effects of (topology-aware) differential privacy.

1 INTRODUCTION

Machine learning is used in a wide array of systems, including smartwatches [50], predictive text [6], and malware detection [41]. These systems require access to large amounts of reliable data in order to function accurately. In practice, the necessary data

usually exist, but are distributed over many data owners. The naive approach for data collection is to have the data owners send their data to a central server, which trains a machine learning model on these data before deploying it. However, sharing private data may result in misuse, for example in the form of targeted advertising or harassment. In an age of increasing privacy awareness, data owners may be reluctant to share their data, threatening the viability of data-intensive machine learning applications.

The emerging field of federated learning, first formalised in [38], addresses these privacy issues by distributing the training process over the data owners. Instead of submitting their data, each data owner first trains a machine learning model on their local data and then submits this model to a central server. This central server, called the aggregator, uses a privacy-preserving summation protocol to combine the received models into a single global model. The central server then sends back the global model to the data owners, who apply another round of training, repeating the entire process until the global model has converged.

A significant drawback of classical federated learning is that communication is a bottleneck, scaling quadratically [6] or poly-logarithmically [2] in the number of users. Decentralised learning, a variant of federated learning [31], removes this bottleneck by distributing both the data and the coordination between users. Training happens in a peer-to-peer fashion, with users exchanging information only with their direct neighbours. This significantly reduces the communication complexity [35], allowing for cost-effective deployments without a central server. Furthermore, because communication is local, it becomes much harder for adversaries to observe the full network [45].

Recently, there has been increased interest in decentralised learning. Though some works do not consider privacy [35, 43, 53], many other works do. Some of these works [3, 46, 53] consider algorithms in which nodes are randomly selected to calculate updates, and protect the private data underlying the models using differential privacy. That is, they apply carefully calibrated random noise to the calculated gradients before sharing them with others. A slight variation of this is to use a random walk through the graph to determine the order in which updates occur [15]. There are also works [10, 40, 42] that use blockchains to facilitate the communication and coordination between nodes, and then similarly use differential privacy. Finally, instead of differential privacy, some works utilise multi-party computation [18, 32, 44], which does not give noisy results, but has higher computational costs.

A common thread in these works is that they apparently assume that if a single summation is secure, then the protocol remains

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secure after multiple summations. However, this requires further scrutiny, as combining information from multiple rounds may reveal previously hidden information. For example, given private records A , B , and C , and a privacy-preserving summation protocol, an adversary could separately query $A + B$, then $B + C$, and finally $A + C$, and use a linear algebra solver to learn all three private records. To defend against such attacks, one must prevent sequences of queries that would reveal private data. Naive restrictions, such as requiring a minimum number of included records per query, are insufficient: The adversary could still first query the sum of all models and then query the sum of all models except one, allowing them to reconstruct the excluded model. As such, designing proper countermeasures requires a formal theory.

Extracting data from outputs traces is known as a reconstruction attack, which has its roots in the theory of statistical disclosure [26]. Many defences have been proposed since the 1970s, including query auditing [13], perturbation [23], and random sampling [21]. However, these works assume either a central database, or otherwise assume a central arbiter that determines which queries are allowed. In decentralised learning, there is no clear leader who can be trusted to audit queries. Instead, decentralised learning requires a decentralised solution. Apart from works on perturbation, to the best of our knowledge, only da Silva et al. [16] have considered reconstruction attacks in peer-to-peer networks, but their work applies only to distributed clustering, and does not propose any countermeasures. When considering perturbation, naively applying user-level differential privacy in a distributed setting results in linearly-scaling noise, severely reducing the protocol’s utility [15, 24, 54]. Intuitively, utility can be increased while keeping the same level of privacy by correlating noise according to topology [23], but to the best of our knowledge only a few works have done this. Guo et al. [28] reduce noise based on the mutual overlaps of neighbours’ neighbourhoods, but do not consider time-series correlations. Cyffers et al. [15] observe that data sensitivity decreases as mutual node distances increase, but their solution does not scale well when adversaries collude.

In this work, we analyse reconstruction attacks by colluding adversaries in peer-to-peer networks. We model the network after decentralised learning, though our analysis is sufficiently generic to describe a sequence of summations in any environment. Summation is a simple protocol, but is sufficient to implement many of the aforementioned decentralised learning protocols, in addition to smart metering [27] and even principal component analysis, singular-value decomposition, and decision tree classifications [4]. We assume a set of nodes, each with a private datum that changes over time, and allow privacy-preserving summation over one’s direct neighbours. We formalise the relation between reconstruction and network topology, and prove that reconstruction is impossible in a specific class of topologies.

Concretely, we begin by showing that reconstruction attacks are practical, and that, in random peer-to-peer subgraphs, three honest-but-curious adversaries with 15 neighbours succeed in finding at least one neighbour’s private datum with an 11.0% success rate, requiring an average of only 8.8 rounds per adversary. The success rate is independent of the size of the full network; it depends only on the adversaries’ local neighbourhood. We then show that the success rate depends on the connectivity of the network rather than

its size. Specifically, we show that reconstruction corresponds to cycles in the graph; if the graph’s shortest cycle has length $2k$, then reconstruction never succeeds if there are fewer than k adversaries. Finally, we briefly evaluate the impact of increasing girth on the convergence of a distributed averaging protocol, and find that dense graphs can be “stretched” to higher girths while retaining faster convergence than sparse graphs.

To the best of our knowledge, our work is the first to propose a topology-based defence to reconstruction attacks. We show that restricting how summations may be composed makes it impossible to reconstruct private data. With the ultimate goal of developing a general theory of structured composition as a distributed reconstruction countermeasure, future work may include finding a condition that is not only sufficient (as seen in this work) but also necessary for reconstruction, generalising these countermeasures to operations beyond summation, stronger notions of privacy rooted in information theory, and investigating the effects of (topology-aware) differentially private noise.

The remainder of this paper is structured as follows. In Section 2, we discuss related work. In Section 3, we describe the preliminaries: We explain basic primitives, formalise our assumptions, and introduce our notation. In Section 4, we formally describe reconstruction attacks, and show that the attack is feasible. In Section 5, we prove that the success rate of the reconstruction attack depends on the graph’s girth, and investigate how girth affects application performance. Finally, in Section 6, we present our conclusions.

2 RELATED WORK

In this work we propose a decentralised reconstruction countermeasure for privacy-preserving summation with dynamic data. To the best of our knowledge, this exact problem has not been treated in literature before. Therefore, in this section, we consider related works from various fields, and describe their similarities and differences.

2.1 Reconstruction Attacks

Consider a database that users can query for statistical information. For example, in a database with employee records, users can query for the sum of salaries of all PhD students. Naturally, the database must ensure that users cannot learn individual employees’ salaries. A naive defence would disallow queries over single records, but a cleverly chosen sequence of queries may still allow the user to reconstruct private data. For example, the user could query the sum of salaries of all employees, and the sum of salaries of all employees except Jay Doe, and reconstruct Jay Doe’s salary from that.

The attack described above is known under various names: *statistical disclosure*¹, the *inference problem*, and the *reconstruction attack*. It has been the subject of research since at least the 1970s [26], originally in the context of releasing census statistics. Since then, many reconstruction defences have been proposed, including random sampling [21], query auditing [13], and perturbation [23].

Most related to our research question are those works that consider sum queries only. Chin [12] studies summation query graphs to determine the exact conditions under which disclosure occurs.

¹Confusingly, the term “statistical disclosure attack” is also a separate attack in peer-to-peer literature [17], but this is an unrelated attack on anonymity rather than confidentiality.

However, his analysis is limited to queries that are over exactly two records each, and cannot easily be generalised. Wang et al. [47] allow queries over more than two records. The authors propose cardinality-based criteria for determining whether reconstruction is possible, and create a whitelist of all summations that can be performed without causing undesired reconstruction.

All aforementioned solutions consider a single trusted database or auditor, making them unsuitable for peer-to-peer protocols, in which the data are spread over many users. Except for perturbation-based techniques, there are very few works that consider reconstruction defences in peer-to-peer settings. In their study on reconstruction attacks in distributed environments, Jebali et al. [30] note only the work by da Silva et al. [16] when discussing peer-to-peer solutions, but the latter applies only to distributed clustering, and does not propose any countermeasures.

Perturbation, on the other hand, has been studied in more detail. Probably the most popular perturbation mechanism for the decentralised setting is local differential privacy [25, 33, 49], a variation of differential privacy [23]. With this technique, when a query is performed over some set of nodes, each node adds a small amount of noise such that the aggregate is relatively accurate, but reconstruction remains impossible even after multiple queries. Various fully-decentralised learning protocols use local differential privacy to allow learning a shared machine learning model without revealing users' private datasets [3, 46, 53]. However, the perturbation is calibrated to protect individual records in users' private datasets, rather than protecting users' entire datasets. As a result, these works are potentially vulnerable to inversion attacks [29, 48]. The level of noise can be increased, but this severely impacts utility [15, 54]. Intuitively, noise can be made more "efficient" by exploiting correlations between users' data [24], which, in peer-to-peer networks, amounts to calibrating noise to the topology. To the best of our knowledge only a few works have done this. Guo et al. [28] reduce noise based on the mutual overlaps of neighbours' neighbourhoods, but do not consider time-series correlations. Cyffers et al. [15] observe that data sensitivity decreases as mutual node distances increase, but their solution does not scale well when adversaries collude.

2.2 Multi-party Computation

In secure multi-party computation, composability [36] is the property of a cryptographic scheme that no additional leakage occurs when it is invoked multiple times, with varying parties, combined with other schemes, and so on. There are numerous frameworks to model composability, including universal composability [8], constructive composability [37], and reactive simulatability [1].

Composability solves a different issue than the one posed in this work. While composability ensures nothing leaks beyond what can be inferred from the outputs, our work is concerned exactly with that which can be inferred from the outputs. Composability does not help when the desired output (implicitly) reveals private data.

In secure multi-party computation literature, this difference is occasionally acknowledged. For example, Bogdanov et al. [5] note that "the composition of ideal functionalities is no longer an ideal functionality", and, before them, Yang et al. [52] made a similar observation. There are more works that consider this difference, but, to the best of our knowledge, these works all resolve the issue

by removing or protecting intermediate values, but do not consider protocols which desire intermediate values, and even then do not consider that reconstruction attacks may be possible after multiple instantiations of the protocol. An exception is the work by Dekker and Erkin [19], which releases intermediate values in a structured manner such that it is not possible to reconstruct all users' values. However, the authors do not prove (or disprove) that it is impossible to find a *single* user's value.

3 PRELIMINARIES

We briefly explain some basics on privacy-preserving summation in Section 3.1 and on bipartite graphs in Section 3.2. After that, we formulate our assumptions and define our notation in Section 3.3.

3.1 Privacy-Preserving Summation

Privacy-preserving summation is a special case of multi-party computation in which an aggregator calculates the sum of users' private values without learning the users' individual values. In this work, we consider privacy-preserving summation to be an information-theoretically secure black-box that reveals only the identities and the sum of the variables.

3.2 Bipartite Graphs

A bipartite graph $H = (U, V, E)$ is a graph with nodes $U \cup V$ and edges E , subject to $U \cap V = \emptyset$ and $\forall (u, v) \in E : u \in U \Leftrightarrow v \in V$.

Furthermore, a bipartite graph $H = (U, V, E)$ can be described by a biadjacency matrix $A \in \{0, 1\}^{|U| \times |V|}$, where $\forall 0 \leq u < |U|, 0 \leq v < |V| : A_{u,v} = 1 \Leftrightarrow (U_u, V_v) \in E$.

In this work, all graphs are undirected.

3.3 Assumptions and Notation

The underlying models and assumptions in this work are based on those seen in the decentralised learning literature [3, 18, 53], but are especially close to the work by Vanhaesebrouck et al. [46].

In general, we denote the first element of a vector v by v_0 , the first row of a matrix A by A_0 , the range of integers $\{0, 1, \dots, n-1\}$ by $\llbracket n \rrbracket$, and the number of elements in a collection S by $|S|$.

3.3.1 User data and objectives. Consider a system of n users V , each with a private datum. Each datum is dynamic; it changes each time the user initiates a round and incorporates new knowledge from their neighbours. (We describe the time model in Section 3.3.4.) Each datum can be a vector of values, though for simplicity we assume scalar values in our notation. Examples of dynamic data are power consumption, GPS coordinates, and machine learning models. In round t , the data of user $i \in \llbracket n \rrbracket$ is denoted $\theta_{i,t}$.

The users want to compute some function over their data without revealing their data to others. Each user regularly runs a privacy-preserving summation protocol to find the sum of their direct neighbours' private data. This sum can be used for principal component analysis, singular-value decomposition, or distributed gradient descent, for example.

3.3.2 Network model. Users communicate with each other in a peer-to-peer network. This can be a physical network, for example based on Bluetooth or Wi-Fi Direct, or an overlay network, in which users are connected through the Internet. We model the network

as an undirected, self-loopless, static graph $G = (V, E)$ in which each node represents a user. (We consider graphs with dynamic edges in Section 5.4.) The direct neighbours of a node $v \in V$ are denoted $N_G(v)$, and for any set of users $U \subseteq V$ we define their shared neighbours $N_G(U) := \bigcup_{u \in U} N_G(u) \setminus U$. The network topology is not private; in fact, users know who their direct neighbours are. Users may run a privacy-preserving summation protocol to learn the sum of their direct neighbours' private values.

3.3.3 Adversarial model. We assume all n users V are honest-but-curious. That is, all users honestly follow the protocol, but may attempt to obtain other users' private data by operating on the data obtained in the protocol in any way they see fit. Additionally, k users $C \subseteq V$ may collude with each other, but we require that each adversary has either zero or at least two non-adversary neighbours, as retrieving private data is trivial otherwise. We give an example of a valid set of adversaries in Figure 1. Colluding users are still honest-but-curious, so their collusion is limited to sharing information outside the protocol.

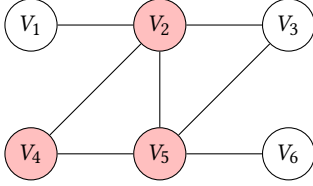


Figure 1: A network with 6 users V . The adversaries $C = \{V_2, V_4, V_5\}$ are shaded. Removing edge (V_2, V_3) would violate our requirements, as adversary V_2 would have exactly one non-adversary neighbour.

While excluding all malicious behaviour is a strenuous assumption in practice, we argue that the challenges in the honest-but-curious model are already sufficiently interesting to warrant investigation. Future work may focus on stronger notions of adversarial behaviour.

3.3.4 Time model. We work in the asynchronous time model [7], in which a global clock ticks whenever a user wakes up and performs some work. Equivalently, each user has their own clock ticking at the speed of a rate-1 Poisson process. When a user's clock ticks, that user wakes up. We denote the current global round number by t (for "time").

4 RECONSTRUCTION IN MULTI-PARTY SUMMATION

In this section we formally define reconstruction attacks in privacy-preserving multi-party dynamic-data summation, and experimentally verify that this attack is feasible. Adversaries passively record the summations they obtain throughout the protocol. Because adversaries know which users are included in which summation, they obtain a system of linear equations. Even if the system has no global solutions, adversaries may still learn the private data of some users.

In Section 4.1, we informally explain reconstruction attacks with examples. In Section 4.2, we give an exact definition of the adversaries' knowledge. In Section 4.3, we formally define reconstruction

on multi-party dynamic-data summation. In Section 4.4, we experimentally verify the feasibility and success rate of reconstruction attacks on random graphs.

4.1 Introduction to Reconstruction Attacks

For this brief introduction, we use somewhat informal notation. We formally define our notation in Section 4.2.

Consider a graph $G = (V, E)$ with users V and a set of k adversaries $C \subseteq V$. If a single adversary $c \in C$ sums their neighbours' values, they learn a linear equation Θ_c over the private values θ of neighbours $N_G(c)$. If multiple adversaries C collude, they share a system of linear equations $A\theta = \Theta$ over the private values θ of $N_G(C)$. If the system of linear equations has a solution, then the adversaries are able to calculate all observed users' private values using linear combinations of the system's rows. For example, given adversaries A, B , and C with observations

$$\begin{aligned} \theta_1 + \theta_2 &= \Theta_A, \\ \theta_1 + \theta_3 &= \Theta_B, \text{ and} \\ \theta_2 + \theta_3 &= \Theta_C, \end{aligned} \tag{1}$$

this is equivalent to the system of linear equations

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \theta = \Theta. \tag{2}$$

Since this system is full rank, adversaries can calculate

$$\begin{aligned} \theta_1 &= \frac{\Theta_A + \Theta_B + \Theta_C}{2} - \Theta_C, \\ \theta_2 &= \Theta_A - \theta_1, \text{ and} \\ \theta_3 &= \Theta_B - \theta_1. \end{aligned} \tag{3}$$

For example, if $\Theta_A = 7$, $\Theta_B = 13$, and $\Theta_C = 8$, the adversaries know with certainty that $\theta_1 = 6$, $\theta_2 = 1$, and $\theta_3 = 7$. Observe that this works even if the summations in Equation 1 are information-theoretically secure.

If the system is rank-deficient, no unique solution exists, but the system may still have partial solutions. That is, even if a system has infinitely many possible solutions, it may be the case that some variables have the same value in all solutions. Even a single user's private value being leaked is a major issue for any privacy-preserving protocol. Consider, for example, the adversarial knowledge consisting of

$$\begin{aligned} \theta_1 + \theta_2 + \theta_3 &= \Theta_A \text{ and} \\ \theta_1 + \theta_2 &= \Theta_B. \end{aligned} \tag{6}$$

Even though there is no unique solution, all solutions have the same value for θ_3 , calculated as $\theta_3 = \Theta_A - \Theta_B$.

The case of Equation 6 is trivial because Θ_B is the sum over a subset of Θ_A . However, there are also rank-deficient systems in which no summation is a subset of another:

$$\begin{aligned} \theta_1 + \theta_2 + \theta_3 &= \Theta_A, \\ \theta_1 + \theta_2 + \theta_4 &= \Theta_B, \text{ and} \\ \theta_3 + \theta_4 &= \Theta_C. \end{aligned} \tag{7}$$

This system, too, has an infinite number of solutions, but each possible solution has the same values

$$\theta_3 = \frac{\Theta_A + \Theta_C - \Theta_B}{2} \text{ and} \quad (8)$$

$$\theta_4 = \frac{\Theta_B + \Theta_C - \Theta_A}{2}. \quad (9)$$

The above examples do not take into account that user data is dynamic; they correspond to cases in which non-adversarial users coincidentally do not update their values in between the adversaries' observations. To model dynamic data, first recall from Section 3.3.1 that users update their values after initiating a summation. Therefore, when an adversary is asked by a neighbour to participate in a privacy-preserving summation, the adversary has learnt that the neighbour will update their value. Adversaries represent the update in the system of equations by adding a new column. For example, consider adversaries C and their neighbours $N_G(C)$ in Figure 2. If these users update their values in the order $(C_1, N_1, C_2, N_2, C_1, C_3)$, each time learning a summation of their neighbours' values, then the adversarial knowledge of C is, with the columns grouped by neighbour,

$$\left[\begin{array}{ccc|ccc|ccc} & \overbrace{N_1} & & & \overbrace{N_2} & & & \overbrace{N_3} & & & \\ \hline 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \theta = \Theta. \quad (10)$$

(The ‘‘useless’’ columns are added to make notation easier later on.) Even in this system with changing variables, adversaries can extract θ_2 , θ_6 , and θ_9 .

Before we give a formal definition of the reconstruction attack, we make two observations:

- (1) Reconstruction does not rely on weaknesses in the summation algorithm; **reconstruction works even if summation is done by a trusted third party**. Instead, reconstruction relies only on the summation revealing both the identities of included variables and the sum of those variables.
- (2) Reconstruction is independent of how users update their private values, and works even if users update their models in random ways or multiple times. **Reconstruction works because adversaries observe multiple summations with at least one unchanged value, and know how the summations are related.**

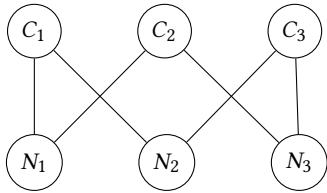


Figure 2: Example graph G with $C = \{C_1, C_2, C_3\}$ and $N = N_G(C) = \{N_1, N_2, N_3\}$

4.2 Obtained Adversarial Knowledge

We give a formal description of adversarial knowledge, which is the system of linear equations that adversaries obtain in a privacy-preserving multi-party dynamic-data summation protocol, and observe two important properties.

Let $G = (V, E)$ be an undirected graph, let $C \subseteq V$ be a collusion of k adversaries, let $n := |N_G(C)|$, and let $t \in \mathbb{N}$ be the number of summations performed by C .

Definition 1 (Adversarial knowledge). The adversarial knowledge over t summations by C is a consistent system of linear equations $A\theta = \Theta$, subject to the conditions that

- $\theta \in \mathbb{R}^{nt \times 1}$ are the private values of neighbours $N_G(C)$, such that θ_{vt+i} is the $i \in \llbracket t \rrbracket$ th unique private value of neighbour $v \in \llbracket n \rrbracket$ that is observed by any adversary in C ,
- $\Theta \in \mathbb{R}^{t \times 1}$ are the sums obtained by the adversaries, where Θ_τ is the $\tau \in \llbracket t \rrbracket$ th such sum, and
- $A \in \{0, 1\}^{t \times nt}$ indicates which private values are observed in which summation, such that $A_{\tau, vt+i} = 1$ if and only if the adversaries' $\tau \in \llbracket t \rrbracket$ th summation includes the $i \in \llbracket n \rrbracket$ th unique private value of neighbour $v \in \llbracket n \rrbracket$.

(Equation 10 satisfies this definition.)

Remark 1. In Theorem 2, we will show that it is not necessary to include adversaries' own private values in $A\theta = \Theta$.

Property 1. Let A be the adversarial knowledge over t summations by C . In each equation, each neighbour in $N_G(C)$ contributes at most one private value:

$$\forall \tau \in \llbracket t \rrbracket, v \in \llbracket n \rrbracket : \sum_{i \in \llbracket t \rrbracket} A_{\tau, vt+i} \in \{0, 1\}. \quad (11)$$

Property 2. Let A be the adversarial knowledge over t summations by C . Since each equation is over all the neighbours of an adversary in C , each row in A corresponds exactly to $N_G(c)$ for some $c \in C$:

$$\forall \tau \in \llbracket t \rrbracket : \exists c \in C : \forall v \in \llbracket n \rrbracket : \left(\sum_{i \in \llbracket t \rrbracket} A_{\tau, vt+i} = 1 \right) \Leftrightarrow (c, N_G(C)_v) \in E. \quad (12)$$

As in Property 1, the summation merely describes whether neighbour v is included in the τ th linear equation.

4.3 Reconstruction from Adversarial Knowledge

Finding a (partial) solution is not trivial. It is well-known that the reduced row echelon form (rref) of a system of linear equations reveals the system's unique solution, if it has one. Clearly, this unique solution is also at least a partial solution. However, if there is no unique solution, there may still be a partial solution, as in Equation 6. We will show in Theorem 1 that finding the reduced row echelon form of the adversarial knowledge is both necessary and sufficient to find all partial solutions. Moreover, we will show in Theorem 2 that this is true even if adversaries' own private values are removed from the adversarial knowledge matrix.

We begin with some definitions. Let $G = (V, E)$ be an undirected graph, let $C \subseteq V$ be a set of k adversaries, let $n := |N_G(C)|$, let $t \in \mathbb{N}$,

and let $A\theta = \Theta$ be the adversarial knowledge over t summations by C ; that is, $A \in \mathbb{R}^{t \times nt}$.

Definition 2 (Solution of a variable). Let $y \in \mathbb{R}^{1 \times t}$ and let $i \in \llbracket nt \rrbracket$. We say that y solves θ_i in $A\theta = \Theta$ if and only if the vector yA contains exactly one non-zero value:

$$(yA)_i \neq 0 \wedge (\forall j \in \llbracket nt \rrbracket \setminus i : (yA)_j = 0). \quad (13)$$

Observe that Equation 13 holds if and only if $(y\Theta)_i = \theta_i$.

Remark 2. Since Equation 13 is independent of θ and Θ , it is equivalent to say that y solves θ_i in A .

Definition 3 (Partial solution). Let $y \in \mathbb{R}^{1 \times t}$. If y solves θ_i in A for any $i \in \llbracket nt \rrbracket$, then we say that y is a partial solution to A .

We proceed with the central theorem of this section, which states that the reduced row echelon form of A describes all partial solutions to A . We remark that a weaker variant of this theorem is given by Wang et al. [47] without a formal proof.

Theorem 1. Let $i \in \llbracket nt \rrbracket$. Then θ_i has a solution in A if and only if there exists $r \in \llbracket t \rrbracket$ such that $\text{rref}(A)_r$ solves θ_i in A .

PROOF. Given $i \in \llbracket nt \rrbracket$, we give a proof for both directions.

We first prove that if a row of $\text{rref}(A)$ solves θ_i in A , then θ_i has a solution in A . Let $B \in \mathbb{R}^{t \times t}$ such that $BA = \text{rref}(A)$, and let $r \in \llbracket t \rrbracket$ such that $\text{rref}(A)_r$ solves θ_i in A . Note that $\text{rref}(A)_{r,i} = 1$ is the only non-zero value of the row $\text{rref}(A)_r$. Since $\text{rref}(A)_r = B_r A$, we find that B_r solves θ_i in A . This proves the first direction of Theorem 1.

We prove the other direction of Theorem 1 by contradiction. Let $y \in \mathbb{R}^{1 \times t}$ be a solution to θ_i in A , so yA has its only non-zero value at $(yA)_i$. For the sake of contradiction, assume that there is no row in $\text{rref}(A)$ that solves θ_i in A . Because $\text{rref}(A)$ has the same row space as A , there exists $y' \in \mathbb{R}^{1 \times t}$ such that $yA = y' \cdot \text{rref}(A)$. Now, y' must have at least two distinct non-zero coefficients y'_r and y'_s such that the rows $\text{rref}(A)_r \neq 0$ and $\text{rref}(A)_s \neq 0$; otherwise, our initial assumption does not hold. Let $\text{rref}(A)_{r,i} = 1$ and $\text{rref}(A)_{s,j} = 1$ be these rows' respective leading coefficients; thus, these are their columns' only non-zero values, and $i \neq j$. Therefore, $(yA)_i = (y' \cdot \text{rref}(A))_i = y'_r \neq 0$, and similarly $(yA)_j = y'_s \neq 0$. However, this contradicts our assumption that yA has a single non-zero value. Therefore, there exists a row in $\text{rref}(A)$ that solves θ_i in A . This proves the other direction of Theorem 1.

Therefore, it is both necessary and sufficient to check the rows of $\text{rref}(A)$ to learn all partial solutions to A . \square

Note that A does not describe that adversaries know each other's private values, since $N_G(C)$ excludes adversaries themselves. We show that including this knowledge does not reveal new partial solutions. Specifically, observe that the adversarial knowledge including self-knowledge over t summations by k adversaries C is

$$A' = \begin{bmatrix} A & R \\ 0 & I_{tk} \end{bmatrix}, \quad (14)$$

where I_{tk} is the $(tk \times tk)$ identity matrix, 0 is an appropriately-sized matrix of 0s, and R is some appropriately-sized binary matrix. The rows of I_{tk} represent that adversaries know each other's values, and R represents the edges between adversaries.

Theorem 2. Let $i < tn$. Then θ_i has a solution in A if and only if θ_i has a solution in A' .

PROOF. Observe that

$$\text{rref}(A') = \begin{bmatrix} \text{rref}(A) & 0 \\ 0 & I_{tk} \end{bmatrix}, \quad (15)$$

ignoring row-switching transformations. The bottom tk rows solve exactly θ_i in A for $i \geq tn$. The upper rows solve θ_i in A for $i < tn$ if and only if the rows of $\text{rref}(A)$ do so. \square

Intuitively, Theorem 2 holds because the linear dependencies that exist within A remain unaffected by R .

4.4 Reconstruction Attack Feasibility

We show that reconstruction is feasible for honest-but-curious adversaries. We run the attack in static graphs with randomly-placed adversaries passively collecting data. We measure both the success rate and the number of rounds until success. Our source code is publicly available [20].

Remark 3. This section pertains only to static graphs. We show a reduction from graphs with dynamic edges to static graphs in Section 5.4.

4.4.1 Experimental setup. By Theorem 1, the success rate of the attack depends only on the adversaries' direct neighbourhood. Therefore, instead of modeling large peer-to-peer networks, it suffices to model only the subgraph that is relevant for the attack. Additionally, by Theorem 2, edges between adversaries can be ignored. Therefore, given any graph $G = (V, E)$ and a set of colluding adversaries $C \subseteq V$, it suffices to model the induced subgraph $G[C]$, minus edges between adversaries. This forms a bipartite graph H . We provide an example of graph induction in Figure 3.

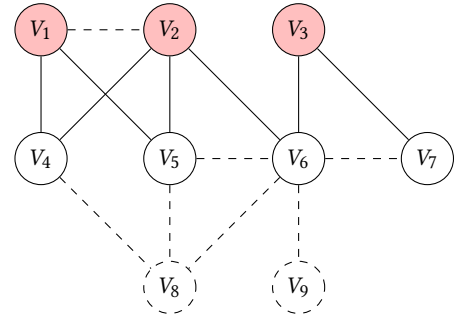


Figure 3: A graph G . Adversaries $C = \{V_1, V_2, V_3\}$ are shaded. The bipartite subgraph $H = G[C]$ consists of exactly the non-dotted nodes and edges.

We emphasise that reconstruction depends only on the adversaries' view, regardless of the remaining graph outside this view. However, the likelihood of obtaining any specific adversarial view does depend on the full graph. For example, the probability that a random adversarial view contains a cycle depends on the connectivity of the full graph. For our experiments, we choose not to make assumptions on the graph's topology, analysing all possible adversarial views equally, so that our results are agnostic to the specific network, application, and adversary.

Bipartite graphs can be parameterised by three variables: the number of adversaries, the number of direct neighbours, and the

number of edges. We generate random graphs according to these parameter, subject to some filtering:

- We exclude graphs in which there is a adversary with only one edge because this would allow trivial attacks, as described in Section 3.3.
- We do *not* exclude graphs in which there is an honest-but-curious user with only one edge, because this user may have more edges in G that are not in H .
- We exclude graphs in which an honest-but-curious user has no neighbours, because these cases do not accurately represent the bipartite graph's parameters.
- We do *not* exclude graphs in which an adversary has no neighbours.
- We do *not* exclude disconnected graphs.

4.4.2 Amount of reconstructed data. For our first experiment, we measure the amount of private data that adversaries can reconstruct. We generate a large amount of random bipartite graphs as described above, and count the number of partial solutions in the biadjacency matrices. This corresponds to the adversarial knowledge if neighbours do not update their values, and thus represents the strongest reconstruction attack that adversaries can perform. In Section 4.4.3 we also consider neighbours updating their values.

Firstly, we look at the proportion of data that can be reconstructed, shown in Figure 4. We see that if the number of adversaries is close to the number of neighbours, the adversary is typically able to reconstruct all neighbours' data. As the number of neighbours increases, fewer data can be reconstructed, unless compensated for by a higher connectivity. If the graph has many neighbours and few edges, adversaries share fewer neighbours, and are thus typically unable to exploit the overlaps in their aggregates.

Secondly, we look at the distribution of how much data can be reconstructed, shown in Figure 5. We see again that adversaries are more successful if they outnumber their neighbours. As the number of neighbours increases, so does the probability of being unable to reconstruct any data. However, even if three adversaries passively observe 15 neighbours, they still have an 11.0% probability of reconstructing at least one neighbour's datum, which is unacceptable for any privacy-preserving scheme.

4.4.3 Rounds until first reconstruction. Some partial solutions are harder to obtain than others. For example, if the graph is such that adversaries must collect many equations without any user updating their value in between, the adversaries may never have the opportunity to collect the necessary information.

In the next experiment, we measure how many rounds adversaries need before reconstruction succeeds. For each of the subgraphs in Figure 4 that were found to be susceptible to the reconstruction attack, we simulate a dynamic-data multi-party summation protocol as follows. Each round, a uniformly random user in the subgraph wakes up. If the user is an adversary, they learn the sum of their neighbours' values, and adds this to the adversarial knowledge. Otherwise, if a non-adversary wakes up, we simulate an update: The next adversarial sum that includes this non-adversary will use a new column in the adversarial knowledge matrix. After every round, the adversaries check whether a partial solution exists. We then repeat this procedure 100 times to control for the order

in which users wake up, truncate instances that have no partial solutions after 250 rounds, and take the mean number of rounds until the first partial solution is found.

We show the mean number of rounds until the reconstruction attack succeeds in Figure 6. We see that the attack is fastest when there are more adversaries, more edges, and fewer neighbours. Intuitively, this means that the required number of summations increases if neighbours can update their values at a higher rate than adversaries can observe them. For example, 3 adversaries against 15 neighbours require on average 8.8 rounds before they can reconstruct private data. In related works such as [11, 14, 46], users run hundreds or thousands of rounds before the protocol terminates.

4.4.4 Conclusion of results. We sampled all possible views of randomly selected adversaries in random graphs, excluding some trivial attack cases. If the reconstruction attack succeeds, the adversaries obtain other users' private inputs to the information-theoretically secure summation operation. Our results show that passive honest-but-curious adversaries are able to obtain private data in this scenario with non-negligible probability. While we note that different classes of graph topologies may have varying susceptibility to reconstruction attacks, we conclude that, in general, individually protecting each summation is insufficient for confidentiality.

5 GIRTH AS A PEER-TO-PEER RECONSTRUCTION COUNTERMEASURE

In a centralised protocol, the single aggregator can track which summations have occurred, and refuse a subsequent summation if it would result in a partial solution. However, in a distributed computation, there is no such aggregator, and simulating the aggregator using a multi-party protocol is impractical as this would require involving all users in each summation. In this section, we show that to prevent reconstruction it is sufficient to increase the network's girth, which is the length of the network's shortest cycle. The network's girth is an established metric for peer-to-peer networks, with various peer-to-peer algorithms for measuring and increasing the girth [9, 22, 34, 39]. Using such an algorithm before running a privacy-preserving dynamic-data multi-party summation protocol is thus sufficient to prevent reconstruction of private data by honest-but-curious adversaries.

We begin in Section 5.1 by showing that reconstruction requires collusion. In Section 5.2, we show that reconstruction does not work in acyclic graphs, regardless of the number of adversaries. In Section 5.3, we generalise our results to determine an upper bound on the number of adversaries. In Section 5.4, we consider graphs with dynamic edges. Finally, in Section 5.5, we briefly evaluate the impact that increasing girth has on distributed convergence.

5.1 Privacy in Static Graphs without Collusion

We begin by considering the special case of $k = 1$, i.e. a setting without collusion. We show that, if the graph is static, the adversary cannot obtain other users' private values regardless of topology, barring trivial attacks.

Assuming a privacy-preserving summation protocol, it is self-evident that repeating the summation over the same set of values does not leak any private data. However, while the set of neighbours

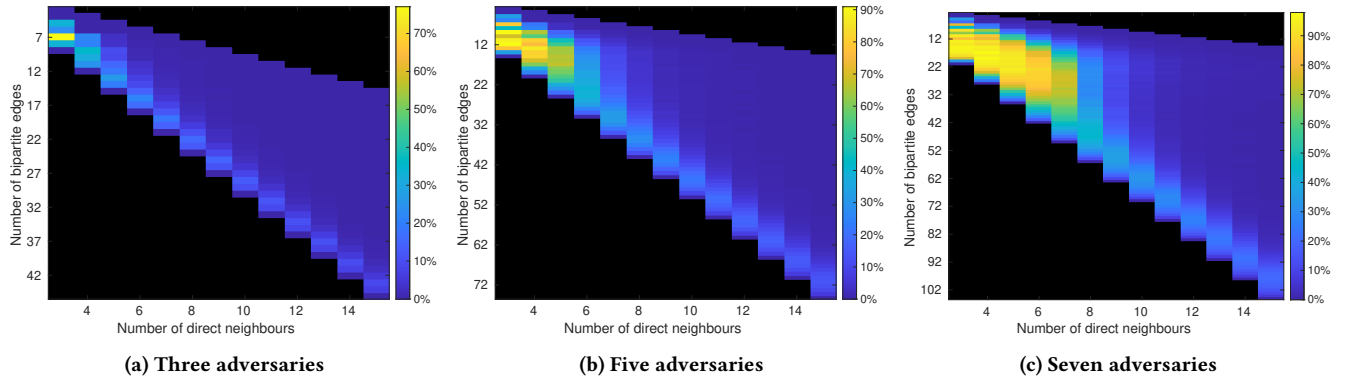


Figure 4: Proportion of neighbours' private data that can be reconstructed by adversaries. Each point represents the mean over 1000 random bipartite graphs. Black points indicate no valid bipartite graphs could be found. Note the different y-axes.

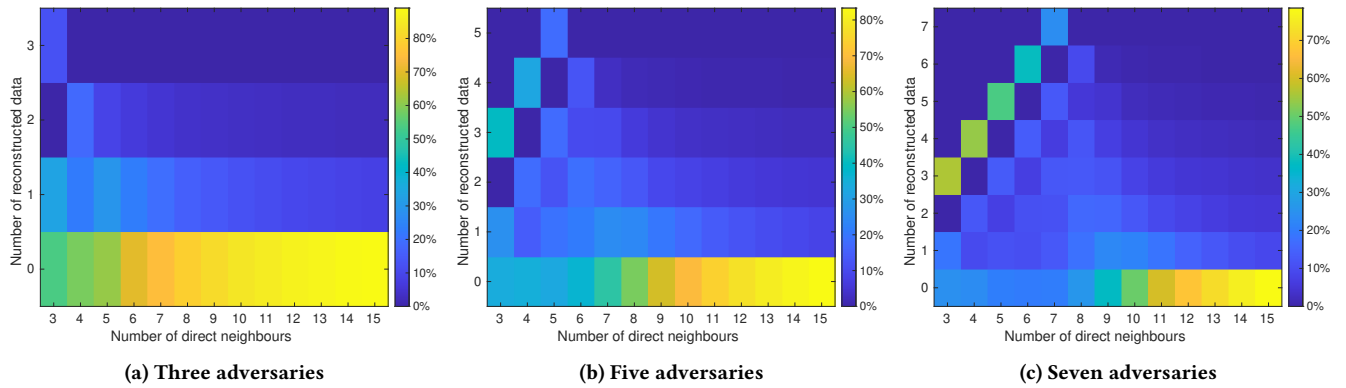


Figure 5: Probability of reconstructing a given number of neighbours' data, ignoring the number of edges. Each column adds up to 100%, and corresponds to a column in Figure 4.

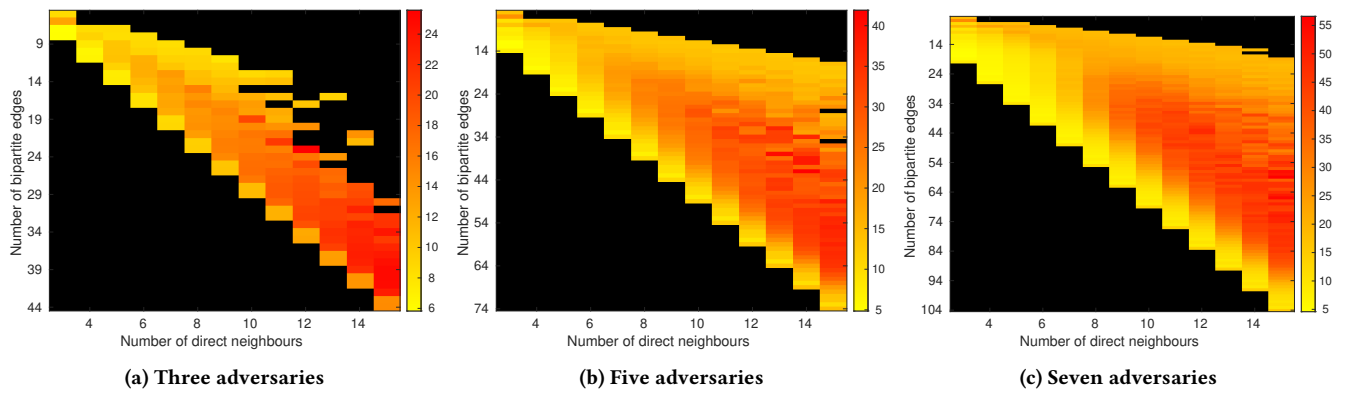


Figure 6: Mean number of adversarial summations needed to obtain private data. Each point corresponds to 100 attacks on each of the solvable graphs from Figure 4.

is always the same in the static no-collusion setting, neighbours still update their local values. Thus, it remains to be shown that no reconstruction is possible with this kind of composition.

Lemma 1. Given adversarial knowledge $A \in \mathbb{R}^{t \times nt}$ of a single adversary with $n \geq 2$ fixed neighbours, we have for any $y \in \mathbb{R}^{1 \times t}$ that

$$\forall \mu, \nu \in \llbracket n \rrbracket : \sum_{i \in \llbracket t \rrbracket} (yA)_{\mu t+i} = \sum_{i \in \llbracket t \rrbracket} (yA)_{\nu t+i}. \quad (16)$$

Here, $\sum_{i \in \llbracket t \rrbracket} (yA)_{vt+i}$ is the sum of components of yA relating to neighbour v . The equation states that in any linear combination yA , every neighbour has the same sum of components.

PROOF. Firstly, because the adversary has fixed neighbours,

$$\forall \tau \in \llbracket t \rrbracket, v \in \llbracket n \rrbracket : \sum_{i \in \llbracket t \rrbracket} A_{vt+i} = 1. \quad (17)$$

In the linear combination yA , the rows of A are scaled according to y and then summed together. Therefore, since each row includes each neighbour exactly once,

$$\forall v \in \llbracket n \rrbracket : \sum_{i \in \llbracket t \rrbracket} (yA)_{vt+i} = \sum_{\tau \in \llbracket t \rrbracket} y_{\tau}. \quad (18)$$

□

Corollary 1. Given adversarial knowledge $A \in \mathbb{R}^{t \times nt}$ of a single adversary with $n \geq 2$ fixed neighbours, there exists no $y \in \mathbb{R}^{1 \times t}$ such that yA has exactly one non-zero value. Therefore, there exist no partial solutions for A .

5.2 Privacy in Static Graphs with Unbounded Collusion

The special case of $k = 1$ provides some insights into the workings of the reconstruction attack, but not allowing any collusion is not realistic, as honest-but-curious collusion in the form of secretly exchanging information is undetectable and there are no strong incentives against it. Therefore, we now proceed to consider the general case of $k \geq 1$.

Partial solutions are linear combinations of the rows of the adversarial knowledge such that all but one column cancels out, as in Equation 1. We already know from Corollary 1 that a partial solution requires multiple adversaries. If two rows in the adversarial knowledge from different adversaries match in multiple columns, then these adversaries share multiple neighbours, and the graph has a cycle. Otherwise, if no two rows from different adversaries overlap in multiple columns, then, since each equation has at least two non-zero columns, each equation introduces new unknowns, taking the adversaries further from a partial solution. In this case, if the adversaries are able to find a partial solution, they must have another row that cancels out the unknowns of multiple other rows; but this, too, introduces a cycle. The intuition thus seems to be that partial solutions require a cyclic graph. We now formally prove that this intuition is correct.

Theorem 3. Let $G = (V_G, E_G)$ be an undirected graph, let $C \subseteq V_G$ be a set of k adversaries, let $n := |N_G(C)|$, let t be the number of summations performed by C , and let $A \in \mathbb{R}^{t \times nt}$ be the adversarial knowledge.

If G is acyclic, then A does not have partial solutions.

PROOF. We give a proof by contraposition: Given a partial solution to A , we show that G is cyclic. Let $y \in \mathbb{R}^{1 \times t}$ be a partial solution to A . We show how to find a bipartite subgraph H of G such that its biadjacency matrix A'' has a partial solution y'' . We then show that this implies the existence of a cycle in G . Our proof works in multiple steps: (1) we combine columns of A to create A' , (2) we combine rows from A' to create A'' , (3) we create the partial solution y'' , and finally (4) we show that G is cyclic.

- (1) *Combine columns.* We merge the t columns in A assigned to each neighbour to obtain A' . Let $y' = y$, and let $A' \in \mathbb{R}^{t \times n}$ such that

$$\forall \tau \in \llbracket t \rrbracket, v \in \llbracket n \rrbracket : A'_{\tau,v} := \sum_{i \in \llbracket t \rrbracket} A_{\tau,vt+i}. \quad (19)$$

It follows from Property 1 that this is a binary matrix, and it follows from Property 2 that no neighbour relations are removed. Furthermore, observe that

$$\forall v \in \llbracket n \rrbracket : (y'A')_v = \sum_{i \in \llbracket t \rrbracket} (yA)_{vt+i}. \quad (20)$$

Since yA contains exactly one non-zero value, so does $y'A'$. Therefore, y' is a partial solution to A' .

- (2) *Combine rows.* We combine duplicate rows and remove unused rows from A' to obtain A'' . We define A'' in terms of a set of rows as

$$A'' := \{A'_i \mid i \in \llbracket t \rrbracket \wedge \quad (21)$$

$$\nexists j \in \llbracket i \rrbracket : A'_i = A'_j \wedge \quad (22)$$

$$\sum \{y'_j \mid j \in \llbracket t \rrbracket \wedge A'_i = A'_j\} \neq 0\}. \quad (23)$$

Here, Equation 22 excludes duplicates by only choosing row A_i if there is no $j < i$ such that $A_i = A_j$, and Equation 23 excludes unused rows by only picking row A_i if the sum of y'_j over identical rows A_j is non-zero.

- (3) *Create partial solution.* We similarly combine and remove the corresponding columns from y' to obtain y'' . To do so, we define a function ϕ that describes how the rows of A'' relate to the rows of A' . Let s be the number of rows in A'' . Then we define $\phi : \llbracket s \rrbracket \rightarrow \llbracket t \rrbracket^*$ such that

$$\forall \tau \in \llbracket t \rrbracket, \sigma \in \llbracket s \rrbracket : \tau \in \phi(\sigma) \Leftrightarrow A'_\tau = A''_\sigma. \quad (24)$$

Using this function, we define $y'' \in \mathbb{R}^{1 \times s}$ as

$$\forall \sigma \in \llbracket s \rrbracket : y''_\sigma := \sum_{\tau \in \phi(\sigma)} y'_\tau. \quad (25)$$

It follows that

$$\forall v \in \llbracket n \rrbracket : (y''A'')_v = \sum_{\sigma \in \llbracket s \rrbracket} (y''_\sigma A''_{\sigma,v}) \quad (26)$$

$$= \sum_{\sigma \in \llbracket s \rrbracket} \sum_{\tau \in \phi(\sigma)} (y'_\tau A''_{\sigma,v}) \quad (27)$$

$$= \sum_{\sigma \in \llbracket s \rrbracket} \sum_{\tau \in \phi(\sigma)} (y'_\tau A'_{\tau,v}) \quad (28)$$

$$= \sum_{\tau \in \llbracket t \rrbracket} (y'_\tau A'_{\tau,v}) \quad (29)$$

$$= (y'A')_v. \quad (30)$$

Therefore, $y''A'' = y'A'$, and, by Definition 3, y'' is a partial solution to A'' .

- (4) *Find cycle.* Note that A'' is the biadjacency matrix of some bipartite subgraph $H = (C', N_G(C), E_H)$ of G , where $C' \subseteq C$ and $E_H \subseteq E_G$. Assume, for the sake of contradiction, that H is acyclic. Then there are at least two distinct nodes i, j in H with degree one. Because each adversary has at least two non-colluding neighbours, and because $\forall c \in C' : N_G(c) = N_H(c)$, it follows that $i, j \notin C'$ but $i, j \in N_G(C)$. Therefore,

the columns in A'' corresponding to nodes i, j each contain exactly one non-zero value. Because y'' does not contain zeroes by Equation 23, it follows that y'' selects the respective rows of A'' in which i and j are non-zero. Therefore, $(y''A'')_i \neq 0$ and $(y''A'')_j \neq 0$. However, this contradicts the observation that y'' is a partial solution to A'' . Therefore, H is cyclic, and so is G .

□

Our proof shows that partial solutions imply the existence of cycles. However, this does not mean that cycles imply the existence of partial solutions. Indeed, we show in Section 5.3 that structured cycles can be introduced without creating partial solutions.

Remark 4. Theorem 3 pertains only to *partial* solutions. In an acyclic topology, adversaries may still find linear relations that do not reveal private information, such as $\theta_2 = 3 \times \theta_4$ or $\theta_4 = \theta_5$. Protecting these relations is left for future work.

5.3 Privacy in Static Graphs with Bounded Collusion

While acyclic graphs resist reconstruction attacks, these graphs are not well-suited for peer-to-peer networks for two reasons. Firstly, if any non-leaf node becomes unavailable, the network becomes disconnected. Secondly, leaf nodes have only one neighbour, and thus cannot initiate summations to learn from their neighbours.

We show that no partial solutions exist given an upper bound on the number of adversaries. This bound depends on the graph’s girth, which is the length of its shortest cycle.

Theorem 4. Let $G = (V_G, E_G)$ be an undirected graph, let $C \subseteq V_G$ be a set of k adversaries, let $n := |N_G(C)|$, let t be the number of summations performed by C , and let $A \in \mathbb{R}^{t \times nt}$ be the adversarial knowledge.

If $\text{girth}(G) > 2k$, then A does not have partial solutions.

PROOF. We give a proof by contraposition: Given a partial solution to A , we show that $\text{girth}(G) \leq 2k$. Let H be as in the proof of Theorem 3. Then H is cyclic. Since H is bipartite, every edge in the cycle is between an adversary and a neighbour. Since each node in the cycle is visited at most once, the cycle length is at most $2k$. This cycle also exists in G . Therefore, $\text{girth}(G) \leq 2k$. □

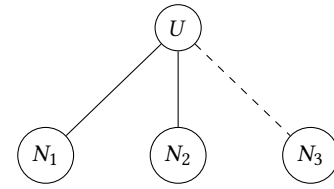
5.4 Privacy in Dynamic Graphs

So far, we have assumed that graphs are static. However, this prevents users from changing their neighbours, which is unrealistic if users move through the network. We briefly show that dynamic graphs can be reduced to static graphs.

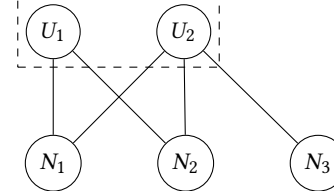
If a single user performs two summations over two sets of neighbours, they learn exactly the same information as two users would over those same sets of neighbours. We show an example in Figure 7. More generally, k users with static neighbours can learn the exact same information as ℓ users with k different sets of neighbours. Our results on reconstruction feasibility in static graphs from Section 4.4 can be translated similarly to dynamic graphs.

We conclude that Theorem 4 implies the following.

Corollary 2. Let $G = (V_G, E_G)$ be a dynamic undirected graph, let $C \subseteq V_G$ be a set of adversaries, let $n := |N_G(C)|$, let t be the



(a) A dynamic graph. The dotted edge is not present in all rounds.



(b) A reduction to a static graph. U has been split into U_1 and U_2 .

Figure 7: Example of how a dynamic graph can be reduced to a static graph. U learns the same as U_1 and U_2 together.

number of summations performed by C , let k be the number of sets of neighbours the adversaries sum over, and let $A \in \mathbb{R}^{t \times nt}$ be the adversarial knowledge.

If $\text{girth}(G) > 2k$, then A does not have partial solutions.

There are several important limitations to this result. Firstly, the upper bound on the number of adversaries depends on the girth, but the girth may not be known beforehand if users move through the network in unpredictable ways. Secondly, even if a minimum girth is guaranteed throughout the protocol, the upper bound implies a maximum number of changes that may occur during the protocol.

5.5 Impact on Convergence

We briefly evaluate the impact of increasing the network’s girth on the convergence of a protocol running over that network. Specifically, we numerically simulate a distributed averaging protocol [51], which is just a non-privacy-preserving form of distributed learning. We intentionally choose a simple, efficient, non-noisy protocol to make the impact of the girth parameter most apparent. The “numerical simulation” part of the description is because we do not actually create separate processes and communication for the nodes. Our source code is publicly available [20].

We use the system model presented in Section 3.3. We create a network by generating a random Erdős–Rényi graph with 50 nodes and with each edge having a probability p of being added. Each node holds a single private scalar value, sampled uniformly from the range $(0, 50)$. In each round, one random node updates their private value to be the unweighted mean of their neighbours’ values and their own value. We say the nodes have converged once all nodes have the same local value, which is when the largest difference between any two nodes’ values is 1. (Altering this threshold does not give fundamentally different results.) We then measure the

mean number of rounds until convergence over 100 repetitions of the protocol.

To measure the effect girth has on convergence, we “stretch” graphs to a given girth by iteratively removing arbitrary edges from cycles shorter than the desired girth until no such cycles exist. With 50 nodes, we stretch up to a girth of 20, which ensures reconstruction attacks are impossible even when 18% of users collude.

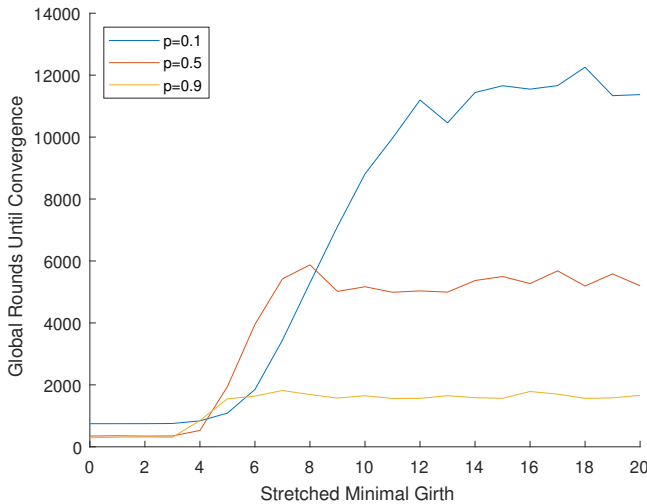


Figure 8: Number of rounds until convergence in distributed averaging in random Erdős–Rényi graphs with 50 nodes and varying edge probabilities p , as a function of the girth to which the graphs are “stretched”.

We show our results in Figure 8. Since undirected graphs always have girth at least 3, no significant changes occur at these low girths. At higher girths, however, the number of required rounds quickly increases until it reaches a ceiling. Higher values of p have a lower ceiling, and reach the ceiling at lower girths. When we look at our experiments in more detail, we see that ceilings occur when all cycles have been removed, and that graphs with high p retain more edges. This matches the intuition that information propagates more efficiently when there are more edges.

Our results show that increasing girth affects convergence speed most in sparse networks. However, the impact of removing all short cycles is significant, and may not be viable for all applications. More efficient methods for selecting which edges to remove may reduce the impact on convergence. Implementing the cycle removal method from our experiment above as a distributed protocol is trivial,² though not necessarily communicationally efficient. To the best of our knowledge, there is no research on communication-efficient distributed “graph stretching”. That said, there are distributed protocols for measuring the network’s girth [9] and for removing all cycles [22, 39]. Therefore, in the context of reconstruction attacks,

²A node can break all cycles of at most length ℓ that they are part of as follows. The node floods a unique random message, paired with a counter starting at ℓ , through the network. Each time a node forwards the message, the counter is decreased. Once the counter reaches zero, nodes stop forwarding the message. If (and only if) the source node receives back their own message, they are part of a cycle of length at most ℓ , and remove the edge on which the message came in.

increasing girth may be a practical solution for dense networks, and determining girth is practical for all networks.

6 CONCLUSION

We investigated reconstruction attacks in the setting of secure multi-party computation. We observed that existing multi-party computation literature does not consider protocols in which intermediate values are intentionally exposed by the ideal functionality, and seemingly assumes that protocols are not self-composed when deployed. In our investigation, we focused on a peer-to-peer setting with privacy-preserving summation in which users’ data change over time. In random subgraphs with 18 users, we found that three passive honest-but-curious adversarial users have an 11.0% success rate at recovering another user’s private data using a reconstruction attack, requiring an average of 8.8 rounds per adversary. We analysed the structural dependencies of the underlying network graph that permit this attack, and proved that acyclic networks are invulnerable to the reconstruction attack. More generally, we showed that the minimum number of adversaries required for the attack depends on the length of the graph’s shortest cycle.

Our work sets the first step towards preventing reconstruction in the peer-to-peer setting as seen in multi-party computation, and opens up multiple questions for future work. Firstly, and most obviously, though we have found a sufficient criterion to determine reconstruction feasibility, finding a criterion that is also necessary would allow using some graphs which our criterion currently forbids. Secondly, future work may introduce stronger notions of privacy by additionally protecting linear relations between data, which is required to protect against adaptive adversaries. Thirdly, though our restriction to the summation operation is already sufficient to analyse decentralised learning, our work could be extended to cover compositions with other operations, such as multiplication or comparison. Finally, we note it may be possible to find tighter bounds on differential privacy budgets on dynamic-data multi-party summation given specific graph topologies.

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