

AVA: AN INTERACTIVE SYSTEM FOR VISUAL AND QUANTITATIVE ANALYSES OF VIBRATO AND PORTAMENTO PERFORMANCE STYLES

Luwei Yang¹ Khalid Z. Rajab² Elaine Chew¹

¹ Centre for Digital Music, Queen Mary University of London

² Antennas & Electromagnetics Group, Queen Mary University of London

{l.yang, k.rajab, elaine.chew}@qmul.ac.uk

ABSTRACT

Vibratos and portamenti are important expressive features for characterizing performance style on instruments capable of continuous pitch variation such as strings and voice. Accurate study of these features is impeded by time consuming manual annotations. We present AVA, an interactive tool for automated detection, analysis, and visualization of vibratos and portamenti. The system implements a Filter Diagonalization Method (FDM)-based and a Hidden Markov Model-based method for vibrato and portamento detection. Vibrato parameters are reported directly from the FDM, and portamento parameters are given by the best fit Logistic Model. The graphical user interface (GUI) allows the user to edit the detection results, to view each vibrato or portamento, and to read the output parameters. The entire set of results can also be written to a text file for further statistical analysis. Applications of AVA include music summarization, similarity assessment, music learning, and musicological analysis. We demonstrate AVA’s utility by using it to analyze vibratos and portamenti in solo performances of two Beijing opera roles and two string instruments, erhu and violin.

1. INTRODUCTION

Vibrato and portamento use are important determinants of performance style across genres and instruments [4, 6, 7, 14, 15]. Vibrato is the systematic, regular, and controlled modulation of frequency, amplitude, or the spectrum [12]. Portamento is the smooth and monotonic increase or decrease in pitch from one note to the next [15]. Both constitute important expressive devices that are manipulated in performances on instruments that allow for continuous variation in pitch, such as string and wind instruments, and voice. The labor intensive task of annotating vibrato and portamento boundaries for further analysis is a major bottleneck in the systematic study of the practice of vibrato and portamento use.

While vibrato analysis and detection methods have been in existence for several decades [2, 10, 11, 13], there is currently no widely available software tool for interactive analysis of vibrato features to assist in performance and musicological research. Portamenti have received far less attention than vibratos due to the inherent ambiguity in what constitutes a portamento—beyond a note transition, a portamento is a perceptual feature that can only exist if it is recognizable by the human ear—some recent work on the modeling of portamenti can be found in [15].

The primary goal of this paper is to introduce the AVA system¹ for interactive vibrato and portamento detection and analysis. AVA seeks to fill the gap in knowledge discovery tools for expressive feature analysis for continuous pitch instruments. The AVA system is built on recent advances in automatic vibrato and portamento detection and analysis. As even the best algorithm sometimes produces erroneous vibrato or portamento detections, the AVA interface allows the user to interactively edit the detection solutions so as to achieve the best possible analysis results.

A second goal of the paper is to demonstrate the utility of the AVA system across instruments and genres using two datasets, one for voice and the other for string instruments. The vocal dataset comprises of monophonic samples of phrases from two Beijing opera roles, one female one male; the string instruments dataset consists of recordings of a well known Chinese piece on erhu and on violin.

Applications of AVA include music pedagogy and musicological analysis. AVA can be used to provide visual and quantitative feedback in instrumental learning, allowing students to inspect their expressive features and adapt accordingly. AVA can also be used to quantify musicians’ vibrato and portamento playing styles for analyses on the ways in which they use these expressive features. It can be used to conduct large-scale comparative studies, for example, of instrumental playing across cultures. AVA’s analysis results can also serve as input to expression synthesis engines, or to transform expressive features in recorded music.

The remainder of the paper is organized as follows: Section 2 presents the AVA system and begins with a description of the vibrato and portamento feature detection and analysis modules; Section 3 follows with details of AVA’s user interface. Section 4 presents two case studies



© Luwei Yang, Khalid Z. Rajab and Elaine Chew. Licensed under a Creative Commons Attribution 4.0 International License (CC BY 4.0). **Attribution:** Luwei Yang, Khalid Z. Rajab and Elaine Chew. “AVA: An Interactive System for Visual and Quantitative Analyses of Vibrato and Portamento Performance Styles”, 17th International Society for Music Information Retrieval Conference, 2016.

¹The beta version of AVA is available at luweiyang.com/research/ava-project.

using AVA to detect and analyse vibratos and portamenti and their properties in two Beijing opera roles, and in violin and erhu recordings. Section 5 closes with discussions and conclusions.

2. FEATURE DETECTION AND ANALYSIS

Figure 1 shows AVA’s system architecture. The system takes monophonic audio as input. The pitch curve, which is given by the fundamental frequency, is extracted from the input using the pYIN method [5]. The first part of the system focuses on vibrato detection and analysis. The pitch curve derived from the audio input is sent to the vibrato detection module, which detects vibrato existence using a Filter Diagonalization Method (FDM). The vibratos extracted are then forwarded to the module for vibrato analysis, which outputs the vibrato statistics.

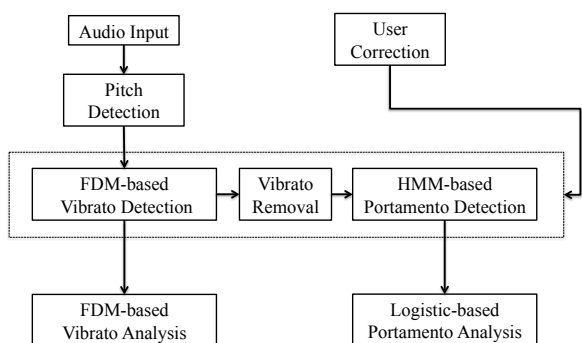


Figure 1. The AVA system architecture.

The next part of the system deals with portamento detection and analysis. The oscillating shapes of the vibratos degrade portamento detection. To ensure the best possible performance for portamento detection, the detected vibratos are flattened using the built-in MATLAB function ‘smooth’. The portamento detection module, which is based on the Hidden Markov Model (HMM), uses this vibrato-free pitch curve to identify potential portamenti. A Logistic Model is fitted to each detected portamento for quantitative analysis.

For both the vibrato and portamento modules, if there are errors in detection, the interface allows the user to mark up missing vibratos or portamenti and delete spurious results. Further details on the AVA interface will be given in Section 3.

2.1 Vibrato Detection and Analysis

We use an FDM-based method described in [16] to analyze the pitch curve and extract the vibrato parameters. The advantage of the FDM is its ability to extract sinusoid frequency and amplitude properties for a short time signal, thus making it possible to determine vibrato presence over the span of a short time frame.

Vibrato detection methods can be classified into note-wise and frame-wise methods. Note-wise methods have a

pre-requisite note segmentation step before they can determine if the note contains a vibrato [8, 10]. Frame-wise methods divide the audio stream, or the extracted f_0 information, into a number of uniform frames. Vibrato existence is then decided based on information in each frame [2, 11, 13, 16]. The FDM approach constitutes one of the newest frame-wise methods.

Fundamentally, the FDM assumes that the time signal (the pitch curve) in each frame is the sum of exponentially decaying sinusoids,

$$f(t) = \sum_{k=1}^K d_k e^{-in\tau\omega_k}, \text{ for } n = 0, 1, \dots, N, \quad (1)$$

where K is the number of sinusoids required to represent the signal to within some tolerance threshold, and the fitting parameters ω_k and d_k are the complex frequency and complex weight, respectively, of the k -th sinusoid. The aim of the FDM is to find the $2K$ unknowns, representing all ω_k and d_k . A brief summary of the steps is described in Algorithm 1. Further details of the algorithm and implementation are given in [16].

Algorithm 1: The Filter Diagonalization Method

Input: Pitch curve (fundamental frequency)

Output: The frequency and amplitude of the sinusoid with the largest amplitude

Set the vibrato frequency range;

Filter out sinusoids having frequency outside the allowable range;

Diagonalize the matrix given by the pitch curve;

for each iteration do

Create a matrix by applying a 2D FFT on the pitch curve;

Diagonalize this matrix;

Get the eigenvalues;

Check that the eigenvalues are within the acceptance range;

end

Compute the frequencies from the eigenvalues;

Calculate the amplitudes from the corresponding eigenvectors;

Return the frequency and amplitude of the sinusoid with the largest amplitude;

Information on vibrato rate and extent fall naturally out of the FDM analysis results. Here, we consider only the frequency and amplitude of the sinusoid having the largest amplitude. The window size is set to 0.125 seconds and step size is one quarter of the window. Given the frequency and amplitude, a Decision Tree determines the likely state of vibrato presence. Any vibrato lasting less than 0.25 seconds is pruned.

A third parameter is reported by the vibrato analysis module, that of sinusoid similarity, which is used to characterize the sinusoid regularity of the shape of the detected

vibrato. The sinusoid similarity is a parameter between 0 and 1 that quantifies the similarity of a vibrato shape to a reference sinusoid using cross correlation (see [14]).

2.2 Portamento Detection and Analysis

Portamenti are continuous variations in pitch connecting two notes. Not all note transitions are portamenti; only pitch slides that are perceptible to the ear are considered portamenti. They are far less well defined in the literature than vibratos, and there is little in the way of formal methods for detecting portamenti automatically.

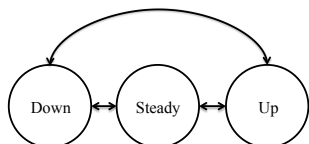


Figure 2. The portamento detection HMM transition network.

To detect portamentos, we create a fully connected three-state HMM using the delta pitch curve as input as shown in Figure 2. The three states are down, steady, and up, which correspond to slide down, steady pitch, and slide up gestures. Empirically, we choose as transitions probabilities the numbers shown in Table 1, which have worked well in practice. Each down state and up state observation

	Down	Steady	Up
Down	0.4	0.4	0.2
Steady	1/3	1/3	1/3
Up	0.2	0.4	0.4

Table 1. Transition probabilities for portamento detection HMM.

is modeled using a Gamma distribution model. The steady pitch observation is modeled as a sharp needle around 0 using a Gaussian function. The best (most likely) path is decoded using the Viterbi algorithm. All state changes are considered to be boundaries, and the minimum note or transition (portamento) duration is set as 0.09 seconds.

To quantitatively describe each portamento, we fit a Logistic Model to the pitch curve in the fashion described in [15]. The choice of model is motivated by the observation that portamenti largely assume S or reverse S shapes. An ascending S shape is characterized by a smooth acceleration in the first half followed by a deceleration in the second half, with an inflection point between the two processes.

The Logistic Model can be described mathematically as

$$P(t) = L + \frac{(U - L)}{(1 + Ae^{-G(t-M)})^{1/B}}, \quad (2)$$

where L and U are the lower and upper horizontal asymptotes, respectively. Musically speaking, L and U are the antecedent and subsequent pitches of the transition. A , B , G , and M are constants. Furthermore, G can be interpreted as the growth rate, indicating the steepness of the transition slope.

The time of the point of inflection is given by

$$t_R = -\frac{1}{G} \ln\left(\frac{B}{A}\right) + M. \quad (3)$$

The pitch of the inflection point can then be calculated by substituting t_R into Eqn (2).

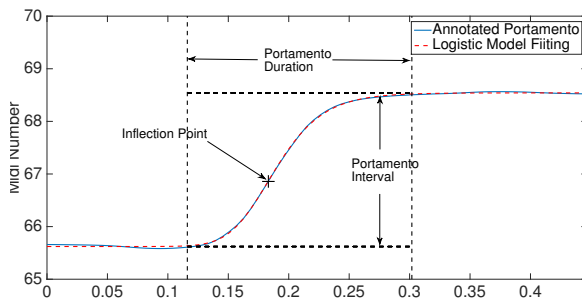


Figure 3. Description of the portamento duration, interval and inflection for a real sample.

Referring to Figure 3, the following portamento parameters are reported by the portamento analysis module and are calculated as follows:

1. Portamento slope: the coefficient G in Eqn (2).
2. Portamento duration (in seconds): the time interval during which the first derivative (slope) of the logistic curve is greater than 0.861 semitones per second (i.e. 0.005 semitones per sample).
3. Portamento interval (in semitones): the absolute difference between the lower (L) and upper (U) asymptotes.
4. Normalized inflection time: time between start of portamento and inflection point time, t_R in Eqn (3), as a fraction of the portamento duration.
5. Normalized inflection pitch: distance between the lower (L) asymptote and the inflection point pitch as a fraction of the portamento interval.

3. THE AVA INTERFACE

The vibrato and portamento detection and analysis methods described above were implemented in AVA using MATLAB.

AVA's Graphical User Interface (GUI) consists of three panels accessed through the tabs: Read Audio, Vibrato Analysis, and Portamento Analysis. The Read Audio panel allows a user to input or record an audio excerpt and obtain the corresponding (fundamental frequency) pitch curve. The Vibrato Analysis and Portamento Analysis panels provide visualizations of vibrato and portamento detection and analysis results, respectively.

Figure 4 shows screenshots of the AVA interface. Figure 4(a) shows the Vibrato Analysis panel analyzing an

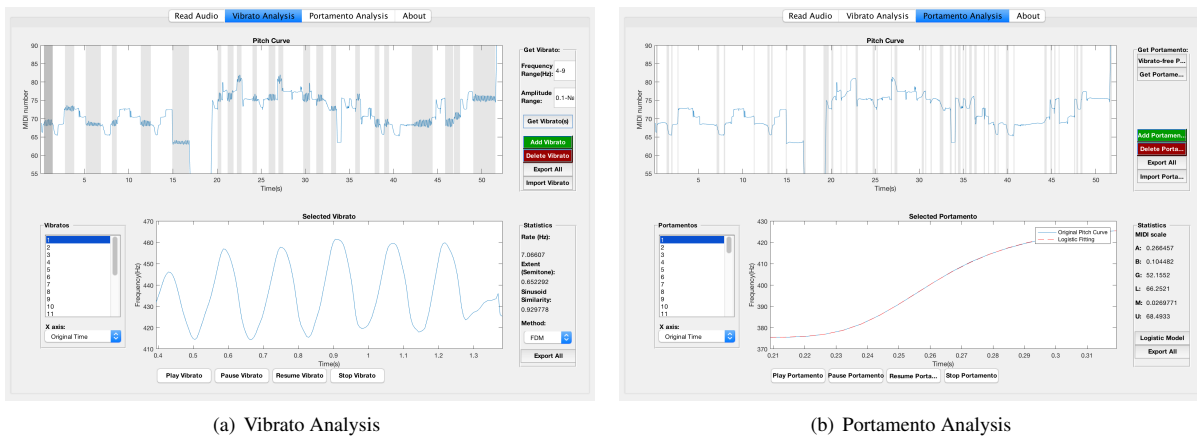


Figure 4. AVA screenshots: the vibrato analysis (left) and portamento analysis (right) panels.

erhu excerpt. Figure 4(b) shows the Portamento Analysis panel analyzing the same excerpt.

Our design principle was to have each panel provide one core functionality while minimizing unnecessary functions having little added value. As vibratos and portamenti relate directly to the pitch curve, each tab shows the entire pitch curve of the excerpt and a selected vibrato or portamento in that pitch curve.

To allow for user input, the Vibrato Analysis and Portamento Analysis panels each have “Add” and “Delete” buttons for creating or deleting highlight windows against the pitch curve. Playback functions allow the user to hear each detected feature so as to inspect and improve detection results. To enable further statistical analysis, AVA can export to a text file all vibrato and portamento annotations and their corresponding parameters.

3.1 Vibrato Analysis Panel

We first describe the Vibrato Analysis Panel, shown in Figure 4(a). The pitch curve of the entire excerpt is presented in the upper part, with the shaded areas marking the detected vibratos. Vibrato existence is determined using the method described in Section 2.1. The computations are triggered using the “Get Vibrato(s)” button in the top right, and the detected vibratos are highlighted by grey boxes on the pitch curve. Users can correct vibrato detection errors using the “Add Vibrato” and “Delete Vibrato” buttons.

The interface allows the user to change the default settings for the vibrato frequency and amplitude ranges; these adaptable limits serve as parameters for the Decision Tree vibrato existence detection process. In this case, with the vibrato frequency range threshold [4, 9] Hz and amplitude range threshold [0.1, ∞] semitones.

On the lower left is a box listing the indices of the detected vibratos. The user can click on each highlighted vibrato on the pitch curve, use the left- or right-arrow keys to navigate from the selected vibrato, or click on one of the indices to select a vibrato. The pitch curve of the vibrato thus selected is presented in the lower plot with corresponding parameters shown to the right of that plot.

In Figure 4(a), the selected vibrato has frequency 7.07 Hz, extent 0.65 semitones, and sinusoid similarity value 0.93. These parameters are obtained using the FDM-based vibrato analysis technique. Alternatively, using the drop down menu currently marked “FDM”, the user can toggle between the FDM-based technique and a more basic Max-min method that computes the vibrato parameters from the peaks and troughs of the vibrato pitch contour.

Another drop down menu, labeled “X axis” under the vibrato indices at the bottom left, lets the user to choose between the original time axis and a normalized time axis for visualizing each detected vibrato. A playback function assists the user in vibrato selection and inspection. All detected vibrato annotations and parameters can be exported to a text file at the click of a button to facilitate further statistical analysis.

3.2 Portamento Analysis Panel

Next, we present the functions available on the Portamento Analysis Panel, shown in Figure 4(b).

In the whole-sample pitch curve of Figure 4(b), the detected vibratos of Figure 4(a) have been flattened to improve portamento detection. Clicking on the “Get Portamentos” button initiates the process of detecting portamenti. The “Logistic Model” button triggers the process of fitting Logistic Models to all the detected portamenti.

Like the Vibrato Analysis panel, the Portamento Analysis panel also provides “Add Portamento” and “Delete Portamento” buttons for the user to correct detection errors. The process for selecting and navigating between detected portamenti is like that for the Vibrato Analysis panel.

When a detected portamento is selected, the best-fit Logistic model is shown as a red line against the original portamento pitch curve. The panel to the right shows the corresponding Logistic Model parameters. In the case of the portamento highlighted in Figure 4(b), the growth rate is 52.15 and the lower and upper asymptotes are 66.25 and 68.49 (in MIDI number), respectively, which could be interpreted as the antecedent and subsequent pitches. From this, we infer that the transition interval is 2.24 semitones.

As with the Vibrato Analysis panel, a playback function assists the user in portamento selection and inspection. Again, all detected portamento annotations and parameters can be exported to a text file at the click of a button to facilitate further statistical analysis.

4. CASE STUDIES

This section demonstrates the application of AVA to two sets of data: one for two major roles in Beijing opera, and one for violin and erhu recordings.

4.1 Case 1: Beijing Opera (Vocal)

Vibrato and portamento are widely and extensively employed in opera; the focus of the study here is on singing in Beijing opera. Investigations into Beijing opera singing include [9]. For the current study, we use selected recordings from the Beijing opera dataset created by Black and Tian [1]; the statistics on the amount of vibrato and portamenti, based on manual annotations, in the sung samples are shown in Table 2. The dataset consists of 16 mono-

No.	Role	Duration(s)	# Vibratos	# Portamenti
1	Laosheng	102	49	94
2		184	62	224
3		91	56	159
4		95	49	166
5		127	26	115
6		147	51	106
7		168	47	144
8		61	19	89
9	Zhengdan	159	47	173
10		80	24	49
11		119	42	176
12		50	24	71
13		155	57	212
14		41	12	48
15		180	59	219
16		70	34	87

Table 2. Beijing opera dataset.

phonic recordings by 6 different Chinese opera singers performing well-known phrases from the Beijing opera roles Laosheng(老生) and Zhengdan(正旦).

Each recording was uploaded to the AVA system for vibrato and portamento detection and analysis. Detection errors were readily corrected using the editing capabilities of AVA. Figure 5 presents the resulting histogram envelopes of the vibrato and portamento parameter values, each normalized to sum to 1, for the Zhengdan (red) and Laosheng (blue) roles. Translucent lines show the parameter’s distributions for individual recordings, and bold lines show the aggregate histogram for each role.

The histograms show the similarities and differences in the underlying probability density functions. Visual inspection shows that the singing of the Zhengdan and

Laosheng roles to be most contrastive in the vibrato extents, with peaks at around 0.5 and 0.8 semitones, respectively. A Kolmogorov-Smirnov (KS) test² shows that the histogram envelopes of vibrato extent from Laosheng and Zheng to be significant different ($p = 2.86 \times 10^{-4}$) at 1% significant level. The same test shows that the distributions for vibrato rate ($p = 0.0536$) and vibrato sinusoid similarity ($p = 0.0205$) are not significant different. Significant differences are found between the singing of the Laosheng and Zhengdan roles for the portamento slope ($p = 1.80 \times 10^{-3}$) and interval ($p = 2.30 \times 10^{-34}$) after testing using the KS test; differences in duration ($p = 0.345$), normalized inflection time ($p = 0.114$) and normalized inflection pitch ($p = 1.00$) are not significant.

4.2 Case 2: Violin vs. Erhu (String)

Here, we demonstrate the usability of the AVA system on the analysis of vibrato and portamento performance styles on erhu and violin. The study centers on a well known Chinese piece *The Moon Reflected on the Second Spring* (二泉映月) [3]. The study uses four recordings, two for erhu and two more for violin. Table 3 lists the details of the test set, which comprises of a total of 23.6 minutes of music; with the help of AVA, 556 vibratos and 527 portamenti were found, verified, and analysed.

No.	Instrument	Duration(s)	# Vibratos	# Portamenti
1	Erhu	446	164	186
2		388	157	169
3	Violin	255	131	91
4		326	104	81

Table 3. Erhu and violin dataset.

The histograms of the vibrato and portamento parameters are summarized in Figure 6. Again, we use the KS test to assess the difference in the histograms between violin and erhu. As with the case for the Beijing opera roles, the most significant difference between the instruments is found in the vibrato extent ($p = 2.70 \times 10^{-3}$), with the vibrato extent for the erhu about twice that for violin (half semitone vs. quarter semitone). There is no significant difference found between erhu and violin for vibrato rate ($p = 0.352$) and sinusoid similarity ($p = 0.261$), although the plots show that the violin recordings have slightly faster vibrato rates and lower sinusoid similarity.

Regarding portamento, the portamento interval histogram has a distinct peak at around three semitones for both violin and erhu, showing that notes separated by this gap is more frequently joined by portamenti. The difference between the histograms is highly insignificant ($p = 0.363$). The most significant difference between violin and erhu portamenti histograms is observed for the slope ($p = 1.51 \times 10^{-4}$). Inspecting the histograms, violinists tend to place the normalized inflection time after the midpoint and erhu players before the midpoint of the portamento duration. However, it is not supported by the KS

² <http://uk.mathworks.com/help/stats/kstest2.html>

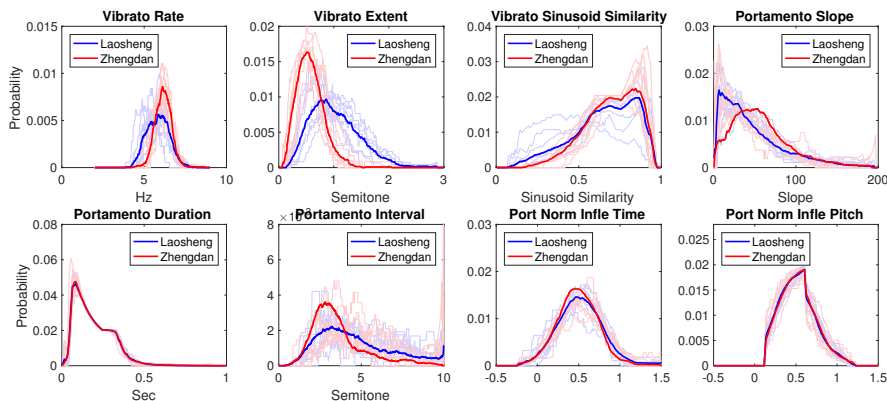


Figure 5. Histogram envelopes of vibrato and portamento parameters for Beijing opera roles: Laosheng (blue) and Zhengdan (red). Translucent lines show histograms for individual singers; bold line shows aggregated histograms for each role.

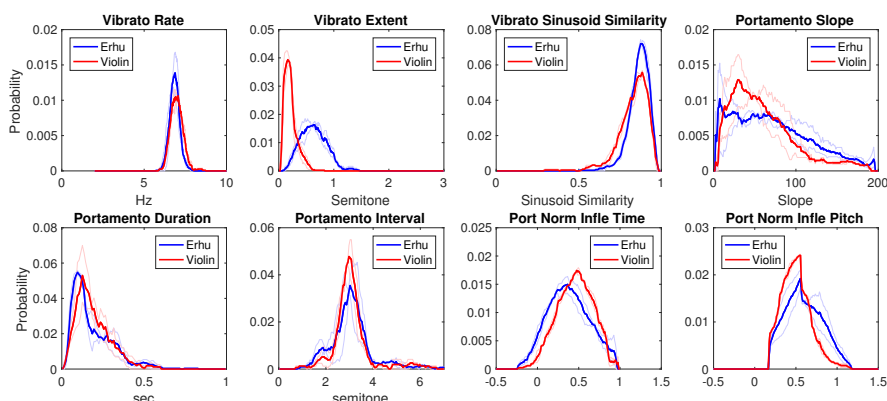


Figure 6. Histogram envelopes of vibrato and portamento parameters for two instruments: erhu (blue) and violin (red). Translucent lines show histograms for individual players; bold line shows aggregated histograms for each instrument.

test ($p = 0.256$). The duration ($p = 0.344$) and normalized inflection pitch ($p = 0.382$) doesn't show significant results.

5. CONCLUSIONS AND DISCUSSIONS

We have presented an interactive vibrato and portamento detection and analysis system, AVA. The system was implemented in MATLAB, and the GUI provides interactive and intuitive visualizations of detected vibratos and portamenti and their properties. We have also demonstrated its use in analyses of Beijing opera and string recordings.

For vibrato detection and analysis, the system implements a Decision Tree for vibrato detection based on FDM output and an FDM-based vibrato analysis method. The system currently uses a Decision Tree method for determining vibrato existence; a more sophisticated Bayesian approach taking advantage of learned vibrato rate and extent distributions is described in [16]. While the Bayesian approach has been shown to give better results, it requires training data; the prior distributions based on training data can be adapted to specific instruments and genres.

For portamento detection and analysis, the system uses

an HMM-based portamento detection method with Logistic Models for portamento analysis. Even though a threshold has been set to guarantee a minimum note transition duration, the portamento detection method sometimes misclassifies normal note transitions as portamenti, often for notes having low intensity (dynamic) values. While there were significant time savings over manual annotation, especially for vibrato boundaries, corrections of the automatically detected portamento boundaries proved to be the most time consuming part of the exercise. Future improvements to the portamento detection method could take into account more features in addition to the delta pitch curve.

For the Beijing opera study, the two roles differed significantly in vibrato extent, and in portamento slope and interval. The violin and erhu study showed the most significant differences in vibrato extent and portamento slope. Finally, the annotations and analyses produced with the help of AVA will be made available for further study.

6. ACKNOWLEDGEMENTS

This project is supported in part by the China Scholarship Council.

7. REFERENCES

- [1] Dawn A. A. Black, Li Ma, and Mi Tian. Automatic identification of emotional cues in Chinese opera singing. In *Proc. of the 13th International Conference on Music Perception and Cognition and the 5th Conference for the Asian-Pacific Society for Cognitive Sciences of Music (ICMPC 13-APSCOM 5)*, 2014.
- [2] Perfecto Herrera and Jordi Bonada. Vibrato extraction and parameterization in the spectral modeling synthesis framework. In *Proc. of the Digital Audio Effects Workshop*, volume 99, 1998.
- [3] Yanjun Hua. *Erquanyingyue*. Zhiruo Ding and Zhanhao He, violin edition, 1958. Musical Score.
- [4] Heejung Lee. Violin portamento: An analysis of its use by master violinists in selected nineteenth-century concerti. In *Proc. of the 9th International Conference on Music Perception and Cognition*, 2006.
- [5] Matthias Mauch and Simon Dixon. pyin: A fundamental frequency estimator using probabilistic threshold distributions. In *Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 2014.
- [6] Tin Lay Nwe and Haizhou Li. Exploring vibrato-motivated acoustic features for singer identification. *IEEE Transactions on Audio, Speech, and Language Processing*, 15(2):519–530, 2007.
- [7] Tan Hakan Özaslan, Xavier Serra, and Josep Lluís Arcos. Characterization of embellishments in ney performances of makam music in turkey. In *Proc. of the International Society for Music Information Retrieval Conference*, 2012.
- [8] Hee-Suk Pang and Doe-Hyun Yoon. Automatic detection of vibrato in monophonic music. *Pattern Recognition*, 38(7):1135–1138, 2005.
- [9] Rafael Caro Repetto, Rong Gong, Nadine Kroher, and Xavier Serra. Comparison of the singing style of two jingju schools. In *Proc. of the International Society for Music Information Retrieval Conference*, 2015.
- [10] Stéphane Rossignol, Philippe Depalle, Joel Soumagne, Xavier Rodet, and Jean-Luc Collette. Vibrato: detection, estimation, extraction, modification. In *Proc. of the Digital Audio Effects Workshop*, 1999.
- [11] Jose Ventura, Ricardo Sousa, and Anibal Ferreira. Accurate analysis and visual feedback of vibrato in singing. In *Proc. of the IEEE 5th International Symposium on Communications, Control and Signal Processing*, pages 1–6, 2012.
- [12] Vincent Verfaille, Catherine Guastavino, and Philippe Depalle. Perceptual evaluation of vibrato models. In *Proc. of the Conference on Interdisciplinary Musicology*, March 2005.
- [13] Henrik von Coler and Axel Roebel. Vibrato detection using cross correlation between temporal energy and fundamental frequency. In *Proc. of the Audio Engineering Society Convention 131*. Audio Engineering Society, 2011.
- [14] Luwei Yang, Elaine Chew, and Khalid Z. Rajab. Vibrato performance style: A case study comparing erhu and violin. In *Proc. of the 10th International Conference on Computer Music Multidisciplinary Research*, 2013.
- [15] Luwei Yang, Elaine Chew, and Khalid Z. Rajab. Logistic modeling of note transitions. In *Mathematics and Computation in Music*, pages 161–172. Springer, 2015.
- [16] Luwei Yang, Khalid Z. Rajab, and Elaine Chew. Filter diagonalisation method for music signal analysis: Frame-wise vibrato detection and estimation. *Journal of Mathematics and Music*, 2016. Under revision.