

#### Autoformer: Decomposition Transformers with Auto-Correlation for Long-Term Series Forecasting

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#### **Time Series Forecasting**



Past Observations

**Future Time Series** 

#### **Long-Term** Time Series Forecasting





Past Observations

**Future Time Series** 

#### Transformers





## **Transformers For Time series Forecasting**





- Modeling the temporal dependencies with point-wise Self-Attention
- Aggregate the representations for forecasting



Informer[Zhou et al. AAAI2021], Reformer[Kitaev et al. ICLR2020], Log Trans[Li et al. NeurIPS19]



Past Observations

Future Time Series

## Transformers For Long-Term Series Forecasting



	Transformers	Autoformer
Intricate Temporal Patterns	Hard to directly find reliable temporal dependencies from series X	Decomposition architecture to ravel out the entangled temporal patterns
Deal with Long Series	<ul> <li>Point-wise Self-Attention is O(L<sup>2</sup>)</li> <li>Adopt sparse version for efficiency</li> <li>Loss information and cause the information utilization bottleneck</li> </ul>	Series-wise Auto-Correlation based on stochastic process theory with inherent O(L log L complexity

#### **Overall Architecture**





#### **Overall Architecture**





Decomposition architecture for intricate temporal patterns.

#### **Overall Architecture**





#### Series-wise Auto-Correlation for information utilization bottleneck.

#### **Decomposition: Pre-processing** Convention



- Limited by the capabilities of decomposition
- Overlooks the potential future interactions among components



#### **Deep Decomposition Architecture**





Progressive decomposition capacity

Decompose the trend from the intermediate "future" and refine it during the decoder.



#### **Deep Decomposition Architecture: Input**





#### **Deep Decomposition Architecture: Encoder**



#### **Deep Decomposition Architecture: Decoder**







# Period-based dependencies The same phase position of different periods

Benefited from the deep decomposition, the **seasonal part** is highlighted with **periodicity**.

Conducts the **dependencies discovery** and **representation aggregation** at the series level.



Discover period-based dependencies with autocorrelation in stochastic process:

$$\mathcal{R}_{\mathcal{X}\mathcal{X}}(\tau) = \lim_{L \to \infty} \frac{1}{L} \sum_{t=0}^{L-1} \mathcal{X}_t \mathcal{X}_{t-\tau}.$$

Autocorrelation reflects the time delay similarity,

and corresponds to the confidence of period estimation.



Larger autocorrelation  $\mathcal{R}(\tau)$  means

- stronger time delay similarity w.r.t.  $\tau$
- more confidence of period length as  $\tau$







#### **1** Discover period-based dependencies

#### **② Aggregate similar sub-processes from different periods**











#### **Auto-Correlation vs Self-Attention Family**



Auto-Correlation can extend the point-wise aggregation to series-wise.

Informer[Zhou et al. AAAI2021], Reformer[Kitaev et al. ICLR2020], Log Trans[Li et al. NeurIPS19]



#### **Experiments: Multivariate setting**

Two works www.

				Tra	ansf	orm	ers			LST	Ms		TCN	
	Models Metric	Autoformer MSE MAE	Informe MSE	er[41] MAE	LogTra MSE	ans[20] MAE	Reform MSE	ner[17] MAE	LSTN MSE	Iet[19] MAE	LSTN MSE	M[13] MAE	TCN[3] MSE MAE	Prediction Accuracy Relative Promotion (In MSE)
_ [	*LL 96 192 336 720	0.255 0.339 0.281 0.340 0.339 0.372 0.422 0.419	0.365 0.533 1.363 3.379	0.453 0.563 0.887 1.388	0.768 0.989 1.334 3.048	0.642 0.757 0.872 1.328	0.658 1.078 1.549 2.631	0.619 0.827 0.972 1.242	3.142 3.154 3.160 3.171	1.365 1.369 1.369 1.368	2.041 2.249 2.568 2.720	1.073 1.112 1.238 1.287	3.0411.3303.0721.3393.1051.3483.1351.354	↑ <b>74%</b> Input-96-predict-336
Energy -	96 192 336 720	0.201 0.317 0.222 0.334 0.231 0.338 0.254 0.361	0.274 0.296 0.300 0.373	0.368 0.386 0.394 0.439	0.258 0.266 0.280 0.283	0.357 0.368 0.380 0.376	0.312 0.348 0.350 0.340	0.402 0.433 0.433 0.420	0.680 0.725 0.828 0.957	0.645 0.676 0.727 0.811	0.375 0.442 0.439 0.980	0.437 0.473 0.473 0.814	0.985 0.813 0.996 0.821 1.000 0.824 1.438 0.784	↑ <b>18%</b> Input-96-predict-336
Economics	96 192 336 720	0.197 0.323 0.300 0.369 0.509 0.524 1.447 0.941	0.847 1.204 1.672 2.478	0.752 0.895 1.036 1.310	0.968 1.040 1.659 1.941	0.812 0.851 1.081 1.127	1.065 1.188 1.357 1.510	0.829 0.906 0.976 1.016	1.551 1.477 1.507 2.285	1.058 1.028 1.031 1.243	1.453 1.846 2.136 2.984	1.049 1.179 1.231 1.427	3.0041.4323.0481.4443.1131.4593.1501.458	↑ <b>61%</b> Input-96-predict-336
Traffic	96 192 336 720	0.613 0.388 0.616 0.382 0.622 0.337 0.660 0.408	0.719 0.696 0.777 0.864	0.391 0.379 0.420 0.472	0.684 0.685 0.733 0.717	0.384 0.390 0.408 0.396	0.732 0.733 0.742 0.755	0.423 0.420 0.420 0.423	1.107 1.157 1.216 1.481	0.685 0.706 0.730 0.805	0.843 0.847 0.853 1.500	0.453 0.453 0.455 0.805	1.438 0.784 1.463 0.794 1.479 0.799 1.499 0.804	↑ <b>15%</b> Input-96-predict-336
Weather	Meather 192 336 720	0.266 0.336 0.307 0.367 0.359 0.395 0.419 0.428	0.300 0.598 0.578 1.059	0.384 0.544 0.523 0.741	0.458 0.658 0.797 0.869	0.490 0.589 0.652 0.675	0.689 0.752 0.639 1.130	0.596 0.638 0.596 0.792	0.594 0.560 0.597 0.618	0.587 0.565 0.587 0.599	0.369 0.416 0.455 0.535	0.406 0.435 0.454 0.520	0.6150.5890.6290.6000.6390.6080.6390.610	↑ <b>21%</b> Input-96-predict-336
Disease	$ \begin{array}{c c} 24 \\ 36 \\ 48 \\ 60 \end{array} $	3.483 1.287 3.103 1.148 2.669 1.085 2.770 1.125	5.764 4.755 4.763 5.264	1.677 1.467 1.469 1.564	4.480 4.799 4.800 5.278	1.444 1.467 1.468 1.560	4.400 4.783 4.832 4.882	1.382 1.448 1.465 1.483	6.026 5.340 6.080 5.548	1.770 1.668 1.787 1.720	5.914 6.631 6.736 6.870	1.734 1.845 1.857 1.879	6.6241.8306.8581.8796.9681.8927.1271.918	↑ <b>43%</b> Input-24-predict-48

*ETT* means the ETTm2. See supplementary materials for the **full benchmark** of ETTh1, ETTh2, ETTm1.

#### **Experiments: ETT benchmark**

		1		1										2
Mo	dels	Autof	ormer	Inform	ner [14]	LogTr	LogTrans [9]		Reformer [7]		let [8]	LSTMa []		
Me	etric	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	. Ro
ETTh1	24 48 168 336 720	0.384 0.392 0.490 0.505 0.498	0.425 0.419 0.481 0.484 0.500	0.577 0.685 0.931 1.128 1.215	0.549 0.625 0.752 0.873 0.896	0.686 0.766 1.002 1.362 1.397	0.604 0.757 0.846 0.952 1.291	0.991 1.313 1.824 2.117 2.415	0.754 0.906 1.138 1.280 1.520	1.293 1.456 1.997 2.655 2.143	0.901 0.960 1.214 1.369 1.380	0.650 0.702 1.212 1.424 1.960	0.624 0.675 0.867 0.994 1.322	· ĸe
ETTh2	24 48 168 336 720	0.261 0.312 0.457 0.471 0.474	0.341 0.373 0.455 0.475 0.484	0.720 1.457 3.489 2.723 3.467	0.665 1.001 1.515 1.340 1.473	0.828 1.806 4.070 3.875 3.913	0.750 1.034 1.681 1.763 1.552	1.531 1.871 4.660 4.028 5.381	1.613 1.735 1.846 1.688 2.015	2.742 3.567 3.242 2.544 4.625	1.457 1.687 2.513 2.591 3.709	1.143 1.671 4.117 3.434 3.963	0.813 1.221 1.674 1.549 1.788	-
ETTm1	24 48 96 288 672	0.383 0.454 0.481 0.634 0.606	0.403 0.453 0.463 0.528 0.542	<b>0.323</b> 0.494 0.678 1.056 1.192	<b>0.369</b> 0.503 0.614 0.786 0.926	0.419 0.507 0.768 1.462 1.669	0.412 0.583 0.792 1.320 1.461	0.724 1.098 1.433 1.820 2.187	0.607 0.777 0.945 1.094 1.232	1.968 1.999 2.762 1.257 1.917	1.170 1.215 1.542 2.076 2.941	0.621 1.392 1.339 1.740 2.736	0.629 0.939 0.913 1.124 1.555	-
ETTm2	24 48 96 288 672	0.153 0.178 0.255 0.342 0.434	0.261 0.280 0.339 0.378 0.430	$\begin{array}{c} 0.173 \\ 0.303 \\ 0.365 \\ 1.047 \\ 3.126 \end{array}$	0.301 0.409 0.453 0.804 1.302	0.211 0.427 0.768 1.090 2.397	0.332 0.487 0.642 0.806 1.214	0.333 0.558 0.658 2.441 3.090	0.429 0.571 0.619 1.190 1.328	1.101 2.619 3.142 2.856 3.409	0.831 1.393 1.365 1.329 1.420	0.580 0.747 2.041 0.969 2.541	0.572 0.630 1.073 0.742 1.239	- -

Prediction Accuracy elative Promotion (In MSE) **55%** 

Input-96-predict-336

↑ **80%** Input-96-predict-336

↑ **40%** Input-96-predict-288

↑ **66%** Input-96-predict-288

#### **Experiments: Univariate setting**



M	odels	Autof	ormer	N-BEA	ATS[23]	Inform	ner[41]	LogTr	ans[20]	Reform	mer[17]	DeepA	AR[28]	Proph	et[33]	ARIN	/IA[1]
Μ	etric	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
	96	0.065	0.189	0.082	0.219	0.088	0.225	0.082	0.217	0.131	0.288	0.099	0.237	0.287	0.456	0.211	0.362
Ę	192	0.118	0.256	0.120	0.268	0.132	0.283	0.133	0.284	0.186	0.354	0.154	0.310	0.312	0.483	0.261	0.406
Ξ	336	0.154	0.305	0.226	0.370	0.180	0.336	0.201	0.361	0.220	0.381	0.277	0.428	0.331	0.474	0.317	0.448
	720	0.182	0.335	0.188	0.338	0.300	0.435	0.268	0.407	0.267	0.430	0.332	0.468	0.534	0.593	0.366	0.487
ge	96	0.241	0.387	0.156	0.299	0.591	0.615	0.279	0.441	1.327	0.944	0.417	0.515	0.828	0.762	0.112	0.245
an	192	0.273	0.403	0.669	0.665	1.183	0.912	1.950	1.048	1.258	0.924	0.813	0.735	0.909	0.974	0.304	0.404
xch	336	0.508	0.539	0.611	0.605	1.367	0.984	2.438	1.262	2.179	1.296	1.331	0.962	1.304	0.988	0.736	0.598
Щ	720	0.991	0.768	1.111	0.860	1.872	1.072	2.010	1.247	1.280	0.953	1.894	1.181	3.238	1.566	1.871	0.935

Competitive baseline N-BEATS







#### Ablation of decomposition architecture



Table 3: Ablation of decomposition in multivariate ETT with MSE metric. **Ours** adopts our progressive architecture into other models. **Sep** employs two models to forecast pre-decomposed seasonal and trend-cyclical components separately. *Promotion* is the MSE reduction compared to **Origin**.

Input-96   Transformer[35]			Info	ormer[4	41]	LogTrans[17]			Ref	ormer[	Promotion			
Predict-O	Origin	Sep	Ours	Origin	Sep	Ours	Origin	Sep	Ours	Origin	Sep	Ours	Sep	Ours
96	0.604	0.311	0.204	0.365	0.490	0.354	0.768	0.862	0.231	0.658	0.445	0.218	0.069	0.347
192	1.060	0.760	0.266	0.533	0.658	0.432	0.989	0.533	0.378	1.078	0.510	0.336	0.300	0.562
336	1.413	0.665	0.375	1.363	1.469	0.481	1.334	0.762	0.362	1.549	1.028	0.366	0.434	1.019
720	2.672	3.200	0.537	3.379	2.766	0.822	3.048	2.601	0.539	2.631	2.845	0.502	0.079	2.332

- Progressive decomposition architecture outperforms separately forecasting convention, especially the long-term setting.
- The decomposition architecture can be generalized to other Transformers with remarkable promotion.

#### Visualization of decomposition architecture





Figure 4: Visualization of learned seasonal  $\mathcal{X}_{de}^{M}$  and trend-cyclical  $\mathcal{T}_{de}^{M}$  of the last decoder layer. We gradually add the decomposition blocks in decoder from left to right. This case is from ETT dataset under input-96-predict-720 setting. For clearness, we add the linear growth to raw data additionally.

• With the increasing of decomposition blocks, the predictions of both seasonal and trend part get better and better progressively.





#### **Auto-Correlation vs. Self-Attention Family**

Table 4: Comparison of Auto-Correlation and self-attention in the multivariate ETT. We **replace** the Auto-Correlation in Autoformer with different self-attentions. The "-" indicates the out-of-memory.

Input Leng		96			192			336		
Prediction Ler	ngth O	336	720	1440	336	720	1440	336	720	1440
Auto-	MSE	0.339	0.422	0.555	0.355	0.429	0.503	0.361	0.425	0.574
Correlation	MAE	0.372	0.419	0.496	0.392	0.430	0.484	0.406	0.440	0.534
Full Attention[35]	MSE MAE	0.375 0.425	0.537 0.502	0.667 0.589	0.450 0.470	0.554 0.533	-	0.501 0.485	0.647 0.491	-
LogSparse	MSE	0.362	0.539	0.582	0.420	0.552	0.958	0.474	0.601	-
Attention[20]	MAE	0.413	0.522	0.529	0.450	0.513	0.736	0.474	0.524	
LSH	MSE	0.366	0.502	0.663	0.407	0.636	1.069	0.442	0.615	-
Attention[17]	MAE	0.404	0.475	0.567	0.421	0.571	0.756	0.476	0.532	
ProbSparse	MSE	0.481	0.822	0.715	0.404	1.148	0.732	0.417	0.631	1.133
Attention[41]	MAE	0.472	0.559	0.586	0.425	0.654	0.602	0.434	0.528	0.691

Under various input-predict settings, Auto-Correlation outperforms the self-attention and their variants.

#### **Visualization of learned dependencies**



Figure 5: Visualization of learned dependencies. For clearness, we select the top-6 time delay sizes  $\tau_1, \dots, \tau_6$  of Auto-Correlation and mark them in raw series (red lines). For self-attentions, top-6 similar points with respect to the last time step (red stars) are also marked by orange points.

#### Auto-Correlation can discover the relevant information

#### more sufficiently and precisely.





#### **Visualization of learned lags**



Figure 6: Statistics of learned lags. For each time series in the test set, we count the top 10 lags learned by decoder for the input-96-predict-336 task. Figure (a)-(d) are the density histograms.

Learned lags can reflect the human-interpretable prediction.

### **Efficiency Analysis**





Figure 6: Efficiency Analysis. For memory, we replace Auto-Correlation with self-attention family in Autoformer and record the memory with input 96. For running time, we run the Auto-Correlation or self-attentions  $10^3$  times to get the execution time per step. The output length increases exponentially.

## Auto-Correlation presents remarkable O(*L* log *L*) complexity in both memory and computation.





#### **Motivation Autoformer** Intricate **Decomposition** architecture **Classic method** to ravel out the entangled Temporal of time series analysis Patterns temporal patterns Deal with Series-wise Auto-Correlation Stochastic process theory Long Series with $O(L \log L)$ complexity

Autoformer achieves the remarkable state-of-the-art on extensive benchmarks.

#### **Open Source**



La thuml / Autoformer Public				● Watch 7 ▼ 😵 F	ork 53 🔶 🛧 Starred 152 💌		
<> Code 💿 Issues 🕴 Pull	requests 🕑 Actions 🖽 Projects	🛙 Wiki 🕕 Security 🖂 Insights	袋 Settings				
	양 main ▾ 양 1 branch ⓒ 0 tags		Go to file Add file - Code -	About 錄			
	wuhaixu2016 update MS setting		7f9ce3c 15 days ago 🕚 26 commits	About Code release for "Autoformer: Decomposition Transformers with Auto-			
	data_provider	update predict	3 months ago	Correlation for Long-Term Series Forecasting" (NeurIPS 2021),			
	exp	update MS setting	15 days ago	https://arxiv.org/abs/2106.13008			
	ayers	init code	4 months ago	C Readme			
	models	using device instead of cuda	last month	MILLicense ☆ 152 stars			
	<b>pi</b> c	init code	4 months ago	<ul> <li>✓ 7 watching</li> <li>✓ 53 forks</li> </ul>			
	scripts	univariate_scripts	15 days ago				
	🖿 utils	add download data script	last month				
	🗅 .gitignore	Initial commit	4 months ago	Releases			
	Dockerfile	add docker, make and conda env	last month	No releases published Create a new release			
	LICENSE	Initial commit	4 months ago				
	🗅 Makefile	add docker, make and conda env	last month	Packages			
	B README.md	Update README.md	last month	No packages published			
	🗅 environment.yml	add docker, make and conda env	last month	Publish your first package			
	🗅 run.py	init code	4 months ago	Contributors			
			R				
			V	wuhaixu2016			
	Autoformer (Neur	IPS 2021)	FedericoGarza fede				
	Autoformer: Decomposition Transfo	rmers with Auto-Correlation for Long-Term	Series Forecasting	Languages			
	Time series forecasting is a critical of stochastic process theory, we propo goes beyond the Transformer fam	<ul> <li>Python 69.4%</li> <li>Shell 29.3%</li> <li>Other 1.3%</li> </ul>					

https://github.com/thuml/Autoformer

Well-organized code and pre-processed dataset



#### Thank You! whx20@mails.tsinghua.edu.cn