

Some Intriguing Consequences of the Quantum Vacuum Fluctuations in the Semi-Classical Formalism

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Abstract

By assuming that vacuum quantum fluctuations are the cause for vacuum electromagnetic polarizability, it is possible to explain the classical and relativistic gravitational effects using a semi-classical formalism, and replicate the results conventionally obtainable only within the framework of General Relativity.

Important Note on the Use of Symbols

In the Maxwell's electromagnetic theory [1] the symbol E is reserved for the electric field strength. But in the General Theory of Relativity [2] the same symbol is used for energy. In order to avoid confusion, we will use the symbol W ('work') for energy, but to respect the historical reasons and preserve the recognizable form of equations like, say, $E = mc^2$, we will keep the symbol E for energy in certain relations, hoping that it will be clear from the context what are we dealing with.

Introduction to the Properties of the Quantum Vacuum

In accordance with classical electrodynamics, a weak electric field influencing a dielectric medium slightly displaces the electron clouds from their distribution in respect with their atomic nuclei, thus creating small dipoles within the material. Averaged over a large number of atoms, the macroscopic effect is that of the appearance of an induced electric field. The classical formalism treats such effects in terms of a polarization vector \mathbf{P} , which is proportional to the electric field \mathbf{E} :

$$\mathbf{P} = \chi \varepsilon_0 \mathbf{E} \quad (1)$$

with χ as the susceptibility of the dielectric medium, whilst ε_0 is the permittivity (also known as the dielectric constant) of the free space. The permittivity ε_0 is usually derived from the constitutional Maxwell's equations, which for linear, isotropic, and homogeneous media, such as the free space is, are reduced from their more general expressions to this simple form:

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} \\ \mathbf{B} &= \mu_0 \mathbf{H} \\ \mathbf{J} &= 0 \end{aligned} \quad (2)$$

By solving the general wave equation using the D'Alembert's solution [3] it can be shown that the electromagnetic wave propagates in free space with the speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad (3)$$

where μ_0 is the magnetic permeability of the free space. In SI units [4], the propagation speed of electromagnetic phenomena, including light, has been derived from the convention about the meter to be exactly $c = 299\,792\,458$ m/s. Likewise, the magnetic permeability has been also assigned the role of a fundamental natural constant, defined to be exactly $\mu_0 = 4\pi \times 10^{-7}$ Vs/Am. Consequently, it is often argued that the role of permittivity, which apparently just happens to have the value of

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} \approx 8.85 \times 10^{-12} \text{ As/Vm} \quad (4)$$

is that of simply providing the matching of measurement units. It is the author's opinion that the role of permittivity is physically of equal fundamental importance as that of the magnetic permeability, as it will be shown here.

One indication that ε_0 must have a more profound meaning can be grasped already from the definition of the electromagnetic impedance of free space:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \Omega \quad (5)$$

Another indication comes from the light refraction index [5] at the border between two optical media of different density. From the energy of a photon [6]:

$$W_\gamma = \hbar\omega = h\nu = h\frac{c}{\lambda} \quad (6)$$

and observing the energy conservation principle, the frequency must also remain unchanged. Therefore both the wavelength λ and the speed of light c must change their values, which results in the equation:

$$\frac{c}{\lambda} = \frac{c'}{\lambda'} \quad (7)$$

and consequently:

$$\frac{\lambda'}{\lambda} = \frac{c'}{c} = \sqrt{\frac{\mu_0 \varepsilon_0}{\mu' \varepsilon'}} \quad (8)$$

In most optically transparent media the magnetic permeability is the same as in vacuum, $\mu' = \mu_0$, which reduces (8) to:

$$\frac{\lambda'}{\lambda} = \frac{c'}{c} = \sqrt{\frac{\varepsilon_0}{\varepsilon'}} = \frac{n_1}{n_2} \quad (9)$$

where n is the refraction index of a particular optical medium.

But there is a more important aspect of ε_0 . From (9) it can be inferred that $\varepsilon' = \varepsilon_r \varepsilon_0$, where with ε_r we have marked the relative permittivity of the medium in

question, compared to the permittivity in vacuum. Usually we define the dielectric displacement as:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_r \varepsilon_0 \mathbf{E} \quad (10)$$

so that the relative permittivity is understood as a function of the susceptibility of the optical medium in question:

$$\varepsilon_r = 1 + \chi \quad (11)$$

Physics textbooks often neglect the importance of dielectric displacement \mathbf{D} , as it combines two seemingly different quantities: the force per unit of charge \mathbf{E} , and the spatial variation of polarization \mathbf{P} (the induced charge). However, it is necessary to realize that both \mathbf{D} and ε_0 have a fundamental meaning. In (10) we can argue that because \mathbf{P} is the polarization of a medium, then $\varepsilon_0 \mathbf{E}$ is the vacuum polarization, and \mathbf{D} is the total effective polarization. Such a view of $\varepsilon_0 \mathbf{E}$ would seem odd in classical electrodynamics, since vacuum is void of any polarizable matter by definition.

However, in quantum electrodynamics [7] the vacuum must also have suitable physical properties. Those properties are governed by ‘virtual’ particles [8] which, in accordance with Heisenberg’s uncertainty principle [9], are created by the vacuum energy random fluctuations as complementary charged pairs (matter and antimatter [10]) and ‘annihilate’ back into pairs of similarly ‘virtual’ photons [11] restoring the vacuum energy. The creation process must always produce complementary pairs, so that all the conservation laws [12], including charge conservation, are obeyed.

That vacuum is not empty has become clear with the Planck’s discovery of the black body radiation spectrum law [13]. In classical theory the black body radiation has been approximated by Wien’s law [14] and the Rayleigh–Jeans’ law [15], which have both been found to diverge, either at low frequencies or at high frequencies. Planck found the exact solution by assuming that matter radiates energy in discrete quanta [16], instead of a continuous spectrum. He calculated the radiated energy as a function of frequency ν of the emitted quanta and the black body’s temperature T :

$$\begin{aligned} W(\nu, T) &= \frac{1}{2} h\nu \frac{e^{\frac{h\nu}{2k_B T}} + e^{-\frac{h\nu}{2k_B T}}}{e^{\frac{h\nu}{2k_B T}} - e^{-\frac{h\nu}{2k_B T}}} \\ &= \frac{1}{2} h\nu \cot \frac{h\nu}{2k_B T} \\ &= \frac{1}{2} h\nu \frac{e^{\frac{h\nu}{k_B T}} + 1}{e^{\frac{h\nu}{k_B T}} - 1} \\ &= \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1} + \frac{h\nu}{2} \end{aligned} \quad (12)$$

This radiation energy has a frequency density (per 1 Hz bandwidth) of:

$$\rho_W(\nu, T) d\nu = 8\pi \frac{\nu^2}{c^3} \left(\frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1} + \frac{h\nu}{2} \right) d\nu \quad (13)$$

This result was also obtained from an experiment performed by Einstein and Stern in 1913. Einstein immediately noted that the second term in parentheses, $h\nu/2$, remains even when the temperature T is reduced to the absolute zero, which prompted him to coin that remaining part as “*Nullpunktsenergie*”, zero-point energy [17].

However, it was Nernst to note in 1916 that there is practically no difference whether the temperature T is a consequence of the thermodynamics of matter particles, or is owed to field oscillations associated with radiation (a certain number of photons, in modern terms). This means that $h\nu/2$ actually represents the lowest (‘ground’) energy state which the vacuum can possibly achieve — without that basic energy level, nothing in this Universe, nor even the Universe itself, could exist.

It is indicative that the same conclusion can be drawn from a purely quantum-mechanical formalism. If we start with the non-commutative representation of the momentum p and position x that a particle can have, we arrive at the Heisenberg’s uncertainty principle:

$$xp - px = \Delta x \cdot \Delta p \geq i\hbar \quad (14)$$

The idea of a ‘photon’ as the quantum of electromagnetic radiation originates from the combination of Maxwell’s equations and Heisenberg’s non-commutativity. By considering the electromagnetic radiation inside a closed metal box, it is possible to decompose the field into Fourier’s components, each proportional to e^{ikx} (assigning a wave number k to every oscillator) and express it as a superposition of a series of oscillators, each with its own frequency ω , its own canonical coordinate q , and a canonical momentum p , within a Hamiltonian operator [18]:

$$\mathcal{H} = \frac{1}{2}(p^2 + \omega^2 q^2) \quad (15)$$

By assigning a unit value to the momentum, the coordinate dependence of the Hamiltonian becomes a function of ω^2 of each oscillator. The energy spectrum of the system is then a superposition of all possible energy states, which satisfy the solutions of the equation:

$$\mathcal{H}\xi = W\xi \quad (16)$$

where ξ is an arbitrarily chosen non-zero eigenstate operator.

It is possible to find the energy spectrum by employing the ‘step-method’: we seek for such a combination of parameters a , q , and p , which satisfies the relation:

$$\mathcal{H}a = a(\mathcal{H} - W_1) \quad (17)$$

Here a decreases the values of W , and consequently \mathcal{H} , in step values of W_1 ; so if $\mathcal{H}\xi = W\xi$ holds, then $\mathcal{H}(a\xi) = (W - W_1)(a\xi)$ must also hold, except for $a\xi = 0$. Such a , if it exists, is called the step operator. In the case of a harmonic oscillator the value of a is:

$$a = 2^{-1/2}(p - jq) \quad (18)$$

The minimal energy step value is:

$$W_1 = \hbar\omega \quad (19)$$

The step operator a can be adjusted so that the Hamiltonian takes the form:

$$\mathcal{H} = \hbar\omega \left(n + \frac{1}{2} \right) \quad (20)$$

with a natural number $n = a^* a$, dividing the range from 1 to n in steps of $a(n - 1)$.

Here we interpret n as the number of excited quantum states within a particular oscillator. Every quantum then supplies $W_1 = \hbar\omega$ to the total energy $W = nW_1$, and similarly the momentum $p_1 = \hbar k$ to the total momentum $p = np_1$. The operator a is sometimes called ‘annihilation operator’, or ‘annihilator’, because it (mathematically) reduces the photon number by 1. The adjugated value a^* is then the ‘creator’, because it increases the number of photons by 1.

It is therefore obvious that in (20) there is always the value $h\nu/2$, even if the number of photons $n = 0$. This justifies the physical existence of the zero-point energy, and a zero-point field within the quantum vacuum, which is associated with it.

There is an important difference, though: quantum–mechanics regards the zero-point energy as a consequence of the oscillations (the infamous Schrödinger’s *Zitterbewegung* [19], an oscillatory motion) of all the particles in the Universe (estimated to something like $N \approx 10^{79}$, but varies widely from model to model). In contrast, in the semiclassical approach the zero-point energy originates from the birth of the Universe, thus assuming a fundamental role in both the creation and the very stability of matter itself. Namely, it is difficult to see the zero-point energy as sourced by particle oscillations, since the cause for those oscillations must then originate within the inner structure or energy of the particles, making them losing energy. Although they would absorb an equivalent amount of energy from other sources, they would be instable, varying their mass and eventually decaying. In contrast, by assuming the cosmological origin of the zero-point energy, the natural oscillation of particles is explained by their need to readjust their internal oscillatory modes to counteract the stochastic nature of the external zero-point energy.

We can now return to the polarizability of the quantum vacuum. It is already well known that very strong fields are required for the spontaneous creation of particle pairs, since intensities of at least:

$$E_s = \frac{m^2 c^3}{q_e \hbar} \approx 10^{18} \text{ V/m} \quad (21)$$

are required to generate them in laboratory conditions. Below that threshold a linear superposition of photons is possible, but above it the quantum vacuum apparently starts to behave non-linearly under such a stress.

As a complementary charged particle pair has been created, for the brief moment of their existence (before the unavoidable annihilation) each pair represents a dipole, which can react with any externally applied field. Thus we have the reason to assume that the physical (electromagnetic) properties of the quantum vacuum are governed by those virtual particle pairs reacting to external fields just like any ordinary material, but with the permittivity and permeability values of ε_0 and μ_0 .

Let us see if we can derive the properties of the quantum vacuum from such semiclassical assumptions and following a classical formalism.

Assume that an external field interacts with a virtual electron-positron pair. The dipole momentum:

$$p = q_e x \quad (22)$$

is induced on that pair, with q_e as the elementary charge, and x as the displacement. We can compute its magnitude by assuming that a virtual pair behaves like a harmonic oscillator in the quasi-static limit:

$$m_e \omega_0^2 x = q_e E \quad (23)$$

with m_e as the electron mass, and ω_0 as the natural resonant frequency. When the externally applied field is small, $E \ll E_s$, it is possible to find the resonant frequency from the energy associated with the quantum transition. If the virtual pair represents the ground energy state of the e^+e^- pair, and the real positronium [20] ‘atom’ represents the excited state, we can assume the existence of an energy gap E_g such that a particular oscillation frequency can be associated with it:

$$\omega_0 = \frac{E_g}{\hbar} \quad (24)$$

This energy gap should be equal to the rest mass of the positronium, $\hbar\omega_0 = 2m_e c^2$. Thus by combining (22) and (23) we find the induced dipole momentum:

$$p = \frac{q_e^2}{m_e \omega_0^2} E \quad (25)$$

Of course, the magnitude of polarization must depend on the effective volume per each dipole. Both theoretical and experimental analyses have shown that the Compton’s wavelength [21] is an appropriate size for a virtual e^+e^- pair:

$$\lambda_C = \frac{\hbar}{m_e c} \quad (26)$$

This brings us to the value of the vacuum polarization:

$$P_0 = \frac{q_e^2}{m_e \omega_0^2 \lambda_C^3} E \quad (27)$$

Because of the similarity with (1), we can assign to the quantity multiplying E the role of an effective permittivity:

$$\tilde{\epsilon}_0 = \frac{q_e^2}{m_e \omega_0^2 \lambda_C^3} \quad (28)$$

By inserting the relevant numbers for various terms in (28) we obtain a numerical value of $\tilde{\epsilon}_0 = 1.62 \times 10^{-12}$ As/Vm, which is much lower than the actual value of ϵ_0 in (4), only some 18% of it. **What went wrong?**

Well, we made one completely arbitrary assumption right from the start, that the dominant virtual particles in the quantum vacuum are e^+e^- pairs.

But what if the zero-point energy actually allows heavier particles to dominate? One possible assumption, inferred from other observations [22], could be that the dominant type of particles are pions, $\pi^+\pi^-$ pairs [23].

However, it is even more plausible that, instead of a single type of particle pairs involved, there is a Gaussian distribution of probabilities of the vacuum energy fluctuations, and consequently a whole range of particle pairs are actually produced, with the center of mass averaged to anywhere in between.

With such a scenario it would be very difficult to calculate the exact permeability value from basic elementary values. But by providing a value well within the correct order of magnitude we have shown that the whole idea is feasible.

The branch of physics which combines quantum phenomena with classical formalism, as just described, is referred to as the ‘**stochastic electrodynamics**’ [24].

The Polarizable Vacuum in Action

The next question we need to ask ourselves is whether the idea of a polarizable physical vacuum [25] can be extended to replicate other effects known from classical physics and the General Theory of Relativity (GTR) with adequate precision.

Our first assumption must regard the polarizability itself. We postulate that additional conditions, such as the presence of ‘real’ EM fields (energy), or massive objects, can modulate the vacuum polarization in a presumably linear fashion. This is of course an *ad hoc* assumption, but it is justified by the analogy of similar effects existing in ordinary dielectric materials. Let us assume the following formal representation of the variable polarization:

$$\mathbf{D} = \varepsilon \mathbf{E} = \kappa \varepsilon_0 \mathbf{E} \quad (29)$$

where the factor κ represents the relatively small amount ($|\kappa - 1| \ll 1$) of the altered permittivity of the free space.

We are going to investigate if there exists a consistent description of various phenomena within the polarizable vacuum paradigm, and in that analysis the factor k will play the key role, as the only variable at the cause of those phenomena.

How the Velocity of Light Varies with κ

Let us start from the velocity of light in the variable polarizability conditions. Our reference is of course the value of c in (3). Although it is questionable if we are allowed to take as the reference the value of c obtained in the vicinity of considerable mass (on the surface of Earth), it is believed that the error made is small, since the Earth’s mass is relatively small, as even the much larger mass of the Sun produces a barely visible relativistic bending of light. And anyway, GTR postulates the constancy of the velocity of light in any inertial frame of reference. Again, it may be argued that the Earth’s gravity field and its daily rotation can hardly represent an inertial frame, but the field is low and the rotation is slow, so the error should be small.

A good starting point could be the well known relation of the fine structure constant α [26]:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0 \hbar c} \quad (30)$$

The conservation of charge ensures the constancy of the value of elementary charge q_e , whilst the conservation of the angular momentum for a circularly polarized photon (even under variable polarization conditions) ensures the constancy of \hbar . Thus we can assume that only c and ε_0 will vary with κ . In accordance with (29):

$$\varepsilon(\kappa) = \kappa\varepsilon_0 \quad (31)$$

whilst c will vary as:

$$c' = \frac{1}{\sqrt{\mu(\kappa)\varepsilon(\kappa)}} \quad (32)$$

By inserting (31), substituting ε_0 from (4), and rearranging a little, we obtain:

$$c' = c\sqrt{\frac{\mu_0}{\kappa\mu(\kappa)}} \quad (33)$$

So what happens with the fine structure constant (30) in altered polarization conditions? By inserting (31) and (33) into (30) we obtain:

$$\alpha' = \frac{q_e^2}{4\pi\kappa\varepsilon_0\hbar c\sqrt{\frac{\mu_0}{\kappa\mu(\kappa)}}} = \frac{q_e^2}{4\pi\varepsilon_0\hbar c}\sqrt{\frac{\kappa\mu(\kappa)}{\kappa^2\mu_0}} = \alpha\sqrt{\frac{\mu(\kappa)}{\kappa\mu_0}} \quad (34)$$

This shows that α' might be a function of κ .

However, astronomical observations have shown that, by comparing the measurements of α in the spectra of stars in galaxies with a high red shift (high z) with the laboratory reference spectra, there is no evidence of α changing [27] with the age of the Universe ($|\Delta\alpha/\alpha| < 10^{-8}$). Because α must remain constant with the change of κ , it is reasonable to assume that the magnetic permeability μ must be affected by κ in the same way as ε , which allows us to write:

$$\mu(\kappa) = \kappa\mu_0 \quad (35)$$

and consequently $\alpha' = \alpha$. Thus from (31) and (35) it is possible to conclude that the impedance of free space will not be affected by altered polarization:

$$Z_0 = \sqrt{\frac{\mu(\kappa)}{\varepsilon(\kappa)}} = \sqrt{\frac{\kappa\mu_0}{\kappa\varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad (36)$$

which is just what is required to maintain the electric (**D**) to magnetic (**B**) energy ratio constant during the adiabatic movement of atoms from a point with lower vacuum polarization to a point with a higher vacuum polarization. This leads us to the conclusion that the velocity of light must vary with different vacuum polarizability values as:

$$c' = \frac{1}{\sqrt{\mu(\kappa)\varepsilon(\kappa)}} = \frac{1}{\sqrt{\kappa^2\mu_0\varepsilon_0}} = \frac{c}{\kappa} \quad (37)$$

It is important to realize that this modified velocity of light resulting from the modified vacuum polarization does not influence the Lorentz transformations [28], nor any other relations in GTR. In fact, we are trying to find a physical justification of

the effects of a variable velocity of light as found in various relativistic situations, such as the bending of light [29] near the Sun, the Shapiro's delay [30] of RF signals passing near the Sun, the gravitational red shift [31] found in massive star spectra, etc.

It is also important to note the difference between the vacuum polarizability (37), where the velocity of light varies inversely with κ , and ordinary dielectric media (9), in which it varies inversely with $\sqrt{\kappa}$. While that may seem strange, we should remember that recently we have been able to construct a new class of so called "meta-materials" [32], in which both μ and ε are modified (within a limiter range of frequencies, though), achieving some previously unattainable optical effects ("slow" light, negative angles of refraction [33], etc.).

So we expect that expressions like $E = mc^2$ are still valid, but in conditions of altered vacuum polarization we must apply c/κ , instead of just c .

We shall continue our discussion by considering how the altered vacuum polarization will affect the time, length, oscillation frequency, energy, and mass.

Time and Length

We know from GTR that the measurement of a time interval Δt , and length Δx are both affected [34] by the velocity of the laboratory reference frame v relative to the velocity of light c . The correction factor in Lorentz transformations is:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (38)$$

so that for events satisfying $\Delta x = 0$:

$$\Delta t' = \gamma \Delta t \quad (39)$$

whilst for events satisfying $\Delta t = 0$:

$$\Delta x' = \frac{\Delta x}{\gamma} \quad (40)$$

Because of the Einstein's equivalence principle [35], similar relations must apply also in the case of a gravitational field.

From this we may infer that if the vicinity of a large mass alters the vacuum polarization, the effects of measuring Δt and Δx as a function of κ must follow the same relations as (39) and (49). The time interval is then altered as:

$$\Delta t' = \Delta t \sqrt{\kappa} \quad (41)$$

and the length is altered as:

$$\Delta x' = \frac{\Delta x}{\sqrt{\kappa}} \quad (42)$$

It must be noted that $\sqrt{\kappa}$ has been assigned the role of the Lorentz transform factor γ , which has been inferred from the assumption of the constancy of the velocity of light. This is not just a coincidence, as will be explained later!

Frequency, Energy, and Mass

It is possible to apply again the equivalence principle and find the energy dependence from the energy expression for the photon, which is a function of frequency (19). Since frequency is defined as the number of periods in a unit of time, we can assign to Δt the value of a period, $\Delta t = 1/\nu = 2\pi/\omega$, and write:

$$\omega' = \frac{2\pi}{\Delta t'} = \frac{2\pi}{\Delta t \sqrt{\kappa}} = \frac{\omega}{\sqrt{\kappa}} \quad (43)$$

This expression is at the core of the gravitational red shift, as we will see later. By referring to (19) we can now find how the energy varies with κ :

$$W' = \hbar\omega' = \hbar \frac{\omega}{\sqrt{\kappa}} = \frac{W}{\sqrt{\kappa}} \quad (44)$$

Finally, by referring to the famous $E = mc^2$, we can show how the value of the mass of a small test body must change in a strong gravitational field. We first express the mass as $m = E/c^2$ and then apply (44) to energy and (37) to the velocity of light, so we obtain:

$$m' = \frac{E'}{(c')^2} = \frac{\frac{E}{\sqrt{\kappa}}}{\left(\frac{c}{\kappa}\right)^2} = \frac{E}{c^2} \cdot \frac{\kappa^2}{\sqrt{\kappa}} = m \kappa^{\frac{3}{2}} \quad (45)$$

Influences of Vacuum Polarization on Matter

Regarding the measuring ‘rigid’ rods in the light of length contraction (42), let us examine the classical Bohr radius [36] of a hydrogen atom in its ground state:

$$r_{B0} = \frac{4\pi\epsilon_0\hbar^2}{m_e q_e^2} = \frac{\hbar}{m_e c \alpha} \quad (46)$$

We have already explained why \hbar and α must remain unchanged, so our only variables are the electron mass m_e and the velocity of light c . Therefore in conditions where $\kappa \neq 1$, the Bohr radius must adapt to:

$$r'_{B0} = \frac{\hbar}{\alpha} \cdot \frac{1}{m'_e c'} = \frac{\hbar}{\alpha} \cdot \frac{1}{m_e \kappa^{\frac{3}{2}} \frac{c}{\kappa}} = \frac{r_{B0}}{\sqrt{\kappa}} \quad (47)$$

which is the same as the length contraction (42). In fact, other similar spatial length relations, like the classical size of the electron, the Compton wavelength, etc., all follow the same relationship, so the expression is universal.

The calculation of the Bohr radius clearly shows that there are no rods rigid enough to serve as a fundamental measure of length. Any material rod varies its length in accordance with the local value of vacuum polarization. From the standpoint of stochastic electrodynamics, the vacuum polarizability is the fundamental cause of the apparent spacetime metric variability in GTR. The only difference is that in GTR all the effects are calculated by assuming a constant velocity of light and then using the

Lorentz transformations in reference frames of different velocity, whilst in stochastic electrodynamics everything is a physical consequence of the variable vacuum polarizability. So we need to verify that the constant velocity of light (the basic GTR postulate) is also being respected in the stochastic electrodynamics interpretation.

The Problem of the Measured Constant Velocity of Light

In accordance with previous calculations the velocity of light, measured by a rod Δx and by a clock indicating an interval Δt between the passage of a light pulse at each end of the rod, should be:

$$\frac{\Delta x}{\Delta t} = \frac{c}{\kappa} \quad (48)$$

However, a rod of length Δx as measured in reference conditions far from any massive body, will contract to $\Delta x\sqrt{\kappa}$ within the gravity potential. Likewise, the clock measuring Δt in reference conditions will slow down in the gravity potential and will display $\Delta t/\sqrt{\kappa}$. Thus the velocity of light measured using the same rod and the same clock within a gravity potential causing $\kappa > 1$ will yield a value:

$$v = \frac{\frac{\Delta x\sqrt{\kappa}}{\Delta t}}{\frac{1}{\sqrt{\kappa}}} = \frac{\Delta x}{\Delta t}\kappa = \frac{c}{\kappa} \cdot \kappa = c \quad (49)$$

So this confirms the assumption of the invariance of measurement of the velocity of light in all reference frames, in the same way as it is customary in GTR.

Influence of a Massive Body on Vacuum Polarization

So far we have been treating only relative changes of various physical properties within a variable vacuum polarization κ . We need to establish the actual form of influence of a massive body on its surrounding vacuum polarization. Let us have a small test particle, with a mass m_0 and charge q , within a relatively weak gravity potential, such as the Earth gravity. The Lorentz force [37] equation, acting on the test particle within an external electrical field \mathbf{E} and the magnetic field \mathbf{B} , and moving with velocity \mathbf{v} , has the form:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (50)$$

Variation of the Lagrangian density $\delta \int L_d dx dy dz dt$ with regard to the particle variables (following standard principle techniques) leads to the equation for particle motion in a variable dielectric vacuum, $\kappa \neq 1$:

$$\frac{d}{dt} \left[\frac{(m_0 \kappa^{\frac{3}{2}}) \mathbf{v}}{\sqrt{1 - \left(\frac{v}{c/\kappa}\right)^2}} \right] = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla \left[\frac{m_0 c^2}{\sqrt{\kappa}} \sqrt{1 - \left(\frac{v}{c/\kappa}\right)^2} \right] \quad (51)$$

This can be rewritten as:

$$\frac{d}{dt} \left[\frac{(m_0 \kappa^{\frac{3}{2}}) \mathbf{v}}{\sqrt{1 - \left(\frac{v}{c/\kappa}\right)^2}} \right] = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{m_0 c^2}{\sqrt{\kappa}} \cdot \frac{1 + \left(\frac{v}{c/\kappa}\right)^2}{2 \sqrt{1 - \left(\frac{v}{c/\kappa}\right)^2}} \cdot \frac{\nabla \kappa}{\kappa} \quad (52)$$

In addition to the Lorentz force, there is a second term representing the dielectric force proportional to the gradient of the vacuum polarization, $\nabla \kappa / \kappa$. This term must be equally effective on any particle, charged or neutral, since it acts on the mass m_0 , thus accounting for the gravitational potential, either in Newtonian or GTR form. It might be interesting to note that with $m_0 \rightarrow 0$ and $v \rightarrow c/\kappa$, as would be the case for a photon, the deflection of the trajectory is twice as the deflection of a slow moving massive particle. This is an important indication of conformity with GTR.

Variation of the Lagrangian density with regard to the κ variable leads to the expression of the generation of vacuum polarization within GTR, owed to the presence of both matter and fields. The equation has three right hand side terms:

$$\nabla^2 \sqrt{\kappa} - \frac{1}{(c/\kappa)^2} \cdot \frac{\partial^2 \sqrt{\kappa}}{\partial t^2} = - \frac{\sqrt{\kappa}}{4\lambda} \left[\mathcal{P}(\kappa) + \mathcal{Q}(\kappa) + \mathcal{R}(\kappa) \right] \quad (53)$$

Here $\mathcal{P}(\kappa)$ represents the change in vacuum polarizability by the mass density, with the vector \mathbf{r} as the distance of the test particle from the system mass center:

$$\mathcal{P}(\kappa) = \frac{m_0 c^2}{\sqrt{\kappa}} \cdot \frac{1 + \left(\frac{v}{c/\kappa}\right)^2}{2 \sqrt{1 - \left(\frac{v}{c/\kappa}\right)^2}} \cdot \delta^3(\mathbf{r} - \bar{\mathbf{r}}) \quad (54)$$

$\mathcal{Q}(\kappa)$ is the change caused by the applied EM field energy density:

$$\mathcal{Q}(\kappa) = \frac{1}{2} \left(\frac{B^2}{\kappa \mu_0} + \kappa \varepsilon_0 E^2 \right) \quad (55)$$

and $\mathcal{R}(\kappa)$ is the change driven by vacuum polarization energy density itself:

$$\mathcal{R}(\kappa) = - \frac{\lambda}{\kappa^2} \left[(\nabla \kappa)^2 + \frac{1}{(c/\kappa)^2} \left(\frac{\partial \kappa}{\partial t} \right)^2 \right] \quad (56)$$

In order to demonstrate the applicability of the general equation (53) in a particular case of a static gravity field of a spherical mass distribution (a planet or a star), we must find the static solution, $\partial \kappa / \partial t = 0$, to equation (53) by solving a suitably modified form:

$$\nabla^2 \sqrt{\kappa} = \frac{1}{4\kappa^{3/2}} (\nabla \kappa)^2 = \frac{1}{\sqrt{\kappa}} (\nabla \sqrt{\kappa})^2 \quad (57)$$

or more simply:

$$\frac{d^2\sqrt{\kappa}}{dr^2} + \frac{2}{r} \cdot \frac{d\sqrt{\kappa}}{dr} = \frac{1}{\sqrt{\kappa}} \left(\frac{d\sqrt{\kappa}}{dr} \right)^2 \quad (58)$$

where the identity $(\nabla\kappa)^2 = 4\kappa(\nabla\sqrt{\kappa})^2$ has been used.

The solution which satisfies the GTR, also in the Newtonian limit, has a simple exponential form:

$$\sqrt{\kappa} = e^{GM/rc^2} \quad (59)$$

which can be approximated by expanding it into a series:

$$\kappa = e^{2GM/rc^2} = 1 + \frac{2GM}{rc^2} + \frac{1}{2} \left(\frac{2GM}{rc^2} \right)^2 + \dots \quad (60)$$

It is possible to verify this solution by substituting it into the equation for particle motion (52). This solution replicates (to the appropriate order) the usual GTR Schwarzschild metric predictions in the weak field limit conditions (i.e. Solar system).

With the obtained solution (59) or (60) we can return to some of the previous questions, say, the gravitational red shift, and find a more detailed form in order to find the frequency shift of the photon emitted by an atom on the surface of a star of mass M and radius R . The photon detected far away from the star will appear red shifted by the following amount:

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} \approx - \frac{GM}{Rc^2} \quad (61)$$

where we have assumed $GM/Rc^2 \ll 1$. The photon, after having climbed up the gravity potential of the star, will retain its acquired frequency unchanged, and the change in frequency can be tested locally by comparing it with photons emitted by the same type of atoms at the same temperature, but within the weak gravity field of the laboratory.

The Bending of Starlight

With that same result it is also possible to analyse the amount of the bending of light rays from a distant star passing near a massive body, like in the classic GTR test performed by the Eddington's expedition during the solar eclipse in May, 1919.

The light ray from a distant star, while passing close to the Sun, will experience a gradual slowing of wavefront velocity coming towards the Sun, and a gradual increasing velocity in leaving the Sun's gravity field. Because κ increases closer to a massive body ($\kappa > 1$), the velocity of light will vary as c/κ . The part of the wavefront closer to the Sun will thus experience a greater slowdown than the part of the wavefront passing further away. This is seen from Earth as an apparent shift of the position of the star close to the Sun's disk edge in the outward direction. In GTR terms, this deflection is a measure of local spacetime curvature. We are interested in calculating the total bending angle.

Because in case of the Sun the total deflection is small ($\varphi < 2$ arc-seconds) we can apply the usual low angle approximations throughout the calculation. And for the same reason we will not make a big mistake if we approximate the variable velocity of light to the first order term of the series expansion (60) of κ :

$$v = \frac{c}{\kappa} \approx \frac{c}{1 + \frac{2GM}{rc^2}} \approx c \left(1 - \frac{2GM}{rc^2} \right) \quad (62)$$

In this relation the radius-vector r denotes the distance of the wavefront from the center of the Sun as it travels by from $-\infty$ to $+\infty$, with the minimum distance of $R + \delta$, where R is the Sun's radius, and δ is the minimum distance from the Sun's surface. By assigning z to the distance of the wavefront along the line of sight (perpendicular to $R + \delta$), the radius-vector becomes $r = \sqrt{(R + \delta)^2 + z^2}$, so the equation (62) can be written as:

$$v \approx c \left[1 - \frac{2GM}{c^2} \cdot \frac{1}{\sqrt{(R + \delta)^2 + z^2}} \right] \quad (63)$$

The differential velocity of light, assuming $\delta \ll R$, is then:

$$\Delta v \approx \frac{2GM}{c^2} \cdot \frac{R\delta}{(R^2 + z^2)^{3/2}} \quad (64)$$

As the wavefront travels a distance $dz \approx v dt$, the differential velocity along the path of light results in an accumulated wavefront path difference Δz :

$$\Delta z = \Delta v dt \approx \frac{2GM}{c^2} \cdot \frac{R\delta}{(R^2 + z^2)^{3/2}} dz \quad (65)$$

This results in an accumulated tilt angle of:

$$d\varphi \approx \tan(d\varphi) = \frac{\Delta z}{\delta} \approx \frac{2GM}{c^2} \cdot \frac{R\delta}{(R^2 + z^2)^{3/2}} dz \quad (66)$$

By integrating (66) over the entire path $-\infty < z < +\infty$ yields:

$$\varphi \approx \frac{4\pi GM}{Rc^2} \quad (67)$$

By inserting $G = 6.672 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$, $M = 1.9891 \times 10^{30} \text{ kg}$, and $R = 696 \times 10^6 \text{ m}$, we obtain $\varphi \approx 1.75$ arc-seconds, which is exactly the value predicted by Einstein's GTR in 1915 [38], and experimentally verified by the Eddington's expedition [39] in 1919 (which was between 1.2 and 1.9 arc-seconds, mainly because of the imperfect optics of the portable telescopes used).

Discussion

Within the formalism of GTR the effects owed to presence of mass and energy are being customarily interpreted in the sense of a change in effective curvature of the local spacetime metric. This has been historically understood mainly in form of a curvature in higher dimensions, with a consequence that the Universe as a whole should also be curved (“closed”), owed to its own mass and energy density.

In more modern interpretations the spacetime curvature is still understood as a variable local metric, but not necessarily as a curvature into higher dimensions. Nevertheless, the idea of the existence of higher dimensions, although ‘compactified’, is still present as a consequence of treating the quantum objects as strings of finite dimensions, within all the versions of String Theories and the ‘Membrane’ Theory.

Astronomical observations of various phenomena during the last 30 years, such as the velocity distribution in the galactic disc rotation [40], the amount of gravitational micro-lensing in galactic clusters [41], etc., have led to the assumption of the existence of large amounts of ‘cold dark matter’ [42] (CDM, non-baryonic, matter moving at non-relativistic velocities, and which does not interact electromagnetically, but only weakly and gravitationally, known in literature also as the WIMPs — weakly interacting massive particles; however, alternative explanations also exist [43]). Their calculated mass density far exceeds the density of ordinary baryonic matter, with a ratio of CDM to baryonic matter of about 82% to 18%, as inferred from observations.

Also, the observations of type Ia supernovae [44] apparent brightness versus distance (the distance calculated on the basis of the Doppler-caused [45] red shift of radiation spectra — the Hubble law) from the mid 1990s onward, has led to the idea that the Universe’s rate of expansion is accelerating. The cause of that acceleration has been attributed to a form of ‘dark energy’ [46], usually interpreted in terms of a ‘cosmological constant’ Λ term in the Einstein–Friedmann equation [47] (although a number of other interpretations exist [48]). The implied ratios of the energy-mass content of Λ to CDM to baryonic matter in the Universe has been estimated to be approximately 74% to 22% to 4% (of this last 4% belonging to ordinary baryonic matter, some 3.6% is in form of interstellar gas, mostly hydrogen, and only 0.4% is associated with stars, planets, asteroids, etc.).

The energy density attributed to dark energy, estimated from the supernova Ia measurements, is about 10^{-120} in reduced Planck’s units [49], equivalent to a mass density of 10^{-32} kg/m³, which is well beyond any possibility of detecting it in laboratory conditions. However, quantum electrodynamics predicts a vacuum energy density of the order of 1 reduced Planck’s unit. This difference of 120 orders of magnitude is the most embarrassing discrepancy in the history of physics, and it needs to be resolved as soon as possible! The approach taken by stochastic electrodynamics may well offer an adequate solution.

Anyway, this $25\times$ higher energy and matter content increases the Universe density, and should strongly influence the general metric curvature of the Universe, thus it has been expected to be observable in several astronomical phenomena. However, the recent studies of the variations in the cosmic microwave background radiation [50] (by experiments COBE, TOCO, BOOMERANG, MAXIMA, WMAP,

and most recently the Planck Surveyor mission), have explicitly shown that the Universe is Euclidean ('flat'), homogeneous, and isotropic to the highest values of the red shift factor ($z \leq 17$). In topological studies this is treated in the form of the relative density parameter Ω , which is a measure of the observed mass-energy density to the critical mass-energy density implied by the theory (the Friedmann–Lemaître–Robertson–Walker metric [51]). The studies show that effectively $\Omega = 1$, implying a radius of the curvature greater than the extent of the observable Universe.

The fact that the effective density of the Universe has practically the same value as the theoretical critical value, indicated by $\Omega = 1$, may be just a coincidence, or it might be the consequence of any of the several possible types of error. Either we are misinterpreting the effective red shift as being purely Doppler in nature (whereas it is actually in part also gravitational, and maybe also caused by other effects), or we are making some measurement errors that we are not aware of, or we have an inadequate working model and the Einstein–Friedmann equation does not apply for this type of Universe, or some other part of the theory is wrong, or we have made some wrong assumptions at the very start in the development of the theory, or the Universe actually is 'flat' and the apparent curvature effects are being misinterpreted by the GTR.

In light of the results that have been obtained from studies within the stochastic electrodynamics formalism, the fact that the Universe metric is essentially 'flat' gives us the reason and justification to reinterpret all the GTR spacetime curvature effects in terms of the change of local vacuum energy density, which affects all electromagnetic and matter phenomena within a flat Euclidean space.

It is the author's belief that the usual interpretation of gravitational phenomena as a consequence of space-time metric curvature in the framework of GTR is unphysical, and must be replaced by the interpretation of variable vacuum polarizability owed to the variable energy density. Still, the true nature of the polarizability κ remains to be found and possibly defined by more basic physical processes. Of course, other people may find the notion of quantum vacuum fluctuations equally unphysical. For now, that remains a matter of personal preference.

Conclusion

We have shown that it is possible to replicate several important results of the General Theory of Relativity (GTR) by starting from the quantum–mechanical prediction of quantum vacuum fluctuations, assuming the polarizability of virtual particle pairs generated by those fluctuations, and employing a classical formalism for deriving the results from classical electrodynamics modified by the of vacuum polarizability, and using the first principles as the starting points of those derivations.

Whilst most of the relations shown are only indicative of the underlying physical processes, the results obtained are identical to those obtained by conventional methods. It is hoped by the author that future research will bring up enough fundamental insight in accordance with experimental measurements that eventually a comprehensive and broadly valid theory may be built.

References

Note 1: The basic ideas explained here date back as far as 1920, and probably earlier, and the references are too many to list completely. The reference list provided is only indicative of the most important physical facts and phenomena, and is intended more as a source for recommended further reading for non-specialists, rather than an actual reference list. However, the interested specialists are forwarded to [24] and [25], where a more complete presentation of stochastic electrodynamics and of the polarizable vacuum are given, with a fairly representative list of contributing authors. Here we will only mention a few in the approximate chronological order: *W. Nernst, T.W. Marshall, A.D. Sakharov, T.H. Boyer, A. Rueda, B. Haisch, H.E. Puthoff, M. Ibison, E.W. Davis, L. de la Peña, G. Cavalleri*. Conditionally, it is possible to count here also *O. Heaviside, N. Tesla, A. Michelson, H. Lorentz*, and even *M. Planck* (his work of 1911, not the one of 1900).

Note 2: The fact that the web-based encyclopedia Wikipedia is often considered as not being always a completely reliable source has prompted the author to investigate several standard physics course textbooks, only to find that many of those textbooks are either hopelessly outdated in certain subjects, or are too vague and superficial in explanations, or have errors of their own. In the situation where a reliable standard reference is not at hand to everyone, a decision has been made to nevertheless rely on Wikipedia as a broadly available and relatively correct and up to date source, suitable enough for most readers. At least, in contrast with printed media, Wikipedia provides warnings on possibly biased reports and the lack of necessary citations. Of course, more demanding readers are encouraged to also look for specialized theoretical articles, as well as beyond.

Note 3: Most of the material discussed in this short review has been already shown elsewhere in one way or another, and the author might not be aware of all the relevant sources. The equations have been checked and found to be substantially free of gross errors; however, any remaining errors or misinterpretations in the presented work is at the responsibility of this author alone.

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