Lectures on

Network Information Theory

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The Early Years

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- The course had some of today's big names in our field:





Syllabus Circa 1983

COURSE OUTLINE

1. Review of basic entropy and mutual information relations.
2. The Asymptotic Equipartition Property, joint typicality.
3. Shannon's source and channel coding theorem.
4. Multiple access channel.
5. The Slepian-Wolf source coding theorem.
6. The multiple access channel with correlated sources.
7. The broadcast channel.
8. Basic rate distortion theory.
9. Source coding with side information.
10. Channel with states.
11. The One Bit Theorem.

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- There was no theoretical or practical interest in these results then

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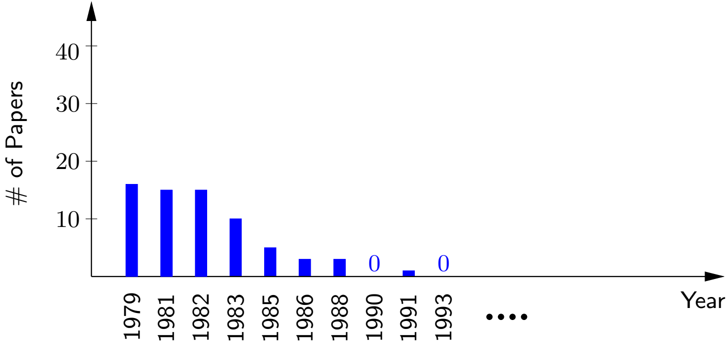

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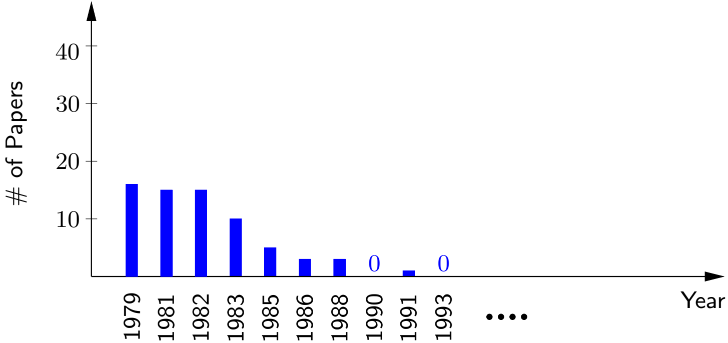

- I stopped teaching the course and moved on to other things

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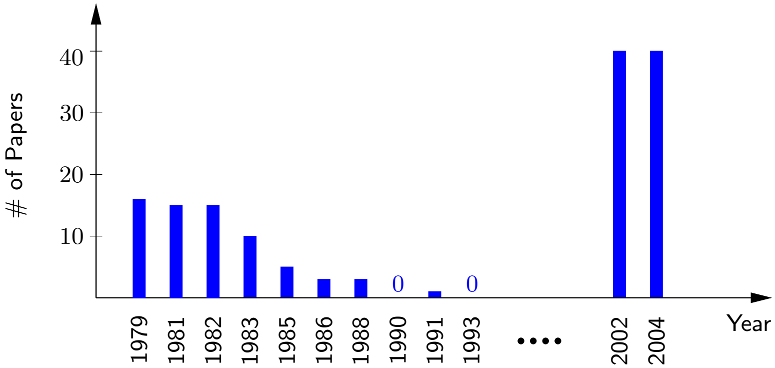
- By late 90s, the Internet and wireless communication began to revive interest in NIT; and by early 2000s, the field was in full swing



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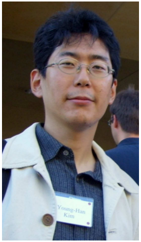

Syllabus Circa 2002

Rough Schedule

Sept 26	Introduction, review
Oct 1	Review
Oct 3	Review, MAC
Oct 8	No lecture
Oct 10	MAC, correlated sources
Oct 15	Correlated sources
Oct 17	Broadcast
Oct 22	Broadcast, interference
Oct 24	Interference, relay
Oct 29	Relay, feedback
Oct 31	Rate distortion
Nov 5	Rate distortion
Nov 7	Side information, multiple descriptions
Nov 12	Channel with state (Project selections)
Nov 14	Channel with state

Technology from Early 80s to 2002

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- Multi-media: From film cameras and Sony Walkman to digital cameras and iPod

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Answer: All of the above

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  Scaling laws

  Deterministic/high SNR approximations (within xx bits)

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  Deterministic/high SNR approximations (within xx bits)
- Attempts to consummate marriage (or at least dating) between IT and networking

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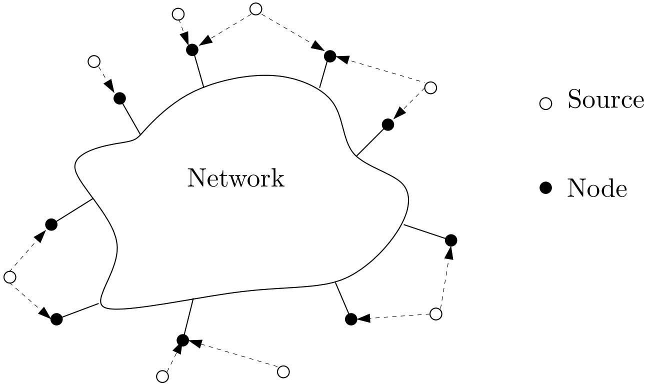
- Emphasize extension to networks
- Use clean and unified notation and terminology

Outline

1. The First Lecture
2. Achievability for DM Sources and Channels
3. Gaussian Sources and Channels
4. Converse
5. Extension to Networks
6. Conclusion

Network Information Flow

- Consider a general networked information processing system:



- Sources: data, speech, music, images, video, sensor data
- Nodes: handsets, base stations, servers, sensor nodes
- Network: wired, wireless, or hybrid

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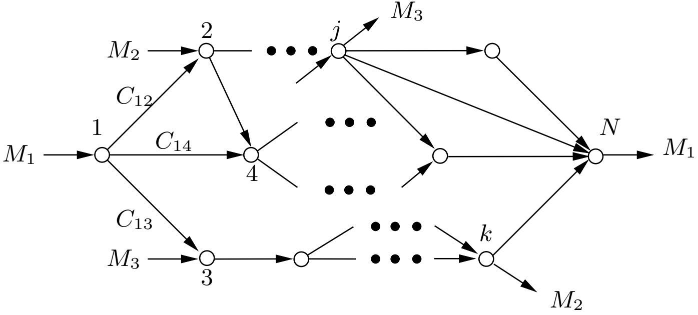
- Information flow questions:

  What are the necessary and sufficient conditions on information flow?

  What are the optimal schemes/techniques needed to achieve them?
- The difficulty in answering these questions depends on:
  - Source and network models
  - Information processing goals
  - Computational capabilities of the nodes

Example: Multi-Commodity Flow

- If the sources are commodities with demands (rates in bits/sec); the nodes are connected by noiseless rate-constrained links; each intermediate node forwards the bits it receives; the goal is to send each commodity to a destination node; the problem reduces to the multi-commodity flow with known conditions on optimal flow



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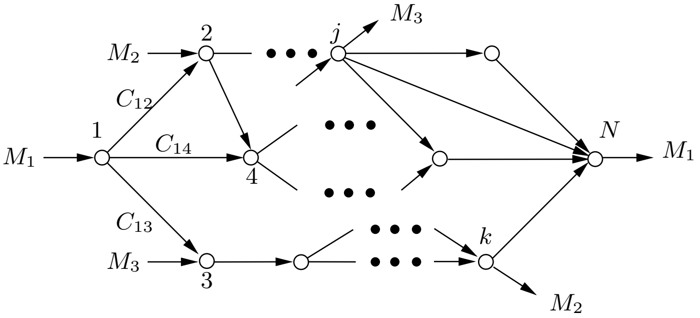

- For single commodity, these conditions reduce to the celebrated max-flow min-cut theorem

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  - Real-world communication nodes may allow for more complex node operations than forwarding
  - The goal in many information processing systems is to partially recover the sources or to compute/make a decision
- Network information theory aims to answer the information flow questions while capturing essential elements of real-world networks in the probabilistic framework of Shannon's information theory

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- Coding techniques developed, e.g., superposition, successive cancellation, Slepian–Wolf, Wyner–Ziv, successive refinement, dirty paper coding, network coding are starting to impact real-world networks
- However, many basic problems remain open and a complete theory is yet to be developed

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  Part II: Single-hop Networks

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- Shannon's point-to-point communication theorems: Random coding; joint typicality encoding/decoding

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  Independent messages over noisy channels:

  Correlated sources over noiseless (wireline) channels:

  Correlated sources over DM channels:

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  Correlated sources over DM channels:

  Separation does not hold in general; common information; sufficient conditions for MAC, BC

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  - Gaussian networks: scaling laws; high SNR approximations

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Correlated sources over noiseless (wireline) channels:

Multiple descriptions networks; interactive source coding

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  Asynchronous communication:

  Random arrivals; asynchronous MAC

Balancing Introduction of Models and Techniques: Broadcast Channel

Degraded broadcast channels:

Channels with state:

Fading channels

General broadcast channels:

Gaussian vector channels:

Balancing Introduction of Models and Techniques: Broadcast Channel

Degraded broadcast channels:

- Superposition coding inner bound
- Degraded broadcast channels
- AWGN broadcast channels
- Less noisy and more capable broadcast channels

Channels with state:

Fading channels

General broadcast channels:

Gaussian vector channels:

Balancing Introduction of Models and Techniques: Broadcast Channel

Degraded broadcast channels:

Channels with state:

- Compound channel
- Arbitrarily varying channel
- Channels with random state
- Causal state information available at encoder
- Noncausal state information available at the encoder
- Writing on dirty paper
- Partial state information

Fading channels

General broadcast channels:

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Balancing Introduction of Models and Techniques: Broadcast Channel

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General broadcast channels:

- DM-BC with degraded message sets
- 3-Receiver multilevel DM-BC with degraded message sets
- Marton inner bound
- Relationship to Gelfand–Pinsker
- Nair–El Gamal outer bound
- Inner bound for more than 2 receivers

Gaussian vector channels:

Balancing Introduction of Models and Techniques: Broadcast Channel

Degraded broadcast channels:

Channels with state:

Fading channels

General broadcast channels:

Gaussian vector channels:

- Gaussian vector channel
- Gaussian vector fading channel
- Gaussian vector multiple access channel
- Spectral Gaussian broadcast channel
- Vector writing on dirty paper
- Gaussian vector broadcast channel

Typicality

- Let (u^n, x^n, y^n) be a triple of sequences with elements drawn from finite alphabets $(\mathcal{U}, \mathcal{X}, \mathcal{Y})$. Define their joint type as

$$\pi(u, x, y | u^n, x^n, y^n) = \frac{|\{i : (u_i, x_i, y_i) = (u, x, y)\}|}{n}$$

for $(u, x, y) \in \mathcal{U} \times \mathcal{X} \times \mathcal{Y}$

- Let $(U, X, Y) \sim p(u, x, y)$. The set $\mathcal{T}_\epsilon^{(n)}(U, X, Y)$ of ϵ-typical n-sequences is defined as

$$\{(u^n, x^n, y^n) : |\pi(u, x, y | u^n, x^n, y^n) - p(u, x, y)| \le \epsilon \cdot p(u, x, y) \text{ for all } (u, x, y) \in \mathcal{U} \times \mathcal{X} \times \mathcal{Y}\}$$

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- Typical average lemma: Let $x^n \in \mathcal{T}_\epsilon^{(n)}(X)$. Then for any $g(x) \geq 0$,

$$(1 - \epsilon) \, \mathsf{E}(g(X)) \leq (1/n) \textstyle\sum_{i=1}^n g(x_i) \leq (1 + \epsilon) \, \mathsf{E}(g(X))$$

Joint Typicality Lemma

- Let $(U, X, Y) \sim p(u, x, y)$.
  1. Let $(u^n, x^n) \in \mathcal{T}_\epsilon^{(n)}(U, X)$ and $\tilde{Y}^n \sim \prod_{i=1}^n p_{Y|U}(\tilde{y}_i|u_i)$. Then

$$\mathsf{P}\{(u^n, x^n, \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)}(U, X, Y)\} \doteq 2^{-nI(X;Y|U)}$$

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  $$\mathsf{P}\{(u^n, x^n, \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)}(U, X, Y)\} \doteq 2^{-nI(X;Y|U)}$$
  2. If $(\tilde{U}^n, \tilde{X}^n) \sim p(\tilde{u}^n, \tilde{x}^n)$ and $\tilde{Y}^n \sim \prod_{i=1}^n p_{Y|U}(\tilde{y}_i|\tilde{u}_i)$. Then
  $$\mathsf{P}\{(\tilde{U}^n, \tilde{X}^n, \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)}(U, X, Y)\} \le 2^{-n(I(X;Y|U)-\delta(\epsilon))}$$

Packing Lemma

Let $(U, X, Y) \sim p(u, x, y)$ and $\tilde{U}^n \sim p(\tilde{u}^n)$. Let $X^n(m)$, $m \in \mathcal{A}$, where $|\mathcal{A}| \leq 2^{nR}$, be random sequences, each distributed according to $\prod_{i=1}^{n} p_{X|U}(x_i|\tilde{u}_i)$ with arbitrary dependence on the rest

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Let $\tilde{Y}^n \in \mathcal{Y}^n$ be another random sequence, conditionally independent of each $X^n(m), m \in \mathcal{A}$, given $\tilde{U}^n$, and distributed according to an arbitrary pmf $p(\tilde{y}^n|\tilde{u}^n)$

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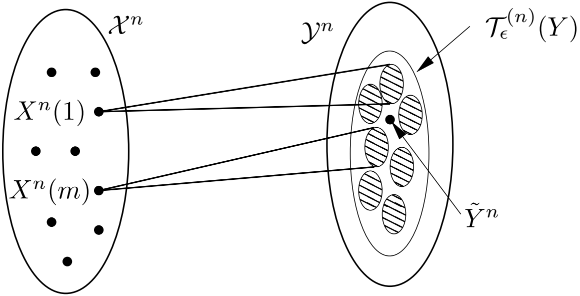
Then, there exists $\delta(\epsilon) \to 0$ as $\epsilon \to 0$ such that

$$\mathsf{P}\{(\tilde{U}^n, X^n(m), \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m \in \mathcal{A}\} \to 0$$

as $n \to \infty$, if $R < I(X; Y|U) - \delta(\epsilon)$

The sequences $X^n(m)$, $m \in \mathcal{A}$, represent codewords. The $\tilde{Y}^n$ sequence represents the received sequence as a result of sending a codeword $\notin \mathcal{A}$



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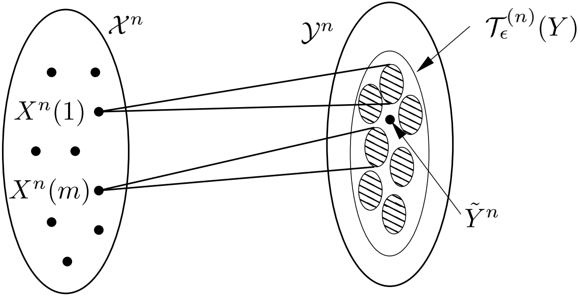

The lemma shows that under any pmf on $\tilde{Y}^n$ the probability that some codeword in $\mathcal{A}$ is jointly typical with $\tilde{Y}^n \to 0$ as $n \to \infty$ if the rate of the code $R < I(X;Y|U)$

Covering Lemma

Let $(U, X, \hat{X}) \sim p(u, x, \hat{x})$. Let $(U^n, X^n) \sim p(u^n, x^n)$ be a pair of arbitrarily distributed random sequences such that $\mathsf{P}\{(U^n, X^n) \in \mathcal{T}_\epsilon^{(n)}(U, X)\} \to 1$ as $n \to \infty$

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Let $\hat{X}^n(m), m \in \mathcal{A}$, where $|\mathcal{A}| \geq 2^{nR}$, be random sequences, conditionally independent of each other and of X^n given U^n, and distributed according to $\prod_{i=1}^n p_{\hat{X}|U}(\hat{x}_i|u_i)$

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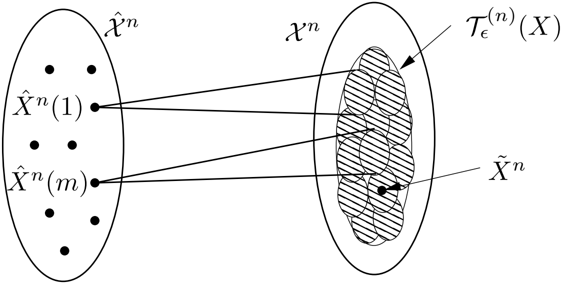
Then, there exists $\delta(\epsilon) \to 0$ as $\epsilon \to 0$ such that

$$\mathsf{P}\{(U^n, X^n, \hat{X}^n(m)) \notin \mathcal{T}_\epsilon^{(n)} \text{ for all } m \in \mathcal{A}\} \to 0$$

as $n \to \infty$, if $R > I(X; \hat{X}|U) + \delta(\epsilon)$

The sequences $\hat{X}^n(m)$, $m \in \mathcal{A}$, represent reproduction sequences and X^n represents the source sequence



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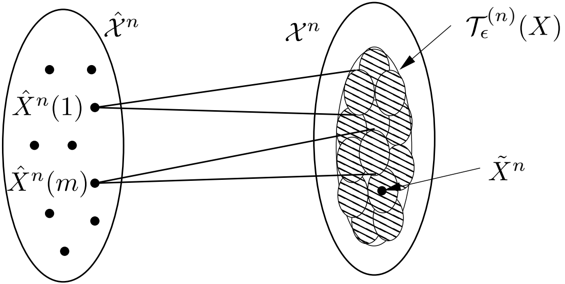

The lemma shows that if $R > I(X; \hat{X}|U)$ then there is at least one reproduction sequence that is jointly typical with $\tilde{X}^n$

Conditional Typicality Lemma

- Let $(X,Y) \sim p(x,y)$, $x^n \in \mathcal{T}_{\epsilon'}^{(n)}(X)$, and $Y^n \sim \prod_{i=1}^n p_{Y|X}(y_i|x_i)$. Then, for every $\epsilon > \epsilon'$,

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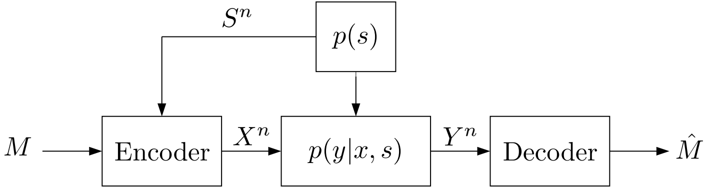
- Markov lemma is a special case: $U \to X \to Y$ form a Markov chain. If $(u^n, x^n) \in \mathcal{T}_{\epsilon'}^{(n)}(U,X)$ and $Y^n \sim \prod_{i=1}^n p_{Y|X}(y_i|x_i)$, then for every $\epsilon > \epsilon'$,

$$\mathsf{P}\{(u^n, x^n, Y^n) \in \mathcal{T}_{\epsilon}^{(n)}(U,X,Y)\} \to 1 \text{ as } n \to \infty$$

Gelfand–Pinsker

- Consider a DMC with DM state $(\mathcal{X} \times \mathcal{S}, p(y|x,s)p(s), \mathcal{Y})$
- The sender X who knows the state sequence S^n noncausally and wishes to send a message $M \in [1 : 2^{nR}]$ to the receiver Y



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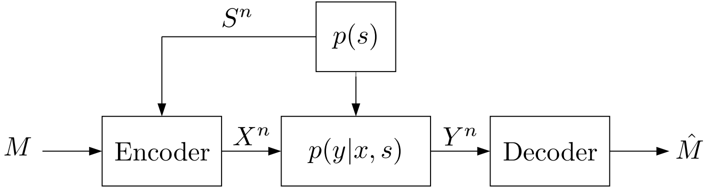


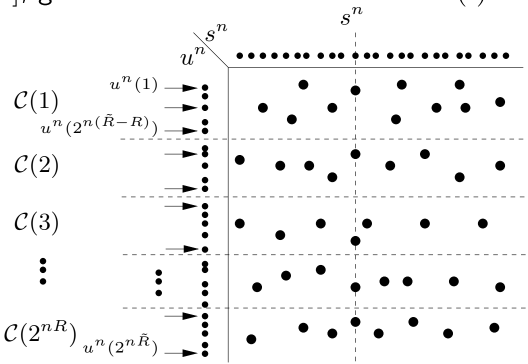
Gelfand–Pinsker Theorem

The capacity of a DMC with DM state available noncausally at the encoder is

$$C_{\mathrm{SI-E}} = \max_{p(u|s),\ x(u,s)} \left(I(U;Y) - I(U;S) \right)$$

Outline of Achievability [Heegard, El Gamal]

- Fix $p(u|s)$, $x(u,s)$ that achieve capacity. For each message $m \in [1:2^{nR}]$, generate a subcode of $2^{n(\tilde{R}-R)}$ $u^n(l)$ sequences



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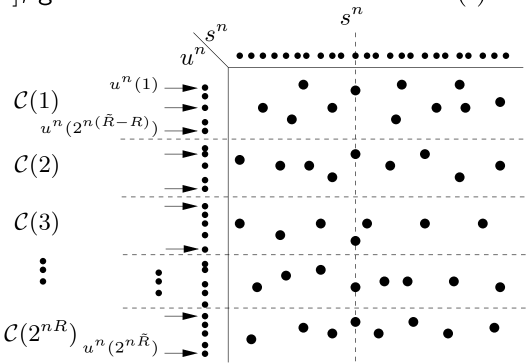

- To send m given s^n, find $u^n(l) \in \mathcal{C}(m)$ that is jointly typical with s^n and transmit $x_i = x(u_i(l), s_i)$ for $i \in [1:n]$
- The receiver finds a jointly typical $\hat{u}^n$ with y^n and declares the subcode index $\hat{m}$ of $\hat{u}^n$ to be the message sent

Analysis of the Probability of Error

- Assume $M = 1$ and let L be the index of the chosen U^n codeword for $M = 1$ and S^n
- We bound each probability of error event:
  - $\mathcal{E}_1 = \{(S^n, U^n(l)) \notin \mathcal{T}_{\epsilon'}^{(n)}$ for all $U^n(l) \in \mathcal{C}(1)\}$:
    By the covering lemma, $\mathsf{P}(\mathcal{E}_1) \to 0$ as $n \to \infty$ if $\tilde{R} - R > I(U;S)$

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  - $\mathcal{E}_3 = \{(U^n(\tilde{l}), Y^n) \in \mathcal{T}_{\epsilon}^{(n)}$ for some $U^n(\tilde{l}) \notin \mathcal{C}(1)\}$
    Since each $U^n(\tilde{l}) \notin \mathcal{C}(1)$ is independent of Y^n and generated according to $\prod_{i=1}^n p_U(u_i)$, by the packing lemma, $\mathsf{P}(\mathcal{E}_3) \to 0$ as $n \to \infty$ if $\tilde{R} < I(U;Y)$

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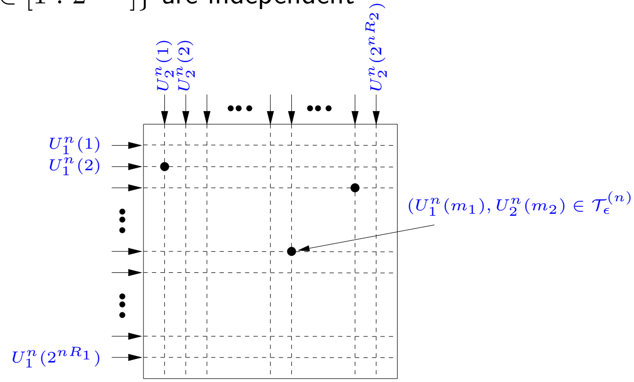
- Thus the probability or error $\to 0$ as $n \to \infty$ if $R < I(U;Y) - I(U;S)$

Mutual Covering Lemma [El Gamal, van der Meulen]

Let $(U_1, U_2) \sim p(u_1, u_2)$. For $j = 1, 2$, let $U_j^n(m_j), m_j \in [1 : 2^{nR_j}]$, be pairwise independent random sequences, each distributed according to $\prod_{i=1}^n p_{U_j}(u_{ji})$. Assume that $\{U_1^n(m_1) : m_1 \in [1 : 2^{nR_1}]\}$ and $\{U_2^n(m_2) : m_2 \in [1 : 2^{nR_2}]\}$ are independent



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- Used in the proof of Marton inner bound for BC
- Can be extended to k variables. Extension used in the proof of El Gamal–Cover inner bound for multiple descriptions and for extending Marton inner bound to k receivers

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Let $(U_1, U_2) \sim p(u_1, u_2)$. For $j = 1, 2$, let $U_j^n(m_j)$, $m_j \in [1 : 2^{nR_j}]$, be random sequences, each distributed according to $\prod_{i=1}^n p_{U_j}(u_{ji})$ with arbitrary dependence on the rest of the $U_j^n(m_j)$ sequences. Assume that $\{U_1^n(m_1) : m_1 \in [1 : 2^{nR_1}]\}$ and $\{U_2^n(m_2) : m_2 \in [1 : 2^{nR_2}]\}$ are independent

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- Used in the proof of the Berger–Tung inner bound for distributed lossy source coding

Gaussian Sources and Channels

- Because Gaussian models are quite popular in wireless communication, we have complete coverage of all basic results

Gaussian Sources and Channels

- Achievability:
  1. Show that Gaussian optimizes mutual information expressions
  2. Prove achievability of optimized expressions via DM counterpart (with cost) by discretization and taking appropriate limits

  The second step is detailed only for AWGN channel and quadratic Gaussian source coding

Gaussian Sources and Channels

- Treatment of Gaussian is interspersed within each lecture,

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  - Capacity region of AWGN-IC under strong interference
  - Han–Kobayashi inner bound for DM-IC

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