

NEMS Coupling

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Abstract

Synchronization is an increasingly important concept in modern physics.[9] Following analytical work,[4] we investigate the possibility of observing synchronization in a system of Nanoelectromechanical oscillators. A pair of doubly clamped mechanical resonators of about 10 μm in length were produced and analyzed. Resonant frequency, nonlinearity, quality factor, frequency tuning, and coupling measurements were taken, employing primarily a bridge measurement scheme using a network analyzer. Results were used to quantify the region of synchronization in terms of physical parameters. It was then attempted to construct a system capable of synchronization and obtain evidence of that fact.

1 Introduction

Synchronization has proved itself to be a ubiquitous and useful concept in physics. As it continues to be applied in more and more technical applications, new types of synchronized systems are being explored. [9]

The field of nanoelectromechanical systems (NEMS) offers a possible realm for the creation of arrays of oscillators. NEMS offer a nice field for synchronization, with their small sizes, very high quality accurate devices can be man-

ufactured, leaving open the possibility for the construction of large arrays. The Roukes group on campus has begun the investigation of the use of nanoelectromechanical cantilevers to create a system of interacting self-sustained oscillators. Much theory has already been developed on the synchronization of oscillators, in particular work has been done by Michael Cross et al. on the synchronization due to reactive coupling inherent in NEMS systems [3, 2].

Furthermore, experimental investigations have begun. The Roukes group has constructed various NEMS resonators, in particular beam resonators with working turning have been constructed, similar to the picture below. These devices have already proven possible of self-sustained oscillation. [8] These are the devices I hope to eventually synchronize.

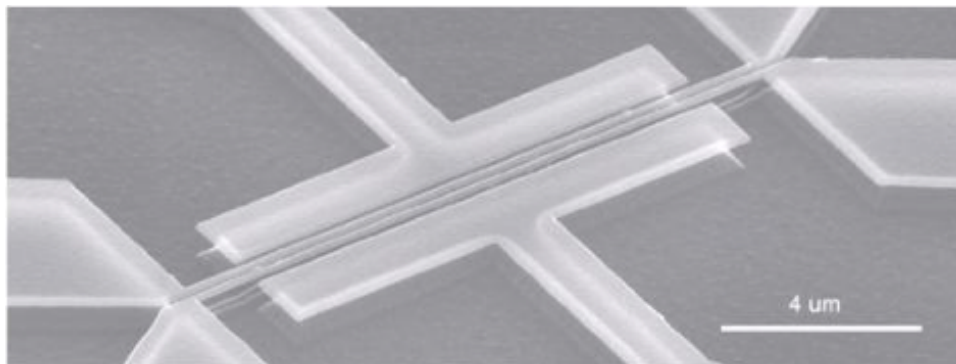


Figure 1: Two-Oscillator System produced by the Roukes group

2 Theory

For most of the theory, I worked off of the model developed by Michael Cross. [3, 2, 4] He models the oscillator as a damped harmonic oscillator, including nonlinear amplifier gains, nonlinear damping, and reactive coupling between the

devices. For the one device we can write this as:

$$\ddot{x}_1 - [G(1 - \tilde{\eta}_G x_1^2) - \gamma(1 - \tilde{\eta}_\gamma x_1^2)] \dot{x}_1 + \omega_0^2 [(1 + \delta_1 - \tilde{a} x_1^2)x_1 - D(x_1 - x_2)] = \frac{f_1}{m}$$

Where G represents the amplifier gain used to sustain oscillations, $\tilde{\eta}_G$ its nonlinear term, γ inherent damping and $\tilde{\eta}_\gamma$ its nonlinear term, ω_0 the devices (planned) intrinsic frequency, δ the true frequency deviation, \tilde{a} the Duffing nonlinearity, D the coupling coefficient, and f_1 the driving force.

This model can then be reduced to a form reminiscent of the Kuramoto model for investigating synchronized oscillators. Matt Grau carried out much of this work as part of a previous SURF,[6] he obtained the model

$$\dot{z}_n = i(\omega_n - \alpha|z_n|^2)z_n + (1 - |z_n|^2)z_n + i \sum_{m=1}^N \beta_{n,m}(z_m - z_n)$$

for a set of coupled nonlinear oscillators, where z now is a complex number representing the oscillator amplitude and phase. He found that they in general will become synchronized in a region determined by the boundary

$$\beta(\Delta\omega) = \sqrt{\frac{1}{8} \left(1 + 3\alpha^2 \pm 4\sqrt{3}\alpha\Delta\omega + 4(\Delta\omega)^2 \right)}$$

In order to determine where we expect to observe synchronization for our NEMS oscillators, it remains to properly transform this region in terms of model parameters into the space of real parameters.

2.1 Coupling

Among the criteria that can affect whether or not two oscillators eventually synchronize are the frequency difference between the devices as well as the strength of the coupling. Unfortunately the particulars of coupling between

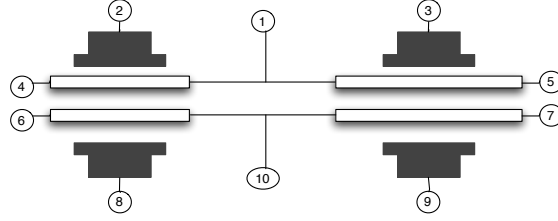


Figure 2: Measurement Scheme

doubly clamped silicon carbide beams with length $\ell = 10\mu\text{m}$ long, $t = 100\text{nm}$ thick and $D = 100\text{nm}$ apart are not properly understood.

Among the possible schemes for coupling between the beams, electrostatic coupling seems a natural avenue to explore.[1] Wherein we have a potential and force of the form:

$$U = -\frac{1}{2}C(x)V^2 \quad F = -\frac{dU}{dx} = \frac{V^2}{2}C'(x)$$

where for thin wires near each other we have

$$C(x) = \frac{\pi\epsilon\ell}{\log\left(\frac{x}{d} + \sqrt{\frac{x^2}{d^2} - 1}\right)}$$

where d is the diameter of the wires. Given a device separation distance of D , lets expand this about the point $x = D + \delta$. We obtain an equation for the coupling of the form:

$$D(x_2 - x_1) = \frac{V_{DC}^2}{2m} [C_0 + C_1(x_2 - x_1)]$$

with

$$\epsilon \approx \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{s}^4\text{A}^2}{\text{m}^3\text{kg}}$$

$$D \approx 200 \text{ nm} \quad d \approx 100 \text{ nm} \quad \ell \approx 10 \mu\text{m} \quad m = 350 \text{ fg}$$

this equates to

$$C_0 \equiv \frac{-l\pi\varepsilon}{\sqrt{\mathcal{D}^2 - d^2} \log^2 \left[\frac{\mathcal{D} + \sqrt{\mathcal{D}^2 - d^2}}{d} \right]} \approx -9.25 \times 10^{-10}$$

$$C_1 \equiv \frac{-\ell\pi\varepsilon \left(\mathcal{D} \log \left(\frac{\mathcal{D} + \sqrt{\mathcal{D}^2 - d^2}}{d} \right) + 2\sqrt{\mathcal{D}^2 - d^2} \right)}{d\sqrt{\frac{\mathcal{D}^2}{d^2} - 1} (d^2 - \mathcal{D}^2) \log^3 \left[\frac{\mathcal{D} + \sqrt{\mathcal{D}^2 - d^2}}{d} \right]} \approx 0.014$$

This should give us an estimate for the observed electrostatic coupling between the two devices. We should expect the coupling to be quadratic in the bias voltage with constants close to those predicted.

3 Experimental Results

In order to investigate we created a chip with two pairs of devices, with each pair as shown in Figure ???. Two pairs were prepared and bridged so as to reduce noise in the measurement. The devices were driven using a magnetomotive technique,[5] and analyzed with a network analyzer. In order to minimize noise a resistance bridge is used to match impedances between the two pairs, while improving the signal to noise ratio this shifts the phase of the response. In addition, the presence of a gate adjacent to each device (Figure 1) allows for the tuning of the devices inherent frequency.[7]

3.1 Nonlinear Behavior

Using this technique and a basic reflection measurement technique, we were able to investigate the first chip. The devices show nice nonlinear response, typified in the graphs below [Fig 3]. As the drive amplitude is increased, our devices response peak turns over. The nonlinear behavior is important. We need our devices to be in the nonlinear regime in order to build effective oscillators and

eventually achieve synchronization.

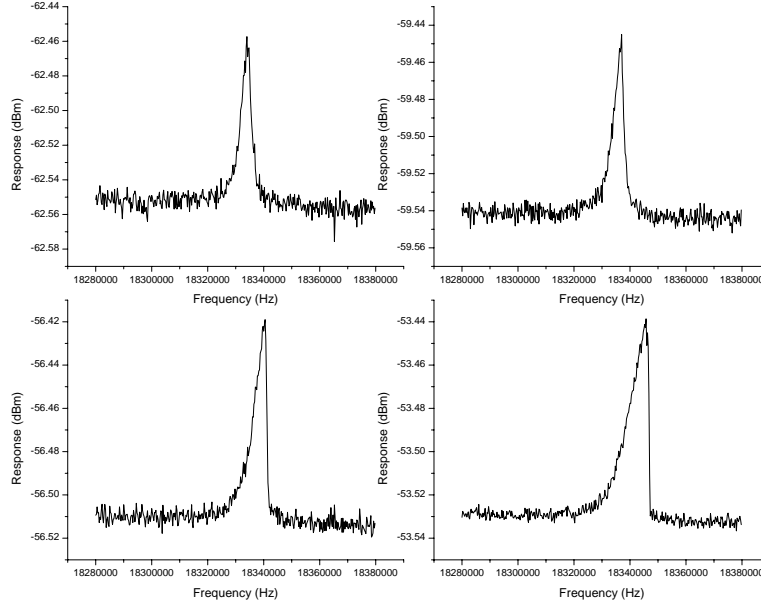


Figure 3: Nonlinear Response (Drive: -84, -81, -78, -75 dBm)

3.2 Tuning

The devices intrinsic frequency can be tuned by putting a bias voltage on the gates (2,3,5,9 above [Fig 2]), which can be used to change the resonant frequency. Initial investigation of tuning, while it worked for the 0 - 2 volt range, yielding a tuning of about 30 kHz per volt, at 2 volts our signal was lost and upon reduction we found the resonant frequency had jumped about a MHz, suggesting that we had burned off some of the aluminum. As such we destroyed the first device.

Having destroyed the first device, we lost confidence in the Si_3N_4 . Next we tried using all metal devices. The initial probe of the resistances looked

promising. We proceeded to attempt to locate the four resonances for the chip. Soon we ran into some problems, losing the signal for all but one of the devices. So, we proceeded to tune this remaining device. All in all we were able to tune this remaining good device a full 1.1 MHz after applying 12 volts. At this point this remaining device died as well.

The all metal devices proved themselves more robust than the Si_3N_4 . Unfortunately, the fabrication of all metal devices had a low yield. Frequently the devices would pull in to the substrate. These considerations led us to adopt the use of silicon carbide with gold deposited on top. These devices proved easy to fabricate as well as robust to tuning.

3.3 Coupling Results

We can measure the coupling directly by tuning one device through the resonance of its partner. If we only drive one beam in a pair, as we tune its frequency through that of its partner, we should instead of seeing the response of a single device, we will begin to excite the coupled modes of the two devices, the separation of which will detail the strength of the coupling independent of other effects. I.e. we could imagine a system described by the following:

$$\begin{aligned}\ddot{x}_1 + \frac{\omega_1}{Q_1}\dot{x}_1 + \omega_1^2 x_1 &= D(x_2 - x_1) + F e^{i\omega t} \\ \ddot{x}_2 + \frac{\omega_2}{Q_2}\dot{x}_2 + \omega_2^2 x_2 &= D(x_1 - x_2)\end{aligned}$$

If we examine the frequency response of x_1 taking ω_1 as a tunable parameter, qualitatively, if ω_1 is far from ω_2 we see a single resonance that moves with ω_1 , as ω_1 approaches ω_2 we begin to see the excitation of a second resonance. At this point the two peaks correspond to the coupled devices operating in the in phase and out of phase modes. As we move through the point at which $\omega_1 = \omega_2$ the two resonances do not cross, always maintaining a minimum separation distance.

This separation distances corresponds directly to the strength of the reactive coupling parameter D in model. In particular we should expect the difference of the squared frequencies for the two modes to equal $2D$, $\omega_s^2 - \omega_a^2 = 2D$.

In fact this is precisely the type of behavior we observe. Driving a single beam and tuning it past the resonance of the other excites two modes as shown in Figure 4. Using a bias tee we can put a DC voltage between the two beams without interfering with the AC drive. Doing this, putting different bias voltages between the two beams and tuning to the point where the peaks are equal in magnitude, we can obtain measurements for the coupling coefficient D as a function of the bias voltage. The data is summarized in Figure 5. We see that there is some inherent coupling even with zero bias voltage between the two beams. This is expected and due probably to elastic coupling through the medium. We also see that the coupling coefficient grows quadratically in the bias voltage, as predicted in our electrostatic coupling model. At higher coupling voltages the curve begins to flatten out, probably due to higher order terms in the electrostatic coupling as well as the possibility of other coupling schemes. A fit was done on the early data points, obtaining a quadratic fit of high quality. The observed coupling coefficient obtained is $D = 0.21719 \text{ Hz}^2/\text{V}^2$.

In order to work towards the production of a system of synchronized NEMS oscillators, the coupling needs to be characterized and understood. We have attempted to explain a possible scheme for coupling as well as estimate its strength. The coupling strength was determined experimentally by observing the coupled mode responses, this gave observed values for the behavior of the coupling through a range of different bias voltages between the beams. With the use of theoretical results obtained by Matt Grau[6], these results can be used to ascertain whether or not synchronization of oscillators produced from these resonators is realistic.

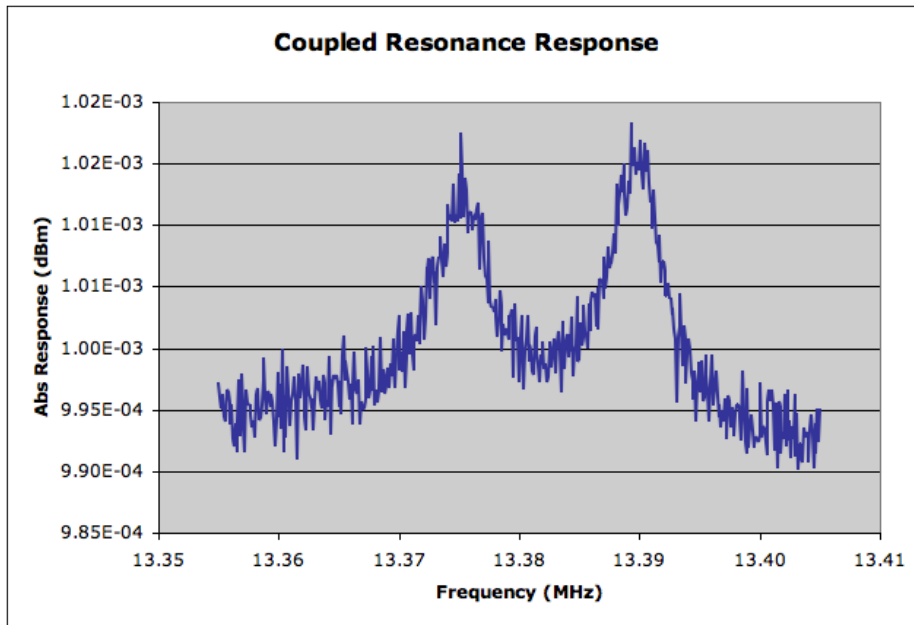


Figure 4: Example Data

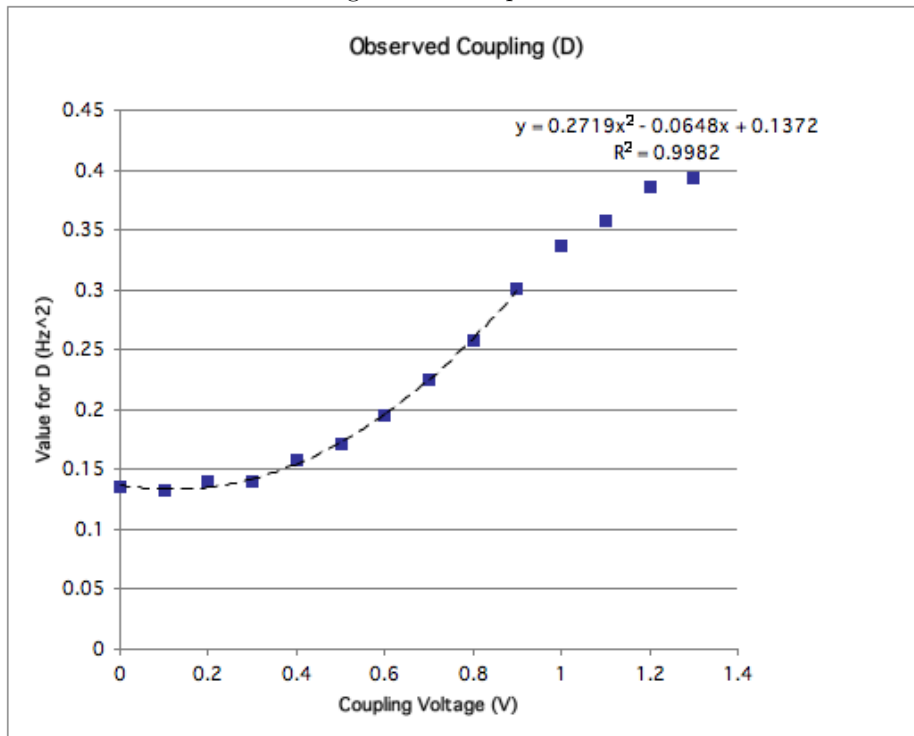


Figure 5: Observed Coupling Data

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