International Journal of Computational Intelligence Systems
Vol. **14(1)**, 2021, *pp.* 1516–1540
DOI: https://doi.org/10.2991/ijcis.d.210413.001; ISSN: 1875-6891; eISSN: 1875-6883
https://www.atlantis-press.com/journals/ijcis/

Research Article

Certain Properties of Single-Valued Neutrosophic Graph With Application in Food and Agriculture Organization

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ARTICLE INFO

Article History

Received 31 Oct 2020
Accepted 31 Mar 2021

Keywords

Single-valued neutrosophic graph
maximal product
rejection
symmetric difference
residue product

ABSTRACT

Fuzzy graph models are present everywhere from natural to artificial structures, embodying the dynamic processes in physical, biological, and social systems. As real-life problems are often uncertain on account of inconsistent and indeterminate information, it seems very demanding for an expert to model those problems using a fuzzy graph. To deal with the uncertainty associated with the inconsistent and indeterminate information of any real-world problems, a neutrosophic graph can be applied, where fuzzy graphs may not bear any fruitful results. The past definitions limitations in fuzzy graphs have directed us to present new definitions in single-valued neutrosophic graph (SVNG). A SVNG has several applications in the fields of physics, bio and connectivity of socialism. It has been an advantageous scope in the recent times for providing such information which is incomplete or uncertain accounting in real problems that gives the direction to describe the relationship between nodes. Operations are conveniently used in many combinatorial applications. In various situations, they present a suitable construction means; therefore, the current study, seeks to present and explore the key features of new operations, including: rejection, maximal product, symmetric difference, and residue product of SVNG. We have discuss the concept of maximal product on two strong-(SVNGS) and maximal product of connected-SVNG with examples. This research article presents the notions of degree of a vertex and total degree of a vertex in SVNG. Moreover, this study summarizes the specific conditions needed for obtaining vertices degrees in SVNG under the operations of maximal product, symmetric difference, residue product, and rejection. In addition, an application was illustrated in the food and agriculture organization with an algorithm to emphasize the contributions of this research article in practical applications.

1. INTRODUCTION

Graph theory is an exceptionally advantageous device in tackling combinatorial issues in different regions including calculation, variable-based math, number hypothesis, geography, and social frameworks. A graph is chiefly a model of relations, and it is applied to speak to the genuine issues including connections between objects. The vertices and edges of the graph are utilized to connote the articles and the relations between objects, individually. In numerous improvement issues, the current data is vague or loose for different reasons, for example, the loss of data, the absence of proof, flawed measurable information, and inadequate data. By and large, the vulnerability, in actuality, issues may show up in the data that characterizes the issue. Fuzzy chart models are important numerical apparatuses for treating the combinatorial issues of different areas enveloping exploration, streamlining, variable-based math, figuring, ecological science, and geography. Fuzzy graphical models are observably more helpful than graphical models due to the common presence of unclearness and equivocalness. Initially, fuzzy set hypothesis is needed to manage numerous perplexing issues including inadequate data. Zadeh [32], firstly exemplified the idea of the set known as the fuzzy set. He described the fuzzy set characterized by true membership function value ranging from closed interval [0, 1]. Fuzzy set theory serves as a very powerful mathematical tool for solving approximate reasoning related problems. These notions effectively illustrate complex phenomena, which are not precisely described by classical mathematics.

The fuzzy graphs idea and concept are discussed by Smarandache and Rosenfeld [27]. The fuzzy graphs application has been extended in few years and it has a scope from 19th century [4,5,10,11,15,16]. It is not necessarily true membership degree of 1, also, the

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nonmembership degree and indeterminacy occur. Nonmembership degree is presented by Atanassove [3] in an intuitionistic fuzzy set. Shao *et al.* [31] labeled new concepts of bondage number in intuitionistic fuzzy graph. Rashmanlou *et al.* [20–26] introduced new concepts in bipolar fuzzy graph and interval-valued fuzzy graphs. Krishna *et al.* [13,14] analyzed the concept of vague set and vague graph. Devi *et al.* [8] investigated new ways in intuitionistic fuzzy labeling graph. Pythagorean fuzzy set also known as IF-set of type-2 [1] is the extension of intuitionistic fuzzy set (IF-set). Parvathi and Karunambigai [19] studied about Intuitionistic fuzzy graphs. After while, Smarandache [31] included the indeterminacy concept in a neutrosophic set. Neutrosophy is the kind of philosophy which analyzes the nature and scope of neutralities. Neutrosophic set is the speculation of fuzzy set and furthermore neutrosophic rationale is the expansion of fuzzy rationale. Smarandache gives the possibility of a neutrosophic set due to introducing the vulnerability in the issues of different fields like clinical science and financial aspects and so forth. He portrayed significant classifications [29] of neutrosophic diagrams from which two classifications are relied upon the strict indeterminacy and other two classes depended [7] on its (t, i, f) parts. Malik and Hassan [12] presented the classification of bipolar single-valued neutrosophic graph (SVNG) classification. Later Malik and Naz *et al.* [17] described new operations on SVNG. Naz *et al.* [17] discussed operations on single-valued neutrosophic graphs with application. Malik *et al.* [18] also investigated some properties of bipolar SVNG. Product operations have applications in different branches, such as coding theory, network designs, chemical graph theory, and others. Many scholars discussed product operations on various generalized FGs. Mordeson and Peng [16] defined some of these product operations on FGs and some new fuzzy models are discussed in [33–38].

In this research, some new properties, including maximal product, symmetric difference, residue product, and rejection of SVNG are presented. Also, the examples of these operations are discussed. We found the degree and the total degree of SVNG. Finally, an application was illustrated in the food and agriculture organization with an algorithm to highlight the contributions of this research article in practical applications.

2. PRELIMINARIES

In this section, the key preliminary notions and definitions that are used in this current research study will be introduced.

Definition 1. [9] A graph $G = (V, E)$ is an ordered pair of set of vertices and set of edges.

Definition 2. [30] Suppose that X is a space of points with generic element in X denoted by x. Then, the neutrosophic set M (NS-M) is defined as $M = < x : T_M(x), I_M(x), F_M(x) >, x \in X$ which obey $0 \leqslant \{T_M(x) + I_M(x) + F_M(x)\} \leqslant 3$. $T_M : V \to [0, 1]$, $I_M: V \to [0, 1]$, and $F_M : V \to [0, 1]$ represents the degree of true membership function, degree of indeterminacy membership function, and degree of false membership function of the element $x \in X$, respectively.

Definition 3. [27] A SVNG $\mathbf{G} = (M, N)$ with underlying set of V is defined to be a pair of $G = (V, E)$ which is defined as (i) $T_M : V \to [0, 1]$, $F_M : V$ $[0, 1]$ and $I_M : V \to [0, 1]$ represents the degree of true membership function, degree of false membership function, and degree of indeterminacy membership function of the element $m \in V$, respectively, where $0 \leqslant T_M(m) + I_M(m) + F_M(m) \leqslant 3, \forall m \in V$.

(ii) The function $T_N : E \to [0, 1]$, $I_N : E \to [0, 1]$ and $F_N : E \to [0, 1]$ are defined by

$$\begin{aligned} T_N(mn) &\leqslant \min\{T_M(m), T_M(n)\} \\ I_N(mn) &\geqslant \max\{I_M(m), I_M(n)\} \\ F_N(mn) &\geqslant \max\{F_M(m), F_M(n)\}. \end{aligned}$$

It is free of any restriction so $0 \leqslant T_N(mn) + I_N(mn) + F_N(mn) \leqslant 3$.

Example 1. Consider the Figure 1 such that $V = \{a, b, c\}$, $E = \{ab, bc, ca\}$, $M =< \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.1}\right) >$, and $N =< \left(\frac{ab}{0.1}, \frac{bc}{0.1}, \frac{ac}{0.2}\right), \left(\frac{ab}{0.7}, \frac{bc}{0.6}, \left(\frac{ac}{0.8}\right), \left(\frac{ab}{0.3}, \frac{bc}{0.2}, \frac{ac}{0.3}\right) >$.

By routine computations, it is easy to show that **G** is a SVNG.

Definition 4. A SVNG **G** is said to be strong if $T_N(mn) = \min(T_M(m), T_M(n))$, $I_N(mn) = \max(I_M(m), I_M(n))$ and $F_N(mn) = \max(F_M(m), F_M(n))$, for all mn in V.

Definition 5. A SVNG **G** is said to be complete if $T_N(mn) = \min(T_M(m), T_M(n))$, $I_N(mn) = \max(I_M(m), I_M(n))$ and $F_N(mn) = \max(F_M(m), F_M(n))$, for all m, n in E.

Definition 6. A SVNG **G** is said to be connected if $T_N^{\infty}(m_i m_j) > 0$, $I_N^{\infty}(m_i m_j) < 1$, $F_N^{\infty}(m_i m_j) < 1$, for all $m_i, m_j \in V$. Also, we have

$$\begin{aligned} T_N^{\infty}(mn) &= \sup\{T_N(mn_1) \wedge T_N(n_1 n_2) \wedge T_N(n_2 n_3) \wedge \\ &\quad \dots \wedge T_N(n_{k-1} n) \mid m, n_1, n_2, \cdots, n_{k-1}, n \in V\}, \end{aligned}$$

$$\begin{aligned} I_N^{\infty}(mn) &= \inf\{I_N(mn_1) \vee I_N(n_1 n_2) \vee I_N(n_2 n_3) \vee \cdots \vee \\ &\quad I_N(n_{k-1} n) \mid m, n_1, n_2, \cdots, n_{k-1}, n \in V\}. \end{aligned}$$

and

$$\begin{aligned} FT_N^{\infty}(mn) &= \inf\{F_N(mn_1) \vee F_N(n_1 n_2) \vee F_N(n_2 n_3) \vee \cdots \vee \\ &\quad F_N(n_{k-1} n) \mid m, n_1, n_2, \cdots, n_{k-1}, n \in V\}. \end{aligned}$$

3. OPERATIONS ON SVNGs

In this section, we define four new kinds of operations on (SVNGs) including maximal product, residue product, rejection, and symmetric difference. We show that maximal product, residue product, and rejection of two (SVNGs) are a SVNG.

Definition 7. The maximal product $\mathbf{G}_1 * \mathbf{G}_2 = (M_1 * M_2, N_1 * N_2)$ of two (SVNGs) $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ is defined as

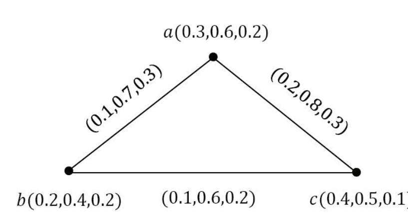

Figure 1 | SVNG(**G**).

(i) $\left(T_{M_1} * T_{M_2}\right)\left(\left(m_1, m_2\right)\right) = \max\left\{T_{M_1}\left(m_1\right), T_{M_2}\left(m_2\right)\right\}$
$\left(I_{M_1} * I_{M_2}\right)\left(\left(m_1, m_2\right)\right) = \min\left\{I_{M_1}\left(m_1\right), I_{M_2}\left(m_2\right)\right\}$
$\left(F_{M_1} * F_{M_2}\right)\left(\left(m_1, m_2\right)\right) = \min\left\{F_{M_1}\left(m_1\right), F_{M_2}\left(m_2\right)\right\}$
$\forall \left(m_1, m_2\right) \in \left(V_1 \times V_2\right)$,

(ii) $\left(T_{M_1} * T_{M_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right) = \max\left\{T_{M_1}(m), T_{N_2}\left(m_2 n_2\right)\right\}$
$\left(I_{M_1} * I_{M_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right) = \min\left\{I_{M_1}(m), I_{N_2}\left(m_2 n_2\right)\right\}$
$\left(F_{M_1} * F_{M_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right) = \min\left\{F_{M_1}(m), F_{N_2}\left(m_2 n_2\right)\right\}$
$\forall\, m \in V_1$ and $m_2 n_2 \in E_2$.

(iii) $\left(T_{M_1} * T_{M_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right) = \max\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}$
$\left(I_{M_1} * I_{M_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right) = \min\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}$
$\left(F_{M_1} * F_{M_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right) = \min\left\{F_{N_1}\left(m_1 n_1\right), F_{M_2}(z)\right\}$
$\forall\, z \in V_2$ and $m_1 n_1 \in E_1$.

Example 2. Consider two (SVNGs) $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$, as shown in Figures 2 and 3. Their maximal product $\mathbf{G}_1 * \mathbf{G}_2$ is shown in Figure 4.

For vertex (e, a), we find membership value, indeterminacy and nonmembership value as follows:

$$\begin{aligned}\left(T_{M_1} * T_{M_2}\right)((e,a)) &= \max\left\{T_{M_1}(e), T_{M_2}(a)\right\}\\ &= \max\{0.3, 0.1\} = 0.3,\\ \left(I_{M_1} * I_{M_2}\right)((e,a)) &= \min\left\{I_{M_1}(e), I_{M_2}(a)\right\}\\ &= \min\{0.4, 0.3\} = 0.3,\\ \left(F_{M_1} * F_{M_2}\right)((e,a)) &= \min\left\{F_{M_1}(e), F_{M_2}(a)\right\}\\ &= \min\{0.5, 0.4\} = 0.4,\end{aligned}$$

for $e \in V_1$ and $a \in V_2$. For edge $(e, a)(e, b)$, we find membership value, indeterminacy, and nonmembership value.

$$\begin{aligned}\left(T_{M_1} * T_{M_2}\right)((e,a)(e,b)) &= \max\left\{T_{M_1}(e), T_{N_2}(ab)\right\}\\ &= \max\{0.3, 0.1\} = 0.3,\\ \left(I_{M_1} * I_{M_2}\right)((e,a)(e,b)) &= \min\left\{I_{M_1}(e), I_{N_2}(ab)\right\}\\ &= \min\{0.4, 0.4\} = 0.4,\\ \left(F_{M_1} * F_{M_2}\right)((e,a)(e,b)) &= \min\left\{F_{M_1}(e), F_{N_2}(ab)\right\}\\ &= \min\{0.5, 0.4\} = 0.4,\end{aligned}$$

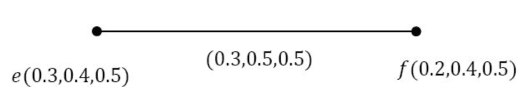

Figure 2 | G_1.

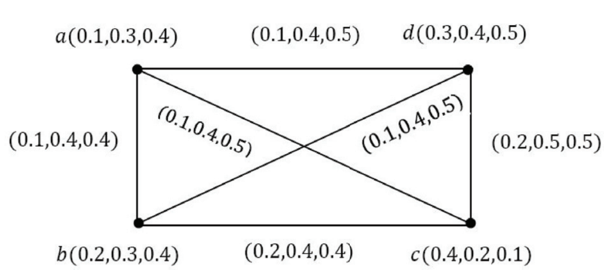

Figure 3 | G_2.

for $e \in V_1$ and $ab \in E_2$. Now, for edge $(e, a)(f, a)$ we have:

$$\begin{aligned}\left(T_{M_1} * T_{M_2}\right)((e,a)(f,b)) &= \max\left\{T_{N_1}(ef), T_{M_2}(a)\right\}\\ &= \max\{0.3, 0.1\} = 0.3,\\ \left(I_{M_1} * I_{M_2}\right)((e,a)(f,b)) &= \min\left\{I_{N_1}(ef), I_{M_2}(a)\right\}\\ &= \min\{0.5, 0.3\} = 0.3,\\ \left(F_{M_1} * F_{M_2}\right)((e,a)(f,b)) &= \min\left\{F_{N_1}(ef), F_{M_2}(a)\right\}\\ &= \min\{0.5, 0.4\} = 0.4,\end{aligned}$$

for $a \in V_2$ and $ef \in E_1$.

Similarly, we can find membership, indeterminacy, and nonmembership value for all remaining vertices and edges.

Proposition 1. *The maximal product of two (SVNGs)* $\mathbf{G}_1$ *and* $\mathbf{G}_2$, *is a SVNG.*

Proof. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs) on crisp graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively and $((m_1, m_2)(n_1, n_2)) \in E_1 \times E$. Then, by Definition 7, we have two cases:

(i) If $m_1 = n_1 = m$

$$\begin{aligned}&(T_{N_1} * T_{N_2})((m, m_2)(m, n_2))\\ &= \max\left\{T_{M_1}(m), T_{N_2}(m_2 n_2)\right\}\\ &\leqslant \max\left\{T_{M_1}(m), \min\left\{T_{M_2}(m_2), T_{M_2}(n_2)\right\}\right\}\\ &= \min\left\{\max\left\{\left\{T_{M_1}(m), T_{M_2}(m_2)\right\}, \right.\right.\\ &\quad \left.\max\left\{\left\{T_{M_1}(m), T_{M_2}(n_2)\right\}\right\}\right.\\ &= \min\left\{(T_{M_1} * T_{M_2})(m, m_2), (T_{M_1} * T_{M_2})(m, n_2)\right\},\end{aligned}$$

$$\begin{aligned}&(I_{N_1} * I_{N_2})((m, m_2)(m, n_2))\\ &= \min\left\{I_{M_1}(m), I_{N_2}(m_2 n_2)\right\}\\ &\geqslant \min\left\{I_{M_1}(m), \max\left\{I_{M_2}(m_2), I_{M_2}(n_2)\right\}\right\}\\ &= \max\left\{\min\left\{\left\{I_{M_1}(m), I_{M_2}(m_2)\right\}, \right.\right.\\ &\quad \left.\min\left\{\left\{I_{M_1}(m), I_{M_2}(n_2)\right\}\right\}\right.\\ &= \max\left\{(I_{M_1} * I_{M_2})(m, m_2), (I_{M_1} * I_{M_2})(m, n_2)\right\},\end{aligned}$$

$$\begin{aligned}&(F_{N_1} * F_{N_2})((m, m_2)(m, n_2))\\ &= \min\left\{F_{M_1}(m), F_{N_2}(m_2 n_2)\right\}\\ &\geqslant \min\left\{F_{M_1}(m), \max\left\{F_{M_2}(m_2), F_{M_2}(n_2)\right\}\right\}\\ &= \max\left\{\min\left\{\left\{F_{M_1}(m), F_{M_2}(m_2)\right\}, \right.\right.\\ &\quad \left.\min\left\{\left\{F_{M_1}(m), F_{M_2}(n_2)\right\}\right\}\right.\\ &= \max\left\{(F_{M_1} * F_{M_2})(m, m_2), (F_{M_1} * F_{M_2})(m, n_2)\right\}.\end{aligned}$$

(ii) If $m_2 = n_2 = z$

$$\begin{aligned}&\left(T_{N_1} * T_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)\\ &= \max\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}\\ &\leqslant \max\left\{\min\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}\\ &= \min\left\{\max\{\{T_{N_1}\left(m_1\right), T_{M_2}(z)\}, \right.\\ &\quad \left.\max\left\{\{T_{M_1}\left(n_1\right), T_{M_2}(z)\right\}\right.\\ &= \min\left\{\left(T_{M_1} * T_{M_2}\right)\left(m_1, z\right), \left(T_{M_1} * T_{M_2}\right)\left(n_1, z\right)\right\},\end{aligned}$$

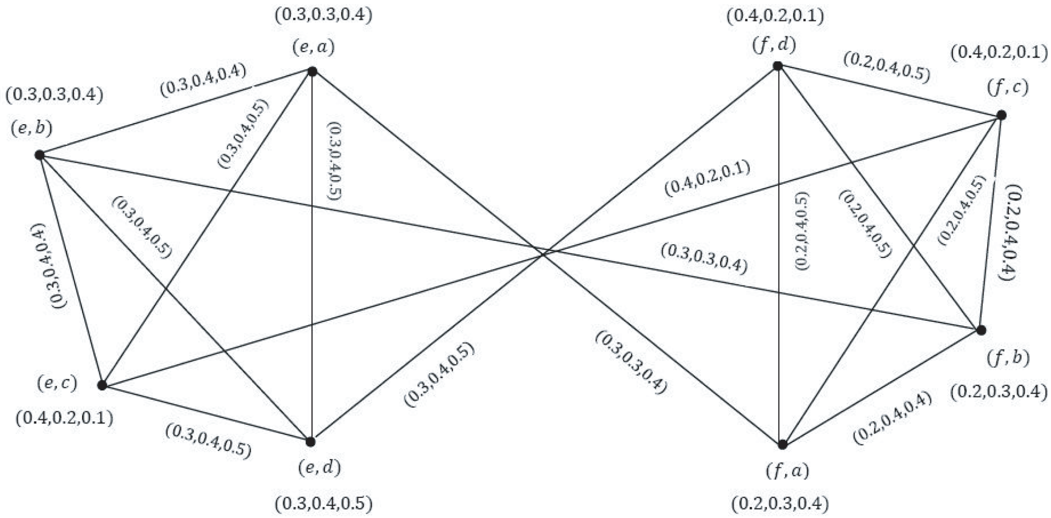

Figure 4 | $G_1 * G_2$.

$$
\begin{aligned}
&\left(I_{N_1} * I_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)\\
&= \min\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}\\
&\geqslant \min\left\{\max\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}\\
&= \max\left\{\min\{\{I_{M_1}\left(m_1\right), I_{M_2}(z)\right\},\\
&\quad \min\left\{\{I_{M_1}\left(n_1\right), I_{M_2}(z)\}\right\}\\
&= \max\left\{\left(I_{M_1} * I_{M_2}\right)\left(m_1, z\right), \left(I_{M_1} * I_{M_2}\right)\left(n_1, z\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(F_{N_1} * F_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)\\
&= \min\left\{F_{N_1}\left(m_1 n_1\right), F_{M_2}(z)\right\}\\
&\geqslant \min\left\{\max\{F_{N_1}\left(m_1 n_1\right), F_{M_2}(z)\right\}\\
&= \max\left\{\min\{\{F_{M_1}\left(m_1\right), F_{M_2}(z)\right\},\\
&\quad \min\left\{\{F_{M_1}\left(n_1\right), F_{M_2}(z)\}\right\}\\
&= \max\left\{\left(F_{M_1} * F_{M_2}\right)\left(m_1, z\right), \left(F_{M_1} * F_{M_2}\right)\left(n_1, z\right)\right\}.
\end{aligned}
$$

Therefore, $\mathbf{G}_1 * \mathbf{G}_2$ is a SVNG. □

Theorem 2. *The maximal product of two strong-(SVNGS) $\mathbf{G}_1$ and $\mathbf{G}_2$, is a strong-SVNG.*

Proof. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two strong-(SVNGS) on crisp graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively and $((m_1, m_2)(n_1, n_2)) \in E_1 \times E_2$. Then by Proposition 1, $\mathbf{G}_1 * \mathbf{G}_2$ is a SVNG. Now we have two cases:

(i) If $m_1 = n_1 = m$

$$
\begin{aligned}
&\left(T_{N_1} * T_{N_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right)\\
&= \max\left\{T_{M_1}(m), T_{N_2}\left(m_2 n_2\right)\right\}\\
&= \max\left\{T_{M_1}(m), \min\left\{T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\}\right\}\\
&= \min\left\{\max\{\{T_{M_1}(m), T_{M_2}\left(m_2\right)\right\}\\
&\quad \max\left\{\{T_{M_1}(m), T_{M_2}\left(n_2\right)\}\right\}\\
&= \min\left\{\left(T_{M_1} * T_{M_2}\right)\left(m, m_2\right), \left(T_{M_1} * T_{M_2}\right)\left(m, n_2\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(I_{N_1} * I_{N_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right)\\
&= \min\left\{I_{M_1}(m), I_{N_2}\left(m_2 n_2\right)\right\}\\
&= \min\left\{I_{M_1}(m), \max\left\{I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\}\right\}\\
&= \max\left\{\min\{\{I_{M_1}(m), I_{M_2}\left(m_2\right)\right\},\\
&\quad \min\left\{\{I_{M_1}(m), I_{M_2}\left(n_2\right)\}\right\}\\
&= \max\left\{\left(I_{M_1} * I_{M_2}\right)\left(m, m_2\right), \left(I_{M_1} * I_{M_2}\right)\left(m, n_2\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(F_{N_1} * F_{N_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right)\\
&= \min\left\{F_{M_1}(m), F_{N_2}\left(m_2 n_2\right)\right\}\\
&= \min\left\{F_{M_1}(m), \max\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}\right\}\\
&\quad \max\left\{\min\left\{\{F_{M_1}(m), F_{M_2}\left(m_2\right)\right\}, \min\left\{\{F_{M_1}(m), F_{M_2}\left(n_2\right)\}\right\}\right\}\\
&= \max\left\{\left(F_{M_1} * F_{M_2}\right)\left(m, m_2\right), \left(F_{M_1} * F_{M_2}\right)\left(m, n_2\right)\right\}.
\end{aligned}
$$

(ii) If $m_2 = n_2 = z$

$$
\begin{aligned}
&\left(T_{N_1} * T_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)\\
&= \max\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}\\
&= \max\left\{\min\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}\\
&= \min\left\{\max\{\{T_{N_1}\left(m_1\right), T_{M_2}(z)\right\},\\
&\quad \max\left\{\{T_{M_1}\left(n_1\right), T_{M_2}(z)\}\right\}\\
&= \min\left\{\left(T_{M_1} * T_{M_2}\right)\left(m_1, z\right), \left(T_{M_1} * T_{M_2}\right)\left(n_1, z\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(I_{N_1} * I_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)\\
&= \min\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}\\
&= \min\left\{\max\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}\\
&= \max\left\{\min\{\{I_{M_1}\left(m_1\right), I_{M_2}(z)\right\},\\
&\quad \min\left\{\{I_{M_1}\left(n_1\right), I_{M_2}(z)\}\right\}\\
&= \max\left\{\left(I_{M_1} * I_{M_2}\right)\left(m_1, z\right), \left(I_{M_1} * I_{M_2}\right)\left(n_1, z\right)\right\},
\end{aligned}
$$

$$\begin{aligned}
&\left(F_{N_1} * F_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right) \\
&= \min\left\{F_{N_1}\left(m_1 n_1\right), F_{M_2}(z)\right\} \\
&= \min\left\{\max\{F_{N_1}\left(m_1 n_1\right), F_{M_2}(z)\right\} \\
&= \max\left\{\min\{\{F_{M_1}\left(m_1\right), F_{M_2}(z)\right\}, \\
&\quad \min\left\{\{F_{M_1}\left(n_1\right), F_{M_2}(z)\}\right\} \\
&= \max\left\{\left(F_{M_1} * F_{M_2}\right)\left(m_1, z\right), \left(F_{M_1} * F_{M_2}\right)\left(n_1, z\right)\right\}.
\end{aligned}$$

Therefore, $\mathbf{G}_1 * \mathbf{G}_2$ is a strong-SVNG. □

Example 3. Consider the strong-(SVNGS) G_1 and G_2 as in Figure 5.

It is easy to see that $G_1 * G_2$ is a strong-SVNG, too.

Remark 1. If the maximal product of two (SVNGs) $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ is strong, then $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ need not to be strong, in general.

Example 4. Consider the (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$ as in Figures 6 and 7. We can see that the maximal product of two (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$, that is $\mathbf{G}_1 * \mathbf{G}_2$ in Figure 8.

Then $\mathbf{G}_1$ and $\mathbf{G}_1 * \mathbf{G}_2$ are strong-(SVNGS), but $\mathbf{G}_2$ is not strong. Since $T_{N_2}\left(m_2, n_2\right) = 0.1$, but

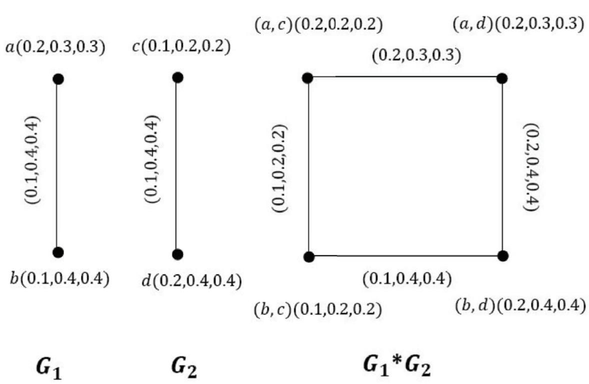

Figure 5 Single-valued neutrosophic graphs.

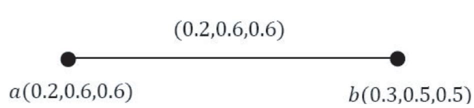

Figure 6 $\mathbf{G}_1$.

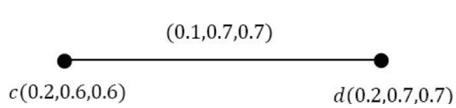

Figure 7 $\mathbf{G}_2$.

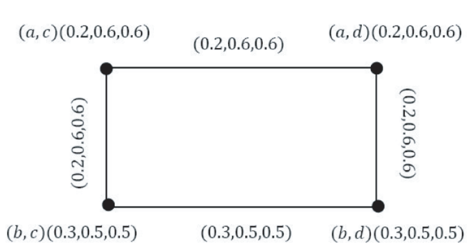

Figure 8 $\mathbf{G}_1 * \mathbf{G}_2$.

$\min\{T_{M_2}\left(m_2\right), T_{M_2}(n_2 = \min\{0.2, 0.2\} = 0.2$. Hence, $T_{N_2}\left(m_2, n_2\right) \neq \min\{T_{M_2}\left(m_2\right), T_{M_2}(n_2$.

Theorem 3. *The maximal product of two connected-(SVNGs) is a connected-SVNG.*

Proof. Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two connected-(SVNGs) on crisp graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively, where $V_1 = \{m_1, m_2, \cdots m_k\}$ and $V_2 = \{n_1, n_2, \cdots n_s\}$. Then $T^{\infty}_{N_1}\left(m_i m_j\right) > 0$, for all m_i, $m_j \in V_1$ and $T^{\infty}_{N_2}\left(n_i n_j\right) > 0$, for all n_i, $n_j \in V_2$ (or $I^{\infty}_{N_1}\left(m_i m_j\right) < 1$, for all m_i, $m_j \in V_1$ and $I^{\infty}_{N_2}\left(n_i n_j\right) < 1$, for all n_i, $n_j \in V_2$ (or $F^{\infty}_{N_1}\left(m_i m_j\right) < 1$, for all m_i, $m_j \in V_1$ and $F^{\infty}_{N_2}\left(n_i n_j\right) < 1$, for all $n_i, n_j \in V_2$. The maximal product of $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ can be taken as $G = (M, N)$. Now, consider the 'k' subgraphs of G with the vertex set $\{(m_i, n_1), (m_i, n_2), \cdots, (m_i, n_s)\}$, for $i = 1, 2, \cdots, k$. Each of these subgraphs of G is connected, since the $m_i's$ are the same and since G_2 is connected, each n_i is adjacent to at least one of the vertices in V_2. Also, since G_1 is connected, each x_i is adjacent to at least one of the vertices in V_1.

Hence, there exists at least one edge between any pair of the above "k" subgraphs. Thus we have $T^{\infty}_{N}\left(\left(m_i, n_j\right)\left(m_m, n_n\right)\right) > 0$ or $I^{\infty}_{N}\left(\left(m_i, n_j\right)\left(m_m, n_n\right)\right) < 1$ (or $F^{\infty}_{N}\left(\left(m_i, n_j\right)\left(m_m, n_n\right)\right) < 1$) for all $((m_i, n_j)(m_m, n_n)) \in E$. Hence, G is a connected-SVNG. □

Remark 2. The maximal product of two complete-(SVNGs) is not a complete-SVNG, in general. Because we do not include the case $(m_1, m_2) \in E_1$ and $(n_1, n_2) \in E_2$ in the definition of the maximal prod-uct of two (SVNGs).

Remark 3. The maximal product of two complete-(SVNGS) is a strong-SVNG.

Example 5. Consider the complete-(SVNGs) G_1 and G_2 as in Figure 5. A simple calculation concludes that $G_1 * G_2$ is a strong-SVNG.

Definition 8. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). $\forall (m_1, m_2) \in V_1 \times V_2$:

$$\begin{aligned}
&\left(d_T\right)_{\mathbf{G}_1 * \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} \left(T_{N_1} * T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \\
&\sum_{m_1 = n_1, m_2 n_2 \in E_2} \max\left\{T_{M_1}\left(m_1\right), T_{N_2}\left(m_2 n_2\right)\right\} + \\
&\sum_{m_1 n_1 \in E_1, m_2 = n_2} \max\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}\left(m_2\right)\right\},
\end{aligned}$$

$$\begin{aligned}
&\left(d_I\right)_{\mathbf{G}_1 * \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} \left(I_{N_1} * I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \\
&\sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\left\{I_{M_1}\left(m_1\right), I_{N_2}\left(m_2 n_2\right)\right\} + \\
&\sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}\left(m_2\right)\right\},
\end{aligned}$$

$$(d_F)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (F_{N_1} * F_{N_2})((m_1,m_2)(n_1,n_2)) = \sum_{m_1=n_1,m_2n_2\in E_2} \min\{F_{M_1}(m_1), F_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1,m_2=n_2} \min\{F_{N_1}(m_1n_1), F_{M_2}(m_2)\}.$$

Theorem 4. *Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ are two (SVNGs). If $T_{M_1} \geqslant T_{N_2}, I_{M_1} \leqslant I_{N_2}, F_{M_1} \leqslant F_{N_2}$ and $T_{M_2} \geqslant T_{N_1}, I_{M_2} \leqslant I_{N_1}, F_{M_2} \leqslant F_{N_1}$ then for every $(m_1, m_2) \in V_1 \times V_2$ we have:*

$$(d_T)_{G_1*G_2}(m_1,m_2) = (d)_{G_2}(m_2)\,T_{M_1}(m_1) + (d)_{G_1}(m_1)\,T_{M_2}(m_2),$$
$$(d_I)_{G_1*G_2}(m_1,m_2) = (d)_{G_2}(m_2)\,I_{M_1}(m_1) + (d)_{G_1}(m_1)\,I_{M_2}(m_2),$$
$$(d_F)_{G_1*G_2}(m_1,m_2) = (d)_{G_2}(m_2)\,F_{M_1}(m_1) + (d)_{G_1}(m_1)\,F_{M_2}(m_2).$$

Proof.

$$(d_T)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (T_{N_1} * T_{N_2})((m_1,m_2)(n_1,n_2))$$
$$= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{T_{M_1}(m_1), T_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1,m_2=n_2} \max\{T_{N_1}(m_1n_1), T_{M_2}(m_2)\}$$
$$= \sum_{m_2n_2\in E_2,m_1=n_1} T_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} T_{N_1}(m_1n_1)$$
$$= (d)_{G_2}(m_2)\,T_{M_1}(m_1) + (d)_{G_1}(m_1)\,T_{M_2}(m_2),$$

$$(d_I)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (I_{N_1} * I_{N_2})((m_1,m_2)(n_1,n_2))$$
$$= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{I_{M_1}(m_1), I_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1,m_2=n_2} \min\{I_{N_1}(m_1n_1), I_{M_2}(m_2)\}$$
$$= \sum_{m_2n_2\in E_2,m_1=n_1} I_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} I_{N_1}(m_1n_1)$$
$$= (d)_{G_2}(m_2)\,I_{M_1}(m_1) + (d)_{G_1}(m_1)\,I_{M_2}(m_2),$$

$$(d_F)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (F_{N_1} * F_{N_2})((m_1,m_2)(n_1,n_2))$$
$$= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{F_{M_1}(m_1), F_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1,m_2=n_2} \min\{F_{N_1}(m_1n_1), F_{M_2}(m_2)\}$$
$$= \sum_{m_2n_2\in E_2,m_1=n_1} F_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} F_{N_1}(m_1n_1)$$
$$= (d)_{G_2}(m_2)\,F_{M_1}(m_1) + (d)_{G_1}(m_1)\,F_{M_2}(m_2).$$

Example 6. Consider the (SVNGs) G_1, G_2, and $G_1 * G_2$ as in Figure 9. Since $T_{M_1} \geqslant T_{N_2}, I_{M_1} \leqslant I_{N_2}, F_{M_1} \leqslant F_{N_2}, T_{M_2} \geqslant T_{N_1} I_{M_2} \leqslant I_{N_1}$ and $F_{M_2} \leqslant F_{N_1}$ by Theorem 4, we have

$$(d_T)_{G_1*G_2}(a,c) = (d)_{G_2}(c)T_{M_1}(a) + (d)_{G_1}(a)T_{M_2}(c) = 1\cdot(0.3) + 1\cdot(0.2) = 0.5,$$
$$(d_I)_{G_1*G_2}(a,c) = (d)_{G_2}(c)I_{M_1}(a) + (d)_{G_1}(a)I_{M_2}(c) = 1\cdot(0.4) + 1\cdot(0.3) = 0.7,$$
$$(d_F)_{G_1*G_2}(a,c) = (d)_{G_2}(c)F_{M_1}(a) + (d)_{G_1}(a)F_{M_2}(c) = 1\cdot(0.4) + 1\cdot(0.3) = 0.7.$$

$$(d_T)_{G_1*G_2}(a,d) = (d)_{G_2}(d)T_{M_1}(a) + (d)_{G_1}(a)T_{M_2}(d) = 1\cdot(0.3) + 1\cdot(0.3) = 0.6,$$
$$(d_I)_{G_1*G_2}(a,d) = (d)_{G_2}(d)I_{M_1}(a) + (d)_{G_1}(a)I_{M_2}(d) = 1\cdot(0.4) + 1\cdot(0.4) = 0.8,$$
$$(d_F)_{G_1*G_2}(a,d) = (d)_{G_2}(d)F_{M_1}(a) + (d)_{G_1}(a)F_{M_2}(d) = 1\cdot(0.4) + 1\cdot(0.4) = 0.8.$$

$$(d_T)_{G_1*G_2}(b,c) = (d)_{G_2}(c)T_{M_1}(b) + (d)_{G_1}(b)T_{M_2}(c) = 1\cdot(0.2) + 1\cdot(0.2) = 0.4,$$
$$(d_I)_{G_1*G_2}(b,c) = (d)_{G_2}(c)I_{M_1}(b) + (d)_{G_1}(b)I_{M_2}(c) = 1\cdot(0.3) + 1\cdot(0.3) = 0.6,$$
$$(d_F)_{G_1*G_2}(b,c) = (d)_{G_2}(c)F_{M_1}(b) + (d)_{G_1}(b)F_{M_2}(c) = 1\cdot(0.3) + 1\cdot(0.3) = 0.6.$$

$$(d_T)_{G_1*G_2}(b,d) = (d)_{G_2}(d)T_{M_1}(b) + (d)_{G_1}(b)T_{M_2}(d) = 1\cdot(0.2) + 1\cdot(0.3) = 0.5,$$
$$(d_I)_{G_1*G_2}(b,d) = (d)_{G_2}(d)I_{M_1}(b) + (d)_{G_1}(b)I_{M_2}(d) = 1\cdot(0.3) + 1\cdot(0.4) = 0.7,$$
$$(d_F)_{G_1*G_2}(b,d) = (d)_{G_2}(d)F_{M_1}(b) + (d)_{G_1}(b)F_{M_2}(d) = 1\cdot(0.3) + 1\cdot(0.4) = 0.7.$$

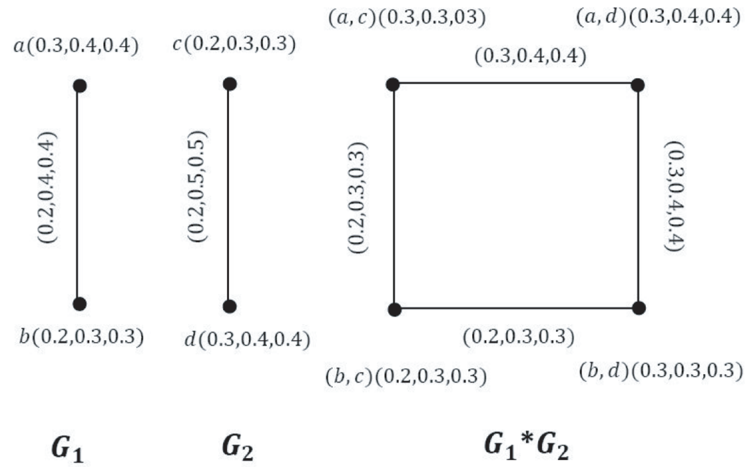

Figure 9 Single-valued neutrosophic graphs.

By direct calculations:

$$
\begin{aligned}
(d_T)_{G_1*G_2}(a,c) &= 0.3+0.2=0.5,\\
(d_I)_{G_1*G_2}(a,c) &= 0.4+0.3=0.7,\\
(d_F)_{G_1*G_2}(a,c) &= 0.4+0.3=0.7,\\
(d_T)_{G_1*G_2}(a,d) &= 0.3+0.3=0.6,\\
(d_I)_{G_1*G_2}(a,d) &= 0.4+0.4=0.8,\\
(d_F)_{G_1*G_2}(a,d) &= 0.4+0.4=0.8,\\
(d_T)_{G_1*G_2}(b,c) &= 0.2+0.2=0.4,\\
(d_I)_{G_1*G_2}(b,c) &= 0.3+0.3=0.6,\\
(d_F)_{G_1*G_2}(b,c) &= 0.3+0.3=0.6,\\
(d_T)_{G_1*G_2}(b,d) &= 0.3+0.2=0.5,\\
(d_I)_{G_1*G_2}(b,d) &= 0.3+0.4=0.7,\\
(d_F)_{G_1*G_2}(b,d) &= 0.3+0.4=0.7.
\end{aligned}
$$

It is clear from the above calculations that the degrees of vertices calculated by using the formula of the above theorem and by directed method are the same.

Definition 9. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). For any vertex $(m_1, m_2) \in V_1 \times V_2$ we have

$$
\begin{aligned}
&(td_T)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) =\\
&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} \left(T_{N_1} * T_{N_2}\right)\left((m_1,m_2)(n_1,n_2)\right)\\
&+\left(T_{M_1} * T_{M_2}(m_1,m_2)\right.\\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\left\{T_{M_1}(m_1), T_{N_2}(m_2n_2)\right\}\\
&\quad + \sum_{m_1n_1\in E_1,m_2=n_2} \max\left\{T_{N_1}(m_1n_1), T_{M_2}(m_2)\right\}\\
&\quad + \max\left\{T_{M_1}(m_1), T_{M_2}(m_2)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&(td_I)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) =\\
&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} \left(I_{N_1} * I_{N_2}\right)\left((m_1,m_2)(n_1,n_2)\right)+\\
&\left(I_{M_1} * I_{M_2}(m_1,m_2)\right)\\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \min\left\{I_{M_1}(m_1), I_{N_2}(m_2n_2)\right\}\\
&\quad + \sum_{m_1n_1\in E_1,m_2=n_2} \min\left\{I_{N_1}(m_1n_1), I_{M_2}(m_2)\right\}\\
&\quad + \min\left\{I_{M_1}(m_1), I_{M_2}(m_2)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&(td_F)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) =\\
&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} \left(F_{N_1} * F_{N_2}\right)\left((m_1,m_2)(n_1,n_2)\right)+\\
&\left(F_{M_1} * F_{M_2}(m_1,m_2)\right)\\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \min\left\{F_{M_1}(m_1), F_{N_2}(m_2n_2)\right\}\\
&\quad + \sum_{m_1n_1\in E_1,m_2=n_2} \min\left\{F_{N_1}(m_1n_1, F_{M_2}(m_2)\right\}\\
&\quad + \min\left\{F_{M_1}(m_1), F_{M_2}(m_2)\right\}.
\end{aligned}
$$

Theorem 5. *Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). If $T_{M_1} \geqslant T_{N_2}, I_{M_1} \leqslant I_{N_2}, F_{M_1} \leqslant F_{N_2}$ and $T_{M_2} \geqslant T_{N_1}, I_{M_2} \leqslant I_{N_1}, F_{M_2} \leqslant F_{N_1}$, then for every $(m_1, m_2) \in V_1 \times V_2$ we have*

$$
\begin{aligned}
(td_T)_{\mathbf{G}_{1*}\mathbf{G}_2}(m_1,m_2) &= (d)_{G_2}(m_2)\,T_{M_1}(m_1) + (d)_{G_1}(m_1)\,T_{M_2}(m_2)\\
&\quad + \max\left\{T_{M_1}(m_1), T_{M_2}(m_2)\right\},\\
(td_I)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) &= (d)_{\mathbf{G}_2}(m_2)\,I_{M_1}(m_1) + (d)_{G_1}(m_1)\,I_{M_2}(m_2)\\
&\quad + \min\left\{I_{M_1}(m_1), I_{M_2}(m_2)\right\},\\
(td_F)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) &= (d)_{G_2}(m_2)\,F_{M_1}(m_1) + (d)_{G_1}(m_1)\,F_{M_2}(m_2)\\
&\quad + \min\left\{F_{M_1}(m_1), F_{M_2}(m_2)\right\}.
\end{aligned}
$$

Proof.

$$(td_T)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (T_{N_1} * T_{N_2})((m_1,m_2)(n_1,n_2)) + (T_{M_1} * T_{M_2})(m_1,m_2)$$

$$= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{T_{M_1}(m_1), T_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1,m_2=n_2} \max\{T_{N_1}(m_1n_1), T_{M_2}(m_2)\} + \max\{T_{M_1}(m_1), T_{M_2}(m_2)\}$$

$$= \sum_{m_2n_2\in E_2,m_1=n_1} T_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} T_{N_1}(m_1n_1) + \max\{T_{M_1}(m_1), T_{M_2}(m_2)\}$$

$$= (d)_{G_2}(m_2)T_{M_1}(m_1) + (d)_{G_1}(m_1)T_{M_2}(m_2) + \max\{T_{M_1}(m_1), T_{M_2}(m_2)\}.$$

$$(td_I)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (I_{N_1} * I_{N_2})((m_1,m_2)(n_1,n_2)) + (I_{M_1} * I_{M_2})(m_1,m_2)$$

$$= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{I_{M_1}(m_1), I_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1,m_2=n_2} \min\{I_{N_1}(m_1n_1), I_{M_2}(m_2)\} + \min\{I_{M_1}(m_1), I_{M_2}(m_2)\}$$

$$= \sum_{m_2n_2\in E_2,m_1=n_1} I_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} I_{N_1}(m_1n_1) + \min\{I_{M_1}(m_1), I_{M_2}(m_2)\}$$

$$= (d)_{G_2}(m_2)I_{M_1}(m_1) + (d)_{G_1}(m_1)I_{M_2}(m_2) + \min\{I_{M_1}(m_1), I_{M_2}(m_2)\}.$$

$$(td_F)_{\mathbf{G}_1*\mathbf{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (F_{N_1} * F_{N_2})((m_1,m_2)(n_1,n_2)) + (F_{M_1} * F_{M_2})(m_1,m_2)$$

$$= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{F_{M_1}(m_1), F_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} \min\{F_{N_1}(m_1n_1), F_{M_2}(m_2)\} + \min\{F_{M_1}(m_1), F_{M_2}(m_2)\}$$

$$= \sum_{m_2n_2\in E_2,m_1=n_1} F_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} F_{N_1}(m_1n_1) + \min\{F_{M_1}(m_1), F_{M_2}(m_2)\}$$

$$= (d)_{G_2}(m_2)F_{M_1}(m_1) + (d)_{G_1}(m_1)F_{M_2}(m_2) + \min\{F_{M_1}(m_1), F_{M_2}(m_2)\}.$$

Example 7. Consider the (SVNGs) $\mathbf{G_1}$, $\mathbf{G_1}$, and $\mathbf{G_1}*\mathbf{G_2}$ as in Figures 2–4. We find the total degree of vertices in maximal product. Hence, we choose vertex (e,a).

$$(td_T)_{\mathbf{G}_1*\mathbf{G}_2}(e,a) = (d)_{G_2}(e)T_{M_1}(a) + (d)_{G_1}(a)T_{M_2}(e) + \max\{T_{M_1}(e), T_{M_2}(a)\}$$
$$= 1(0.1) + 3(0.3) + \max(0.1, 0.3)$$
$$= 0.1 + 0.9 + 0.3 = 1.3$$

$$(td_I)_{\mathbf{G}_1*\mathbf{G}_2}(e,a) = (d)_{G_2}(e)I_{M_1}(a) + (d)_{G_1}(a)I_{M_2}(e) + \min\{I_{M_1}(e), I_{M_2}(a)\}$$
$$= 1(0.3) + 3(0.4) + \min(0.3, 0.4)$$
$$= 0.3 + 1.2 + 0.3 = 1.8$$

$$(td_F)_{\mathbf{G}_1*\mathbf{G}_2}(e,a) = (d)_{G_2}(e)F_{M_1}(a) + (d)_{G_1}(a)F_{M_2}(e) + \min\{F_{M_1}(e), F_{M_2}(a)\}$$
$$= 1(0.4) + 3(0.5) + \min(0.4, 0.5)$$
$$= 0.4 + 1.5 + 0.4 = 2.3.$$

In the same way we can find the total degree for all remaining vertices.

Definition 10. The rejection $\mathbf{G_1}|\mathbf{G_2} = (M_1|M_2, N_1|N_2)$ of two (SVNGs) $\mathbf{G_1} = (M_1, N_1)$ and $\mathbf{G_2} = (M_2, N_2)$ is defined as

(i) $(T_{M_1} \mid T_{M_2})((m_1,m_2)) = \min\{T_{M_1}(m_1), T_{M_2}(m_2)\}$

$(I_{M_1} \mid I_{M_2})((m_1,m_2)) = \max\{I_{M_1}(m_1), I_{M_2}(m_2)\}$

$(F_{M_1} \mid F_{M_2})((m_1,m_2)) = \max\{F_{M_1}(m_1), F_{M_2}(m_2)\}$

$\forall (m_1,m_2) \in (V_1 \times V_2)$,

(ii) $(T_{N_1} \mid T_{N_2})((m,m_2)(m,n_2)) = \min\{T_{M_1}(m), \forall m \in T_{M_2}(m_2), T_{M_2}(n_2)\}$

$(I_{N_1} \mid I_{N_2})((m,m_2)(m,n_2)) = \max\{I_{M_1}(m), I_{M_2}(m_2), I_{M_2}(n_2)\}$

$(F_{N_1} \mid F_{N_2})((m,m_2)(m,n_2)) = \max\{F_{M_1}(m), \{F_{M_2}(m_2), F_{M_2}(n_2)\}$

V_2 *and* $m_2n_2 \notin E_2$,

(iii) $(T_{N_1} \mid T_{N_2})((m,m_2)(m,n_2)) = \min\{T_{M_1}(m), \forall z \in T_{M_2}(m_2), T_{M_2}(n_2)\}$

$(I_{N_1} \mid I_{N_2})((m,m_2)(m,n_2)) = \max\{I_{M_1}(m), I_{M_2}(m_2), I_{M_2}(n_2)\}$

$(F_{N_1} \mid F_{N_2})((m,m_2)(m,n_2)) = \max\{F_{M_1}(m), F_{M_2}(m_2), F_{M_2}(n_2)\}$

V_2 *and* $m_1n_1 \notin E_1$,

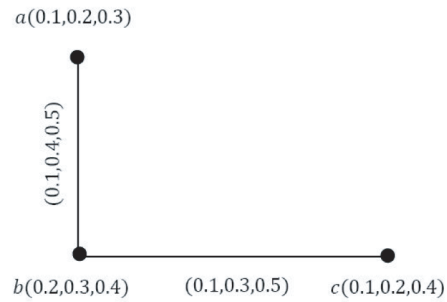

Figure 10 | G_1.

(iv) $$\left(T_{N_1} \mid T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \min\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right), T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\}$$
$$\left(I_{N_1} \mid I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), I_{M_2}\left(m_2\right), I_{N_2}\left(n_2\right)\right\}$$
$$\left(F_{N_1} \mid F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}$$
$\forall\, m_1 n_1 \notin E_1$ *and* $m_2 n_2 \notin E_2$.

Example 8. Consider the (SVNGs) $\mathbf{G_1}$ and $\mathbf{G_2}$ as in Figures 10 and 11. We can see that the rejection of two (SVNGs) $\mathbf{G_1}$ and $\mathbf{G_2}$, that is $\mathbf{G_1}|\mathbf{G_2}$ in Figure 12.

For vertex (e, a), we find true membership value, indeterminacy, and false membership value as follows:

$$\left(T_{M_1} \mid T_{M_2}\right)((e,a)) = \min\left\{T_{M_1}(e), T_{M_2}(a)\right\} = \min\{0.3, 0.1\} = 0.1,$$
$$\left(I_{M_1} \mid I_{M_2}\right)((e,a)) = \max\left\{I_{M_1}(e), I_{M_2}(a)\right\} = \max\{0.2, 0.2\} = 0.2,$$
$$\left(F_{M_1} \mid F_{M_2}\right)((e,a)) = \max\left\{F_{M_1}(e), F_{M_2}(a)\right\} = \max\{0.4, 0.3\} = 0.4,$$

for $a \in V_1$ and $e \in V_2$. For edge $(e, c)(e, a)$, we calculate true membership value, indeterminacy, and false membership value, also.

$$\left(T_{N_1} \mid T_{N_2}\right)((e,c)(e,a)) = \min\left\{T_{M_1}(e), T_{M_2}(c), T_{M_2}(a)\right\} = \min\{0.3, 0.1, 0.1\} = 0.1,$$
$$\left(I_{N_1} \mid I_{N_2}\right)((e,c)(e,a)) = \max\left\{I_{M_1}(e), I_{M_2}(c), I_{M_2}(a)\right\} = \max\{0.2, 0.2, 0.2\} = 0.2,$$
$$\left(F_{N_1} \mid F_{N_2}\right)((e,c)(e,a)) = \max\left\{F_{M_1}(e), F_{M_2}(c), F_{M_2}(a)\right\} = \max\{0.4, 0.4, 0.3\} = 0.4,$$

for $e \in V_2$ and $ac \notin E_1$.

Similarly, we can find both membership and non-membership value for all remaining vertices and edges.

Proposition 6. *The rejection of two (SVNGs)* $\mathbf{G}_1$ *and* $\mathbf{G}_2$*, is a SVNG.*

Proof. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs) on crisp graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively and $(m_1, m_2)(n_1, n_2)) \in E_1 \times E_2$. Then by Definition 10, we have

$$\left(I_{N_1} \mid I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\}$$
$$= \max\left\{\max\{I_{M_1}\left(m_1\right), I_{M_2}\left(m_2\right)\}, \max\left\{I_{M_1}\left(n_1\right), I_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \max\left\{\left(I_{M_1} \mid I_{M_2}\right)\left(m_1, m_2\right), \left(I_{M_1} \mid I_{M_2}\right)\left(n_1, n_2\right)\right\},$$

$$\left(F_{N_1} \mid F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}$$
$$= \max\left\{\max\{F_{M_1}\left(m_1\right), F_{M_2}\left(m_2\right)\}, \max\left\{F_{M_1}\left(n_1\right), F_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \max\left\{\left(F_{M_1} \mid F_{M_2}\right)\left(m_1, m_2\right), \left(F_{M_1} \mid F_{M_2}\right)\left(n_1, n_2\right)\right\}.$$

(i) If $m_1 = n_1$, $m_2 n_2 \notin E_2$

$$\left(T_{N_1} \mid T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \min\left\{T_{M_1}\left(m_1\right), T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\}$$
$$= \min\left\{\min\{T_{M_1}\left(m_1\right), T_{M_2}\left(m_2\right)\}, \min\left\{\{T_{M_1}\left(n_1\right), T_{M_2}\left(n_2\right)\}\right\}\right.$$
$$= \min\left\{\left(T_{M_1} \mid T_{M_2}\right)\left(m_1, m_2\right), \left(T_{M_1} \mid T_{M_2}\right)\left(n_1, n_2\right)\right\},$$

$$\left(I_{N_1} \mid I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\left\{I_{M_1}\left(m_1\right), I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\}$$
$$= \max\left\{\max\{I_{M_1}\left(m_1\right), I_{M_2}\left(m_2\right)\}, \max\left\{I_{M_1}\left(n_1\right), I_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \max\left\{\left(I_{M_1} \mid I_{M_2}\right)\left(m_1, m_2\right), \left(I_{M_1} \mid I_{M_2}\right)\left(n_1, n_2\right)\right\},$$

$$\left(F_{N_1} \mid F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\{F_{M_1}\left(m_1\right), F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)$$
$$= \max\{\max\{F_{M_1}\left(m_1\right), F_{M_2}\left(m_2\right), \max\{F_{M_1}\left(n_1\right), F_{M_2}\left(n_2\right)$$
$$= \max\{\left(F_{M_1} \mid F_{M_2}\right)\left(m_1, m_2\right), \left(F_{M_1} \mid F_{M_2}\right)\left(n_1, n_2\right).$$

(ii) If $m_2 = n_2$, $m_1 n_1 \notin E_1$

$$\left(T_{N_1} \mid T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \min\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right), T_{M_2}\left(m_2\right)\right\}$$
$$= \min\left\{\min\{T_{M_1}\left(m_1\right), T_{M_2}\left(m_2\right)\}, \min\left\{T_{M_1}\left(n_1\right), T_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \min\left\{\left(T_{M_1} \mid T_{M_2}\right)\left(m_1, m_2\right), \left(T_{M_1} \mid T_{M_2}\right)\left(n_1, n_2\right)\right\},$$

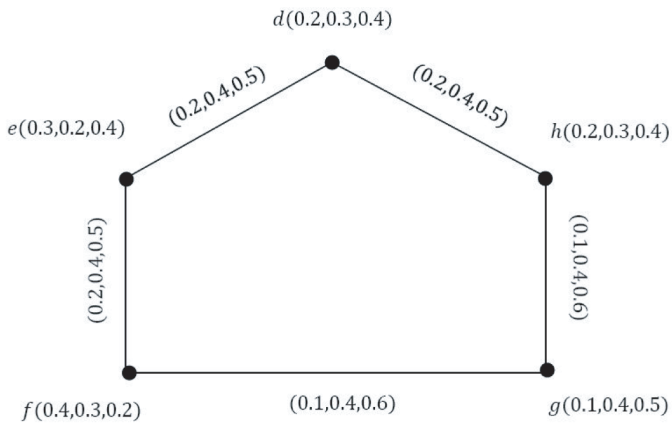

Figure 11 | G_2.

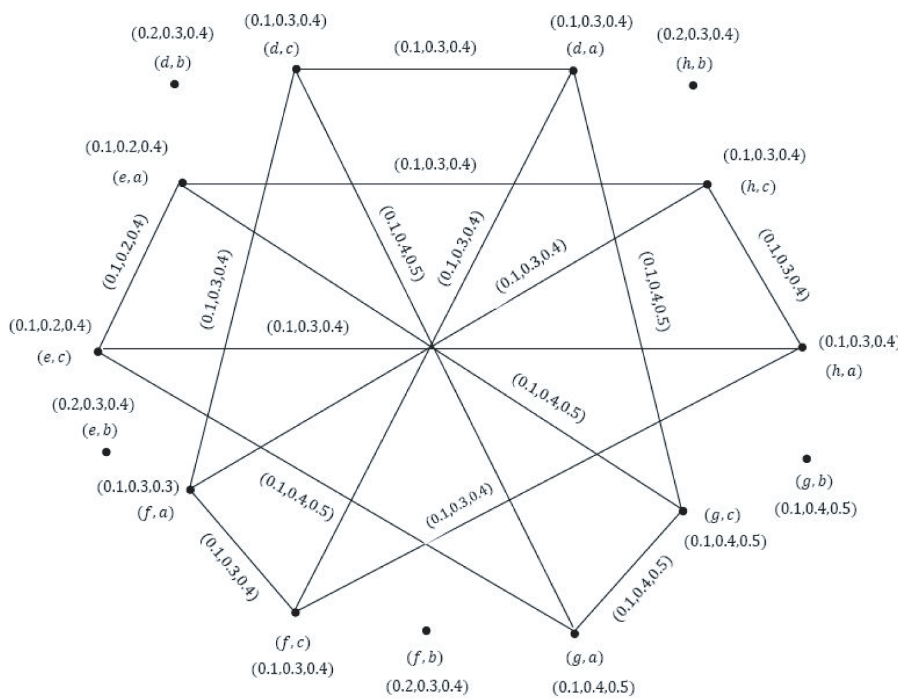

Figure 12 | $G_1|G_2$.

$$
\begin{aligned}
&\left(I_{N_1} \mid I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)\\
&= \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), I_{M_2}\left(m_2\right)\right\}\\
&= \max\left\{\max\{I_{M_1}\left(m_1\right), I_{M_2}\left(m_2\right)\right\}\\
&\max\left\{\left\{I_{M_1}\left(n_1\right), I_{M_2}\left(n_2\right)\right\}\right\}\\
&= \max\left\{\left(I_{M_1} \mid I_{M_2}\right)\left(m_1, m_2\right), \left(I_{M_1} \mid I_{M_2}\right)\left(n_1, n_2\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\left(F_{N_1} \mid F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)\\
&= \min\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), F_{M_2}\left(m_2\right)\right\}\\
&= \min\left\{\min\{F_{M_1}\left(m_1\right), F_{M_2}\left(m_2\right)\right\},\\
&\min\left\{F_{M_1}\left(n_1\right), F_{M_2}\left(n_2\right)\right\}\right\}\\
&= \min\left\{\left(F_{M_1} \mid F_{M_2}\right)\left(m_1, m_2\right), \left(F_{M_1} \mid F_{M_2}\right)\left(n_1, n_2\right)\right\},
\end{aligned}
$$

(iii) If $m_1n_1 \notin E_1$ and $m_2n_2 \notin E_2$

$$\begin{aligned}&\left(T_{N_1} \mid T_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)\\&=\min\left\{T_{M_1}\left(m_1\right),T_{M_1}\left(n_1\right),T_{M_2}\left(m_2\right),T_{M_2}\left(n_2\right)\right\}\\&=\min\left\{\min\{T_{M_1}\left(m_1\right),T_{M_2}\left(m_2\right)\},\right.\\&\quad\left.\min\left\{T_{M_1}\left(n_1\right),T_{M_2}\left(n_2\right)\right\}\right\}\\&=\min\left\{\left(T_{M_1}\mid T_{M_2}\right)\left(m_1,m_2\right),\left(T_{M_1}\mid T_{M_2}\right)\left(n_1,n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(I_{N_1} \mid I_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)=\\&\max\left\{I_{M_1}\left(m_1\right),I_{M_1}\left(n_1\right),I_{M_2}\left(m_2\right),I_{M_2}\left(n_2\right)\right\}\\&=\max\left\{\max\{I_{M_1}\left(m_1\right),I_{M_2}\left(m_2\right)\},\right.\\&\quad\left.\max\left\{I_{M_1}\left(n_1\right),I_{M_2}\left(n_2\right)\right\}\right\}\\&=\max\left\{\left(I_{M_1}\mid I_{M_2}\right)\left(m_1,m_2\right),\left(I_{M_1}\mid I_{M_2}\right)\left(n_1,n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(F_{N_1} \mid F_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)=\\&\max\left\{F_{M_1}\left(m_1\right),F_{M_1}\left(n_1\right),F_{M_2}\left(m_2\right),F_{M_2}\left(n_2\right)\right\}\\&=\max\left\{\max\{F_{M_1}\left(m_1\right),F_{M_2}\left(m_2\right)\},\right.\\&\quad\left.\max\left\{F_{M_1}\left(n_1\right),F_{M_2}\left(n_2\right)\right\}\right\}\\&=\max\left\{\left(F_{M_1}\mid F_{M_2}\right)\left(m_1,m_2\right),\left(F_{M_1}\mid F_{M_2}\right)\left(n_1,n_2\right)\right\},\end{aligned}$$

Therefore, $\mathbf{G}_1|\mathbf{G}_2 = (M_1|M_2, N_1|N_2)$ is a SVNG. □

Remark 4. The rejection of two complete (SVNGs) $\mathbf{G_1} = (M_1, N_1)$ and $\mathbf{G_2} = (M_2, N_2)$ is a complete-SVNG.

Definition 11. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). For any vertex $(m_1, m_2) \in V_1 \times V_2$ we have

$$\begin{aligned}&\left(d_T\right)_{\mathbf{G}_1|\mathbf{G}_2}\left(m_1,m_2\right)=\\&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(T_{N_1}\mid T_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)\\&=\sum_{m_1=n_1,m_2n_2\notin E_2}\min\left\{T_{M_1}\left(m_1\right),T_{M_2}\left(m_2\right)|,T_{M_2}\left(n_2\right)\right\}\\&+\sum_{m_2=n_2,m_1n_1\notin E_1}\min\left\{T_{M_1}\left(m_1\right),T_{M_1}\left(n_1\right),T_{M_2}\left(m_2\right)\right\}\\&+\sum_{m_1n_1\notin E_1 \, and \, m_2n_2\notin E_2}\min\left\{T_{M_1}\left(m_1\right),T_{M_1}\left(n_1\right),\right.\\&\left.T_{M_2}\left(m_2\right),T_{M_2}\left(n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(d_I\right)_{\mathbf{G}_1|\mathbf{G}_2}\left(m_1,m_2\right)=\\&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(I_{N_1}\mid I_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)\\&=\sum_{m_1=n_1,m_2n_2\notin E_2}\max\left\{I_{M_1}\left(m_1\right),I_{M_2}\left(m_2\right),I_{M_2}\left(n_2\right)\right\}\\&+\sum_{m_2=n_2,m_1n_1\notin E_1}\max\left\{I_{M_1}\left(m_1\right),I_{M_1}\left(n_1\right),I_{M_2}\left(m_2\right)\right\}\\&+\sum_{m_1n_1\notin E_1 \, and \, m_2n_2\notin E_2}\max\left\{I_{M_1}\left(m_1\right),I_{M_1}\left(n_1\right),\right.\\&\left.I_{M_2}\left(m_2\right),I_{M_2}\left(n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(d_F\right)_{\mathbf{G}_1|\mathbf{G}_2}\left(m_1,m_2\right)=\\&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(F_{N_1}\mid F_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)\\&=\sum_{m_1=n_1,m_2n_2\notin E_2}\max\left\{F_{M_1}\left(m_1\right),F_{M_2}\left(m_2\right),F_{M_2}\left(n_2\right)\right\}\\&+\sum_{m_2=n_2,m_1n_1\notin E_1}\max\left\{F_{M_1}\left(m_1\right),F_{M_1}\left(n_1\right),F_{M_2}\left(m_2\right)\right\}\\&+\sum_{m_1n_1\notin E_1 \, and \, m_2n_2\notin E_2}\max\left\{F_{M_1}\left(m_1\right),F_{M_1}\left(n_1\right),\right.\\&\left.F_{M_2}\left(m_2\right),F_{M_2}\left(n_2\right)\right\}.\end{aligned}$$

Definition 12. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, Y_2)$ be two (SVNGs). $\forall (m_1, m_2) \in V_1 \times V_2$

$$\begin{aligned}&\left(td_T\right)_{\mathbf{G}_1|\mathbf{G}_2}\left(m_1,m_2\right)=\\&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(T_{N_1}\mid T_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)+\\&\left(T_{M_1}\mid T_{M_2}\right)\left(m_1,m_2\right)\\&=\sum_{m_1=n_1,m_2n_2\notin E_2}\min\left\{T_{M_1}\left(m_1\right),T_{M_2}\left(m_2\right),T_{M_2}\left(n_2\right)\right\}\\&+\sum_{m_2=n_2,m_1n_1\notin E_1}\min\left\{T_{M_1}\left(m_1\right),T_{M_1}\left(n_1\right),T_{M_2}\left(m_2\right)\right\}\\&+\sum_{\mathrm{m_1n_1.\in E_1} \, and \, \mathrm{m_2n_2\in E_2}}\min\left\{T_{M_1}\left(m_1\right),T_{M_1}\left(n_1\right),\right.\\&\left.T_{M_2}\left(m_2\right),T_{M_2}\left(n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(td_I\right)_{\mathbf{G}_1|\mathbf{G}_2}\left(m_1,m_2\right)=\\&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(I_{N_1}\mid I_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)+\\&\left(I_{M_1}\mid I_{M_2}\right)\left(m_1,m_2\right)\\&=\sum_{m_1=n_1,m_2n_2\notin E_2}\max\left\{I_{M_1}\left(m_1\right),I_{M_2}\left(m_2\right),I_{M_2}\left(n_2\right)\right\}\\&+\sum_{m_2=n_2,m_1n_1\notin E_1}\max\left\{I_{M_1}\left(m_1\right),I_{M_1}\left(n_1\right),I_{M_2}\left(m_2\right)\right\}\\&+\sum_{m_1n_1\notin E_1 \text{ and } m_2n_2\notin E_2}\max\left\{I_{M_1}\left(m_1\right),I_{M_1}\left(n_1\right),\right.\\&\left.I_{M_2}\left(m_2\right),I_{M_2}\left(n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(td_F\right)_{\mathbf{G}_1|\mathbf{G}_2}\left(m_1,m_2\right)=\\&\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(F_{N_1}\mid F_{N_2}\right)\left(\left(m_1,m_2\right)\left(n_1,n_2\right)\right)+\\&\left(F_{M_1}\mid F_{M_2}\right)\left(m_1,m_2\right)\\&=\sum_{m_1=n_1,m_2n_2\notin E_2}\max\left\{F_{M_1}\left(m_1\right),F_{M_2}\left(m_2\right),F_{M_2}\left(n_2\right)\right\}\\&+\sum_{m_2=n_2,m_1n_1\notin E_1}\max\left\{F_{M_1}\left(m_1\right),F_{M_1}\left(n_1\right),F_{M_2}\left(m_2\right)\right\}\end{aligned}$$

$$+\sum_{m_1n_1\notin E_1 \text{ and } m_2n_2\notin E_2} \max\left\{F_{M_1}(m_1), F_{M_1}(n_1), F_{M_2}(m_2), F_{M_2}(n_2)\right\}.$$

Example 9. In this example we find the degree and the total degree of vertex (*d, a*) in Example 8.

$$(d_T)_{\mathbf{G}_1|\mathbf{G}_2}(d,a) = \min\left\{T_{M_2}(d), T_{M_1}(a), T_{M_1}(c)\right\} + \min\left\{T_{M_2}(d), T_{M_1}(a), T_{M_2}(f), T_{M_1}(c)\right\} + \min\left\{T_{M_2}(d), T_{M_1}(a), T_{M_2}(g), T_{M_1}(c)\right\}$$
$$= \min\{0.2, 0.1, 0.1\} + \min\{0.2, 0.1, 0.4, 0.1\} + \min\{0.2, 0.1, 0.1, 0.1\} = 0.1 + 0.1 + 0.1 = 0.3,$$

$$(d_I)_{\mathrm{G}_1|\mathrm{G}_2}(d,a) = \max\{I_{M_2}(d), I_{M_1}(a), I_{M_1}(c) + \max\left\{I_{M_2}(d), I_{M_1}(a), I_{M_2}(f), I_{M_1}(c)\right\} + \max\left\{I_{M_2}(d), I_{M_1}(a), I_{M_2}(g), I_{M_1}(c)\right\}$$
$$= \max\{0.3, 0.2, 0.3\} + \max\{0.3, 0.2, 0.3, 0.2\} + \max\{0.3, 0.2, 0.4, 0.2\} = 0.3 + 0.3 + 0.4 = 1.0,$$

$$(d_F)_{\mathbf{G}_1|\mathbf{G}_2}(d,a) = \max\left\{F_{M_2}(d), F_{M_1}(a), F_{M_1}(c)\right\} + \max\left\{F_{M_2}(d), F_{M_1}(a), F_{M_2}(f), F_{M_1}(c)\right\} + \max\left\{F_{M_2}(d), F_{M_1}(a), F_{M_2}(g), F_{M_1}(c)\right\}$$
$$= \max\{0.4, 0.3, 0.4\} + \max\{0.4, 0.3, 0.2, 0.4\} + \max\{0.4, 0.3, 0.5, 0.4\} = 0.4 + 0.4 + 0.5 = 1.3.$$

Hence, $d_{\mathbf{G}_1|\mathbf{G}_2}(a,c) = (0.3, 1.0, 1.3)$.

In addition, by definition of total vertex degree in rejection,

$$(td_T)_{\mathbf{G}_1|\mathbf{G}_2}(d,a) = \min\left\{T_{M_2}(d), T_{M_1}(a), T_{M_1}(c)\right\} + \min\left\{T_{M_2}(d), T_{M_1}(a), T_{M_2}(f), T_{M_1}(c)\right\} + \min\left\{T_{M_2}(d), T_{M_1}(a), T_{M_2}(g), T_{M_1}(c)\right\} + \min\{T_{M_2}(d), T_{M_1}(a)$$
$$= \min\{0.2, 0.1, 0.1\} + \min\{0.2, 0.1, 0.4, 0.1\} + \min\{0.2, 0.1, 0.1, 0.1\} + \min\{0.2, 0.1\} = 0.1 + 0.1 + 0.1 + 0.1 = 0.4,$$

$$(td_I)_{\mathbf{G}_1|\mathbf{G}_2}(d,a) = \max\left\{I_{M_2}(d), I_{M_1}(a), I_{M_1}(c)\right\} + \max\left\{I_{M_2}(d), I_{M_1}(a), I_{M_2}(f), I_{M_1}(c)\right\} + \max\left\{I_{M_2}(d), I_{M_1}(a), I_{M_2}(g), I_{M_1}(c)\right\} + \max\left\{I_{M_2}(d), I_{M_1}(a)\right\}$$
$$= \max\{0.3, 0.2, 0.3\} + \max\{0.3, 0.2, 0.3, 0.3\} + \max\{0.3, 0.2, 0.4, 0.3\} + \max\{0.3, 0.2\}$$
$$= 0.3 + 0.3 + 0.4 + 0.3 = 1.3,$$

$$(td_F)_{\mathbf{G}_1|\mathbf{G}_2}(d,a) = \max\left\{F_{M_2}(d), F_{M_1}(a), F_{M_1}(c)\right\} + \max\left\{F_{M_2}(d), F_{M_1}(a), F_{M_2}(f), F_{M_1}(c)\right\} + \max\left\{F_{M_2}(d), F_{M_1}(a), F_{M_2}(g), F_{M_1}(c)\right\} + \max\left\{F_{M_2}(d), F_{M_1}(a)\right\}$$
$$= \max\{0.4, 0.3, 0.4\} + \max\{0.4, 0.3, 0.2, 0.4\} + \max\{0.4, 0.3, 0.5, 0.4\} + \max\{0.4, 0.3\}$$
$$= 0.4 + 0.4 + 0.5 + 0.4 = 1.7.$$

So, $td_{\mathbf{G}_1|\mathbf{G}_2}(a,c) = (0.4, 1.3, 1.7)$.

Similarly, we can find the degree and the total degree of all vertices in $\mathbf{G}_1|\mathbf{G}_2$.

Definition 13. The symmetric difference $\mathbf{G}_1 \oplus \mathbf{G}_2 = (M_1 \oplus M_2, N_1 \oplus N_2)$ of two (SVNGs) $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ is defined as

(i) $$\left(T_{M_1} \oplus T_{M_2}\right)\left((m_1, m_2)\right) = \min\left\{T_{M_1}(m_1), T_{M_2}(m_2)\right\}$$
$$\left(I_{M_1} \oplus I_{M_2}\right)\left((m_1, m_2)\right) = \max\left\{I_{M_1}(m_1), I_{M_2}(m_2)\right\}$$
$$\left(F_{M_1} \oplus F_{M_2}\right)\left((m_1, m_2)\right) = \max\left\{F_{M_1}(m_1), F_{M_2}(m_2)\right\}$$
$\forall (m_1, m_2) \in (V_1 \times V_2)$,

(ii) $$\left(T_{N_1} \oplus T_{N_2}\right)\left((m, m_2)(m, n_2)\right) = \min\left\{T_{M_1}(m), T_{N_2}(m_2n_2)\right\}$$
$$\left(I_{N_1} \oplus I_{N_2}\right)\left((m, m_2)(m, n_2)\right) = \max\left\{I_{M_1}(m), I_{N_2}(m_2n_2)\right\}$$
$$\left(F_{N_1} \oplus F_{N_2}\right)\left((m, m_2)(m, n_2)\right) = \max\left\{F_{M_1}(m), F_{N_2}(m_2n_2)\right\}$$
$\forall m \in \mathrm{V}_1 \text{ and } m_2n_2 \in E_2$,

(iii) $$\left(T_{N_1} \oplus T_{N_2}\right)\left((m_1, z)(n_1, z)\right) = \min\left\{T_{N_1}(m_1n_1), T_{M_2}(z)\right\}$$
$$\left(I_{N_1} \oplus I_{N_2}\right)\left((m_1, z)(n_1, z)\right) = \max\left\{I_{N_1}(m_1n_1), I_{M_2}(z)\right\}$$
$$\left(F_{N_1} \oplus F_{N_2}\right)\left((m_1, z)(n_1, z)\right) = \max\left\{F_{N_1}(m_1n_1), F_{M_2}(z)\right\}$$
$\forall z \in V_2 \text{ and } m_1n_1 \in E_1$,

(iv) $(T_{N1} \oplus T_{N2})((m_1, m_2)(n_1, n_2)) = \min\{T_{M1}(m_1), T_{M1}(n_1), T_{N2}(m_2n_2)\}$ *forall* $m_1n_1 \notin E_1$ *and* $m_2n_2 \in E_2$
or
$= \min\{T_{M2}(m_2), T_{M2}(n_2), T_{N1}(m_1n_1)\}$
forall $m_1n_1 \in E_1$ *and* $m_2n_2 \notin E_2$,
$(I_{N1} \oplus I_{N2})((m_1, m_2)(n_1, n_2)) = \max\{I_{M1}(m_1), I_{M1}(n_1), I_{N2}(m_2n_2)\}$ *forall* $m_1n_1 \notin E_1$ *and* $m_2n_2 \in E_2$
or
$= \max\{I_{M2}(m_2), I_{M2}(n_2), I_{N1}(m_1n_1)\}$

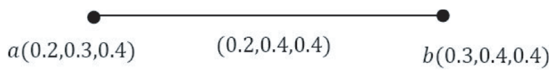

Figure 13 | G_1.

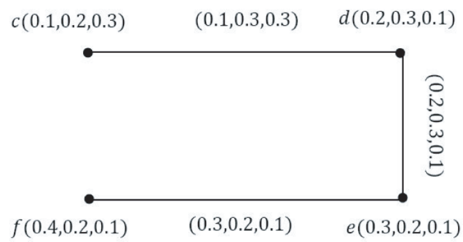

Figure 14 | G_2.

forall $m_1n_1 \in E_1$ *and* $m_2n_2 \notin E_2$,

$$\left(F_{N_1} \oplus F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), F_{N_2}\left(m_2n_2\right)\right\} \textit{for all } m_1n_1 \notin E_1 \textit{ and } m_2n_2 \in E_2$$

or

$$= \max\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right), F_{N_1}\left(m_1n_1\right)\right\} \textit{for all } m_1n_1 \in E_1 \textit{ and } m_2n_2 \notin E_2.$$

Example 10. Consider the (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$ as in Figures 13 and 14. We can see the symmetric difference of two (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$, that is $\mathbf{G}_1 \oplus \mathbf{G}_2$ in Figure 15.

For vertex (a, f), we find the true membership value, indeterminacy, and the false membership value as follows:

$$\left(T_{M_1} \oplus T_{M_2}\right)((a,f)) = \min\left\{T_{M_1}(a), T_{M_2}(f)\right\}$$
$$= \min\{0.2, 0.4\} = 0.2,$$
$$\left(I_{M_1} \oplus I_{M_2}\right)((a,f)) = \max\left\{I_{M_1}(a), I_{M_2}(f)\right\}$$
$$= \max\{0.3, 0.2\} = 0.3,$$
$$\left(F_{M_1} \oplus F_{M_2}\right)((a,f)) = \max\left\{F_{M_1}(a), F_{M_2}(f)\right\}$$
$$= \max\{0.4, 0.1\} = 0.4,$$

for $a \in V_1$ and $f \in V_2$.

For edge $(a, d)(a, e)$, we calculate the true membership value, indeterminacy, and the false membership value.

$$\left(T_{N_1} \oplus T_{N_2}\right)((a,d)(a,e)) = \min\left\{T_{M_1}(a), T_{N_2}(de)\right\}$$
$$= \min\{0.2, 0.2\} = 0.2,$$
$$\left(I_{N_1} \oplus I_{N_2}\right)((a,d)(a,e)) = \max\left\{I_{M_1}(a), I_{N_2}(de)\right\},$$
$$= \max\{0.3, 0.3\} = 0.3,$$
$$\left(F_{N_1} \oplus F_{N_2}\right)((a,d)(a,e)) = \max\left\{F_{M_1}(a), F_{N_2}(de)\right\}$$
$$= \max\{0.4, 0.1\} = 0.4.$$

for $a \in V_1$ and $de \in E_2$.

Now, for edge $(a, d)(b, d)$ we have

$$\left(T_{N_1} \oplus T_{N_2}\right)((a,d)(b,d)) = \min\left\{T_{N_1}(ab), T_{M_2}(d)\right\}$$
$$= \min\{0.2, 0.2\} = 0.2,$$
$$\left(I_{N_1} \oplus I_{N_2}\right)((a,d)(b,d)) = \max\left\{I_{N_1}(ab), I_{M_2}(d)\right\}$$
$$= \max\{0.4, 0.3\} = 0.4,$$
$$\left(F_{N_1} \oplus F_{N_2}\right)((a,d)(b,d)) = \max\left\{F_{N_1}(ab), F_{M_2}(d)\right\}$$
$$= \max\{0.4, 0.1\} = 0.4,$$

for $ab \in E_1$ and $d \in V_2$.

Finally, for edge $(a, c)(b, f)$ we can find the true membership value, indeterminacy, and the false membership value as follows:

$$\left(T_{N_1} \oplus T_{N_2}\right)((a,c)(b,f)) = \min\{T_{M_2}(c), T_{M_2}(f).$$
$$T_{N_1}(ab)\} = \min\{0.1, 0.4, 0.2\} = 0.1,$$
$$\left(I_{N_1} \oplus I_{N_2}\right)((a,c)(b,f)) = \max\{I_{M_2}(c), F_{M_2}(f),$$
$$I_{N_1}(ab)\} = \max\{0.2, 0.2, 0.4\} = 0.4,$$
$$\left(F_{N_1} \oplus F_{N_2}\right)((a,c)(b,f)) = \max\{F_{M_2}(c), F_{M_2}(f),$$
$$\left\{F_{N_1}(ab)\right\} = \max\{0.3, 0.4, 0.4\} = 0.4,$$

for $ab \in E_1$ and $cf \notin E_2$. In the same way, we can find the true membership value, indeterminacy, and the false membership value for all remaining vertices and edges.

Proposition 7. *The symmetric difference of two (SVNGs)* $\mathbf{G}_1$ *and* $\mathbf{G}_2$, *is a SVNG.*

Proof. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs) on crisp graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively and $((m_1, m_2)(n_1, n_2)) \in E_1 \times E_2$. Then by Definition 3.21 we have

(i) If $m_1 = n_1 = m$

$$\left(T_{N_1} \oplus T_{N_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right)$$
$$= \min\left\{T_{M_1}(m), T_{N_2}\left(m_2n_2\right)\right\}$$
$$\leqslant \min\left\{T_{M_1}(m), \min\left\{T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \min\left\{\min\left\{\left\{T_{M_1}(m), T_{M_2}\left(m_2\right)\right\}, \min\left\{\left\{T_{M_1}(m), T_{M_2}\left(n_2\right)\right\}\right\}\right.$$
$$= \min\left\{\left(T_{M_1} \oplus T_{M_2}\right)\left(m, m_2\right), \left(T_{M_1} \oplus T_{M_2}\right)\left(m, n_2\right)\right\},$$
$$\left(I_{N_1} \oplus I_{N_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right)$$
$$= \max\left\{I_{M_1}(m), I_{N_2}\left(m_2n_2\right)\right\}$$
$$\geqslant \max\left\{I_{M_1}(m), \max\left\{I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \max\left\{\max\left\{\left\{I_{M_1}(m), I_{M_2}\left(m_2\right)\right\}, \max\left\{\left\{I_{M_1}(m), I_{M_2}\left(n_2\right)\right\}\right\}\right.$$
$$= \max\left\{\left(I_{M_1} \oplus I_{M_2}\right)\left(m, m_2\right), \left(I_{M_1} \oplus I_{M_2}\right)\left(m, n_2\right)\right\},$$

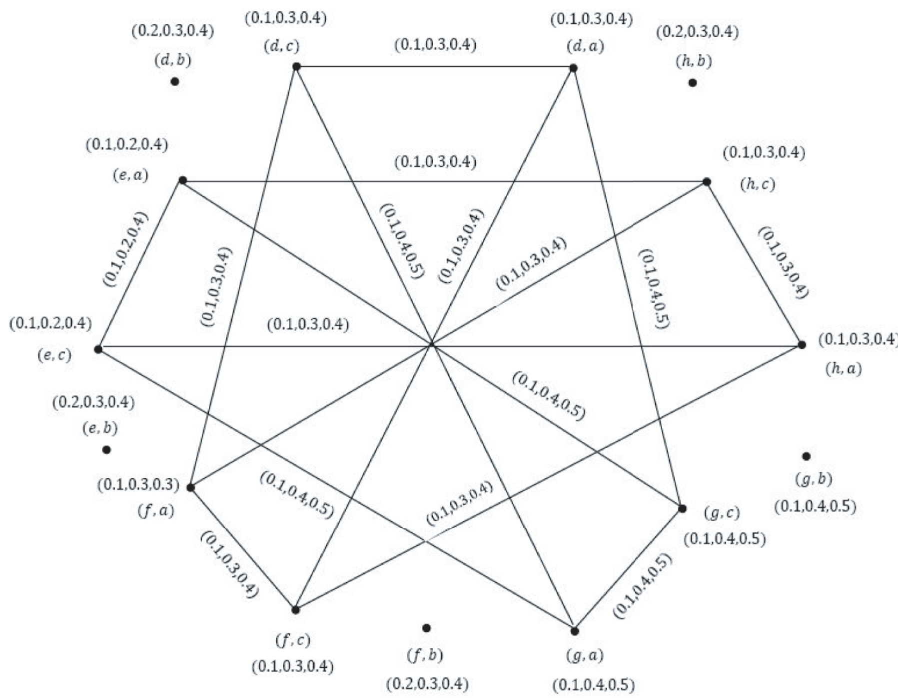

Figure 15 $G_1 \oplus G_2$.

$$\left(F_{N_1} \oplus F_{N_2}\right)\left(\left(m, m_2\right)\left(m, n_2\right)\right)$$
$$= \max\left\{F_{M_1}(m), F_{N_2}\left(m_2 n_2\right)\right\}$$
$$\geqslant \max\left\{F_{M_1}(m), \max\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \max\left\{\max\left\{\left\{F_{M_1}(m), F_{M_2}\left(m_2\right)\right\}, \max\left\{\left\{F_{M_1}(m), F_{M_2}\left(n_2\right)\right\}\right\}\right.$$
$$= \max\left\{\left(F_{M_1} \oplus F_{M_2}\right)\left(m, m_2\right), \left(F_{M_1} \oplus F_{M_2}\right)\left(m, n_2\right)\right\}.$$

(ii) If $m_2 = n_2 = z$

$$\left(T_{N_1} \oplus T_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)$$
$$= \min\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}$$
$$\leqslant \min\left\{\min\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}\right.$$
$$= \min\left\{\min\left\{\left\{T_{M_1}\left(m_1\right), T_{M_2}(z)\right\}, \min\left\{\left\{T_{M_1}\left(n_1\right), T_{M_2}(z)\right\}\right\}\right.$$
$$= \min\left\{\left(T_{M_1} \oplus T_{M_2}\right)\left(m_1, z\right), \left(T_{M_1} \oplus T_{M_2}\right)\left(n_1, z\right)\right\},$$

$$\left(I_{N_1} \oplus I_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)$$
$$= \max\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}$$
$$\geqslant \max\left\{\max\{I_{N_1}\left(m_1 n_1\right), I_{M_2}(z)\right\}$$
$$= \max\left\{\max\left\{\left\{I_{M_1}\left(m_1\right), I_{M_2}(z)\right\}, \max\left\{\left\{I_{M_1}\left(n_1\right), I_{M_2}(z)\right\}\right.\right.$$
$$= \max\left\{\left(I_{M_1} \oplus I_{M_2}\right)\left(m_1, z\right), \left(I_{M_1} \oplus I_{M_2}\right)\left(n_1, z\right)\right\},$$

$$\left(F_{N_1} \oplus F_{N_2}\right)\left(\left(m_1, z\right)\left(n_1, z\right)\right)$$
$$= \max\left\{F_{N_1}\left(m_1 n_1\right), F_{M_2}(z)\right\}$$
$$\geqslant \max\left\{\max\{F_{N_1}\left(m_1 n_1\right), T_{M_2}(z)\right\}$$
$$= \max\left\{\max\left\{\left\{F_{M_1}\left(m_1\right), F_{M_2}(z)\right\}, \max\left\{\left\{F_{M_1}\left(n_1\right), F_{M_2}(z)\right\}\right\}\right.$$
$$= \max\left\{\left(F_{M_1} \oplus F_{M_2}\right)\left(m_1, z\right), \left(F_{M_1} \oplus F_{M_2}\right)\left(n_1, z\right)\right\}.$$

(iii) *If* $m_1 n_1 \in E_1$ *and* $m_2 n_2 \notin E_2$

$$\left(T_{N_1} \oplus T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)$$
$$= \min\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right), T_{N_2}\left(m_2 n_2\right)\right\}$$
$$\leqslant \min\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right), \min\left\{T_{M_2}\left(m_2\right) T_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \min\left\{\min\{T_{M_1}\left(m_1\right), T_{M_2}\left(m_2\right)\right\}, \min\{T_{M_1}\left(m_1\right), T_{M_2}\left(n_2\right)$$
$$= \min\left\{\left(T_{M_1} \oplus T_{M_2}\right)\left(m_1, m_2\right), \left(T_{M_1} \oplus T_{M_2}\right)\left(n_1, n_2\right)\right\},$$
$$\left(I_{N_1} \oplus I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)$$
$$= \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), I_{N_2}\left(m_2 n_2\right)\right\}$$
$$\geqslant \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), \max\{I_{M_2}\left(m_2\right) I_{M_2}\left(n_2\right)\right\}$$
$$= \max\left\{\max\{I_{M_1}\left(m_1\right), I_{M_2}\left(m_2\right)\right\}, \max\{I_{M_1}\left(m_1\right), I_{M_2}\left(n_2\right)$$
$$= \max\left\{\left(I_{M_1} \oplus I_{M_2}\right)\left(m_1, m_2\right), \left(I_{M_1} \oplus I_{M_2}\right)\left(n_1, n_2\right)\right\},$$
$$\left(F_{N_1} \oplus F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)$$
$$= \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), F_{N_2}\left(m_2 n_2\right)\right\}$$
$$\geqslant \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), \max\left\{F_{M_2}\left(m_2\right) F_{M_2}\left(n_2\right)\right\}\right\}$$
$$= \max\left\{\max\left\{F_{M_1}\left(m_1\right), F_{M_2}\left(m_2\right)\right\}, \max\left\{F_{M_1}\left(m_1\right), F_{M_2}\left(n_2\right)\right\}\right.$$
$$= \max\left\{\left(F_{M_1} \oplus F_{M_2}\right)\left(m_1, m_2\right), \left(F_{M_1} \oplus F_{M_2}\right)\left(n_1, n_2\right)\right\}.$$

(iv) If $m_1 n_1 \in E_1$ *and* $m_2 n_2 \notin E_2$

$$\left(T_{N_1} \oplus T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)$$
$$= \min\left\{T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right), T_{N_1}\left(m_1 n_1\right)\right\}$$
$$\leqslant \min\left\{T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right), \min\left\{T_{M_1}\left(m_1\right) T_{M_1}\left(n_1\right)\right\}\right\}$$
$$= \min\left\{\min\left\{T_{M_1}\left(m_1\right), T_{M_2}\left(m_2\right)\right\}, \min\{T_{M_1}\left(n_1\right), T_{M_2}\left(n_2\right)\right\}$$
$$= \min\left\{\left(T_{M_1} \oplus T_{M_2}\right)\left(m_1, m_2\right), \left(T_{M_1} \oplus T_{M_2}\right)\left(n_1, n_2\right)\right\},$$

$$
\begin{aligned}
&\left(I_{N_1} \oplus I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \max\left\{I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right), I_{N_1}\left(m_1 n_1\right)\right\} \\
&\geqslant \max\left\{I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right), \max\left\{I_{M_1}\left(m_1\right) I_{M_1}\left(n_1\right)\right\}\right\} \\
&= \max\left\{\max\left\{I_{M_2}\left(m_2\right), I_{M_1}\left(m_1\right)\right\},\right. \\
&\quad \left.\max\left\{I_{M_2}\left(m_2\right), I_{M_1}\left(n_1\right)\right\}\right\} \\
&= \max\left\{\left(I_{M_1} \oplus I_{M_2}\right)\left(m_1, m_2\right), \left(I_{M_1} \oplus I_{M_2}\right)\left(n_1, n_2\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(F_{N_1} \oplus F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \max\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right), F_{N_1}\left(m_1 n_1\right)\right\} \\
&\geqslant \max\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right), \max\left\{F_{M_1}\left(m_1\right) F_{M_1}\left(n_1\right)\right\}\right\} \\
&= \max\left\{\max\left\{F_{M_2}\left(m_2\right), F_{M_1}\left(m_1\right)\right\},\right. \\
&\quad \left.\max\left\{F_{M_2}\left(m_2\right), F_{M_1}\left(n_1\right)\right\}\right\} \\
&= \max\left\{\left(F_{M_1} \oplus F_{M_2}\right)\left(m_1, m_2\right), \left(F_{M_1} \oplus F_{M_2}\right)\left(n_1, n_2\right)\right\}.
\end{aligned}
$$

Hence, $\mathbf{G}_1 \oplus \mathbf{G}_2$ is a SVNG. □

Remark 5. The symmetric difference of two connected-(SVNGs) $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ is connected. Because we include the case $(m_1, m_2) \in E_1$ and $(n_1, n_2) \in E_2$ in the definition of the symmetric difference of two (SVNGs).

Definition 14. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). For any vertex $(m_1, m_2) \in V_1 \times V_2$ we have

$$
\begin{aligned}
&\left(d_T\right)_{\mathbf{G}_1 \oplus \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{\left(m_1, m_2\right)\left(n_1, n_2\right) \in E_1 \times E_2.} \left(T_{N_1} \oplus T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\left\{T_{M_1}\left(m_1\right), T_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\left\{T_{N_1}(m_1 n_1, T_{M_2}\left(m_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{ and } m_2 n_2 \in E_2} \min\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right), T_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{ and } m_2 n_2 \in E_2} \min\left\{T_{N_1}\left(m_1 n_1\right), T_{M_1}\left(m_2\right), T_{M_2}\left(n_2\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(d_I\right)_{\mathbf{G}_1 \oplus \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{\left(m_1, m_2\right)\left(n_1, n_2\right) \in E_1 \times E_2.} \left(I_{N_1} \oplus I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\left\{I_{M_1}\left(m_1\right), I_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\left\{I_{N_1}(m_1 n_1, I_{M_2}\left(m_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{ and } m_2 n_2 \in E_2} \min\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), I_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1 \text{ and } m_2 n_2 \notin E_2} \min\left\{I_{N_1}\left(m_1 n_1\right), I_{M_1}\left(m_2\right), I_{M_2}\left(n_2\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left(d_F\right)_{\mathbf{G}_1 \oplus \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{\left(m_1, m_2\right)\left(n_1, n_2\right) \in E_1 \times E_2} \left(F_{N_1} \oplus F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \max\left\{F_{M_1}\left(m_1\right), F_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \max\left\{F_{N_1}(m_1 n_1, F_{M_2}\left(m_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{ and } m_2 n_2 \in E_2} \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right), F_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1 \text{ and } m_2 n_2 \notin'_k E_2} \max\left\{F_{N_1}\left(m_1 n_1\right), F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}.
\end{aligned}
$$

Theorem 8. *Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, Y_2)$ be two (SVNGs). If $T_{M_1} \geqslant T_{N_2}, I_{M_1} \leqslant I_{N_2}, F_{M_1} \leqslant F_{N_2}$ and $T_{M_2} \geqslant T_{N_1}, I_{M_2} \leqslant I_{N_1}, F_{M_2} \leqslant F_{N_1}$, then for every $(m_1, m_2) \in V_1 \times V_2$ we have*

$(d)_{\mathbf{G}_1 \oplus \mathbf{G}_2}\left(m_1, m_2\right) = q(d)_{\mathbf{G}_1}\left(m_1\right) + s(d)_{\mathbf{G}_2}\left(m_2\right)$ where $s = |V_1| - (d)_{\mathbf{G}_1}\left(m_1\right)$ and $q = |V_2| - (d)_{\mathbf{G}_2}\left(m_2\right)$.

Proof.

$$
\begin{aligned}
&\left(d_T\right)_{\mathbf{G}_1 \oplus \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{\left(m_1, m_2\right)\left(n_1, n_2\right) \in E_1 \times E_2} \left(T_{N_1} \oplus T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\{T_{M_1}\left(m_1\right), T_{N_2}\left(m_2 n_2\right) \\
&\quad + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\{T_{N_1}\left(m_1 n_1\right), T_{M_2}\left(m_2\right) \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{and } m_2 n_2 \in E_2} \min\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right), T_{N_2}\left(m_2 n_2\right) \\
&\quad + \sum_{m_1 n_1 \in E_1 \text{and } m_2 n_2 \notin E_2} \min\left\{T_{N_1}\left(m_1 n_1\right), T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\} \\
&= \sum_{m_2 n_2 \in E_2} T_{N_2}\left(m_2 n_2\right) + \sum_{m_1 n_1 \in E_1} T_{N_1}\left(m_1 n_1\right) \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{and } m_2 n_2 \in E_2} T_{N_2}\left(m_2 n_2\right)\} + \\
&\sum_{m_1 n_1 \in E_1 \text{ and } m_2 n_2 \notin E_2} T_{N_1}\left(m_1 n_1\right) \\
&= q\left(d_T\right)_{\mathbf{G}_1}\left(m_1\right) + s\left(d_T\right)_{\mathbf{G}_2}\left(m_2\right),
\end{aligned}
$$

$$
\begin{aligned}
&\left(d_I\right)_{\mathbf{G}_1 \oplus \mathbf{G}_2}\left(m_1, m_2\right) = \\
&\sum_{\left(m_1, m_2\right)\left(n_1, n_2\right) \in E_1 \times E_2} \left(I_{N_1} \oplus I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \max\left\{I_{M_1}\left(m_1\right), I_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \max\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}\left(m_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{and } m_2 n_2 \in E_2} \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right), I_{N_2}\left(m_2 n_2\right)\right\} \\
&\quad + \sum_{m_1 n_1 \in E_1 \text{and } m_2 n_2 \notin E_2} \max\left\{I_{N_1}\left(m_1 n_1\right), I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\} \\
&= \sum_{m_2 n_2 \in E_2} I_{N_2}\left(m_2 n_2\right) + \sum_{m_1 n_1 \in E_1} I_{N_1}\left(m_1 n_1\right) \\
&\quad + \sum_{m_1 n_1 \notin E_1 \text{and } m_2 n_2 \in E_2} I_{N_2}\left(m_2 n_2\right) + \\
&\sum_{m_1 n_1 \in E_1 \text{ and } m_2 n_2 \notin E_2} I_{N_1}\left(m_1 n_1\right) \\
&= q\left(d_I\right)_{\mathbf{G}_1}\left(m_1\right) + s\left(d_I\right)_{\mathbf{G}_2}\left(m_2\right),
\end{aligned}
$$

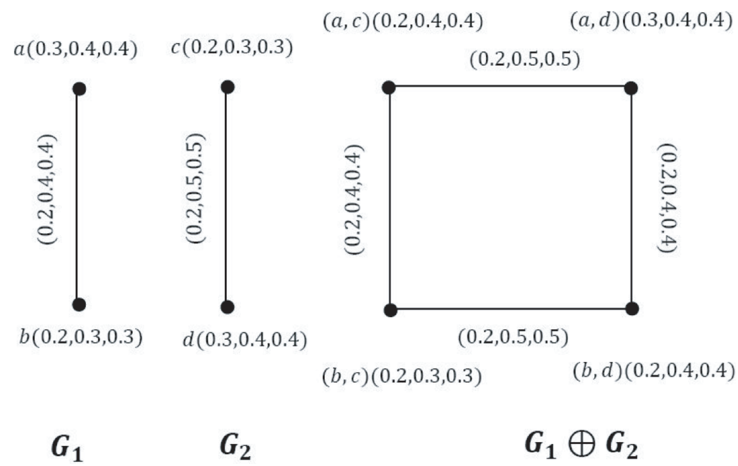

Figure 16 | Symmetric difference.

$$
\begin{aligned}
&(d_F)_{\mathbf{G}_1 \oplus \mathbf{G}_2}(m_1, m_2) \\
&= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2} \left(F_{N_1} \oplus F_{N_2}\right)\left((m_1, m_2)(n_1, n_2)\right) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \max\left\{F_{M_1}(m_1), F_{N_2}(m_2n_2)\right\} \\
&\quad + \sum_{m_1n_1\in E_1, m_2=n_2} \max\left\{F_{N_1}(m_1n_1), F_{M_2}(m_2)\right\} \\
&\quad + \sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} \max\left\{F_{M_1}(m_1), F_{M_1}(n_1), F_{N_2}(m_2n_2)\right\} \\
&\quad + \sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} \max\left\{F_{N_1}(m_1n_1), F_{M_2}(m_2), F_{M_2}(n_2)\right\} \\
&= \sum_{m_2n_2\in E_2} F_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1} F_{N_1}(m_1n_1) \\
&\quad + \sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} F_{N_2}(m_2n_2) \\
&\sum_{m_1n_1\in E_1 \text{ and } m_2n_2\notin E_2} F_{N_1}(m_1n_1) \\
&= q\,(d_F)_{\mathbf{G}_1}(m_1) + s\,(d_F)_{\mathbf{G}_2}(m_2).
\end{aligned}
$$

We conclude that $(d)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1, m_2) = q(d)_{\mathbf{G}_1}(m_1) + s(d)_{\mathbf{G}_2}(m_2)$ where $s = |V_1| - (d)_{G_1}(m_1)$ and $q = |V_2| - (d)_{G_2}(m_2)$. □

Example 11. In Figure 16, $T_{M_1} \geqslant T_{N_2}, F_{M_1} \leqslant F_{N_2}, T_{M_2} \geqslant T_{N_1}$, and $F_{M_2} \leqslant F_{N_1}$. So, the total degree of vertex in symmetric difference is calculated by using the following formula:

$$
\begin{aligned}
(d_T)_{G_1\oplus G_2}(m_1, m_2) &= q\,(d_T)_{G_1}(m_1) + s\,(d_T)_{G_2}(m_2), \\
(d_I)_{G_1\oplus G_2}(m_1, m_2) &= q\,(d_I)_{G_1}(m_1) + s\,(d_I)_{G_2}(m_2), \\
(d_F)_{G_1\oplus G_2}(m_1, m_2) &= q\,(d_F)_{G_1}(m_1) + s\,(d_F)_{G_2}(m_2).
\end{aligned}
$$

$$
\begin{aligned}
(d_T)_{G_1\oplus G_2}(a, c) &= 1\cdot(0.2) + 1\cdot(0.1) = 0.3, \\
(d_I)_{G_1\oplus G_2}(a, c) &= 1\cdot(0.4) + 1\cdot(0.3) = 0.7, \\
(d_F)_{G_1\oplus G_2}(a, c) &= 1\cdot(0.4) + 1\cdot(0.3) = 0.7, \\
(d_T)_{G_1\oplus G_2}(a, d) &= 1\cdot(0.2) + 1\cdot(0.1 + 0.2) = 0.5, \\
(d_I)_{G_1\oplus G_2}(a, d) &= 1\cdot(0.4) + 1\cdot(0.3 + 0.3) = 1.0, \\
(d_F)_{G_1\oplus G_2}(a, d) &= 1\cdot(0.4) + 1\cdot(0.3 + 0.1) = 0.8.
\end{aligned}
$$

Hence, $(d)_{G_1\oplus G_2}(a, c) = (0.3, 0.7, 0.7)$ and $(d)_{G_1\oplus G_2}(a, d) = (0.5, 1.0, 0.8)$. In the same way, we can show that $(d)_{G_1\oplus G_2}(b, c) = (d)_{G_1\oplus G_2}(b, d) = (0.4, 0.9, 0.9)$. By direct calculations:

$$
\begin{aligned}
(d_T)_{G_1\oplus G_2}(a, c) &= 0.3, \\
(d_I)_{G_1\oplus G_2}(a, c) &= 0.7, \\
(d_F)_{G_1\oplus G_2}(a, c) &= 0.7, \\
(d_T)_{G_1\oplus G_2}(a, d) &= 0.5, \\
(d_I)_{G_1\oplus G_2}(a, d) &= 1.0, \\
(d_F)_{G_1\oplus G_2}(a, d) &= 0.8, \\
(d_T)_{G_1\oplus G_2}(b, c) &= 0.3, \\
(d_I)_{G_1\oplus G_2}(b, c) &= 0.7, \\
(d_F)_{G_1\oplus G_2}(b, c) &= 0.7, \\
(d_T)_{G_1\oplus G_2}(b, d) &= 0.5, \\
(d_I)_{G_1\oplus G_2}(b, d) &= 1.0, \\
(d_F)_{G_1\oplus G_2}(b, d) &= 0.8.
\end{aligned}
$$

It is obvious from the above calculations that the degrees of vertices calculated by using the formula of the above theorem and by direct method are the same.

Definition 15. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). For any vertex $(m_1, m_2) \in V_1 \times V_2$ we have

$$\begin{aligned}
&(td_T)_{\mathbf{G}_1 \oplus \mathbf{G}_2}(m_1, m_2) \\
&= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (T_{N_1} \oplus T_{N_2})((m_1, m_2)(n_1, n_2)) + \\
&(T_{M_1} \oplus T_{M_2}(m_1, m_2) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{T_{M_1}(m_1), T_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1, m_2=n_2} \min\{T_{N_1}(m_1n_1, T_{M_2}(m_2)\} \\
&\quad + \sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} \min\{T_{M_1}(m_1), T_{M_1}(n_1), T_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} \min\{T_{N_1}(m_1n_1), T_{M_2}(m_2), T_{M_2}(n_2)\} \\
&\quad + \min\{T_{M_1}(m_1), T_{M_2}(m_2)\},
\end{aligned}$$

$$\begin{aligned}
&(td_I)_{\mathbf{G}_1 \oplus \mathbf{G}_2}(m_1, m_2) \\
&= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (I_{N_1} \oplus I_{N_2})((m_1, m_2)(n_1, n_2)) + \\
&(I_{M_1} \oplus I_{M_2}(m_1, m_2) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{I_{M_1}(m_1), I_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1, m_2=n_2} \max\{I_{N_1}(m_1n_1, I_{M_2}(m_2)\} \\
&\quad + \sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} \max\{I_{M_1}(m_1), I_{M_1}(n_1), I_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} \max\{I_{N_1}(m_1n_1), I_{M_2}(m_2), I_{M_2}(n_2)\} \\
&\quad + \max\{I_{M_1}(m_1), I_{M_2}(m_2)\},
\end{aligned}$$

$$\begin{aligned}
&(td_F)_{\mathbf{G}_1 \oplus \mathbf{G}_2}(m_1, m_2) \\
&= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (F_{N_1} \oplus F_{N_2})((m_1, m_2)(n_1, n_2)) + \\
&(F_{M_1} \oplus F_{M_2}(m_1, m_2) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{F_{M_1}(m_1), F_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \max\{F_{N_1}(m_1n_1, F_{M_2}(m_2)\} \\
&\quad + \sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} \max\{F_{M_1}(m_1), F_{M_1}(n_1), F_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} \max\{F_{N_1}(m_1n_1), F_{M_2}(m_2), F_{M_2}(n_2)\} \\
&+ \max\{F_{M_1}(m_1), F_{M_2}(m_2)\}.
\end{aligned}$$

Theorem 9. *Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, Y_2)$ be two (SVNGs).*

(i) If $T_{M_1} \geqslant T_{N_2}$ and $T_{M_2} \geqslant T_{N_1}$, then $\forall (m_1, m_2) \in V_1 \times V_2$:

$$\begin{aligned}
&(td_T)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1, m_2) = q\,(td_T)_{\mathbf{G}_1}(m_1) + s\,(td_T)_{\mathbf{G}_2}(m_2) \\
&-(q-1)T_{\mathbf{G}_1}(m_1) - \max\{T_{\mathbf{G}_1}(m_1), T_{\mathbf{G}_1}(m_1)\}.
\end{aligned}$$

(ii) If $I_{M_1} \leqslant I_{N_2}$ and $I_{M_2} \leqslant I_{N_1}$, then $\forall (m_1, m_2) \in V_1 \times V_2$:

$$\begin{aligned}
&(td_I)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1, m_2) = q\,(td_I)_{\mathbf{G}_1}(m_1) + s\,(td_I)_{\mathbf{G}_2}(m_2) \\
&-(q-1)I_{\mathbf{G}_1}(m_1) - \min\{I_{\mathbf{G}_1}(m_1), I_{\mathbf{G}_1}(m_1)\}.
\end{aligned}$$

(iii) If $F_{M_1} \leqslant F_{N_2}$ and $F_{M_2} \geqslant F_{N_1}$, then $\forall (m_1, m_2) \in V_1 \times V_2$:

$$\begin{aligned}
&(td_F)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1, m_2) = q\,(td_F)_{\mathbf{G}_1}(m_1) + s\,(td_F)_{\mathbf{G}_2}(m_2) \\
&-(q-1)F_{\mathbf{G}_1}(m_1) - \min\{F_{\mathbf{G}_1}(m_1), F_{\mathbf{G}_1}(m_1)\}.
\end{aligned}$$

$\forall (m_1, m_2) \in V_1 \times V_2, s = |V_1| - (d)_{G_1}(m_1)$ *and* $q = |V_2| - (d)_{G_2}(m_2)$.

Proof. $\forall (m_1, m_2) \in V_1 \times V_2$ we have

$$\begin{aligned}
&(td_T)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1, m_2) \\
&= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2} (T_{N_1} \oplus T_{N_2})((m_1, m_2)(n_1, n_2)) + \\
&(T_{M_1} \oplus T_{M_2})(m_1, m_2) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{T_{M_1}(m_1), T_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1, m_2=n_2} \min\{T_{N_1}(m_1n_1), T_{M_2}(m_2)\} \\
&\quad + \sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} \min\{T_{M_1}(m_1), T_{M_1}(n_1), T_{N_2}(m_2n_2)\} \\
&\quad + \sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} \min\{T_{N_1}(m_1n_1), T_{M_2}(m_2), T_{M_2}(n_2)\} \\
&\quad + \max\{T_{M_1}(m_1), T_{M_2}(m_2)\} \\
&= \sum_{m_2n_2\in E_2} T_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1} T_{N_1}(m_1n_1) \\
&\quad + \sum_{m_1n_1\in E_1 \text{ and } m_2n_2\notin E_2} T_{N_2}(m_2n_2)\} + \\
&\sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} T_{N_1}(m_1n_1) + \max\{T_{M_1}(m_1), T_{M_2}(m_2)\} \\
&= \sum_{m_2n_2\in E_2} T_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1} T_{N_1}(m_1n_1) + \\
&\sum_{m_1n_1\notin E_1 \, and \, m_2n_2\in E_2} T_{N_2}(m_2n_2)\} \\
&\sum_{m_1n_1\in E_1 \, and \, m_2n_2\notin E_2} T_{N_1}(m_1n_1) + T_{M_1}(m_1) + T_{M_2}(m_2) - \\
&\max\{T_{M_1}(m_1), T_{M_2}(m_2)\} \\
&= q\,(td_T)_{\mathbf{G}_1}(m_1) + s\,(td_T)_{\mathbf{G}_2}(m_2) \\
&-(q-1)T_{\mathbf{G}_1}(m_1) - \max\{T_{\mathbf{G}_1}(m_1), T_{\mathbf{G}_1}(m_1)\},
\end{aligned}$$

$$
\begin{aligned}
&(td_I)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1,m_2)\\
&=\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(I_{N_1}\oplus I_{N_2}\right)\left((m_1,m_2)(n_1,n_2)\right)\\
&\quad+\left(I_{M_1}\oplus I_{M_2}\right)(m_1,m_2)\\
&=\sum_{m_1=n_1,m_2n_2\in E_2}\max\left\{I_{M_1}(m_1),I_{N_2}(m_2n_2)\right\}\\
&\quad+\sum_{m_1n_1\in E_1,m_2=n_2}\max\left\{I_{N_1}(m_1n_1),I_{M_2}(m_2)\right\}\\
&\quad+\sum_{m_1n_1\notin E_1\ and\ m_2n_2\in E_2}\max\left\{I_{M_1}(m_1),I_{M_1}(n_1),I_{N_2}(m_2n_2)\right\}\\
&\quad+\sum_{m_1n_1\in E_1\ and\ m_2n_2\notin E_2}\max\left\{I_{N_1}(m_1n_1),I_{M_2}(m_2),I_{M_2}(n_2)\right\}\\
&\quad+\min\left\{I_{M_1}(m_1),I_{M_2}(m_2)\right\}\\
&=\sum_{m_2n_2\in E_2}I_{N_2}(m_2n_2)+\sum_{m_1n_1\in E_1}I_{N_1}(m_1n_1)\\
&\quad+\sum_{m_1n_1\notin E_1\ and\ m_2n_2\in E_2}I_{N_2}(m_2n_2)+\\
&\quad\sum_{m_1n_1\in E_1\ and\ m_2n_2\notin E_2}I_{N_1}(m_1n_1)+\min\left\{I_{M_1}(m_1),I_{M_2}(m_2)\right\}\\
&=\sum_{m_2n_2\in E_2}I_{N_2}(m_2n_2)+\sum_{m_1n_1\in E_1}I_{N_1}(m_1n_1)\\
&\quad+\sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2}I_{N_2}(m_2n_2)\}\\
&\quad+\sum_{m_1n_1\in E_1\ and\ m_2n_2\notin E_2}I_{N_1}(m_1n_1)+I_{M_1}(m_1)+I_{M_2}(m_2)\\
&\quad-\min\left\{I_{M_1}(m_1),I_{M_2}(m_2)\right\}\\
&=q\,(td_I)_{\mathbf{G}_1}(m_1)+s\,(td_I)_{\mathbf{G}_2}(m_2)\\
&\quad-(q-1)I_{\mathbf{G}_1}(m_1)-\min\left\{I_{\mathbf{G}_1}(m_1),I_{\mathbf{G}_1}(m_1)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&(td_F)_{\mathbf{G}_1\oplus\mathbf{G}_2}(m_1,m_2)\\
&=\sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.}\left(F_{N_1}\oplus F_{N_2}\right)\left((m_1,m_2)(n_1,n_2)\right)\\
&\quad+\left(F_{M_1}\oplus F_{M_2}\right)(m_1,m_2)\\
&=\sum_{m_1=n_1,m_2n_2\in E_2}\max\left\{F_{M_1}(m_1),F_{N_2}(m_2n_2)\right\}\\
&\quad+\sum_{m_1n_1\in E_1,m_2=n_2}\max\left\{F_{N_1}(m_1n_1),F_{M_2}(m_2)\right\}\\
&\quad+\sum_{m_1n_1\notin E_1\text{ and }m_2n_2\in E_2}\max\left\{F_{M_1}(m_1),F_{M_1}(n_1),F_{N_2}(m_2n_2)\right\}\\
&\quad+\sum_{m_1n_1\in E_1\text{ and }m_2n_2\notin E_2}\max\left\{F_{N_1}(m_1n_1),F_{M_2}(m_2),F_{M_2}(n_2)\right\}\\
&\quad+\min\left\{F_{M_1}(m_1),F_{M_2}(m_2)\right\}\\
&=\sum_{m_2n_2\in E_2}F_{N_2}(m_2n_2)+\sum_{m_1n_1\in E_1}F_{N_1}(m_1n_1)\\
&\quad+\sum_{m_1n_1\notin E_1\text{ and }m_2n_2\in E_2}F_{N_2}(m_2n_2)\}\\
&\quad+\sum_{m_1n_1\in E_1\text{ and }m_2n_2\notin E_2}F_{N_1}(m_1n_1)+\min\left\{F_{M_1}(m_1),F_{M_2}(m_2)\right\}\\
&=\sum_{m_2n_2\in E_2}F_{N_2}(m_2n_2)+\sum_{m_1n_1\in E_1}F_{N_1}(m_1n_1)\\
&\quad+\sum_{m_1n_1\notin E_1\text{ and }m_2n_2\in E_2}F_{N_2}(m_2n_2)\}\\
&\quad+\sum_{m_1n_1\in E_1\text{ and }m_2n_2\notin E_2}F_{N_1}(m_1n_1)+\{F_{M_1}(m_1)+F_{M_2}(m_2)\\
&\quad-\min\left\{F_{M_1}(m_1),F_{M_2}(m_2)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&=q\,(td_F)_{\mathbf{G}_1}(m_1)+s\,(td_F)_{\mathbf{G}_2}(m_2)\\
&\quad-(q-1)F_{\mathbf{G}_1}(m_1)-\min\left\{F_{\mathbf{G}_1}(m_1),F_{\mathbf{G}_1}(m_1)\right\},
\end{aligned}
$$

where $s=|V_1|-(d)_{G_1}(m_1)$ and $q=|V_2|-(d)_{G_2}(m_2)$. □

Example 12. In this example, we calculate the total degree of vertices in Example 10.

$$(d_T)_{\mathbf{G}_1\oplus\mathbf{G}_2}(a,e)=q\,(d_T)_{\mathbf{G}_1}(a)+s\,(d_T)_{\mathbf{G}_2}(e),$$

where $s=|V_1|-(d)_{G_1}(a)$ and $q=|V_2|-(d)_{G_2}(e)$.

$$s=|V_1|-(d)_{G_1}(a)=2-1=1.$$

Similarly,

$$q=|V_2|-(d)_{G_2}(e)=4-2=2.$$

$$
\begin{aligned}
&(td_T)_{\mathbf{G}_1\oplus\mathbf{G}_2}(a,e)=q\,(td_T)_{\mathbf{G}_1}(a)+s\,(td_T)_{\mathbf{G}_2}(e)\\
&\quad-(s-1)T_{\mathbf{G}_2}(e)-(q-1)T_{\mathbf{G}_1}(a)-\max\left\{T_{\mathbf{G}_1}(a),T_{\mathbf{G}_2}(e)\right\}\\
&=2(0.2+0.2)+1(0.3+0.3+0.2)\\
&\quad-(1-1)(0.3)-(2-1)(0.2)-\max\{0.2,0.3\}\\
&=2(0.4)+0.8-0.2-0.3=1.1,\\
&(td_I)_{\mathbf{G}_1\oplus\mathbf{G}_2}(a,e)=q\,(td_I)_{\mathbf{G}_1}(a)+s\,(td_I)_{\mathbf{G}_2}(e)\\
&\quad-(s-1)I_{\mathbf{G}_2}(e)-(q-1)I_{\mathbf{G}_1}(a)-\min\left\{I_{\mathbf{G}_1}(a),I_{\mathbf{G}_2}(e)\right\}\\
&=2(0.3+0.4)+1(0.2+0.2+0.3)\\
&\quad-(1-1)(0.2)-(2-1)(0.3)-\min\{0.3,0.2\}\\
&=2(0.7)+0.7-0.3-0.2=1.6,
\end{aligned}
$$

$$
\begin{aligned}
&(td_F)_{\mathbf{G}_1\oplus\mathbf{G}_2}(a,e)=q\,(td_F)_{\mathbf{G}_1}(a)+s\,(td_F)_{\mathbf{G}_2}(e)\\
&\quad-(s-1)F_{\mathbf{G}_2}(e)-(q-1)F_{\mathbf{G}_1}(a)-\min\left\{F_{\mathbf{G}_1}(a),F_{\mathbf{G}_2}(e)\right\}\\
&=2(0.4+0.4)+1(0.1+0.1+0.1)\\
&\quad-(1-1)(0.1)-(2-1)(0.4)-\min\{0.4,0.1\}\\
&=2(0.8)+0.3-0.4-0.1=0.6,
\end{aligned}
$$

and

$$(td)_{\mathbf{G}_1\oplus\mathbf{G}_2}(a,e)=(1.1,1.6,0.6).$$

It is clear from the above calculations that total degrees of vertices calculated by using the formula of the above theorem and by direct method are same.

Definition 16. The residue product $\mathbf{G}_1\bullet\mathbf{G}_2=(M_1\bullet M_2,N_1\bullet N_2)$ of two (SVNGs) $\mathbf{G}_1=(M_1,N_1)$ and $\mathbf{G}_2=(M_2,N_2)$ is defined as

(i) $\left(T_{M_1}\bullet T_{M_2}\right)((m_1,m_2))=\max\left\{T_{M_1}(m_1),T_{M_2}(m_2)\right\}$
$\left(I_{M_1}\bullet I_{M_2}\right)((m_1,m_2))=\min\left\{I_{M_1}(m_1),I_{M_2}(m_2)\right\}$
$\left(F_{M_1}\bullet F_{M_2}\right)((m_1,m_2))=\min\left\{F_{M_1}(m_1),F_{M_2}(m_2)\right\}$

$$\forall(m_1,m_2)\in(V_1\times V_2),$$

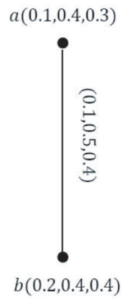

Figure 17 G_1.

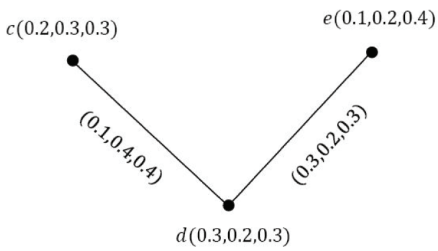

Figure 18 G_2.

(ii) $$\left(T_{N_1} \bullet T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = T_{N_1}\left(m_1 n_1\right)$$
$$\left(I_{N_1} \bullet I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = I_{N_1}\left(m_1 n_1\right)$$
$$\left(F_{N_1} \bullet F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = F_{N_1}\left(m_1 n_1\right)$$
$$\forall m_1 n_1 \in E_1, m_2 \neq n_2.$$

Example 13. Consider the (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$ as in Figures 17 and 18. We can see the residue product of two (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$, that is $\mathbf{G}_1 \bullet \mathbf{G}_2$ in Figure 19.

For vertex (b, e), we find the true membership value, indeterminacy, and the false membership value as follows:

$$\begin{aligned}\left(T_{M_1} \bullet T_{M_2}\right)((b, e)) &= \max\left\{T_{M_1}(b), T_{M_2}(e)\right\}\\ &= \max\{0.2, 0.1\} = 0.2,\\ \left(I_{M_1} \bullet I_{M_2}\right)((b, e)) &= \min\left\{I_{M_1}(b), I_{M_2}(e)\right\}\\ &= \min\{0.4, 0.2\} = 0.2,\\ \left(F_{M_1} \bullet F_{M_2}\right)((b, e)) &= \min\left\{F_{M_1}(b), F_{M_2}(e)\right\}\\ &= \min\{0.4, 0.4\} = 0.4,\end{aligned}$$

for $b \in V_1$ and $e \in V_2$.

For edge $(a, c)\,(b, d)$, we calculate the true membership value, indeterminacy, and the false membership value as follows:

$$\begin{aligned}\left(T_{N_1} \bullet T_{N_2}\right)((a, c)(b, d)) &= T_{N_1}(ab) = 0.1,\\ \left(I_{N_1} \bullet I_{N_2}\right)((a, c)(b, d)) &= F_{N_1}(ab) = 0.5,\\ \left(F_{N_1} \bullet F_{N_2}\right)((a, c)(b, d)) &= F_{N_1}(ab) = 0.4,\end{aligned}$$

for $ab \in E_1$ and $c \neq d$.

Similarly, we can find the true membership value, indeterminacy, and the false membership value for all remaining vertices and edges.

Proposition 10. *The residue product of two (SVNGs) $\mathbf{G}_1$ and $\mathbf{G}_2$, is a SVNG.*

Proof. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs) on crisp graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively and $((m_1, m_2)(n_1, n_2)) \in E_1 \times E_2$. If $m_1 n_1 \in E_1$ *and* $m_2 \neq n_2$ then we have

$$\begin{aligned}&\left(T_{N_1} \bullet T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = T_{N_1}\left(m_1 n_1\right)\\ &\leqslant \min\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right)\right\}\\ &\leqslant \max\left\{\min\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right)\},\right.\\ &\qquad \left.\min\left\{T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\}\right\}\\ &= \min\left\{\max\left\{T_{M_1}\left(m_1\right), T_{M_1}\left(n_1\right)\right\},\right.\\ &\qquad \left.\max\left\{T_{M_2}\left(m_2\right), T_{M_2}\left(n_2\right)\right\}\right\}\\ &= \min\left\{\left(T_{M_1} \bullet T_{M_2}\right)\left(m_1, m_2\right), \left(T_{M_1} \bullet T_{M_2}\right)\left(n_1, n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(I_{N_1} \bullet I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = I_{N_1}\left(m_1 n_1\right)\\ &\geqslant \max\left\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right)\right\}\\ &\geqslant \min\left\{\max\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right)\},\right.\\ &\qquad \left.\max\left\{I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\}\right\}\\ &= \max\left\{\min\{I_{M_1}\left(m_1\right), I_{M_1}\left(n_1\right)\},\right.\\ &\qquad \left.\min\left\{I_{M_2}\left(m_2\right), I_{M_2}\left(n_2\right)\right\}\right\}\\ &= \max\left\{\left(I_{M_1} \bullet I_{M_2}\right)\left(m_1, m_2\right), \left(I_{M_1} \bullet I_{M_2}\right)\left(n_1, n_2\right)\right\},\end{aligned}$$

$$\begin{aligned}&\left(F_{N_1} \bullet F_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right) = F_{N_1}\left(m_1 n_1\right)\\ &\geqslant \max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right)\right\}\\ &\geqslant \min\{\max\left\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right)\right\},\\ &\qquad \left.\max\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}\right\}\end{aligned}$$

$$\begin{aligned}&= \max\left\{\min\{F_{M_1}\left(m_1\right), F_{M_1}\left(n_1\right)\},\right.\\ &\qquad \left.\min\left\{F_{M_2}\left(m_2\right), F_{M_2}\left(n_2\right)\right\}\right\}\\ &= \max\left\{\left(F_{M_1} \bullet F_{M_2}\right)\left(m_1, m_2\right), \left(F_{M_1} \bullet F_{M_2}\right)\left(n_1, n_2\right)\right\}.\end{aligned}$$

Definition 17. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). For any vertex $(m_1, m_2) \in V_1 \times V_2$ we have

$$\begin{aligned}&\left(d_T\right)_{\mathbf{G}_1 \bullet \mathbf{G}_2}\left(m_1, m_2\right) =\\ &\sum_{(m_1, m_2)(n_1, n_2) \in E_1 \times E_2.} \left(T_{N_1} \bullet T_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} T_{N_1}\left(m_1 n_1\right)\\ &= \left(d_T\right)_{\mathbf{G}_1}\left(m_1\right),\end{aligned}$$

$$\begin{aligned}&\left(d_I\right)_{\mathbf{G}_1 \bullet \mathbf{G}_2}\left(m_1, m_2\right) =\\ &\sum_{(m_1, m_2)(n_1, n_2) \in E_1 \times E_2.} \left(I_{N_1} \bullet I_{N_2}\right)\left(\left(m_1, m_2\right)\left(n_1, n_2\right)\right)\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} I_{N_1}\left(m_1 n_1\right)\\ &= \left(d_I\right)_{\mathbf{G}_1}\left(m_1\right),\end{aligned}$$

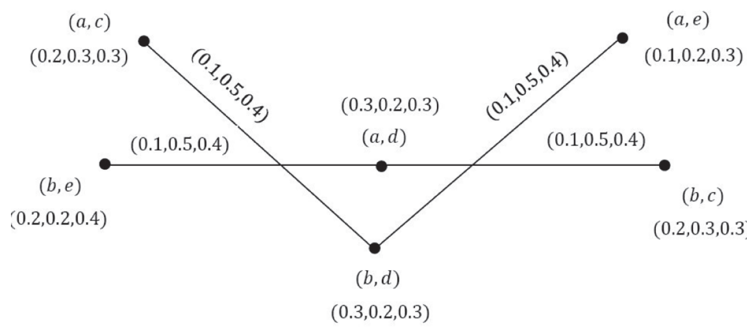

Figure 19 $G_1 \oplus G_2$.

$$\begin{aligned}(d_F)_{\mathbf{G}_1\bullet\mathbf{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (F_{N_1}\bullet F_{N_2})((m_1,m_2)(n_1,n_2)) \\ &= \sum_{m_1n_1\in E_1, m_2\neq n_2} F_{N_1}(m_1n_1) \\ &= (d_F)_{\mathbf{G}_1}(m_1).\end{aligned}$$

Definition 18. Let $\mathbf{G}_1 = (M_1, N_1)$ and $\mathbf{G}_2 = (M_2, N_2)$ be two (SVNGs). For any vertex$(m_1, m_2) \in V_1 \times V_2$ we have

$$\begin{aligned}(td_T)_{\mathbf{G}_1\bullet\mathbf{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (T_{N_1}\bullet T_{N_2})((m_1,m_2)(n_1,n_2)) + (T_{M_1}\bullet T_{M_2})(m_1,m_2)\end{aligned}$$

$$\begin{aligned}&= \sum_{m_1n_1\in E_1, m_2\neq n_2} T_{N_1}(m_1n_1) + \min\{T_{M_1}(m_1), T_{M_2}(m_2)\} \\ &= \sum_{m_1n_1\in E_1, m_2\neq n_2} T_{N_1}(m_1n_1) + T_{M_1}(m_1) + T_{M_2}(m_2) - \max\{T_{M_1}(m_1), T_{M_2}(m_2)\} \\ &= (td_T)_{\mathbf{G}_1}(m_1) + T_{M_2}(m_2) - \max\{T_{M_1}(m_1), T_{M_2}(m_2)\},\end{aligned}$$

$$\begin{aligned}(td_I)_{\mathbf{G}_1\bullet\mathbf{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (I_{N_1}\bullet I_{N_2})((m_1,m_2)(n_1,n_2)) + (I_{M_1}\bullet I_{M_2}(m_1,m_2))\end{aligned}$$

$$\begin{aligned}&= \sum_{m_1n_1\in E_1, m_2\neq n_2} I_{N_1}(m_1n_1) + \max\{I_{M_1}(m_1), I_{M_2}(m_2)\} \\ &= \sum_{m_1n_1\in E_1, m_2\neq n_2} I_{N_1}(m_1n_1) + I_{M_1}(m_1) + I_{M_2}(m_2) - \min\{I_{M_1}(m_1), I_{M_2}(m_2)\} \\ &= (td_I)_{\mathbf{G}_1}(m_1) + I_{M_2}(m_2) - \min\{I_{M_1}(m_1), I_{M_2}(m_2)\},\end{aligned}$$

$$\begin{aligned}(td_F)_{\mathbf{G}_1\bullet\mathbf{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (F_{N_1}\bullet F_{N_2})((m_1,m_2)(n_1,n_2)) + (F_{M_1}\bullet F_{M_2}(m_1,m_2)) \\ &= \sum_{m_1n_1\in E_1, m_2\neq n_2} F_{N_1}(m_1n_1) + \max\{F_{M_1}(m_1), F_{M_2}(m_2)\} \\ &= \sum_{m_1n_1\in E_1, m_2\neq n_2} F_{N_1}(m_1n_1) + F_{M_1}(m_1) + F_{M_2}(m_2) - \min\{F_{M_1}(m_1), F_{M_2}(m_2)\} \\ &= (td_F)_{\mathbf{G}_1}(m_1) + F_{M_2}(m_2) - \min\{F_{M_1}(m_1), F_{M_2}(m_2)\}.\end{aligned}$$

Example 14. In this example we find the degree and the total degree of vertex (b, e) in Example 13.

$$(d_T)_{\mathbf{G}_1\bullet\mathbf{G}_2}(b,e) = (d_T)_{\mathbf{G}_1}(b) = 0.1,$$
$$(d_I)_{\mathbf{G}_1\bullet\mathbf{G}_2}(b,e) = (d_I)_{\mathbf{G}_1}(b) = 0.5,$$
$$(d_F)_{\mathbf{G}_1\bullet\mathbf{G}_2}(b,e) = (d_F)_{\mathbf{G}_1}(b) = 0.4.$$

Therefore,

$$(d)_{\mathbf{G}_1\bullet\mathbf{G}_2}(b,e) = (0,1,0,5,0,7,4).$$

Also, total degree of vertex (a, e) is given by

$$\begin{aligned}(td_T)_{\mathbf{G}_1\bullet\mathbf{G}_2}(a,e) &= (td_T)_{\mathbf{G}_1}(a) + T_{M_2}(e) - \max\{T_{M_1}(a), T_{M_2}(e)\} \\ &= (0.2+0.1) + 0.1 - \max(0.2, 0.1) = 0.2,\end{aligned}$$

$$\begin{aligned}(td_I)\,\mathbf{G}_1\bullet\mathbf{G}_2(a,e) &= (td_I)_{\mathbf{G}_1}(a) + I_{M_2}(e) - \min\{I_{M_1}(a), I_{M_2}(e)\} \\ &= (0.4+0.5) + 0.2 - \min(0.4, 0.2) = 0.9,\end{aligned}$$

$$\begin{aligned}(td_F)_{\mathbf{G}_1\bullet\mathbf{G}_2}(a,e) &= (td_F)_{\mathbf{G}_1}(a) + F_{M_2}(e) - \min\{F_{M_1}(a), F_{M_2}(e)\} \\ &= (0.4+0.4) + 0.4 - \min(0.4, 0.4) = 0.8.\end{aligned}$$

Table 1 SVNPR of the exporter from Pakistan.

R_1	b_1	b_2	b_3	b_4	b_5
b_1	<0.5, 0.5, 0.5>	<0.2, 0.8, 0.1>	<0.1, 0.6, 0.2>	<0.2, 0.3, 0.6>	<0.1, 0.2, 0.4>
b_2	<0.1, 0.2, 0.2>	<0.5, 0.5, 0.5>	<0.2, 0.4, 0.7>	<0.1, 0.4, 0.2>	<0.9, 0.3, 0.4>
b_3	<0.1, 0.4, 0.2>	<0.7, 0.6, 0.2>	<0.5, 0.5, 0.5>	<0.6, 0.3, 0.2>	<0.4, 0.2, 0.6>
b_4	<0.6, 0.7, 0.1>	<0.2, 0.6, 0.1>	<0.2, 0.7, 0.6>	<0.5; 0.5; 0.5>	<0.3; 0.2; 0.7>
b_5	<0.4, 0.8, 0.1>	<0.4, 0.7, 0.9>	<0.6, 0.8, 0.4>	<0.7, 0.8, 0.3>	<0.5; 0.5; 0.5>

Table 2 SVNPR of the exporter from India.

R_2	b_1	b_2	b_3	b_4	b_5
b_1	<0.5, 0.5, 0.5>	<0.4, 0.6, 0.3>	<0.9, 0.4, 0.3>	<0.2, 0.1, 0.6>	<0.8, 0.3, 0.4>
b_2	<0.3, 0.4, 0.4>	<0.5, 0.5, 0.5>	<0.4, 0.8, 0.2>	<0.2, 0.1, 0.8>	<0.6, 0.3, 0.4>
b_3	<0.3, 0.6, 0.9>	<0., 20.2, 0.4>	<0.5, 0.5, 0.5>	<0.4, 0.2, 0.6>	<0.3, 0.2, 0.7>
b_4	<0.6, 0.9, 0.2>	<0.8, 0.9, 0.2>	<0.6, 0.8, 0.4>	<0.5, 0.5, 0.5>	<0.2, 0.1, 0.6>
b_5	<0.4, 0.7, 0.8>	<0.4, 0.7, 0.6>	<0.7, 0.8, 0.3>	<0.6, 0.9, 0.2>	<0.5, 0.5, 0.5>

Table 3 SVNPR of the exporter from America.

R_3	b_1	b_2	b_3	b_4	b_5
b_1	<0.5, 0.5, 0.5>	<0.6, 0.4, 0.3>	<0.5, 0.3, 0.2>	<0.4, 0.3, 0.9>	<0.2, 0.1, 0.6>
b_2	<0.3, 0.6, 0.6>	<0.5, 0.5, 0.5>	<0.4, 0.3, 0.2>	<0.5, 0.1, 0.6>	<0.2, 0.3, 0.1>
b_3	<0.2, 0.7, 0.5>	<0.2, 0.7, 0.4>	<0.5, 0.5, 0.5>	<0.4, 0.3, 0.9>	<0.2, 0.6, 0.1>
b_4	<0.9, 0.7, 0.4>	<0.6, 0.9, 0.5>	<0.9, 0.7, 0.4>	<0.5, 0.5, 0.5>	<0.4, 0.3, 0.6>
b_5	<0.6, 0.9, 0.2>	<0.1, 0.7, 0.2>	<0.1, 0.4, 0.2>	<0.6, 0.7, 0.4>	<0.5, 0.5, 0.5>

Table 4 Collective SVNPR of all above individuals SVNPRs.

R	b_1	b_2	b_3	b_4	b_5
b_1	<0.500, 0.5000, 0.5000>	<0.4231, 0.5769, 0.2080>	<0.6443, 0.4160, 0.2289>	<0.2732, 0.2080, 0.6868>	<0.4759, 0.1817, 0.4579>
b_2	<0.2388, 0.3634, 0.3634>	<0.5000, 0.5000, 0.5000>	<0.3396, 0.4579, 0.3037>	<0.2886, 0.1587, 0.4579>	<0.6825, 0.3000, 0.2520>
b_3	<0.2042, 0.5518, 0.4481>	<0.4231, 0.4380, 0.3175>	<0.5000, 0.5000, 0.5000>	<0.4759, 0.2621, 0.4762>	<0.3048, 0.2885, 0.3476>
b_4	<0.7480, 0.7612, 0.2000>	<0.6000, 0.7862, 0.2154>	<0.6825, 0.7319, 0.4579>	<0.5000, 0.5000, 0.5000>	<0.3048, 0.1817, 0.6316>
b_5	<0.4759, 0.7958, 0.2520>	<0.3132, 0.7000, 0.4762>	<0.5238, 0.6350, 0.2885>	<0.6366, 0.7958, 0.2885>	<0.5000, 0.5000, 0.5000>

Hence,

$$(td)_{\mathbf{G}_1 \bullet \mathbf{G}_2}(a, e) = (0.2, 0.9, 0.8).$$

Similarly, the degree and the total degree of all vertices can be defined in $\mathbf{G}_1 \bullet \mathbf{G}_2$.

4. APPLICATION OF SVNG IN GROUP DECISION-MAKING

Definition 19. Let [2] $Q = \{q_1, q_2, \ldots, q_n$ be the set on which single-valued neutrosophic preference relation (SVNPR) is defined. It can be denoted by a matrix of $R = (m_{st})_{n \times n}$ where $m_{st} = < q_s q_t, T(q_s q_t), I(q_s q_t), F(q_s q_t) >$ for all s and t varies from 1 to n.

4.1. Food and Agriculture Organization of United Nation Select a Most Suitable Company

FAO is attempting to help in the disposal of yearning, food instability, and creation strength the executives. Objectives can be accomplished when this association chooses the most reasonable organization for formers and works together with it which can assist Former with developing more food, offer types of assistance, and suitable item. There are five organizations of Syngenta b_1, Bayers b_2, Investment organization Institute (ICI) b_3, Agria Corporation Company (ACC) b_4, and Fazal Mahmood Company (FMC) b_5. Three exporters from various nations are welcome to partake in the choice examination. One exporter is from Pakistan, the second is from India, and the third is from America. These exporters use SVNPRs $R_i = \left(q_{xy}^{(i)}\right)_{5\times5}$ SVNDGs D_i comparing to SVNPRs $R_i (i = 1, 2, 3)$ are given in Table 1–3.

By using the aggregation operator to find all SVNPRs $R_i = \left(q_{xy}^{i}\right)_{5\times5}$, where i=1,2,3 into total SVNPR $R = (q_{st})_{5\times5}$ which is shown in Table 4. For SVNPR, we use operator SVNWA [6]. SVNWA $\left(q_{st}^{(1)}, q_{st}^{(2)}, \ldots, q_{st}^{(k)}\right) =< 1 - \prod_{i=1}^{k}\left(1 - T_{st}^{(i)}\right)^{\frac{1}{k}}, < \prod_{i=1}^{k}\left(I_{st}^{(i)}\right)^{\frac{1}{k}}, \prod_{i=1}^{s}\left(F_{st}^{(i)}\right)^{\frac{1}{k}} >$.

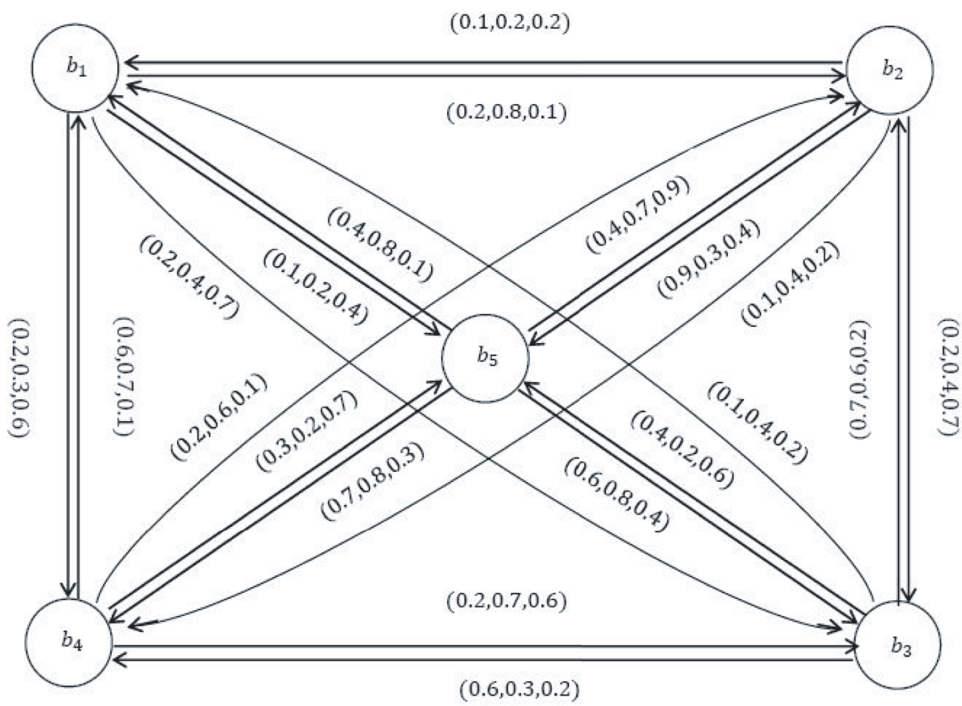

Figure 20 Single-valued neutrosophic diagraph D_1.

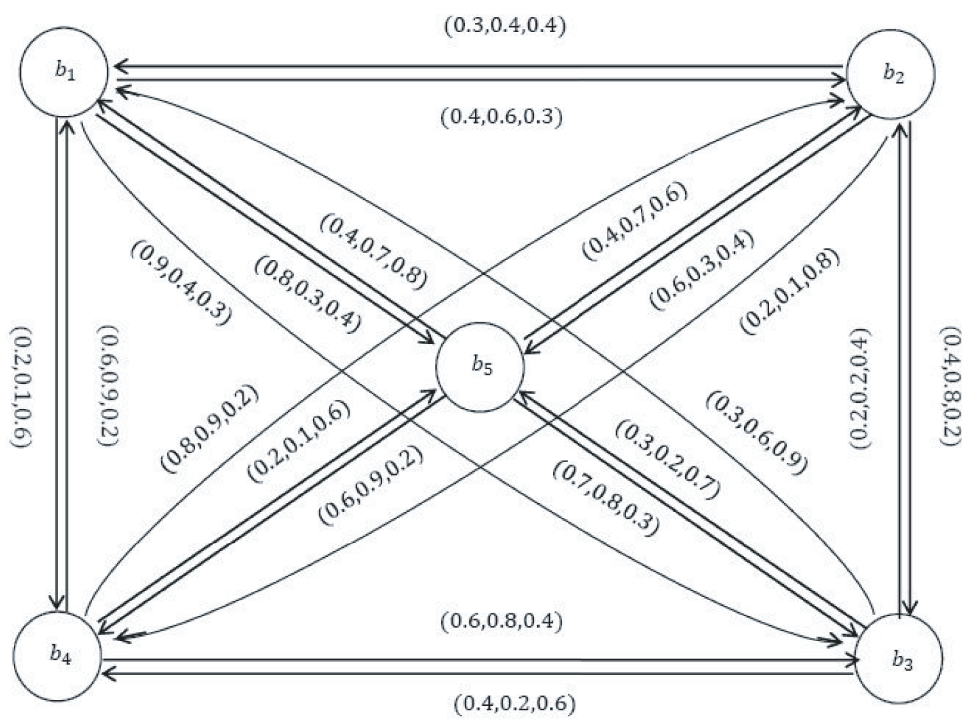

Figure 21 Single-valued neutrosophic diagraph D_2.

Data is converted in digraphs which shown in Figures 20–22. We can draw directed network corresponding to a collective SVNPR above, which is already shown in Figure 23. Under some conditions, $T_{xy} > 0.5$, where x and y ranges from 1 to 5. Likewise, we have a partial diagram of all fused SVNPR which shown in Figure 24.

We will find out the degrees which are denoted by $out - dout - d\,(b_x)$ with $x = 1,2,3,4,5$ of the whole criteria in a partial directed network as follows:

$out - d(b_1) = (0.0000, 0.0000, 0.0000)$

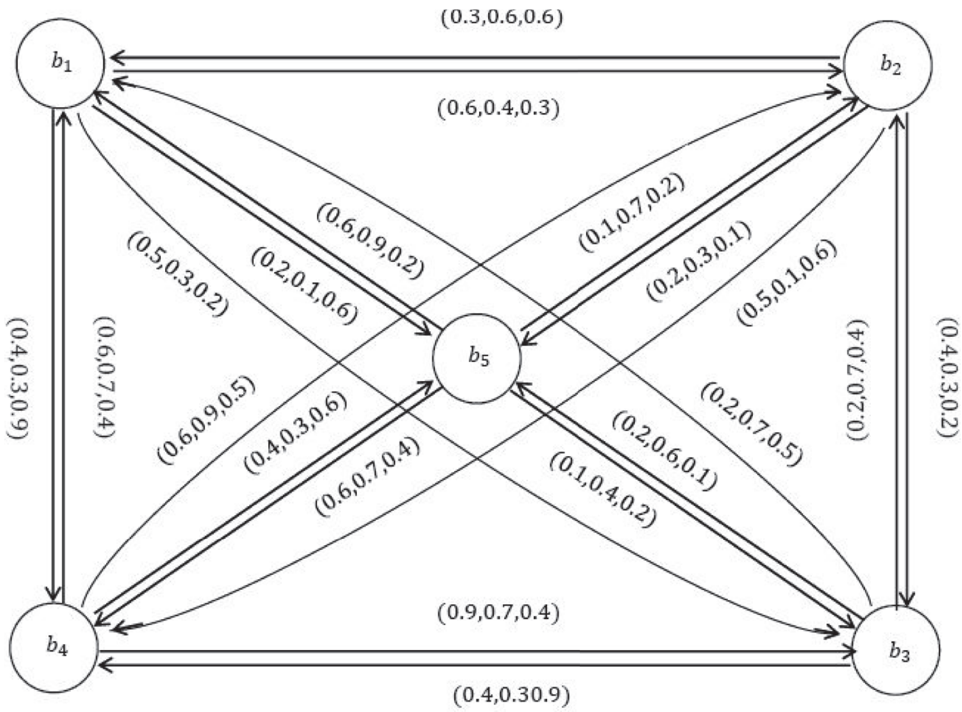

Figure 22 | Single-valued neutrosophic diagraph D_3.

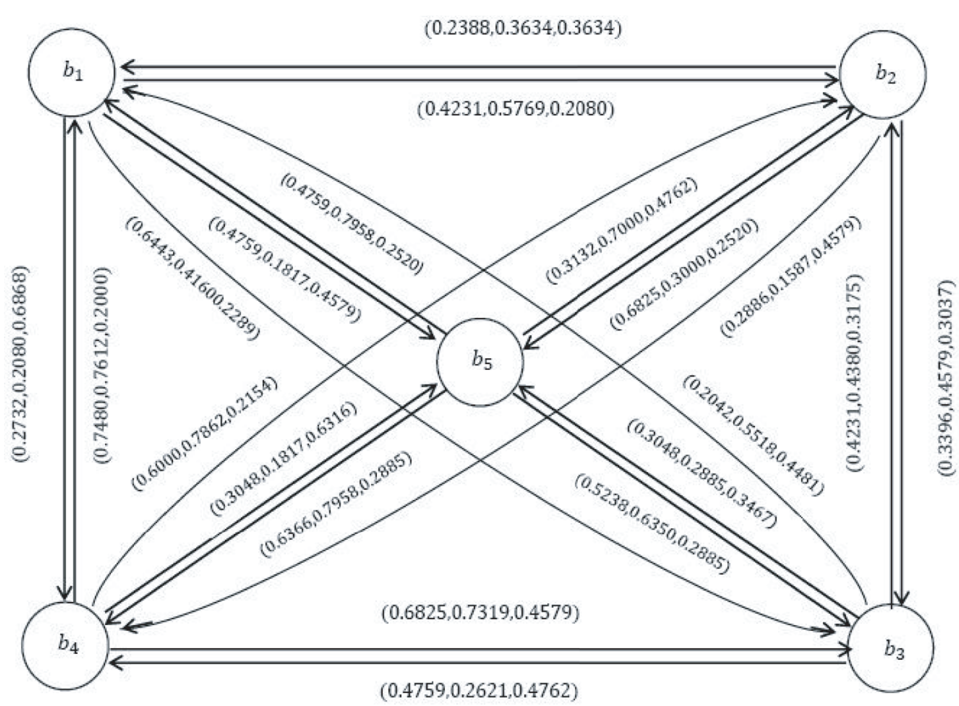

Figure 23 | Directed network of all fused SVNPR.

$out - d(b_2) = (0.6825, 0.3000, 0.2520)$

$out - d(b_3) = (0.0000, 0.0000, 0.0000)$

$out - d(b_4) = (2.0305, 2.2793, 0.6733)$

$out - d(b_5) = (1.1604, 1.4308, 0.5770)$ according to the membership degree rule of $out - d(b_x)$, $x = 1, 2, 3, 4, 5$, a ranking factors which is given below is obtained

$b_4 \succ b_5 \succ b_2 \succ b_1 \sim b_3$. So the ranking of b_5 is higher and serves as the best choice ACC b_4. To discuss the application, we give an algorithm as follows:

5. CONCLUSION

The adaptability and equivalence of neutrosophic models are higher than fluffy models and intuitionistic fluffy models. A SVNG is

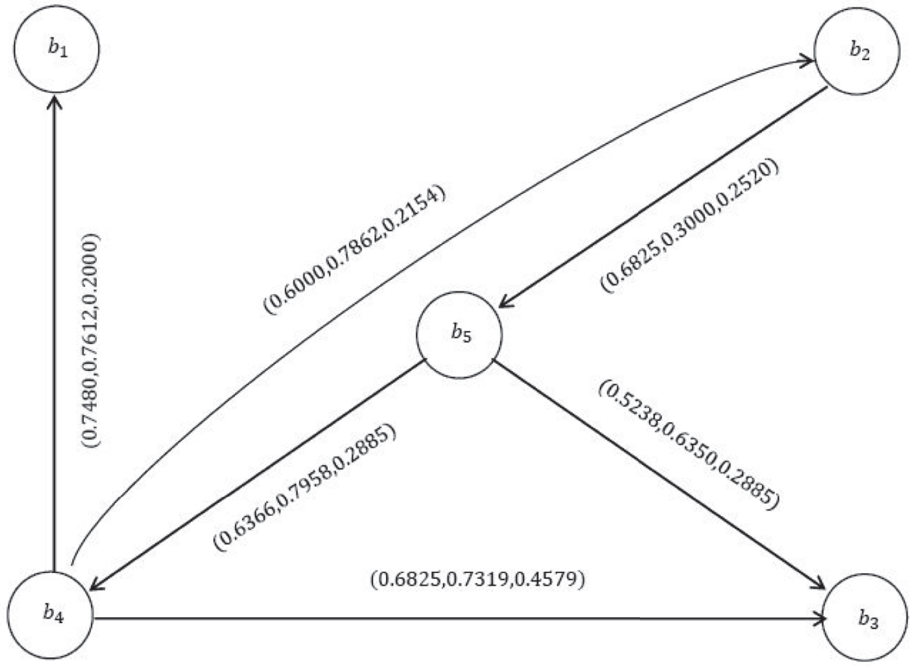

Figure 24 | Partial directed network of all fused SVNPR.

broadly utilized in clinical sciences, financial matters, and logical designing. At the point when faltering happens in a genuine issue then the SVNG has a fundamental part to investigate the vulnerability since chart and the fluffy diagram don't think about the vulnerability among the relationship of the articles. We have examined the new properties on a SVNG known as the buildup item, maximal item, symmetric distinction, and dismissal of a chart. We likewise examined the thought with guides to discover the degree and absolute level of vertices of some specific charts. A few hypotheses of these diagrams were recently settled by utilizing the idea of degree and complete level of a vertex of a chart. Additionally, the hypotheses which were identified with these properties were demonstrated. Additionally, the fascinating and helpful use of a SVNG was examined which was a choice of reasonable organization by FAO. At last, a calculation which is the strategy of our application was introduced. Next, our motivation in future work is to introduce this idea on (1) complex bipolar-SVNG, (2) complex bipolar fuzzy graph, and (3) complex interval-valued fuzzy graph with their connected applications.

CONFLICTS OF INTEREST

The authors declare of no conflicts of interest.

AUTHORS' CONTRIBUTIONS

All authors have equal contribution.

ACKNOWLEDGMENTS

This work is supported by the Social Sciences Planning Projects of Zhejiang (21QNYC11ZD).

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