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The linguistic intuitionistic fuzzy set TOPSIS method for linguistic multi-criteria decision makings

Yue Ou [1], Liangzhong Yi [2], Bin Zou [3], Zheng Pei [3]

[1] *School of Mechanical Engineering, Xihua University, Chengdu, Sichuan 610039, China.*
[2] *Sichuan Police College, Luzhou, Sichuan 646000, China.*
[3] *Center for Radio Administration & Technology Development, Xihua University, Chengdu, Sichuan 610039, China.*
E-mail: pqyz@263.net

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Abstract

In the paper, we express uncertain assessments information in linguistic multi-criteria decision makings (LMCDMs) as linguistic intuitionistic fuzzy sets, *i.e.*, the decision maker provides membership and non-membership fuzzy linguistic terms to represent uncertain assessments information of alternatives in LMCDMs, and present Hamming distance between two linguistic intuitionistic fuzzy sets. Then we propose the linguistic intuitionistic fuzzy set TOPSIS method for LMCDMs, compared with the traditional TOPSIS method, we provide different the positive ideal solution, the negative ideal solution and the relative closeness degrees of alternatives, in addition, we design an algorithm to finish the linguistic intuitionistic fuzzy set TOPSIS method for LMCDMs. We utilize a LMCDM problem to illustrate the performance, usefulness and effectiveness of the linguistic intuitionistic fuzzy set TOPSIS method, and compare it with the hesitant fuzzy linguistic multi-criteria optimization and compromise solution (HFL-VIKOR) method, the symbolic aggregation-based method and the hesitant fuzzy linguistic TOPSIS (HFL-TOPSIS) method in the example, results show that the linguistic intuitionistic fuzzy set TOPSIS method is a useful and alternative method for LMCDMs.

Keywords: The TOPSIS method; The 2-tuple linguistic model; Hesitant fuzzy linguistic term set; Intuitionistic fuzzy sets; Linguistic multi-criteria decision makings.

1. Introduction

We always face tasks and activities in which it is necessary to use decision making processes in our daily lives. Generally, decision making is a cognitive process based on different mental and reasoning processes that lead to the choice of a suitable alternative from a set of possible alternatives in a decision situation[1−4]. Despite the existence of different decision making processes in the literature composed of different phases, the TOPSIS method proposed in[5] is useful, important and widely studied multiple attribute group decision making method, formally, the TOPSIS method originates from the concept that the selected alternative should have the shortest distance from the positive ideal solution and the farthest from the negative ideal solution, it's decision making process can be expressed in the following five steps[6]: 1) normalization of decision matrix; 2) construction of weighted normalized decision matrix; 3) determination of positive and negative ideal solutions; 4) calculation of separation measures and relative closeness; 5) ranking of alternatives. After then, many extended TOPSIS methods have been applied to multiple attribute decision makings[7−13], such as, Chen[14] proposed an extended TOPSIS method for multiple attribute decision makings by considering triangular fuzzy numbers and defining crisp Euclidean distance between two fuzzy numbers. Similarly, Ashtiani et al.[15] extended the TOPSIS method to solve multiple attribute decision making problem with the interval-valued fuzzy sets. He and Gong[16] proposed a natural generalization of the TOPSIS method to solve multiple attribute decision making problem with intuitionistic fuzzy sets. Liu, et al.[17] developed a new TOPSIS method for decision making problems with interval-valued intuitionistic fuzzy data. Yue[18] developed a method for decision making problems with interval number and extended his method to intuitionistic fuzzy sets.

Because of the inherent complexity and uncertainty of the decision situation or the existence of multiple and conflicting objectives in decision making problems, human beings often use fuzzy linguistic values to express complex or uncertain information in decision making process, and decision makings with fuzzy linguistic information attract many researchers[19−29], in which, the 2-tuple linguistic model[30] is a useful and important tool for expressing and dealing with linguistic information, which provides a continuous fuzzy representation for linguistic values by the translation of the linguistic value obtained from the symbolic computation to the closest linguistic value in the initial linguistic value set. Formally, the 2-tuple linguistic model consists of modeling the linguistic information by means of a pair of elements, one element is a linguistic value similar to the fuzzy linguistic approach whose semantics is provided by a fuzzy membership function and the syntax chosen according to the choices offered by the fuzzy linguistic approach, another element is a numerical value, also called symbolic translation, that indicates the translation of the fuzzy membership function which represents the closest linguistic value if it does not match exactly the computed linguistic information. Up to now, many new symbolic representation models to improve different aspects of the 2-tuple linguistic model have been developed and many different real-world decision makings based on 2-tuple linguistic model have been applied, such as, Xu[31] introduced the extended linguistic variable based on the concept of virtual linguistic values to improve the operational laws of symbolic operations. Wang and Hao[32] proposed the linguistic proportional 2-tuple model to represent linguistic information that is a generalization and extension of the 2-tuple linguistic model, Guo et al.[33] later extended the linguistic proportional 2-tuple model by using a third parameter to deal with incomplete linguistic preferences. Dong et al.[34] presented the concept of numerical scale with the aim of completing the 2-tuple linguistic model and proportional 2-tuple models and making the elicitation of information more consistent in different decision situations. Li[35] proposed an extended 2-tuple linguistic model that fuses the use of virtual linguistic values and 2-tuple linguistic values. Yang[36] developed the counted linguistic variable for representing and aggregating linguistic information with the aim of providing better results and being easier to understand. Wei[37] investigated the 2-tuple linguistic multiple attribute group decision making problems in which the information about attribute weights is partially known. Cables et al.[38] proposed a decision making method in which the decision makers provided their assessment information to represent their qualitative preferences under 2-tuple linguistic environment. Moreover, many 2-tuple linguistic aggregation operators have been proposed in literatures[39−44] to make the aggregation of linguistic information much more flexible, we refer[4] for more details about 2-tuple linguistic model and decision makings based on 2-tuple linguistic model.

Recently, inspired by intuitionistic fuzzy sets and 2-tuple linguistic values, Chen, et al.[45] developed the concept of linguistic intuitionistic fuzzy numbers where membership and and nonmembership are represented as 2-tuple linguistic values. In order to process the multiple attribute decision making with linguistic intuitionistic fuzzy numbers, they provided the linguistic score index and linguistic accuracy index of number, analyzed the operation laws for linguistic intuitionistic fuzzy numbers and the related properties of the operation laws. Furthermore, they developed the linguistic intuitionistic fuzzy weighted averaging operator, linguistic intuitionistic fuzzy ordered weighted averaging operator and linguistic intuitionistic fuzzy hybrid averaging (LIFHA) operator, which can be utilized to aggregate the linguistic intuitionistic fuzzy information. Based on linguistic intuitionistic fuzzy numbers, this paper propose the linguistic intuitionistic fuzzy set TOPSIS method for linguistic multi-criteria decision makings, in which, we present Hamming distance between two linguistic intuitionistic fuzzy sets and analyze it's several properties, then we provide the positive ideal solution, the negative ideal solution and the relative closeness degree of alternatives to solve LMCDMs. In numerical example, we compare the linguistic intuitionistic fuzzy set TOPSIS method with the HFL-VIKOR method[46], the symbolic aggregation-based method[47] and the HFL-TOPSIS method[48]. The rest of this paper is structured as follows: In Section 2, we briefly review some basic concepts and linguistic intuitionistic fuzzy numbers. In Section 3, we define Hamming distance between two linguistic intuitionistic fuzzy sets and analyze their several properties. In Section 4, we present the framework of the linguistic intuitionistic fuzzy set TOPSIS method, and provide an algorithm to deal with LMCDMs in linguistic intuitionistic fuzzy set environment. In Section 5, we utilize a LMCDM problem to illustrate the practicality of the linguistic intuitionistic fuzzy set TOPSIS method and comparison results. We conclude the paper in Section 6.

2. Preliminaries

In this section, we briefly review some basic concepts in 2-tuple linguistic model and linguistic intuitionistic fuzzy numbers.

The 2-tuple fuzzy linguistic representation model consists of a pair of elements [30], which is explained as follows: 1) Let $s_i \in S = \{s_0, s_1, s_2, \dots, s_g\}$ be a initial linguistic term set, whose semantics is provided by a fuzzy membership function on a universe of discourse, an ordered structure provided in the linguistic term set is a total order, *i.e.*, for any $s_i, s_j \in S$, $s_i \leqslant s_j$ if and only if $i \leqslant j$, moreover, a negation operator is $Neg(s_i) = s_j$ such that $j = g - i$ ($g+1$ is the cardinality), the maximal and minimal operators are $max\{s_i, s_j\} = s_j$ and $min\{s_i, s_j\} = s_i$ if $s_i \leqslant s_j$; 2) α is a numerical value that indicates error when a fuzzy membership function is translated to the closest linguistic term, *i.e.*,

$$\alpha = \begin{cases} [-0.5, 0.5), & \text{if } s_i \in \{s_1, s_2, \dots, s_{g-1}\}, \\ [0, 0.5), & \text{if } s_i = s_0, \\ [-0.5, 0), & \text{if } s_i = s_g. \end{cases}$$

Accordingly, the 2-tuple linguistic value is a pair of elements noted as (s_i, α), which can be used to express the linguistic information on a universe of discourse, for example, a set of nine linguistic terms S on $[0,1]$ is $S = \{s_0$ (extremely poor), s_1 (very poor), s_2 (poor), s_3 (slightly poor), s_4 (fair), s_5 (slightly good), s_6 (good), s_7 (very good), s_8 (extremely good)$\}$ and their fuzzy sets on $[0,1]$ are graphically shown in Fig.1.

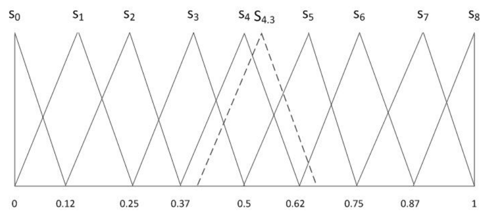

Fig. 1. Fuzzy sets of S on $[0,1]$

In which, such as the 2-tuple linguistic value $(s_4, 0.3) = s_{4.3}$ corresponds to fuzzy set $s_{4.3}$ in Fig.1, formally, using the 2-tuple linguistic model can effectively avoid the loss and distortion of information in linguistic information processing, for simplify, we denote $S_{[0,g]} = \{s_\alpha | s_0 \leqslant s_\alpha \leqslant s_g\}$ as all 2-tuple linguistic values on $[0,1]$ [31].

Definition 1. [45] Let $s_\alpha, s_\beta \in S_{[0,g]}$ and $\gamma = (s_\alpha, s_\beta)$, if $\alpha + \beta \leqslant g$, then we call γ the linguistic intuitionistic fuzzy numbers defined on $S_{[0,g]}$. If $s_\alpha, s_\beta \in S$, then we call γ the original linguistic intuitionistic fuzzy numbers, otherwise, we call γ the virtual linguistic intuitionistic fuzzy numbers.

In the paper, we denote $\Gamma_{[0,g]} = \{(s_\alpha, s_\beta) | s_\alpha, s_\beta \in S_{[0,g]}\}$ as all linguistic intuitionistic fuzzy numbers defined on $S_{[0,g]}$. For any (s_α, s_β), $(s_{\alpha_1}, s_{\beta_1})$, $(s_{\alpha_2}, s_{\beta_2}) \in \Gamma_{[0,g]}$, we have the following operators inspired by operations of intuitionistic fuzzy sets:

- $(s_{\alpha_1}, s_{\beta_1}) \cup (s_{\alpha_2}, s_{\beta_2}) = (max(s_{\alpha_1}, s_{\alpha_2}), min(s_{\beta_1}, s_{\beta_2}))$;
- $(s_{\alpha_1}, s_{\beta_1}) \cap (s_{\alpha_2}, s_{\beta_2}) = (min(s_{\alpha_1}, s_{\alpha_2}), max(s_{\beta_1}, s_{\beta_2}))$;
- $(s_\alpha, s_\beta)^c = (s_\beta, s_\alpha)$;
- $(s_{\alpha_1}, s_{\beta_1}) \subseteq (s_{\alpha_2}, s_{\beta_2})$ iff $s_{\alpha_1} \leqslant s_{\alpha_2}$, and $s_{\beta_1} \geqslant s_{\beta_2}$;
- $(s_{\alpha_1}, s_{\beta_1}) = (s_{\alpha_2}, s_{\beta_2})$ iff $s_{\alpha_1} = s_{\alpha_2}$, and $s_{\beta_1} = s_{\beta_2}$.

Let $(s_\alpha, s_\beta), (s_{\alpha_1}, s_{\beta_1}), (s_{\alpha_2}, s_{\beta_2}) \in \Gamma_{[0,g]}$, $\lambda > 0$, the following operations of linguistic intuitionistic fuzzy numbers have been defined [45]:

- $(s_{\alpha_1}, s_{\beta_1}) \oplus (s_{\alpha_2}, s_{\beta_2}) = (s_{\alpha_1+\alpha_2-\frac{\alpha_1\alpha_2}{g}}, s_{\frac{\beta_1\beta_2}{g}})$;
- $(s_{\alpha_1}, s_{\beta_1}) \otimes (s_{\alpha_2}, s_{\beta_2}) = (s_{\frac{\alpha_1\alpha_2}{g}}, s_{\beta_1+\beta_2-\frac{\beta_1\beta_2}{g}})$;
- $\lambda(s_\alpha, s_\beta) = (s_{g-g(1-\frac{\alpha}{g})^\lambda}, s_{g(\frac{\beta}{g})^\lambda})$;
- $(s_\alpha, s_\beta)^\lambda = (s_{g(\frac{\alpha}{g})^\lambda}, s_{g-g(1-\frac{\beta}{g})^\lambda})$.

Accordingly, the linguistic intuitionistic fuzzy weighted averaging operator have been proposed: Let $\gamma_j = (s_{\alpha_j}, s_{\beta_j}) \in \Gamma_{[0,g]} (j = 1, \ldots, n)$ and w_j be the weight of γ_j, satisfying $0 \leqslant w_j \leqslant 1 (j = 1, \ldots, n)$ and $\sum_{j=1}^{n} w_j = 1$. Then the linguistic intuitionistic fuzzy weighted averaging operator is

$$\begin{aligned} & LIFWA(\gamma_1, \ldots, \gamma_n) \\ = \ & (s_{g-g\prod_{j=1}^n (1-\frac{\alpha_j}{g})^{\omega_j}}, s_{g\prod_{j=1}^n (\frac{\beta_j}{g})^{\omega_j}}). \end{aligned} \quad (1)$$

As a special case, if $w = (\frac{1}{n}, \ldots, \frac{1}{n})$, then Eq.(1) is reduced to the arithmetic aggregation operator:

$$\begin{aligned} & LIFWA(\gamma_1, \ldots, \gamma_n) \\ = \ & (s_{g-g\prod_{j=1}^n (1-\frac{\alpha_j}{g})^{\frac{1}{n}}}, s_{g\prod_{j=1}^n (\frac{\beta_j}{g})^{\frac{1}{n}}}). \end{aligned} \quad (2)$$

Let a LMCDM problem: The decision maker is asked to assess a set of alternatives $X = \{x_1, \ldots, x_m\}$ with respect to criteria $C = \{c_1, \cdots, c_n\}$, where the initial linguistic term set is $S = \{s_0, \cdots, s_g\}$, then the linguistic intuitionistic fuzzy assessment of alternative x_i with respect to the criterion c_j provided by the decision maker is represented as

$$A_j = < x_i, s_\alpha^j(x_i), s_\beta^j(x_i) >, \quad (3)$$

where $s_\alpha^j(x_i), s_\beta^j(x_i) \in S_{[0,g]}$ and $0 \leqslant \alpha + \beta \leqslant g$, $s_\alpha^j(x_i)$ represents the membership fuzzy linguistic assessment of x_i provided by the decision maker with respect to the criterion c_j, $s_\beta^j(x_i)$ represents the nonmembership fuzzy linguistic assessment of x_i provided by the decision maker with respect to the criterion c_j. Here, linguistic intuitionistic fuzzy assessments of alternatives are obviously linguistic intuitionistic fuzzy numbers on S.

For any A_j of x_i, due to $\alpha + \beta \leqslant g$, we have $s_\alpha^j(x_i) \leqslant s_{g-\beta}^j(x_i)$, inspired by the intuitionistic fuzzy hesitation degree of intuitionistic fuzzy set, we call $s_i \in S$ as a linguistic intuitionistic fuzzy hesitation assessment of x_i if $s_\alpha^j(x_i) \leqslant s_i \leqslant s_{g-\beta}^j(x_i)$, and all linguistic intuitionistic fuzzy hesitation assessments of x_i are formed as a hesitant fuzzy linguistic term set [47]: $H_S^j(x_i) = \{s_i \in S | s_\alpha^j(x_i) \leqslant s_i \leqslant s_{g-\beta}^j(x_i)\}$. In the following, we denote $A_j =< x_i, s_\alpha^j(x_i), s_\beta^j(x_i) >$ as $A_j = (s_\alpha, s_\beta)$ when x_i is clear.

Example 1. Let alternatives $X = \{x_1, x_2, x_3\}$, criteria $C = \{c_1, c_2, c_3\}$ and the initial linguistic term set $S = \{s_0$ (nothing), s_1 (very low), s_2 (low), s_3 (medium), s_4 (high), s_5 (very high), s_6 (perfect)$\}$, linguistic intuitionistic fuzzy assessments provided by the decision maker d are shown in Table 1, in which, such as for (s_1, s_3) of x_1 with respect to c_1, the hesitant fuzzy linguistic term set is $H_S^1(x_1) = \{s_i \in S | s_1 \leqslant s_i \leqslant s_{6-3}\} = \{s_1, s_2, s_3\}$. All hesitant fuzzy linguistic term sets of three alternatives with respect to criteria are shown in Table 2.

Table 1. Linguistic intuitionistic assessments of alternatives.

		c_1	c_2	c_3
	x_1	(s_1, s_3)	(s_4, s_1)	(s_4, s_2)
d	x_2	(s_2, s_3)	(s_3, s_3)	(s_0, s_4)
	x_3	(s_4, s_0)	(s_1, s_4)	(s_4, s_0)

Table 2. The hesitant fuzzy linguistic term sets of alternatives based on Table 1.

		c_1	c_2	c_3
	x_1	$\{s_1, s_2, s_3\}$	$\{s_4, s_5\}$	$\{s_4\}$
d	x_2	$\{s_2, s_3\}$	$\{s_3\}$	$\{s_0, s_1, s_2\}$
	x_3	$\{s_4, s_5, s_6\}$	$\{s_1, s_2\}$	$\{s_4, s_5, s_6\}$

3. The distance between two linguistic intuitionistic fuzzy numbers

The distance measure is an important notion in existed TOPSIS methods. Inspired by the distance measure of intuitionistic fuzzy sets[27], we can define the following Hamming distance between two linguistic intuitionistic fuzzy sets.

Definition 2. Let $A = (s_{\alpha_1}, s_{\beta_1})$ and $B = (s_{\alpha_2}, s_{\beta_2})$ be two linguistic intuitionistic fuzzy numbers. Then Hamming distance between A and B is as follows:

$$d(A,B) = \frac{|\alpha_1 - \alpha_2| + |\beta_1 - \beta_2| + |\pi_1 - \pi_2|}{2}, \quad (4)$$

where $\pi_1 = g - \alpha_1 - \beta_1, \pi_2 = g - \alpha_2 - \beta_2$.

Proposition 1. *The distance $d(A,B)$ between A and B satisfies: (1) $0 \leqslant d(A,B) \leqslant g$; (2) $A = B$ iff $d(A,B) = 0$; (3) $d(A,B) = d(B,A)$;(4) If $A \subseteq B \subseteq C$ for $A,B,C \in \Gamma_{[0,g]}$, then $d(A,B) \leqslant d(A,C)$ and $d(B,C) \leqslant d(A,C)$.*

Proof. (2) and (3) are obvious. We only prove (1) and (4).

According to $0 \leqslant \alpha_i, \beta_i, \pi_i \leqslant g$ and $\alpha_i + \beta_i + \pi_i = g$ $(i = 1,2)$, we can obtain $0 \leqslant d(A,B) = \frac{|\alpha_1-\alpha_2|+|\beta_1-\beta_2|+|\pi_1-\pi_2|}{2} \leqslant \frac{\alpha_1+\alpha_2+\beta_1+\beta_2+\pi_1+\pi_2}{2} = g$, *i.e.*, (1) holds.

We have $\alpha_1 \leqslant \alpha_2 \leqslant \alpha_3$ and $\beta_1 \geqslant \beta_2 \geqslant \beta_3$ due to $A \subseteq B \subseteq C$. Then $d(A,C) - d(A,B) = \frac{|\alpha_1-\alpha_3|+|\beta_1-\beta_3|+|\pi_1-\pi_3|}{2} - \frac{|\alpha_1-\alpha_2|+|\beta_1-\beta_2|+|\pi_1-\pi_2|}{2} = \frac{\alpha_3-\alpha_1+\beta_1-\beta_3+|\pi_1-\pi_3|}{2} - \frac{\alpha_2-\alpha_1+\beta_1-\beta_2+|\pi_1-\pi_2|}{2} = \frac{\alpha_3-\alpha_2+\beta_2-\beta_3+|\pi_1-\pi_3|-|\pi_1-\pi_2|}{2} \geqslant \frac{\alpha_3-\alpha_2+\beta_2-\beta_3-|\pi_2-\pi_3|}{2} = \frac{\alpha_3-\alpha_2+\beta_2-\beta_3-|\alpha_3-\alpha_2+\beta_3-\beta_2|}{2} \geqslant \frac{\alpha_3-\alpha_2+\beta_2-\beta_3-|\alpha_3-\alpha_2|-|\beta_3-\beta_2|}{2} = \frac{\alpha_3-\alpha_2+\beta_2-\beta_3-\alpha_3+\alpha_2-\beta_2+\beta_3}{2} = 0$, *i.e.*, $d(A,B) \leqslant d(A,C)$ holds, similarly, $d(B,C) \leqslant d(A,C)$ holds. □

Example 2. In Example 1, for $A_1 = (s_1, s_3)$ of x_1, $A_1 = (s_2, s_3)$ of x_2 and $A_3 = (s_4, s_0)$ of x_3, due to $s_1 < s_2 < s_4$ and $s_3 = s_3 > s_0$, we have $A_1 \subseteq A_2 \subseteq A_3$, in addition, $\pi_1 = 6 - 1 - 3 = 3$, $\pi_2 = 6 - 2 - 3 = 1$ and $\pi_3 = 6 - 4 - 0 = 2$,

$$\begin{aligned} d(A_1,A_2) &= \frac{|1-2|+|3-3|+|3-1|}{2} = 1, \\ d(A_1,A_3) &= \frac{|1-4|+|3-0|+|3-2|}{2} = 3.5, \\ d(A_2,A_3) &= \frac{|2-4|+|3-0|+|1-2|}{2} = 3. \end{aligned}$$

Hence, $d(A_1,A_2) < d(A_1,A_3)$ and $d(A_2,A_3) < d(A_1,A_3)$.

4. The linguistic intuitionistic fuzzy set TOPSIS method

In this section, we present the linguistic intuitionistic fuzzy set TOPSIS method to solve LMCDMs, the method is mainly consisted of the four phases, *i.e.*,

1. construct the linguistic intuitionistic fuzzy decision matrix;
2. determine the positive and negative ideal solutions of alternatives;
3. calculate the relative closeness degree of every alternative;
4. rank alternatives according to their relative closenesses degree.

Where, we use the linguistic intuitionistic fuzzy decision matrix to express linguistic assessments of alternatives, *i.e.*, in LMCDMs, we utilize linguistic intuitionistic fuzzy sets on S and weights of criteria to represent uncertain linguistic assessments of alternatives provided by decision maker with respect to criteria, which is described in Subsection 4.1. In Subsection 4.2, based on the linguistic intuitionistic fuzzy decision matrix, we adopt $\cup$ and $\cap$ operations of linguistic intuitionistic fuzzy sets and the linguistic intuitionistic fuzzy weighted averaging operator to determine the positive and negative ideal solutions of alternatives. In Subsection 4.3, we use Hamming distance between linguistic intuitionistic fuzzy assessment of each alternative and the positive and negative ideal solutions to calculate the relative closeness degree of each alternative, then according to relative closeness degrees, we rank the preference order of all alternatives. We also provide Algorithm 1 in the end of Subsection 4.3 to automatically deal with LMCDM problems.

4.1. *The linguistic intuitionistic fuzzy decision matrix*

Let $W = \{w_1, \ldots, w_n\}$ be weights of criteria $C = \{c_1, \cdots, c_n\}$, $A_{ij} = (s_{\alpha_{ij}}, s_{\beta_{ij}})$ be linguistic intuitionistic fuzzy assessment of the alternative x_i provided by decision maker with respect to the criterion c_j. Then the linguistic intuitionistic fuzzy decision matrix of a LMCDM can be formed as follows:

$$D = (A_{ij})_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{ccc} c_1(w_1) & \ldots & c_n(w_n) \end{array} \\ \begin{pmatrix} (s_{\alpha_{11}}, s_{\beta_{11}}) & \ldots & (s_{\alpha_{1n}}, s_{\beta_{1n}}) \\ (s_{\alpha_{21}}, s_{\beta_{21}}) & \ldots & (s_{\alpha_{2n}}, s_{\beta_{2n}}) \\ \vdots & \cdots & \vdots \\ (s_{\alpha_{m1}}, s_{\beta_{m1}}) & \cdots & (s_{\alpha_{mn}}, s_{\beta_{mn}}) \end{pmatrix} \end{array}, \quad (5)$$

Example 3. In Example 1, suppose that weights of criteria $C = \{c_1, c_2, c_3\}$ is $W = \{0.3, 0.4, 0.3\}$, then the linguistic intuitionistic fuzzy decision matrix of the LMCDM is

$$D = (A_{ij})_{3\times 3} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} c_1(0.3) & c_2(0.4) & c_3(0.3) \end{array} \\ \begin{pmatrix} (s_1, s_3) & (s_4, s_1) & (s_4, s_2) \\ (s_2, s_3) & (s_3, s_3) & (s_0, s_4) \\ (s_4, s_0) & (s_1, s_4) & (s_4, s_0) \end{pmatrix} \end{array}.$$

4.2. *The positive and negative ideal solutions of alternatives*

According to the linguistic intuitionistic fuzzy decision matrix D (Eq.(5)), we use $\cup$ and $\cap$ operations of linguistic intuitionistic fuzzy numbers and the linguistic intuitionistic fuzzy weighted averaging operator to determine the positive and negative ideal solutions of alternatives, formally, for each column of D, $\cup$ and $\cap$ of linguistic intuitionistic fuzzy sets are as follows:

$$\begin{aligned} \vee c_j &= (s_{\vee\alpha_j}, s_{\vee\beta_j}) = \bigcup_{i=1}^{m} A_{ij} = \bigcup_{i=1}^{m} (s_{\alpha_{ij}}, s_{\beta_{ij}}) \\ &= (max\{s_{\alpha_{1j}}, \ldots, s_{\alpha_{mj}}\}, min\{s_{\beta_{1j}}, \ldots, s_{\beta_{mj}}\}), \\ \wedge c_j &= (s_{\wedge\alpha_j}, s_{\wedge\beta_j}) = \bigcap_{i=1}^{m} A_{ij} = \bigcap_{i=1}^{m} (s_{\alpha_{ij}}, s_{\beta_{ij}}) \\ &= (min\{s_{\alpha_{1j}}, \ldots, s_{\alpha_{mj}}\}, max\{s_{\beta_{1j}}, \ldots, s_{\beta_{mj}}\}). \end{aligned}$$

Based on $\vee c_j$, $\wedge c_j$ and weights $W = \{w_1, \ldots, w_n\}$ of criteria $C = \{c_1, \cdots, c_n\}$, we utilize the linguistic intuitionistic fuzzy weighted averaging operator to determine the positive ideal solution (PIS) and the negative ideal solution (NIS) of alternatives, *i.e.*,

$$\begin{aligned} \text{PIS} &= (s_{\alpha_p}, s_{\beta_p}) = LIFWA(\vee c_1, \ldots, \vee c_n) \\ &= (s_{g - g\prod_{j=1}^{n}(1-\frac{\vee\alpha_j}{g})^{w_j}}, s_{g\prod_{j=1}^{n}(\frac{\vee\beta_j}{g})^{w_j}}), \quad (6) \\ \text{NIS} &= (s_{\alpha_n}, s_{\beta_n}) = LIFWA(\wedge c_1, \ldots, \wedge c_n) \\ &= (s_{g - g\prod_{j=1}^{n}(1-\frac{\wedge\alpha_j}{g})^{w_j}}, s_{g\prod_{j=1}^{n}(\frac{\wedge\beta_j}{g})^{w_j}}). \quad (7) \end{aligned}$$

Example 4. According to D in Example 1, we have $\vee c_1 = \bigcup_{i=1}^{3} A_{i1} = (max\{s_1, s_2, s_4\}, min\{s_3, s_3, s_0\}) = (s_4, s_0)$, $\wedge c_1 = \bigcap_{i=1}^{3} A_{i1} = (min\{s_1, s_2, s_4\}, max\{s_3, s_3, s_0\}) = (s_1, s_3)$, $\vee c_2 = (s_4, s_1)$, $\wedge c_2 = (s_1, s_4)$, $\vee c_3 = (s_4, s_0)$ and $\wedge c_2 = (s_0, s_4)$. Based on Eqs.(6) and (7), we can obtain PIS and NIS of D,

$$\begin{aligned} \text{PIS} &= LIFWA(\vee c_1, \vee c_2, \vee c_3) \\ &= (s_{6-6\prod_{j=1}^{3}(1-\frac{\vee\alpha_j}{6})^{w_j}}, s_{6\prod_{j=1}^{3}(\frac{\vee\beta_j}{6})^{w_j}}), \\ &= (s_{6-6\times(1-\frac{4}{6})}, s_{6\times 0\times\frac{1}{6}\times 0}) \\ &= (s_4, s_0), \\ \text{NIS} &= LIFWA(\wedge c_1, \wedge c_2, \wedge c_3) \\ &= (s_{6-6\prod_{j=1}^{3}(1-\frac{\wedge\alpha_j}{6})^{w_j}}, s_{6\prod_{j=1}^{3}(\frac{\wedge\beta_j}{6})^{w_j}}) \\ &= (s_{6-6\times(1-\frac{1}{6})^{0.7}\times 1}, s_{6\times(\frac{3}{6})^{0.3}\times(\frac{4}{6})^{0.7}}) \\ &\doteq (s_{0.72}, s_{3.65}). \end{aligned}$$

4.3. *The relative closeness and the ranking of alternatives*

To rank alternatives, we first calculate the relative closeness degree of every alternative, theoretically, the relative closeness degree is determined by distances between the linguistic intuitionistic fuzzy assessment of each alternative and the positive and negative ideal solutions. Based on Eq.(5), we use the linguistic intuitionistic fuzzy weighted averaging operator to obtain the linguistic intuitionistic fuzzy set of each alternative in D, *i.e.*, for each row of D, the linguistic intuitionistic fuzzy set A_i of x_i is

$$\begin{aligned} A_i &= (s_{\alpha_i}, s_{\beta_i}) = LIFWA(A_{i1},\dots,A_{in}) \\ &= (s_{g-g\prod_{j=1}^{n}(1-\frac{\alpha_{ij}}{g})^{w_j}}, s_{g\prod_{j=1}^{n}(\frac{\beta_{ij}}{g})^{w_j}}). \quad (8) \end{aligned}$$

Based on A_i of x_i, PIS (Eq.(6)), NIS (Eq.(7)) and Eq.(4), Hamming distances between the linguistic intuitionistic fuzzy set of each alternative and the positive and negative ideal solutions are

$$d(A_i,\text{PIS}) = \frac{|\alpha_i-\alpha_p|+|\beta_i-\beta_p|+|\pi_i-\pi_p|}{2}, \quad (9)$$

$$d(A_i,\text{NIS}) = \frac{|\alpha_i-\alpha_n|+|\beta_i-\beta_n|+|\pi_i-\pi_n|}{2}, \quad (10)$$

where, $\pi_i = g-\alpha_i-\beta_i$, $\pi_p = g-\alpha_p-\beta_p$ and $\pi_n = g-\alpha_n-\beta_n$.

Originated from the TOPSIS method, the ranking of alternatives is based on "the shortest distance from the positive ideal solution and the farthest from the negative ideal solution", formally, this is also fulfilled by the relative closeness degree of each alternative in existed TOPSIS methods. Based on Hamming distance between the linguistic intuitionistic fuzzy set of each alternative and the positive and negative ideal solutions (Eqs.(9) and (10)), we provide the following relative closeness degree $C(x_i)$ of each alternative x_i,

$$d^-_{max} = max\{d(A_1,\text{NIS}),\dots,d(A_m,\text{NIS})\}, \quad (11)$$

$$d^+_{min} = min\{d(A_1,\text{PIS}),\dots,d(A_m,\text{PIS})\}, \quad (12)$$

$$C(x_i) = \frac{1}{2}(\frac{d(A_i,\text{NIS})}{d^-_{max}} + \frac{d^+_{min}}{d(A_i,\text{PIS})}). \quad (13)$$

Formally, $C(x_i)$ is in $[0,1]$ for any alternative x_i and a monotone function in its components, *i.e.*, $C(x_i)$ is increasing for $d(A_i,\text{NIS})$, and decreasing for $d(A_i,\text{PIS})$. Based on relative closeness degrees of alternatives, we can obtain the ranking of alternatives as follows: $\forall x_i, x_{i'} \in X$,

$$x_i \prec x_{i'} \text{ if and only if } C(x_i) \leqslant C(x_{i'}). \quad (14)$$

Example 5. According to D in Example 1 and Eq.(8), we obtain the linguistic intuitionistic fuzzy assessment of each alternative as follows:

$$\begin{aligned} A_1 &= LIFWA((s_1,s_3),(s_4,s_1),(s_4,s_2)) \\ &= (s_{6-6\times(1-\frac{1}{6})^{0.3}\times(1-\frac{4}{6})^{0.7}}, s_{6\times(\frac{3}{6})^{0.3}\times(\frac{1}{6})^{0.4}\times(\frac{2}{6})^{0.3}}) \\ &\doteq (s_{3.38}, s_{1.71}), \\ A_2 &= LIFWA((s_2,s_3),(s_3,s_3),(s_0,s_4)) \\ &\doteq (s_{1.98}, s_{3.31}), \\ A_3 &= LIFWA((s_4,s_0),(s_1,s_4),(s_4,s_0)) \\ &= (s_{6-6\times(1-\frac{4}{6})^{0.6}\times(1-\frac{1}{6})^{0.4}}, s_{6\times(\frac{0}{6})^{0.3}\times(\frac{1}{6})^{0.4}\times(\frac{2}{6})^{0.3}}) \\ &\doteq (s_{3.10}, s_0), \end{aligned}$$

Based on PIS and NIS in Example 4 and Eqs.(9) and (10), we can obtain $d(A_1,\text{PIS}) = \frac{|3.38-4|+|1.71-0|+|0.91-2|}{2}$=1.71 and $d(A_1,\text{NIS}) = \frac{|3.38-0.72|+|1.71-3.65|+|0.91-1.63|}{2}$=2.16, similarly, $d(A_2,\text{PIS}) = 3.31$ and $d(A_2,\text{NIS}) = 1.26$, $d(A_3,\text{PIS}) = 0.795$ and $d(A_3,\text{NIS}) = 3.545$. According to Eqs.(11), (12) and (13), we obtain

$$\begin{aligned} d^-_{max} &= max\{2.16, 1.26, 3.545\} = 3.545, \\ d^+_{min} &= min\{1.71, 3.31, 0.795\} = 0.795, \\ C(x_1) &= \frac{1}{2}(\frac{2.16}{3.545} + \frac{0.795}{1.71}) = 0.535, \\ C(x_2) &= \frac{1}{2}(\frac{1.26}{3.545} + \frac{0.795}{3.31}) = 0.30, \\ C(x_3) &= \frac{1}{2}(\frac{3.545}{3.545} + \frac{0.795}{0.795}) = 1. \end{aligned}$$

Hence, according to Eq.(14), the ranking of $\{x_1,x_2,x_3\}$ is $x_2 \prec x_1 \prec x_3$, *i.e.*, x_3 is the most satisfying alternative.

Based on discussions in Subsections 4.1, 4.2 and 4.3, we provide the following algorithm to carry out the linguistic intuitionistic fuzzy set TOPSIS method to solve LMCDMs.

Algorithm 1

Input: The numbers of alternatives (m) and criteria (n), the linguistic term set $S = \{s_0, s_1, \cdots, s_g\}$.

Output: The ranking of alternatives and the most satisfying alternative.

Begin

Step 1: According to the membership and nonmembership fuzzy linguistic assessments and Eq.(5), the linguistic intuitionistic fuzzy decision matrix D of the LMCDM problem can be constructed;
Step 2: For each column of D, calculate $\vee c_j$ and $\wedge c_j$, then the linguistic intuitionistic fuzzy weighted averaging operator is used to determine PIS (Eq.(6)) and NIS (Eq.(7));
Step 3: For each row of D, the linguistic intuitionistic fuzzy weighted averaging operator is used to obtain the linguistic intuitionistic fuzzy assessment A_i (Eq.(8)) of each alternative x_i;
Step 4: For each alternative, Hamming distances $d(A_i,\text{PIS})$ and $d(A_i,\text{NIS})$ between each A_i and PIS (or NIS) are calculated by Eqs.(9) and (10), the maximum Hamming distance d^-_{max} (Eq.(11)) of all $d(A_i,\text{NIS})(i=1,\ldots,m)$ and the minimum Hamming distance d^+_{min} (Eq.(12)) of all $d(A_i,\text{PIS})(i=1,\ldots,m)$ are obtained, then the relative closeness degree $C(x_i)$ of each alternative x_i is calculated by Eq.(13);
Output: $x_i \prec x_{i'}$ if and only if $C(x_i) \leqslant C(x_{i'})$ by using Eq.(14).
end

5. Numerical example

In this section, we utilize an example to illustrate the practicality of the linguistic intuitionistic fuzzy set TOPSIS method, and compare the linguistic intuitionistic fuzzy set TOPSIS method with the HFL-VIKOR method[46], the symbolic aggregation-based method[47] and the HFL-TOPSIS method[48]. The example initially used in reference[46] to show the HFL-VIKOR method.

Example 6. [46] A company intends to select an ERP system to implement from three candidates $A=\{a_1,a_2,a_3\}$. To make a more reasonable decision, the chief information officer (CIO) of the company assesses the candidate ERP systems in terms of three criteria, i.e., c_1 (potential cost), c_2 (function), and c_3 (operation complexity). The weights of these criteria are 0.3, 0.5 and 0.2. Since the three criteria are qualitative, the CIO gives his assessment values in linguistic expressions (shown in Table 3). Reference [46] used $S=\{$none (s_{-3}), very low (s_{-2}), low (s_{-1}), medium (s_0), high (s_1), very high (s_2), perfect (s_2)$\}$. Here, we use the linguistic term set $S'=\{$none (s_0), very low (s_1), low (s_2), medium (s_3), high (s_4), very high (s_5), perfect (s_6)$\}$.

Table 3. The hesitant fuzzy linguistic term sets of alternatives.

	c_1	c_2	c_3
a_1	$\{s_1,s_2,s_3\}$	$\{s_2,s_3\}$	$\{s_1,s_2,s_3\}$
a_2	$\{s_1,s_2,s_3\}$	$\{s_1,s_2,s_3\}$	$\{s_{-2},s_{-1},s_0\}$
a_3	$\{s_2,s_3\}$	$\{s_1,s_2,s_3\}$	$\{s_3\}$

By using the linguistic intuitionistic fuzzy set TOPSIS method (Algorithm 1), the LMCDM problem can be carried out as follows:

Step 1: According to Table 3 and Eq.(5), we use S' to obtain the linguistic intuitionistic fuzzy decision matrix D, *i.e.*,

$$D = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{pmatrix} (s_4,s_0) & (s_5,s_0) & (s_4,s_0) \\ (s_4,s_0) & (s_4,s_0) & (s_1,s_3) \\ (s_5,s_0) & (s_4,s_0) & (s_3,s_3) \end{pmatrix} \end{array}$$

In Table 3, such as for hesitant fuzzy linguistic term sets $\{s_1,s_2,s_3\}$ of a_1 with respect to c_1, in S', $\{s_1,s_2,s_3\}$ is transformed as $\{s_4,s_5,s_6\}$, then we select $s_4=min\{s_4,s_5,s_6\}$ as the membership fuzzy linguistic assessment of a_1 with respect to c_1 and $s_0=s_{g-max\{4,5,6\}}$ as the nonmembership fuzzy linguistic assessment of a_1 with respect to c_1, *i.e.*, the linguistic intuitionistic fuzzy set of a_1 with respect to c_1 is (s_4,s_0), others linguistic intuitionistic fuzzy sets in D can be similarly obtained. Formally, for any hesitant fuzzy linguistic term set $\{s_i,\ldots,s_j\}(i\leqslant\ldots\leqslant j)$ on $S=\{s_0,\ldots,s_g\}$, (s_i,s_{g-j}) is a linguistic intuitionistic fuzzy set on S due to $i+g-j=g-(j-i)\leqslant g$. This does not mean that linguistic intuitionistic fuzzy sets and hesitant fuzzy linguistic term sets on S are equal to each other, because in real world practice, a decision maker provides a hesitant fuzzy linguistic term set $\{s_i,\ldots,s_j\}$, it does not imply that s_i is membership fuzzy linguistic assessment and s_{g-j} is nonmembership fuzzy linguistic assessment. Here, we only limit discussion in a formal form, and we think that hesitant fuzzy linguistic term set and linguistic intuitionistic fuzzy set are two alternative tools for representing linguistic assessments of alternatives in linguistic decision making.

Step 2: For each column of D, we have

$$\begin{aligned}
\vee c_1 &= (max\{s_4,s_4,s_5\},min\{s_0,s_0,s_0\}) = (s_5,s_0),\\
\vee c_2 &= (max\{s_5,s_4,s_4\},min\{s_0,s_0,s_0\}) = (s_5,s_0),\\
\vee c_3 &= (max\{s_4,s_1,s_3\},min\{s_0,s_3,s_3\}) = (s_4,s_0),\\
\wedge c_1 &= (min\{s_4,s_4,s_5\},max\{s_0,s_0,s_0\}) = (s_4,s_0),\\
\wedge c_2 &= (min\{s_5,s_4,s_4\},max\{s_0,s_0,s_0\}) = (s_4,s_0),\\
\wedge c_3 &= (min\{s_4,s_1,s_3\},max\{s_0,s_3,s_3\}) = (s_1,s_3).
\end{aligned}$$

Based on Eqs.(6) and (7), we obtain the positive and negative ideal solutions are

$$\begin{aligned}
\text{PIS} &= LIFWA(\vee c_1,\vee c_2,\vee c_3)\\
&= (s_{6-6\times(1-\frac{5}{6})^{0.8}\times(1-\frac{4}{6})^{0.2}}, s_{6\times(\frac{0}{6})^{0.3+0.5+0.2}})\\
&\doteq (s_{4.86},s_0),
\end{aligned}$$

$$\begin{aligned}
\text{NIS} &= (s_{\alpha_n},s_{\beta_n}) = LIFWA(\wedge c_1,\wedge c_2,\wedge c_3)\\
&= (s_{6-6\times(1-\frac{4}{6})^{0.8}\times(1-\frac{1}{6})^{0.2}}, s_{6\times(\frac{0}{6})^{0.8}\times(\frac{3}{6})^{0.2}})\\
&\doteq (s_{3.6},s_0).
\end{aligned}$$

Step 3: For each row of D, according to Eq.(8), we have

$$\begin{aligned}
A_1 &= LIFWA((s_4,s_0),(s_5,s_0),(s_4,s_0))\\
&= (s_{6-6\times(1-\frac{4}{6})^{0.5}\times(1-\frac{5}{6})^{0.5}}, s_{6\times(\frac{0}{6})^{0.3+0.5+0.2}})\\
&\doteq (s_{4.58},s_0),\\
A_2 &= LIFWA((s_4,s_0),(s_4,s_0),(s_1,s_3))\\
&\doteq (s_{3.6},s_0),\\
A_3 &= LIFWA((s_5,s_0),(s_4,s_0),(s_3,s_3))\\
&\doteq (s_{4.24},s_0),
\end{aligned}$$

Step 4: For each alternative, according to Eqs.(9) and (10), we obtain the following Hamming distances

$$\begin{aligned}
d(A_1,\text{PIS}) &= \frac{|4.58-4.86|+|0-0|+|1.42-1.14|}{2}\\
&= 0.28,\\
d(A_1,\text{NIS}) &= \frac{|4.58-3.6|+|0-0|+|1.42-2.4|}{2}\\
&= 0.98,\\
d(A_2,\text{PIS}) &= 1.26, d(A_2,\text{NIS}) = 0,\\
d(A_3,\text{PIS}) &= 0.62, d(A_2,\text{NIS}) = 0.64,
\end{aligned}$$

Based on Eqs.(11), (12) and (13), the maximum Hamming distance, the minimum Hamming distance and the relative closeness degree of each alternative are as follows:

$$\begin{aligned}
d^-_{max} &= max\{0.98,0,0.64\} = 0.98,\\
d^+_{min} &= min\{0.28,1.26,0.618\} = 0.28,\\
C(x_1) &= \frac{1}{2}(\frac{0.98}{0.98}+\frac{0.28}{0.28}) = 1,\\
C(x_2) &= \frac{1}{2}(\frac{0}{0.98}+\frac{0.28}{1.26}) \doteq 0.22,\\
C(x_3) &= \frac{1}{2}(\frac{0.64}{0.98}+\frac{0.28}{0.62}) \doteq 0.55.
\end{aligned}$$

Accordingly, the ranking of three alternatives is $a_2 \prec a_3 \prec a_1$ according to Eq.(14), and the set of most satisfying alternatives is $A_{MS} = \{a_1\}$.

In the following, we compare the linguistic intuitionistic fuzzy set TOPSIS method with the HFL-VIKOR method, the symbolic aggregation-based method and the HFL-TOPSIS method, in which, the HFL-VIKOR method, the symbolic aggregation-based method and the HFL-TOPSIS method transform assessments of alternatives into HFLTSs based on Table 3, however, the linguistic intuitionistic fuzzy set TOPSIS method transforms assessments of alternatives into linguistic intuitionistic fuzzy sets, we carry out comparison of four methods as follows.

a) The positive and negative ideal solutions: The HFL-VIKOR method[46] and the HFL-TOPSIS method[48] utilized the score function, the variance function, the Max and Min operators of HFLTSs to obtain PIS and NIS, this means that their PIS or NIS are one of HFLTSs of alternatives, such as in Table 4, $\{s_3\}$ of PIS is HFLTS of c_3, $\{s_{-2},s_{-1}, s_0\}$ of NIS is HFLTS of c_3. Its drawback is that PIS or NIS may be a HFLTS of an alternative, such as NIS $(\{s_1,s_2,s_3\},\{s_1,s_2, s_3\},\{s_{-2},s_{-1},s_0\})$ is the HFLTS of a_2, *i.e.*, this makes that a_2 is the worst alternative.

The symbolic aggregation-based method[47] utilized Min-upper and Max-lower operators to obtain the upper bound and the lower bound of each HFLTS and construct the core information of alternatives (CIA), such as for c_1, Min bounds of a_1, a_2 and a_3 are s_1, s_1 and s_2, so the Min-upper of c_1 is s_2. Max bounds of a_1, a_2 and a_3 are s_3, s_3 and s_3, so the Max-lower of c_1 is s_3, hence CIA for c_1 is $[s_2,s_3]$.

Intuitively, CIA reduces HFLTSs into linguistic intervals, especially, if the Min-upper is equal to the Max-lower for each criterion, then CIA contain certain linguistic information, such as $[s_3, s_3]$ for c_3 in Table 4, which maybe loss a lot of useful information, and there is no any hesitant fuzzy linguistic information in CIA.

In the linguistic intuitionistic fuzzy set TOPSIS method, we use linguistic intuitionistic fuzzy numbers on the linguistic term set S to represent assessments of alternatives, and utilize Union and Intersection of linguistic intuitionistic fuzzy numbers and the linguistic intuitionistic fuzzy weighted averaging operator to obtain PIS and NIS, which are still linguistic intuitionistic fuzzy numbers, *i.e.*, hesitant fuzzy linguistic information PIS are contained in PIS and NIS, because we use the linguistic intuitionistic fuzzy weighted averaging operator, our PIS and NIS are different with PIS and NIS in methods [46,47,48], this can be seen from Table 4.

b) The ranking of alternatives: In the HFL-VIKOR method[46], the hesitant fuzzy linguistic group utility measure HFLGU_i and the hesitant fuzzy individual regret measure HFLIR_i for the alternative a_i are defined by the hesitant fuzzy linguistic Euclidean L_p-metric, then the hesitant fuzzy linguistic compromise measure HFLC_i is established,

$$\mathrm{HFLC}_i = \theta \frac{\mathrm{HFLGU}_i - \mathrm{HFLGU}^+}{\mathrm{HFLGU}^- - \mathrm{HFLGU}^+} + (1-\theta)\frac{\mathrm{HFLIR}_1 - \mathrm{HFLIR}^+}{\mathrm{HFLIR}^- - \mathrm{HFLIR}^+},$$

in which, $\mathrm{HFLGU}^+ = min\{\mathrm{HFLGU}_1, \mathrm{HFLGU}_2, \mathrm{HFLGU}_3\}$, $\mathrm{HFLGU}^- = max\{\mathrm{HFLGU}_1, \mathrm{HFLGU}_2, \mathrm{HFLGU}_3\}$, $\mathrm{HFLIR}^+ = min\{\mathrm{HFLIR}_1, \mathrm{HFLIR}_2, \mathrm{HFLIR}_3\}$ and $\mathrm{HFLIR}^- = max\{\mathrm{HFLIR}_1, \mathrm{HFLIR}_2, \mathrm{HFLIR}_3\}$, and $\theta \in [0,1]$ is the weight of the strategy of the majority of criteria or the maximum overall utility. By ranking HFLGU_i, HFLIR_i and HFLC_i in descending order, the final optimal solution should be the one that makes those measures attain the minimum values. Formally, the HFL-VIKOR method is very effective in handling the noncommensurable criteria, derives the compromise solution(s) which consider not only maximizing the group utility for the majority but mini- mizing individual regret for the opponent as well and takes different weights of the criteria into account.

In the symbolic aggregation-based method[47], based on the CIA of each alternative, a binary preference relation between two alternatives is calculated, and the nondominance degree (NDD) of each alternative is used to obtain the set of nondominated alternatives, which indicates the degree to which alternative a_i is not dominated by the remaining ones. In the HFL-TOPSIS method[48], the Euclidean distance measure is used to obtain distances between alternatives and the positive and negative ideal solutions, then the relative closeness degree (RC) of each alternative is calculated to rank alternatives, where weights of criteria are not used in the symbolic aggregation-based method and the HFL-TOPSIS method.

In the linguistic intuitionistic fuzzy set TOPSIS method, we use Hamming distances between alternatives and the positive and negative ideal solutions to calculate the relative closeness of each alternative. As shown in Table 5, the ranking of alternatives are the same in the linguistic intuitionistic fuzzy set TOPSIS method and the HFL-VIKOR method[46], however, calculation of the linguistic intuitionistic fuzzy set TOPSIS method are simpler than the HFL-VIKOR method. We notice that the ranking results of the symbolic aggregation-based method and the HFL-TOPSIS method are different from that produced by the linguistic intuitionistic fuzzy set TOPSIS method and the HFL-VIKOR method, The reasons leading to this unconvincing result can be set out as follows: for one thing, with the symbolic aggregation-based method and the HFL-TOPSIS method, the weights of different criterion do not take into consideration; For another thing, when aggregating the HFLTSs by the symbolic aggregation operators, including the Min-upper operator and the Max-lower operator, the HFLTSs are reduced into linguistic intervals, which losses quite a lot of useful information. When aggregating the HFLTSs by Euclidean distance measure, including the normalized hesitant fuzzy linguistic terms, it only considers the distances from the ideal solution and from the negative-ideal solution, without considering their relative importance.

Table 4: Main results of four methods

	PIS	NIS	CIA
The method[46]	$(\{s_2,s_3\},\{s_2,s_3\},\{s_3\})$	$(\{s_1,s_2,s_3\},\{s_1,s_2,s_3\},\{s_{-2},s_{-1},s_0\})$	$-$
The method[47]	$-$	$-$	$([s_2,s_3],[s_0,s_1],[s_3,s_3])$
The method[48]	$(\{s_2,s_3\},\{s_2,s_3\},\{s_3\})$	$(\{s_1,s_2,s_3\},\{s_1,s_2,s_3\},\{s_{-2},s_{-1},s_0\})$	$-$
Our method	$(s_{4.86},s_0)$	$(s_{3.6},s_0)$	$-$

Table 5: The ranking of four methods

	Using wights	HFLC (NDDs or RC)	The ranking	The best one
The method[46]	$\surd$	$(0^*,1^-,0.6074)$	$a_2 \prec a_3 \prec a_1$	a_1
The method[47]	$-$	$(0,0.5,1)$	$a_1 \prec a_2 \prec a_3$	a_3
The method[48]	$-$	$(0.6531,0,0.8799)$	$a_2 \prec a_1 \prec a_3$	a_3
Our method	$\surd$	$(1,0.22,0.55)$	$a_2 \prec a_3 \prec a_1$	a_1

Summary, in Example 6, we respectively use hesitant fuzzy linguistic term sets and linguistic intuitionistic fuzzy sets to represent linguistic assessments of the LMCDM problem, then we compare the linguistic intuitionistic fuzzy set TOPSIS method with the HFL-VIKOR method, the symbolic aggregation-based method and the HFL-TOPSIS method, which are based on hesitant fuzzy linguistic term sets. An intriguing problem is which method is more reasonable among the above mentioned four methods or which representation of the above mentioned four methods is more reasonable? In real world practice, it is not possible to determine which one is the best suitable alternative for a given decision problem, this means that it is difficult to answer the intriguing problem. Theoretically, the testing criteria to evaluate the validity of MCDM methods in the same numerical data has been established such as in [49], which can help us for our future research works.

6. Conclusions

Motivated by linguistic intuitionistic fuzzy numbers, in the paper, uncertain assessments information in linguistic multi-criteria decision makings are express by linguistic intuitionistic fuzzy sets on linguistic terms set, then Hamming distance between two linguistic intuitionistic fuzzy sets and their properties are presented and analyzed. Accordingly, the linguistic intuitionistic fuzzy set TOPSIS method for LMCDM problems is proposed, compared with the traditional TOPSIS methods, different the positive ideal solution, the negative ideal solution and the relative closeness degrees of alternatives are provided, based on the designed algorithm, LMCDM problems with linguistic intuitionistic fuzzy sets can be automatically carried out. An example is also utilized to illustrate the performance, usefulness and effectiveness of the linguistic intuitionistic fuzzy set TOPSIS method, and compare the method with the HFL-VIKOR method, the symbolic aggregation-based method and the HFL-TOPSIS method.

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