

New applications of dipole Monte Carlo implementations

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Introduction

- Everywhere we look, we find heavy ion behavior!
- Monte Carlos for pp physics have had:
 1. No space–time structure.
 2. No heavy ion collisions.
 3. No collective effects.
- And that was a problem!



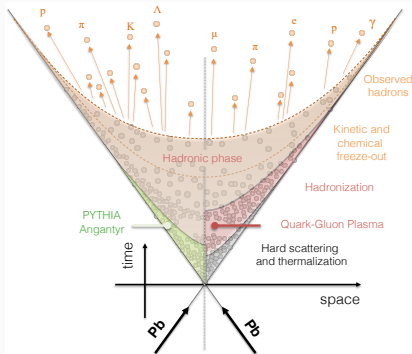
At least do enough for a non-QGP baseline.



But if it works, how far can we go?

The key differences between standard approaches

- Standard MC approach: Matrix element, parton shower + string hadronization.
- Note different time-scales.



(Figure: D. D. Chinellato)

- Initial state geometry with Monte Carlo.
 1. From Mueller dipoles to event geometry.
 2. Fluctuating cross sections.
 3. Towards EIC.
- Matching to a multi-parton interactions.
 1. Pythia and the Angantyr model.
 2. Fluctuations in parton level geometry.
- From geometry to collectivity.
 1. The string shoving model.
 2. Shoving and Angantyr.
 3. Response to geometry.

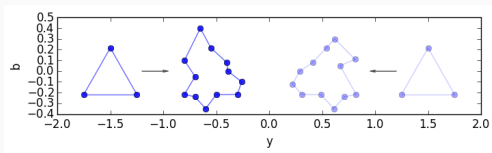
Mueller dipole initial states

The aim and the means

A reasonable calculation of initial state geometry.

Fluctuating nucleon–nucleon cross sections.

MC implementation of Mueller dipoles.

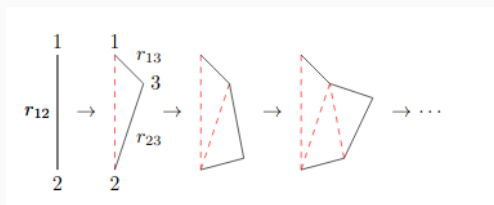


- Projectile and target cascades evolved for each event.
- Formalism in impact parameter and rapidity.
- Single-event spatial structure.

A step back, BFKL, B-JIMWLK and all that...

- Start with Mueller dipole branching probability:

$$\frac{d\mathcal{P}}{dy} = d^2\vec{r}_3 \frac{N_c\alpha_s}{2\pi^2} \frac{r_{12}^2}{r_{13}^2 r_{23}^2} \equiv d^2\vec{r}_3 \kappa_3.$$



- Evolve any observable $O(y) \rightarrow O(y + dy)$ in rapidity:

$$\begin{aligned} \bar{O}(y+dy) &= dy \int d^2\vec{r}_3 \kappa_3 [O(r_{13}) \otimes O(r_{23})] + O(r_{12}) \left[1 - dy \int d^2\vec{r}_3 \kappa_3 \right] \\ &\rightarrow \frac{\partial \bar{O}}{\partial y} = \int d^2\vec{r}_3 \kappa_3 [O(r_{13}) \otimes O(r_{23}) - O(r_{12})]. \end{aligned}$$

A powerful formalism!

- Example: S -matrix (eikonal approximation, b -space):

$$O(r_{13}) \otimes O(r_{23}) \rightarrow S(r_{13})S(r_{23})$$

- Change to $T \equiv 1 - S$:

$$\frac{\partial \langle \overline{T} \rangle}{\partial y} = \int d^2 \vec{r}_3 \kappa_3 [\langle T_{13} \rangle + \langle T_{23} \rangle - \langle T_{12} \rangle - \langle T_{13} T_{23} \rangle].$$

- B-JIMWLK equation, but could be written with other observables.
- Example: Average dipole coordinate ($\langle z \rangle$):

$$\frac{\partial \langle \overline{z} \rangle}{\partial y} = \int d^2 \vec{r}_3 \kappa_3 \left(\frac{1}{3} z_3 - \frac{1}{6} (z_1 + z_2) \right).$$

Monte Carlo implementation

Drawbacks to analytic approach

Involved observables are hard!

Not obvious how to include sub-leading effects.

Not obvious how to treat exclusive final states.

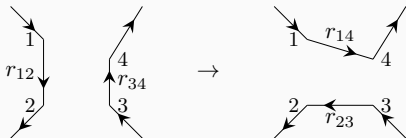
- The MC way is a tradeoff: formal precision vs. pragmatism.
- Get for free: Rest of the MC infrastructure.
- Practically a parton shower-like implementation.
- Step 1: Modify splitting kernel with Sudakov:

$$\frac{d\mathcal{P}}{dy d^2\vec{r}_3} = \frac{N_c \alpha_s}{2\pi^2} \frac{r_{12}^2}{r_{13}^2 r_{23}^2} \exp\left(-\int_{y_{\min}}^y dy d^2\vec{r}_3 \frac{N_c \alpha_s}{2\pi^2} \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\right)$$

- Winner-takes-it-all algorithm generates emission up to maximal rapidity.
- Throws away the non-linear term in the cascade.

Colliding dipole chains & unitarity

- Have: Evolved dipole chain á la BFKL.
- Dipole cross section in large- N_c limit (consistency with evolution):



$$\frac{d\sigma_{\text{dip}}}{d^2\vec{b}} = \frac{\alpha_s^2 C_F}{N_c} \log^2 \left[\frac{r_{13} r_{24}}{r_{14} r_{23}} \right]$$
$$\rightarrow \frac{\alpha_s^2}{2} \log^2 \left[\frac{r_{13} r_{24}}{r_{14} r_{23}} \right] \equiv f_{ij}$$

- Unitarized scattering amplitude: $T(\vec{b}) = 1 - \exp \left(- \sum_{ij} f_{ij} \right)$

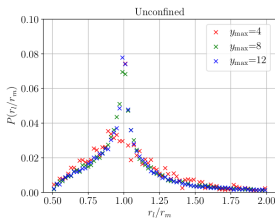
Some details

A dipole has a rapidity y , and a p_{\perp} related to its size $p_{\perp} \hbar/r$. Thus its lightcone momenta is $p_{\pm} = p_{\perp} \exp(\pm y)$.

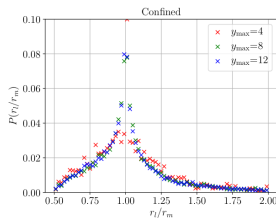
- Energy-momentum conservation from bounded p_{\perp} translate to upper bound on dipole sizes.
- Running α_s : Easily included per-splitting.
- Non-eikonal effects: recoil distributed on emitters in p_+ , p_{\perp} , and thus also y .
- Confinement: Explicit confinement scale (or fictitious gluon mass) entering evolution and collision.
- Unitarized scattering amplitude resums $1/N_c^2$ terms in interaction, equivalent to multi-pomeron exchanges in interaction frame.

Example: confinement \rightarrow hot-spots

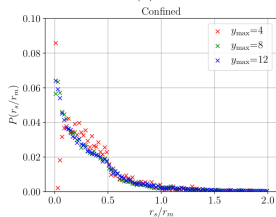
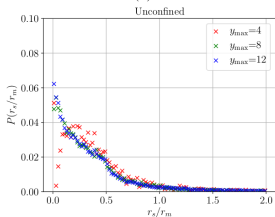
- MC makes it easy to switch physics effects on and off.
- More activity around end-points: Hot-spots!
- Initial triangle by hand. Less important at high energies, but deserves more thought.



(a)

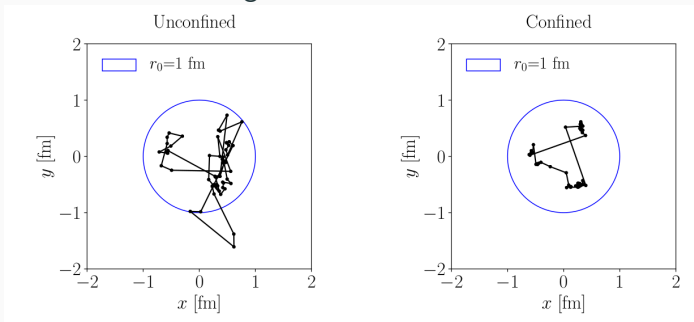


(b)



Example: confinement \rightarrow hot-spots

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- Dynamically generated!

Good-Walker & cross sections

- Cross sections from $T(\vec{b})$ with normalizable particle wave functions:

$$\sigma_{\text{tot}} = 2 \int d^2\vec{b} \Gamma(\vec{b}) = 2 \int d^2\vec{b} \langle T(\vec{b}) \rangle_{p,t}$$

$$\sigma_{\text{el}} = \int d^2\vec{b} |\Gamma(\vec{b})|^2 = \int d^2\vec{b} \langle T(\vec{b}) \rangle_{p,t}^2$$

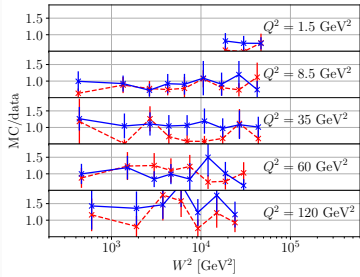
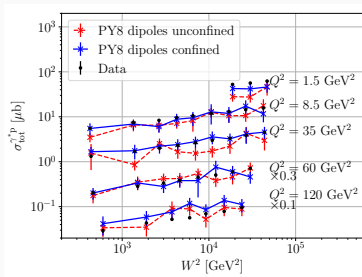
$$B_{\text{el}} = \frac{\partial}{\partial t} \log \left(\frac{d\sigma_{\text{el}}}{dt} \right) \Big|_{t=0} = \frac{\int d^2\vec{b} b^2/2 \langle T(\vec{b}) \rangle_{p,t}}{\int d^2\vec{b} \langle T(\vec{b}) \rangle_{p,t}}$$

- Or with photon wave function:

$$\sigma^{\gamma^*P}(s) = \int_0^1 dz \int_0^{r_{\text{max}}} r dr \int_0^{2\pi} d\phi (|\psi_L(z, r)|^2 + |\psi_T(z, r)|^2) \sigma_{\text{tot}}(z, \vec{r})$$

Model parameters

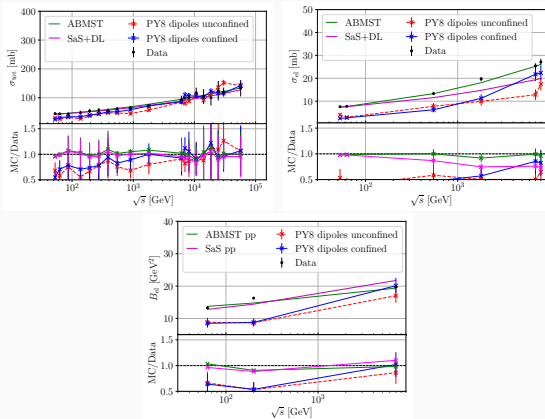
- This means that all parameters (4) can be tuned to cross sections



- Could constrain better in ep with eg. vector meson production.

Model parameters II

- Same parameters should describe pp, adds more data to the tuning.



- Not as good as dedicated (Regge-based) models.
- Accuracy not the point, control of physics features is!

Cross section colour fluctuations

- Cross section fluctuates event by event: important for pA , γ^*A and less AA .
- Projectile remains frozen through the passage of the nucleus.
- Consider fixed state (k) projectile scattered on single target nucleon:

$$\begin{aligned}\Gamma_k(\vec{b}) &= \langle \psi_S | \psi_I \rangle = \langle \psi_k, \psi_t | \hat{T}(\vec{b}) | \psi_k, \psi_t \rangle = \\ &= (c_k)^2 \sum_t |c_t|^2 T_{tk}(\vec{b}) \langle \psi_k, \psi_t | \psi_k, \psi_t \rangle = \\ &= (c_k)^2 \sum_t |c_t|^2 T_{tk}(\vec{b}) \equiv \langle T_{tk}(\vec{b}) \rangle_t\end{aligned}$$

- And the relevant amplitude becomes $\langle T_{t_i, k}^{(nN_i)}(\vec{b}_{ni}) \rangle_t$

Fluctuating nucleon-nucleon cross sections

- Let nucleons collide with total cross section $2\langle T \rangle_{p,t}$
- Inserting frozen projectile recovers total cross section.
- Consider instead inelastic collisions only (color exchange, particle production):

$$\frac{d\sigma_{\text{inel}}}{d^2\vec{b}} = 2\langle T(\vec{b}) \rangle_{p,t} - \langle T(\vec{b}) \rangle_{p,t}^2.$$

- Frozen projectile will not recover original expression, but require target average first.

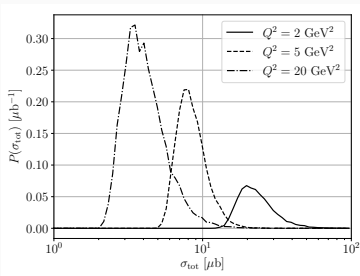
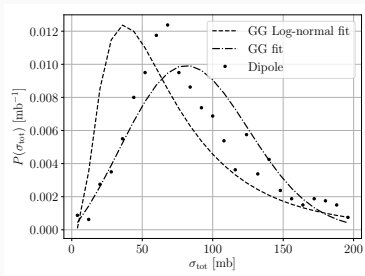
$$\frac{d\sigma_w}{d^2\vec{b}} = 2\langle T_k(\vec{b}) \rangle_p - \langle T_k^2(\vec{b}) \rangle_p = 2\langle T(\vec{b}) \rangle_{t,p} - \langle \langle T(\vec{b}) \rangle_t^2 \rangle_p$$

- Increases fluctuations! But pp can be parametrized.

EIC adds more complications

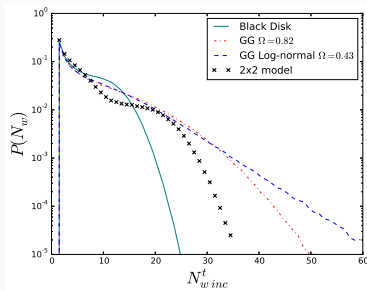
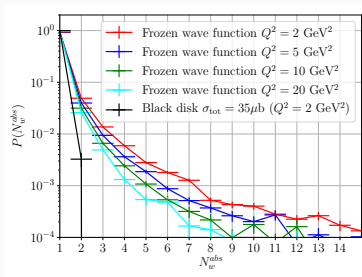
- For γ^*A collisions the trick can be repeated.
- But photon wave function collapse to previous result at first hit.

$$\frac{d\sigma_w}{d^2\vec{b}} = \int dz \int d^2\vec{r} (|\psi_L(z, \vec{r})|^2 + |\psi_T(z, \vec{r})|^2) (2\langle T(\vec{b}) \rangle_{t,p} - \langle \langle T(\vec{b}) \rangle_t^2 \rangle_p).$$



Drastic for number of wounded nucleons

- More multi-hit events, meaning more background.
- Clearly non-negligible, lesson already learned in p-Pb at LHC.



The story so far

- Mueller dipole MC for fluctuations and impact parameter space.
- Drastic consequences for wounded nucleons.
- Must be coupled to particle production.
- ...and to initial spatial parton density.

- Several partons taken from the PDF.
- Hard subcollisions with $2 \rightarrow 2$ ME:

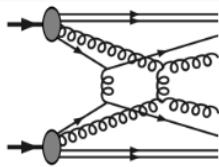


Figure T. Sjöstrand

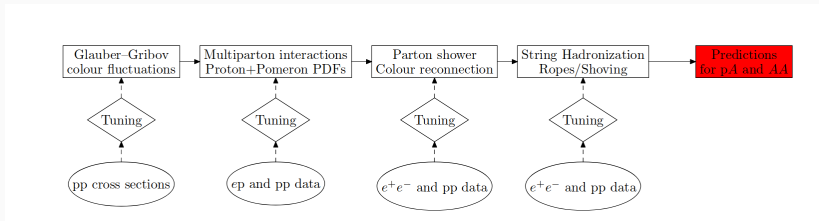
$$\frac{d\sigma_{2 \rightarrow 2}}{dp_{\perp}^2} \propto \frac{\alpha_s^2(p_{\perp}^2)}{p_{\perp}^4} \rightarrow \frac{\alpha_s^2(p_{\perp}^2 + p_{\perp 0}^2)}{(p_{\perp}^2 + p_{\perp 0}^2)^2}.$$

- Momentum conservation and PDF scaling.
- Ordered emissions: $p_{\perp 1} > p_{\perp 2} > p_{\perp 4} > \dots$ from:

$$\mathcal{P}(p_{\perp} = p_{\perp i}) = \frac{1}{\sigma_{nd}} \frac{d\sigma_{2 \rightarrow 2}}{dp_{\perp}} \exp \left[- \int_{p_{\perp}}^{p_{\perp i-1}} \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp'_{\perp}} dp'_{\perp} \right]$$

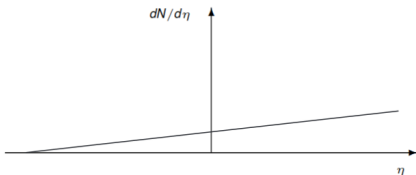
- Picture blurred by CR, but holds in general.

- Pythia MPI model extended to heavy ions since v. 8.235.
 1. Glauber geometry with Gribov colour fluctuations.
 2. Attention to diffractive excitation & forward production.
 3. Hadronize with Lund strings.



Particle production: Wounded nucleons

- Simple model by Białas and Czyz.
- Wounded nucleons contribute equally to multiplicity in η .
- Originally: Emission function $F(\eta)$ fitted to data.

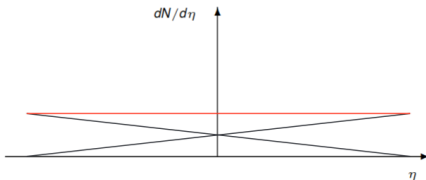


$$\frac{dN}{d\eta} = F(\eta) \quad (\text{single wounded nucleon})$$

- Angantyr: No fitting to HI data, but include model for emission function.
- Model fitted to reproduce pp case, high \sqrt{s} , can be retuned down to 10 GeV.

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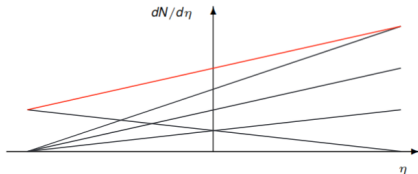


$$\frac{dN}{d\eta} = F(\eta) + F(-\eta) \quad (\text{pp})$$

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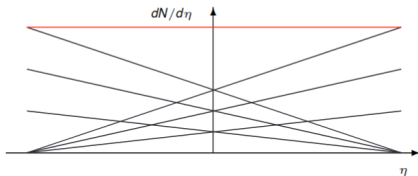


$$\frac{dN}{d\eta} = w_t F(\eta) + F(-\eta) \quad (\text{pA})$$

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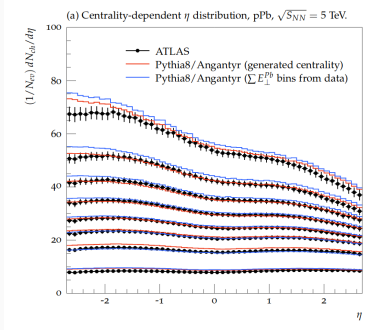
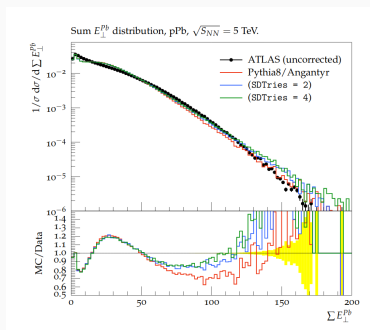


$$\frac{dN}{d\eta} = w_t F(\eta) + w_p F(-\eta) \quad (AA)$$

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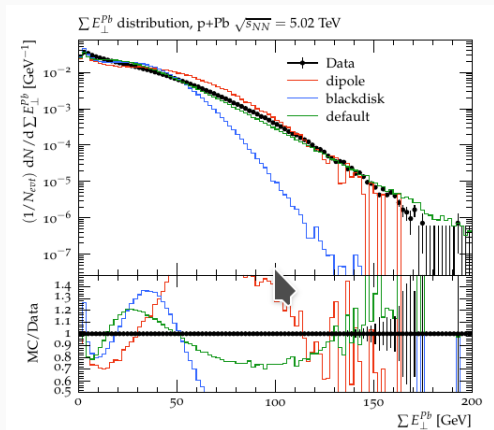
Some results - pPb

- Centrality measures are delicate, but well reproduced.
- So is charged multiplicity.



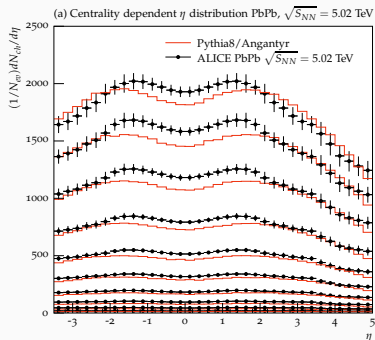
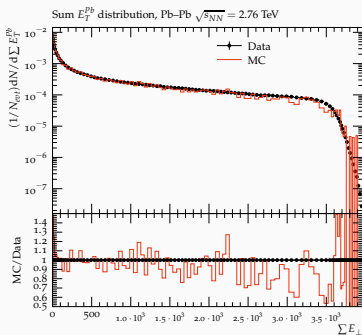
Direct dipole fluctuations

- Comparison between direct calculation (slow), and parametrized (fast).



Basic quantities in AA

- Reduces to normal Pythia in pp. In pA in AA:
 1. Good reproduction of centrality measure.
 2. Particle density at mid-rapidity.



- Geometric quantities from matching to dipole calculations coming up.

Parton vertices

- Parton vertices assigned according to dipole calculation.
- Cannot be done from first principles!

Principles of vertex assignments

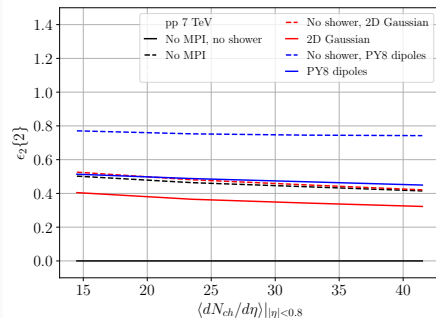
- ◇ Dipole cascade branches go on-shell *iff* colliding with another.
 - ◇ Partonic sub-collisions ordered in importance, *i.e.* contribution to cross section.
 - ◇ Further emissions by parton shower smears and recoils with a Gaussian.
-
- Default model, for comparison, is proton mass distribution = 2D Gaussian.

Eccentricities

- Initial state anisotropy quantified:

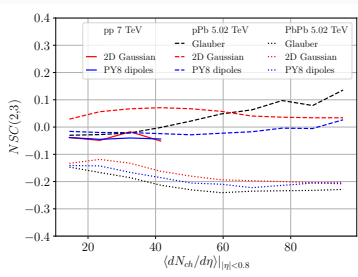
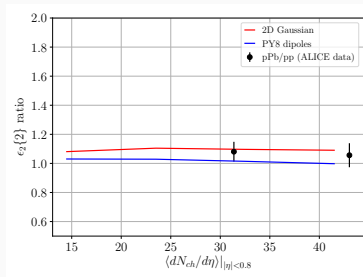
$$\epsilon_n = \frac{\sqrt{\langle r^2 \cos(n\phi) \rangle^2 + \langle r^2 \sin(n\phi) \rangle^2}}{\langle r^2 \rangle}.$$

- ...and the usual higher moments.
- Beware infrared safety! $\rightarrow p_{\perp}/(p_{\perp} + p_{\perp,min})$



Could differences be measured?

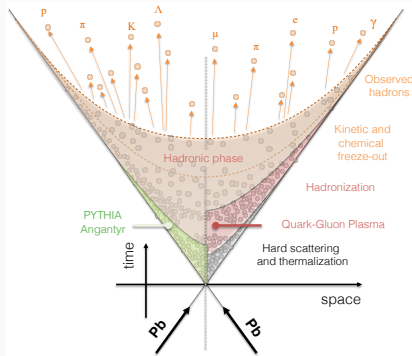
- Differences visible, but p-Pb might be the best!



- NSC correlated flow coefficients, and scale out the magnitude.
- For p-Pb: Only negative in dipole picture.

Adding transport to final state

- The left side has now been established.
- Emphasis on colour fluctuations, forward production and partonic vertices.
- Rest of the talk: Transporting anisotropy to final state.



(Figure: D. D. Chinellato)

Microscopic final state collectivity

- Proposal: Model microscopic dynamics with interacting Lund strings
- Additional input fixed or inspired by lattice, few tunable parameters.

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$\tau \approx 0$ **fm**: Strings no transverse extension. No interactions, partons may propagate.

$\tau \approx 0.6$ **fm**: Parton shower ends. Depending on "diluteness", strings may shove each other around.

$\tau \approx 1$ **fm**: Strings at full transverse extension. Shoving effect maximal.

$\tau \approx 2$ **fm**: Strings will hadronize. Possibly as a colour rope.

$\tau > 2$ **fm**: Possibility of hadronic rescatterings.

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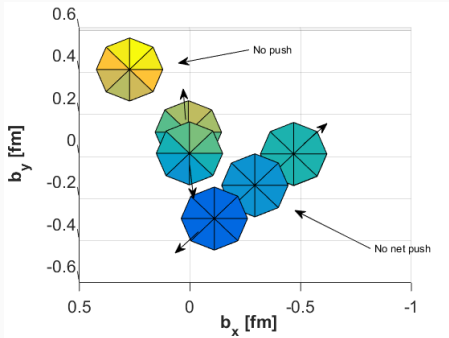
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The cartoon picture

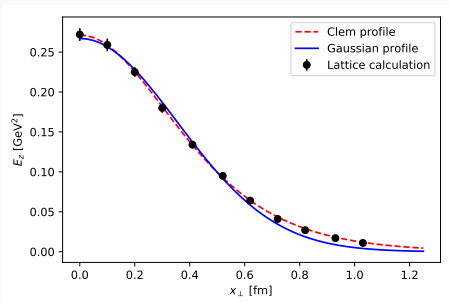
- Strings push each other in transverse space.
- Colour-electric fields \rightarrow classical force.



- 👍 Transverse-space geometry.
- 👍 Particle production mechanism.
- ?? String radius and shoving force

MIT bag model, dual superconductor or lattice?

- Easier analytic approaches, eg. bag model:
$$\kappa = \pi R^2 [(\Phi/\pi R^2)^2/2 + B]$$
- Bad R 1.7 and dual sc. 0.95 respectively, shape of field is input.
- Lattice can provide shape, but uncertain R .



- Solution: Keep shape fixed, but R ballpark-free.

The shoving force

- Energy in field, in condensate and in magnetic flux.
- Let g determine fraction in field, and normalization N is given:

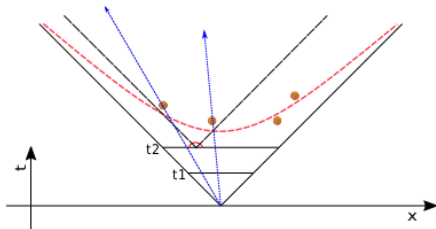
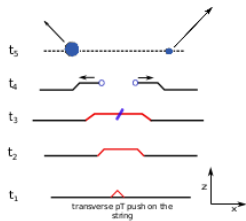
$$E = N \exp(-\rho^2/2R^2)$$

- Interaction energy calculated for transverse separation d_{\perp} , giving a force:

$$f(d_{\perp}) = \frac{g\kappa d_{\perp}}{R^2} \exp\left(-\frac{d_{\perp}^2}{4R^2}\right)$$

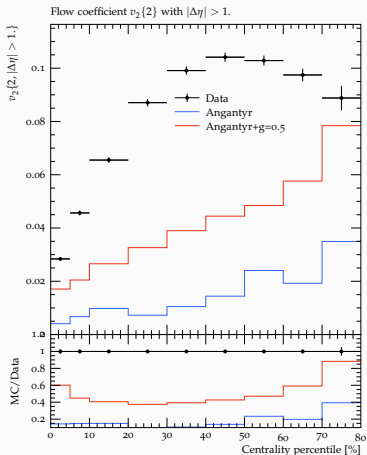
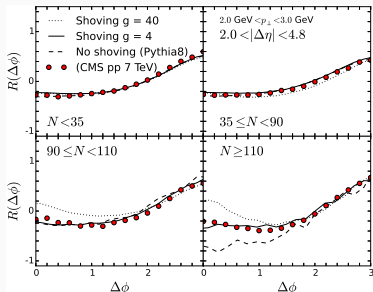
Monte Carlo details

- Distance d_{\perp} calculated in a frame where strings lie in parallel planes.
- Everything is two-string interactions.
- The shoving action implemented as a parton shower (again!)
- Push propagated along string, and distributed on final state hadrons.



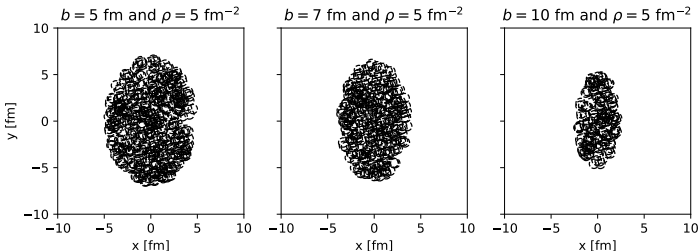
Directly: varying results

- Results are ok in pp, but off in AA.
- This is with the full initial state machinery = many things can go wrong.



Simpler toy initial state

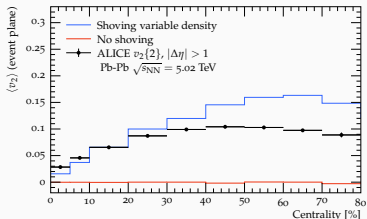
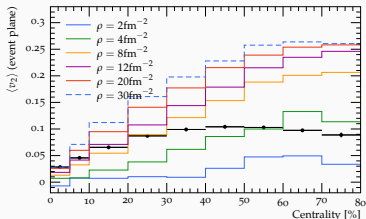
- Problematic: many soft gluons in final state.
- Corrections to string hadronization, saturation scale...
- Set up toy system of straight strings, study response to geometry.



Fixed density close to reality

- Choose array of fixed densities, insert long strings, and calculate v_n :

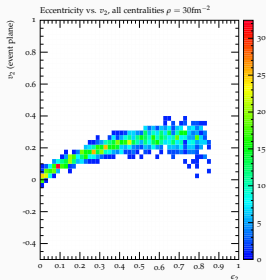
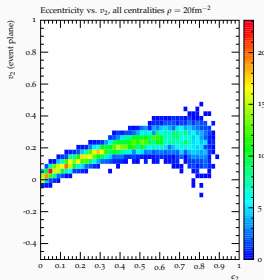
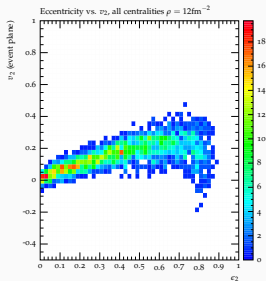
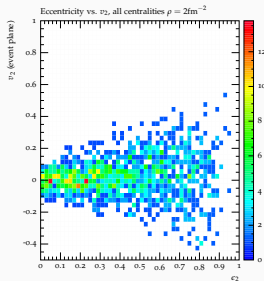
$$v_n = \langle \cos(n(\phi - \Psi_n)) \rangle, \Psi_n = \frac{1}{n} \arctan \left(\frac{\langle p_\perp \sin(n\phi) \rangle}{\langle p_\perp \cos(n\phi) \rangle} \right)$$



- Closer to data is nice, gives a path forward.

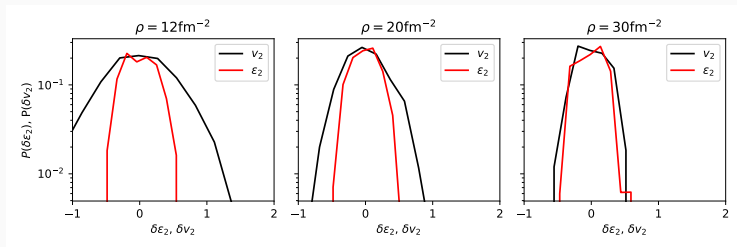
Scaling with initial eccentricity

Critical density? Critical for what?



Better: Rescaled variables

$$\delta\epsilon_2 = \frac{\epsilon_2 - \langle\epsilon_2\rangle}{\langle\epsilon_2\rangle} \quad \text{and} \quad \delta v_2 = \frac{v_2 - \langle v_2\rangle}{\langle v_2\rangle}$$



- Scaling like hydro for large densities.
- ...but more fluctuations for low densities!

Summary: Dipoles and string interactions

- Mueller dipoles for geometry and IS fluctuations.
- Mapping to Pythia/Angantyr for particle production.
- Angantyr = p-A and AA final states, eA are coming.
- Huge opportunity: Control geometry and density at EIC.
- String shoving: interactions to generate transverse pressure.
- Interface to Angantyr still not perfect.
- Behaves like hydro in simple, high-density systems.

*Thank you for the invitation!
Exciting times are still ahead!*