Scattered thoughts about initial state geometry

Christian Bierlich, bierlich@thep.lu.se
Lund University
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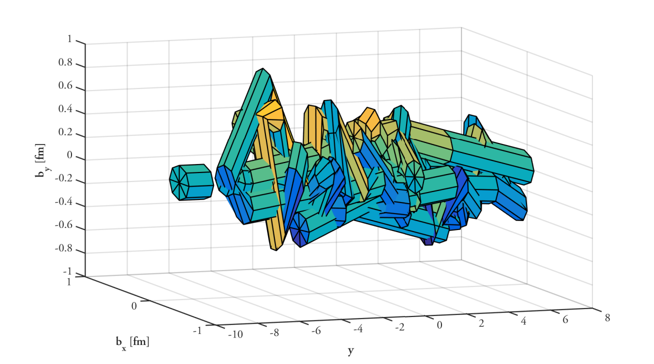
Importance of geometry



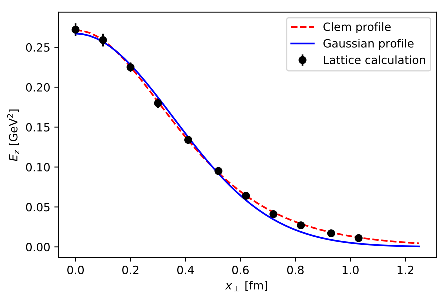
- For ropes and shoving, good models for parton level geometry is crucial.
- It is, however, mostly *ad hoc*.
- This talk will concern recent work, some unpublished.

Ropes and shoving, in case you forgot

- Shoving: Pair-wise string interactions with:

$$f(d_\perp) = \frac{g\kappa d_\perp}{R^2} \exp\left(-\frac{d_\perp^2}{4R^2}\right)$$

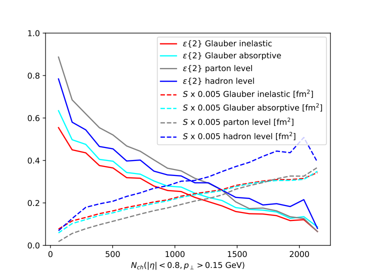
- Ropes: Enhancing string tension according to *string overlaps*.
- String profile "known" from lattice. Geometry provides the initial conditions.

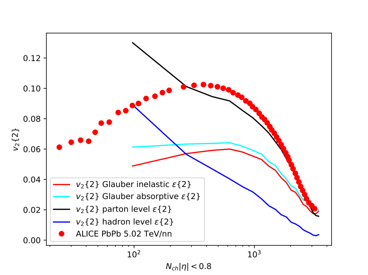


AA is probably easiest

- Assuming that $d_{\mathrm{AA}} \ll d_{\mathrm{MPI}}$, Glauber + Gaussian smearing will suffice.
- See also: TRenTo, IP-Glasma, GLISSANDO + wounded quarks etc.
- Fluctuations/parton shower may play a larger role, here:

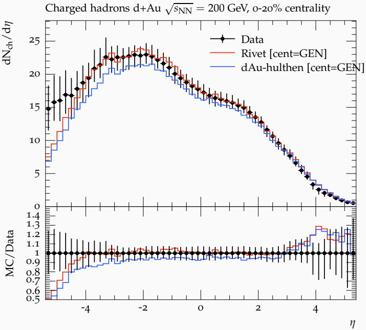
$$v_2 = \epsilon_2 \left(\frac{v_2}{\epsilon_2}\right)_{\mathrm{hydro}} \frac{1}{1 + \frac{\lambda}{K_0}\frac{\langle S \rangle}{\langle \frac{dN}{d\eta} \rangle}}$$

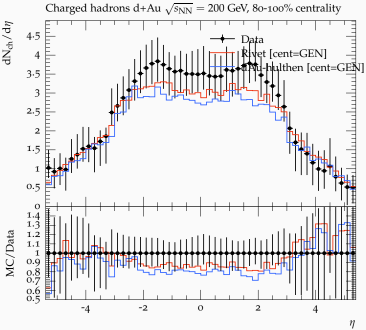




Light nuclei: hairs in the soup

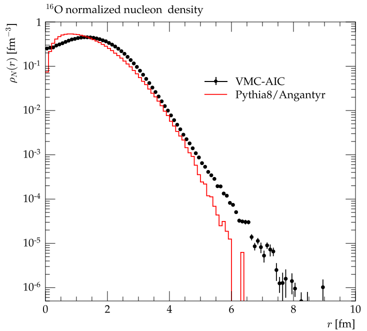
- Does not follow Woods-Saxon, other potentials must be used.
- Deuteron: Hulthen form, well known, calculable in QM.
- Parameters can be estimated from data.
- 10-20% effects in basic quantities (d-Au 200 GeV), probably more for flow.

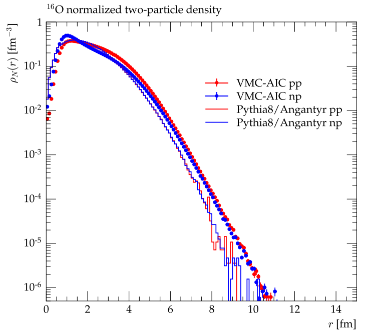




Oxygen

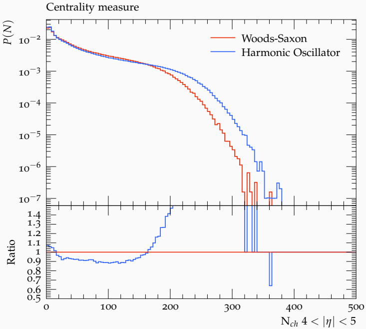
- Potential and parameters unclear. Preciously little data.
- Possibility: Using multibody QM calculation directly or indirectly (*ab initio* initial condittions)
- Harmonic oscillator shell model or simple Gaussian ansatz? How to be sure?

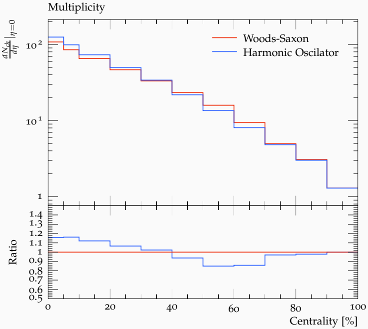




Look forward!

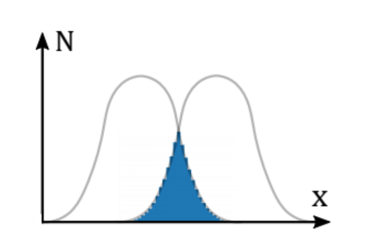
- The forward region is most sensitive to small changes in geometry.
- Could be discriminating factor, in particular with RHIC+LHC energies.
- Easy test of α-clustering? (Ba. of Aliaksei Kuzmenka)

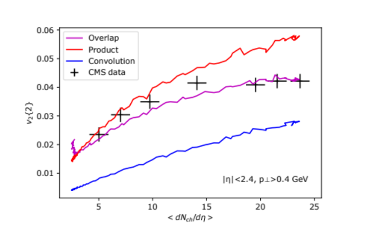




Protons are more difficult

- Currently in PYTHIA: Convolution of two Gaussians, no b-dependence of eccentricity.
- Naive approach could be useful: Ovelapping Gaussians (Ba. of Johannes Holst)
- Similar to IP-Glasma? Re: Jarkko.

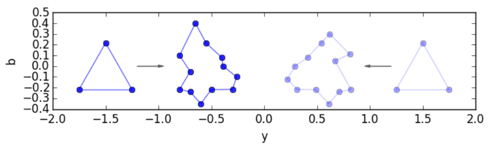




More advanced using dipoles (1907.12871 + many(!) DIPSY papers, LL & GG)

The aim and the means

A reasonable calculation of initial state geometry.
Fluctuating nucleon–nucleon cross sections.
MC implementation of Mueller dipoles.

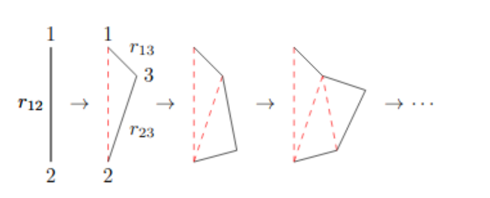


- Projectile and target cascades evolved for each event.
- Formalism in impact parameter and rapidity.
- Single-event spatial structure.

A step back, BFKL, B-JIMWLK and all that...

- Start with Mueller dipole branching probability:

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}y} = \mathrm{d}^2\vec{r}_3\ \frac{N_c\alpha_s}{2\pi^2}\frac{r_{12}^2}{r_{13}^2 r_{23}^2} \equiv \mathrm{d}^2\vec{r}_3\ \kappa_3.$$



- Evolve any observable $O(y) \rightarrow O(y + \mathrm{d}y)$ in rapidity:

$$\bar{O}(y+\mathrm{d}y) = \mathrm{d}y \int \mathrm{d}^2\vec{r}_3\ \kappa_3 \left[O(r_{13}) \otimes O(r_{23})\right] + O(r_{12}) \left[1 - \mathrm{d}y \int \mathrm{d}^2\vec{r}_3\ \kappa_3\right]$$

$$\rightarrow \frac{\partial \bar{O}}{\partial y} = \int \mathrm{d}^2\vec{r}_3\ \kappa_3 \left[O(r_{13}) \otimes O(r_{23}) - O(r_{12})\right].$$

A powerful formalism!

- Example: S-matrix (eikonal approximation, b-space):

$$O(r_{13}) \otimes O(r_{23}) \to S(r_{13})S(r_{23})$$

- Change to $T \equiv 1 - S$:

$$\frac{\partial \overline{\langle T \rangle}}{\partial y} = \int \mathrm{d}^2 \vec{r}_3 \ \kappa_3 \left[\langle T_{13} \rangle + \langle T_{23} \rangle - \langle T_{12} \rangle - \langle T_{13} T_{23} \rangle \right].$$

- B-JIMWLK equation, but could be written with other observables.
- Example: Average dipole coordinate ($\langle z \rangle$):

$$\frac{\partial \overline{\langle z \rangle}}{\partial y} = \int \mathrm{d}^2 \vec{r}_3 \kappa_3 \left(\frac{1}{3} z_3 - \frac{1}{6}(z_1 + z_2) \right).$$

Monte Carlo implementation

Drawbacks to analytic approach

Involved observables are hard!
Not obvious how to include sub-leading effects.
Not obvious how to treat exclusive final states.

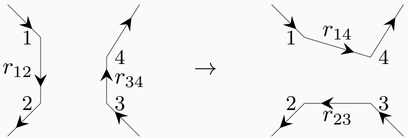
- The MC way is a tradeoff: formal precision vs. pragmatism.
- Get for free: Rest of the MC infrastructure.
- Practically a parton shower-like implementation.
- Step 1: Modify splitting kernel with Sudakov:

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}y\ \mathrm{d}^2\vec{r}_3} = \frac{N_c\alpha_s}{2\pi^2}\frac{r_{12}^2}{r_{13}^2 r_{23}^2}\exp\left(-\int_{y_{\min}}^{y}\mathrm{d}y\mathrm{d}^2\vec{r}_3\ \frac{N_c\alpha_s}{2\pi^2}\frac{r_{12}^2}{r_{13}^2 r_{23}^2}\right)$$

- Winner-takes-it-all algorithm generates emission up to maximal rapidity.
- Throws away the non-linear term in the cascade.

Colliding dipole chains & unitarity

- Have: Evolved dipole chain á la BFKL.
- Dipole cross section in large-N_c limit (consistency with evolution):



$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\vec{b}} = \frac{\alpha_s^2 C_F}{N_c} \log^2\left[\frac{r_{13}r_{24}}{r_{14}r_{23}}\right]$$
$$\to \frac{\alpha_s^2}{2}\log^2\left[\frac{r_{13}r_{24}}{r_{14}r_{23}}\right] \equiv f_{ij}$$

- Unitarized scattering amplitude: $T(\vec{b}) = 1 - \exp\left(-\sum_{ij} f_{ij}\right)$

Effects beyond leading log

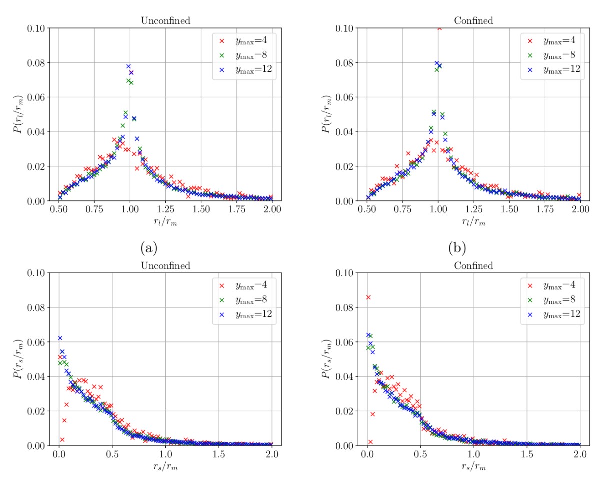
Some details

A dipole has a rapidity y, and a $p_\perp$ related to its size $p_\perp\ \hbar/r$. Thus its lightcone momenta is $p_\pm = p_\perp \exp(\pm y)$.

- Energy-momentum conservation from bounded p_- translate to upper bound on dipole sizes.
- Running α_s: Easily included per-splitting.
- Non-eikonal effects: recoil distributed on emitters in $p_+, p_\perp$, and thus also y.
- Confinement: Explicit confinement scale (or fictitious gluon mass) entering evolution and collision.
- Unitarized scattering amplitude resums $1/N_c^2$ terms in interaction, equivalent to multi-pomeron exchanges in interaction frame.

Example: confinement → hot-spots

- MC makes it easy to switch physics effects on and off.
- More activity around end-points: Hot-spots!
- Initial triangle by hand. Less important at high energies, but deserves more thought.



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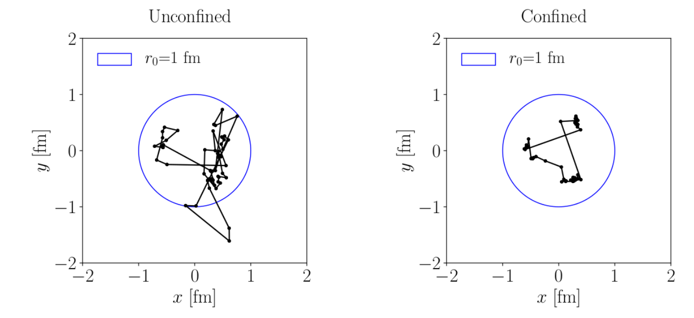


- Dynamically generated!

Good–Walker & cross sections

- Cross sections from $T(\vec{b})$ with normalizable particle wave functions:

$$\sigma_{\mathrm{tot}} = 2\int \mathrm{d}^2\vec{b}\Gamma(\vec{b}) = 2\int \mathrm{d}^2\vec{b}\ \langle T(\vec{b})\rangle_{p,t}$$

$$\sigma_{\mathrm{el}} = \int \mathrm{d}^2\vec{b}|\Gamma(\vec{b})|^2 = \int \mathrm{d}^2\vec{b}\ \langle T(\vec{b})\rangle_{p,t}^2$$

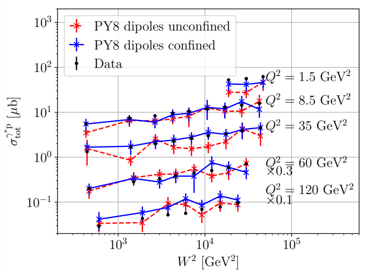
$$B_{\mathrm{el}} = \frac{\partial}{\partial t}\log\left(\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}t}\right)\Big|_{t=0} = \frac{\int \mathrm{d}^2\vec{b}\ b^2/2\ \langle T(\vec{b})\rangle_{p,t}}{\int \mathrm{d}^2\vec{b}\ \langle T(\vec{b})\rangle_{p,t}}$$

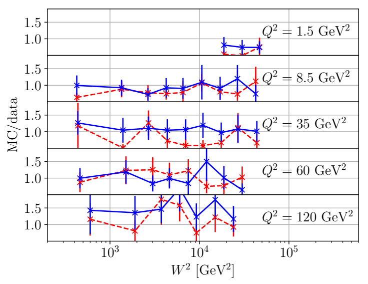
- Or with photon wave function:

$$\sigma^{\gamma^*\mathrm{p}}(s) = \int_0^1 \mathrm{d}z \int_0^{r_{\max}} r\mathrm{d}r \int_0^{2\pi} \mathrm{d}\phi\left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right)\sigma_{\mathrm{tot}}(z,\vec{r})$$

Model parameters

- This means that all parameters (4) can be tuned to cross sections

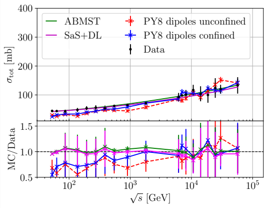


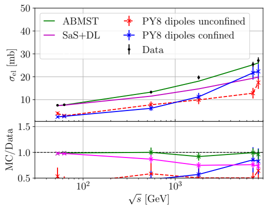


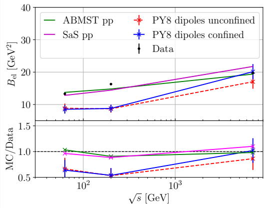
- Could constrain better in *ep* with eg. vector meson production.

Model parameters II

- Same parameters should describe pp, adds more data to the tuning.



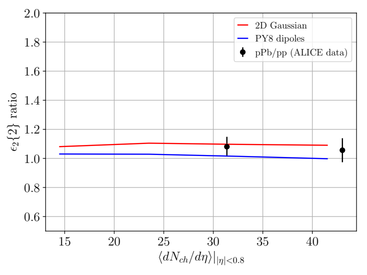


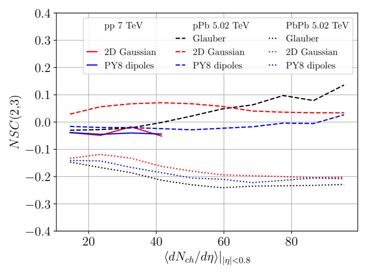


- Not as good as dedicated (Regge-based) models.
- Accuracy not the point, control of physics features is!

When does substructure start to matter?

- Differences visible, but p-Pb might be the best!





- NSC correlated flow coefficients, and scale out the magnitude.
- For p-Pb: Only negative in dipole picture.

Future: stuff I want to do (instead of a summary)

- We have: A good MPI model (PYTHIA) extendible to AA (Angantyr) with possibility of adding nuclear geometries.
- We have: The dipole picture giving a motivated calculation of substructure, parameters connected to cross sections.
- I want:
  - Tests of nuclear geometry to be carried out at LHC and RHIC. Requires *ab initio* estimates of model parameters at least.
  - A combined MPI model, getting rid of the PYTHIA $p_{\perp 0}$ parameter in place of an event-by-event physical quantity.
  - To see how the low-$p_\perp$ behaviour of such a model differs from scattering of CYM fields.
  - To use it for ep and eA collision, with all that entails of vector meson states.
  - As many observables as possible to connect the model to data – what could ropes and jet physics do? what could HBT do? does this have effects on the rescattering phase $\rightarrow$ deuteron production