Hadronization and Soft QCD
…with a bias towards Lund strings

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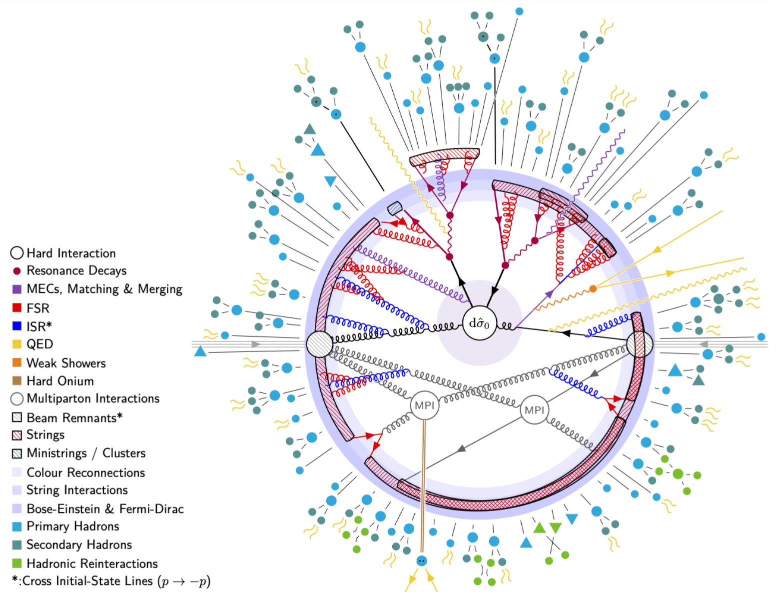
Hadronization: What? (PYTHIA manual: arXiv:2203.11601)



Hadronization: Why?

- *Because the world is colourless!*
- Quarks and gluons from pQCD and showers cannot be observed.
- Need “some way transform”, or at least calculate corrections.
- We cannot use pQCD, and lattice QCD has no dynamics.
- Must “rely on models”, whatever that means.

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- Need “some way transform”, or at least calculate corrections.
- We cannot use pQCD, and lattice QCD has no dynamics.
- Must “rely on models”, whatever that means.
- Opportunity to model physics which cannot be solved.
- Good models also have predictive power = fruitful.
- Intruiging LHC discoveries based on our non-understanding.

Lecture(s) overview

- Part I: The overview.
  1. Local Parton Hadron Duality & Independent fragmentation.
  2. Cluster hadronization.
  3. The (Lund) string in brief overview.
- Part II: A closer look at Lund strings.
  1. String motion.
  2. String motivation.
  3. String decay.
- Part III: Thinking for yourself.
  1. Some (concept) exercises.
- Part IV: Heavy ion collisions and collectivity
  1. Are pp and AA really that different?
  2. Interactions between Lund strings.
- *What does it mean that "hadronization relies on models"*?

Local Parton Hadron Duality (Poggio, Quinn and Weinberg, PRD (1976))

- Inclusive hadronic cross sections co-incides with (pertubative) quark-gluon cross sections.

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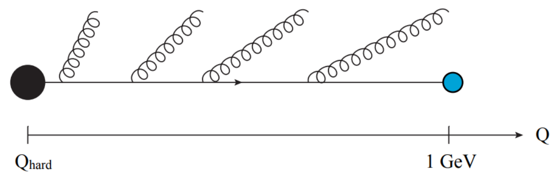
  - *Approximately* coincides.
  - Not clear if input should include non-perturbative effects.



Describes momentum specta rather well, but few redeeming factors for event generation. At this point mostly a historical artefact.

Problems with the simple approach

- Motivates “independent fragmentation”, basically:

$$q \to h, ..., h$$

- Exclusive model → event generatation (Feynman–Field model).
- Can even apply “correction factors” to describe string effects

  (Ballochi & Odorico: Nucl. Phys.B 345 (1990) 173-185)

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😈 Misses the physics of confinement:
  1. Partons are coloured.
  2. Hadronization neutralises the colour.

😈 *Unphysical* to let single parton fragment to hadrons.

😈 LPHD too naive to motivate exclusive fragmentation model.

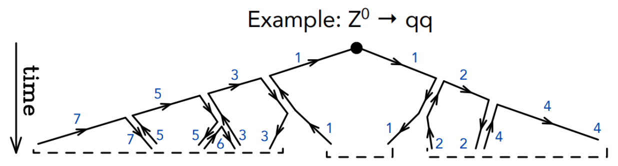
😇 Might be fine if hadronization is just a nuisance, and your goal is to parametrize.

Colour flow & Preconfinement

- Hadronization should involve *at least* two partons with "opposite colour".
- Think of this as $r\bar{r}$, $b\bar{b}$ or $g\bar{g}$ but really a singlet state:

$$\frac{1}{\sqrt{3}}(|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle).$$

- In *leading colour* (ie. $N_c \to \infty$) in $\mathrm{e}^+\mathrm{e}^-$ (cleanest) we get a sense of *preconfinement*:



Universal property of parton shower.

The cluster spectrum

- The *Preconfinement property of Parton Showers* (Amati & Veneziano: Phys.Lett.B83 (1979) 87)
    1. Colour singlet clusters can be formed at *any* evolution scale Q_0.
    2. *Asymptotically universal invariant* mass distribution.
    3. Meaning: $P = P(M, Q_0, \Lambda_{\mathrm{QCD}})$, $Q_0 \ll Q$.

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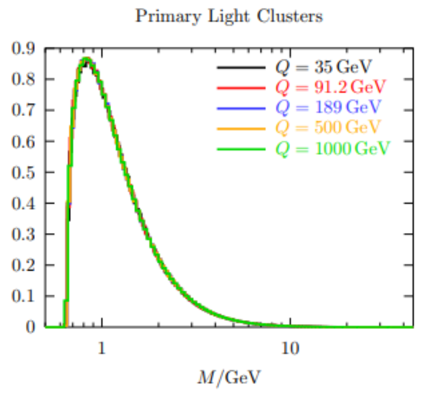
Modelling:

a) Enforce non-perturbative splitting of $g \to q\bar{q}$.

b) Quark (and diquark!) flavours must be imposed somwhow.



Cluster decay

- 😇 Low-mass clusters → spectrum of mesons.
  → Isotropic two-body decay.
- 😈 High-mass clusters must decay → proto-hadrons?
- 😈 Is $g \rightarrow s\bar{s}$ (implicitly higher scale) breaking universal property?

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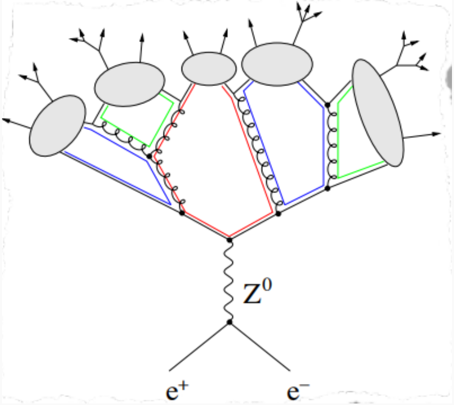
a) Probably simplest, still well-motivated model.

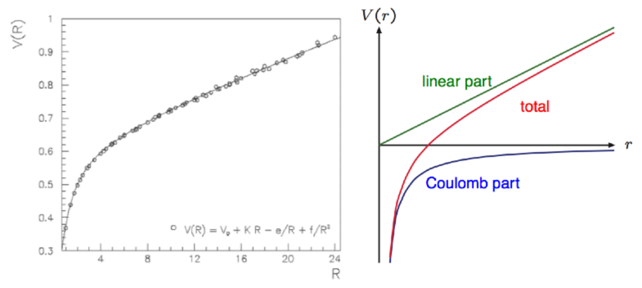
b) Used in HERWIG and SHERPA (PYTHIA adding the option).

c) Physics picture may be exhausted at some point (?)



Strings: The QCD potential

- Maybe we can start somewhere else? A model of *dynamics*?
- Can draw inspiration from Lattice QCD.



(Figure credit: Torbjörn Sjöstrand)

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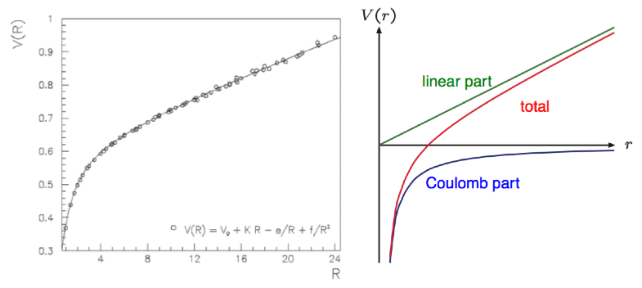


(Figure credit: Torbjörn Sjöstrand)

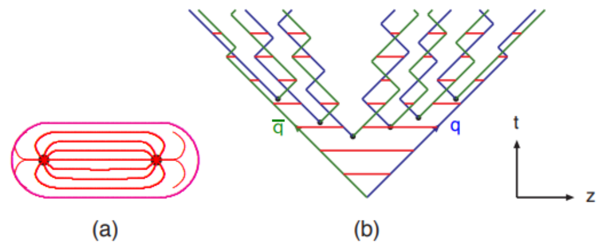
- Small distances: "Coulomb": Here we use pQCD.
- Large distances: Which system has a linear potential?

$$V(r) \approx \kappa r;\ \text{Force} = \text{const} = \kappa \approx 1\text{GeV/fm}$$

String motion (more on this later) and basics

- Simple, but powerful, dynamical picture:
  A 3 GeV quark can move 3 fm before all energy is tranferred to the string.
- String *breaks* to produce hadrons (yo-yo modes).
- Constant particle density in rapidity.
- Maximal string length (all E_q to single pion):

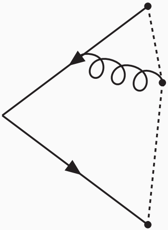
$$y_{\max} \approx \log\left(\frac{2E_q}{m_\pi}\right) \rightarrow \text{rapidity plateau}$$



(a) (b)

String decay (also more on this later)

- Microscopic decay laws for string breaking.
- Produces yo-yo's with incoming $q\bar{q}$ ends. Or diquarks.



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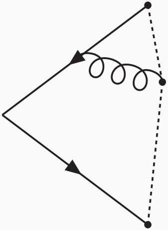

Lund symmetric fragmentation function

$$f(z) \propto z^{-1}(1-z)^a \exp\left(\frac{-bm_\perp}{z}\right).$$

a and b related to total multiplicity.

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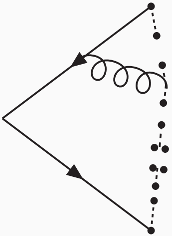

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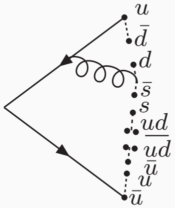

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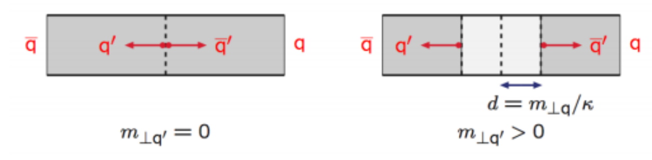
Flavours by relative probabilities

$$\rho = \frac{\mathcal{P}_{\text{strange}}}{\mathcal{P}_{\text{u or d}}}, \xi = \frac{\mathcal{P}_{\text{diquark}}}{\mathcal{P}_{\text{quark}}}$$

The tunneling equation

- Tunneling a QM phenomenon. Treated in WKB approximation (given assumptions) or in analogy with QED.
- In overview (see Andersson *et. al.*: Phys. Rept. 97 (1983) 31-145 for details)



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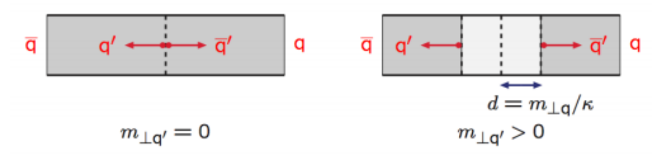

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp,q}^2}{\kappa}\right) = \exp\left(-\frac{\pi p_{\perp,q}^2}{\kappa}\right)\exp\left(-\frac{\pi m_q^2}{\kappa}\right).$$

- Directly: q and $\bar{q}$ opposite, compensating kicks:

$$\langle p_{\perp,q}^2\rangle = \kappa/\pi \approx (0.25\mathrm{GeV})^2$$

Tunneling equation cont'd

$p_\perp$ kick not enough to describe data!

- Also directly:
  Current m_q: $m_s \approx 0.1$ GeV $m_{u,d} \approx 0.$
  $\rightarrow$ Too many $s\bar{s}$.
  Constituent m_q: $m_s \approx 0.51$ GeV $m_{u,d} \approx 0.33$ GeV.
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Also cannot describe data!

- Solution: Free parameters. Motivation:
  $p_\perp$: soft gluon emissions below the shower cut-off.
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Well motivated parametrizations based on limited physics understanding. Parameters are not evil.

Combining quarks to hadrons

- Hadrons in general are superpositions, eg:

$$\rho^0 = \frac{1}{\sqrt{2}}\left(|\mathrm{u}\bar{\mathrm{u}}\rangle + |\mathrm{d}\bar{\mathrm{d}}\rangle\right), \quad \pi^0 = \frac{1}{\sqrt{2}}\left(|\mathrm{u}\bar{\mathrm{u}}\rangle - |\mathrm{d}\bar{\mathrm{d}}\rangle\right).$$

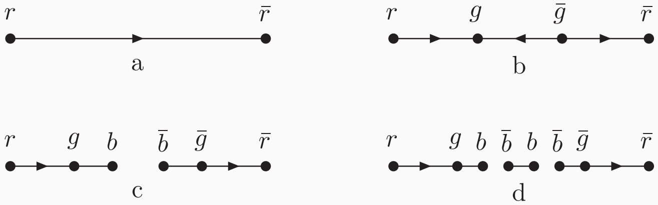
- “Ingoing” quarks must be combined using other rules:
  1. Spin counting: V/PS $= 3{:}1$, but $m_\rho \gg m_\pi$, empirically 1:1 $=$ parameter.
  2. Also for same spin: $m_{\eta'} \gg m_\eta \gg m_{\pi^0}$ gives mass suppression $=$ parameters.
- Worse for baryons:
  1. **SU**(**6**) (flavour × spin) Clebsch-Gordans.
  2. And simple baryon production model severely lacking.

Around 20 parameters/“material constants” neccesary.

And these are not the only possible choices (CB *et. al.*: arXiv:2201.06316)

Popcorn model

- Dynamical model for baryon production, improving "simple diquark".
- Problem: $B\bar{B}$-pairs produced too close in phase space (rapidity).



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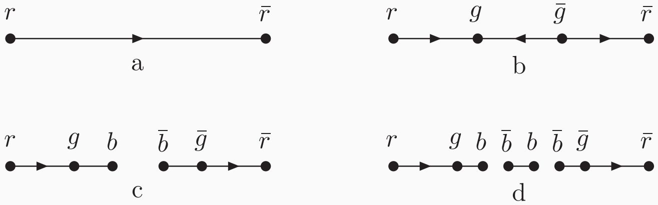
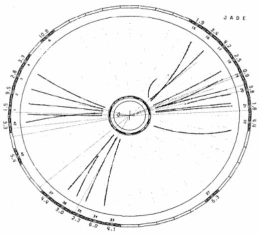


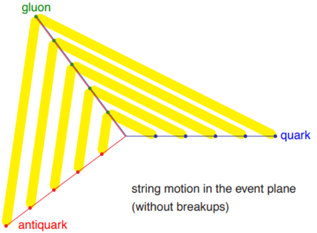
- Effect confirmed at LEP, intermediate mesons observed.

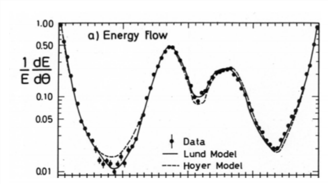
Modelling can teach us lessons, even with parameters!

Lund string gluons

- Benefit of dynamical picture: Dynamics!
- Historically *the* most characteristic feature of Lund strings.







- Unique event structure *between* jets!
- Instrumental for MC generators as a whole.

Strings vs. clusters

Clusters:

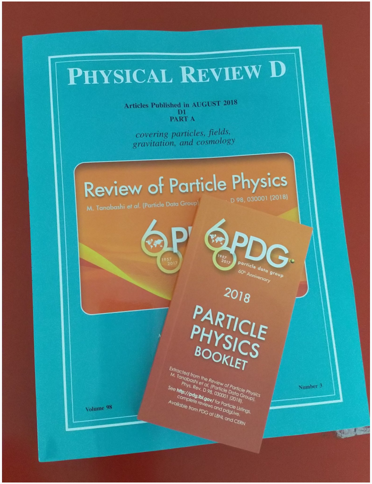
- *Focus on perturbative physics.*
- Simple energy–momentum picture.
- Unpredictive.
- Large clusters fragment “string–like”.
- Simple flavour composition.
- Few parameters.
- Difficult to extend.

Strings:

- *Hadrons should be produced by hadronization.*
- Powerful energy-momentum picture.
- Small strings fragment “cluster–like”.
- Messy flavour composition.
- Many parameters.
- Easy to extend, but beware of *ad hoc* modelling!

Decays

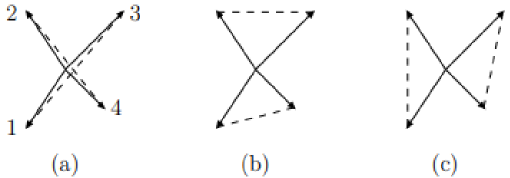
- Not a sexy task, but someone has to do it.



- Properties provided in machine–readable form.
- But most still must be done “by hand”.
- Recently developments towards *final state rescattering*.
- Known physics, but possible large effects.
- Most important for heavy ion physics.
- Also raises questions about transition region.

Before hadronization: From shower to strings

- All is well for a single string.
- But what if you have many? In pp min bias you have tens of MPIs!
- Even in $\mathrm{e^+e^-} \to W^+W^- \to \mathrm{q\bar{q}q\bar{q}}$ you have a choice.



(a) (b) (c)

Figure E. Nörbin

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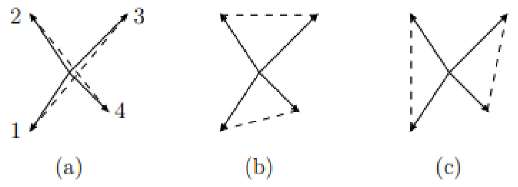

Figure E. Nörbin

- The effect is, however, rather small here.

Colour reconnection models

- In pp handled by “colour reconnection”.
- Practical solution, clearly *ad hoc*.
- Easy to merge low-$p_\perp$ systems, hard to merge two hard-$p_\perp$.

$$\mathcal{P}_{merge} = \frac{(\gamma p_{\perp 0})^2}{(\gamma p_{\perp 0})^2 + p_\perp^2}$$

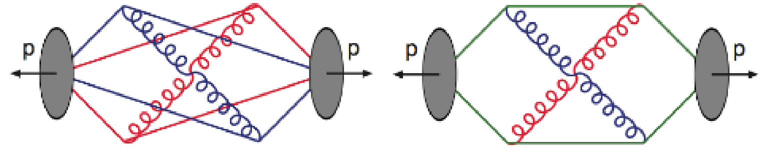

Figure T. Sjöstrand

- Actual merging by minimization of ”potential energy”:

$$\lambda = \sum_{dipoles} \log(1 + \sqrt{2}E/m_0)$$

Concluding the summary

- Hadronization is neccesary if you want to produce full events.
- Simple models give simple results. Some not well motivated physically, but works for their purpose.
- Better motivated models like strings or clusters are used in generators.
- Beware: Your initial assumptions can only take you so far!
- *Are strings more than a model? Is this how Nature works, or are we just parametrizing data?*
- Next: Lund strings – back to basics.
- Tomorrow: Collective effects from string interactions.
- Now: More details on Lund strings!