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Sovereign credit and exchange rate risks: Evidence from Asia-Pacific local currency bonds*

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Abstract

We study the dynamic properties of sovereign bonds in emerging market economies and their associated risk premiums. We focus on the properties of credit spreads, exchange rates, and their interaction. Relying on the term structure of local currency bonds issued by Asia-Pacific sovereigns, we find that local variables are significant in the dynamics of currency and credit risk, and the components of bond risk premiums reflecting these risks. Local currency bonds dramatically improve the investment frontier.

JEL Classification Codes: F31, G12, G15.

Keywords: emerging bond markets, credit risk, currency risk, Twin Ds, affine model.

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1 Introduction

This paper establishes evidence on how financial markets perceive and value the joint risk of sovereign default and its currency devaluation, known as the Twin Ds. Understanding how the markets perceive the Twin Ds is important as credit and exchange rate risks may be reinforcing each other on the way towards economic and financial distress. Furthermore, devaluation could be used as a tool to mitigate the impact of sovereign default, a point made in the theoretical literature as early as by Calvo (1988) and more recently by Na, Schmitt-Grohe, Uribe, and Yue (2018). Typically, such risk arises in the context of emerging market economies, and to date the measurement was constrained to realized outcomes during sovereign distress (Na, Schmitt-Grohe, Uribe, and Yue, 2018, Reinhart, 2002). The advantage of using prices of assets that are sensitive to this risk is that they are informative about the whole distribution of outcomes, not only the distress itself, and how these risks are valued in the marketplace.

Local currency (LC) bonds are the ideal source of information about the Twin Ds. This is because a US dollar (USD) investor is exposed to both credit and exchange rate risks. The availability of LC debt is a relatively recent phenomenon. Du and Schreger (2016) emphasize the steady growth of LC debt issued by emerging market economies since the early 2000s accompanied by a decline in the debt issued in foreign currency. Amongst all the regions, the Asia-Pacific (AP) boasts the most developed LC debt markets. In part spurred by policy initiatives in response to the 1997-98 Asian financial crisis, local currency government bond markets have grown from less than a quarter of a trillion USD in 1995 (or 10% of GDP) to $10.1 trillion in 2018 (corresponding to 48.5% of GDP). That is in contrast to, for example, many Latin American countries, where insufficient macroeconomic stability has hampered the growth of local currency bond markets. As a result, the amount of outstanding Latin American LC bonds has barely increased over the past decade: there was a total of $917 billion in 2018, compared to $808 billion in 2005.[1]

The focus on the AP region offers another point of departure from the traditional analysis of sovereign credit risk. Usually, it is studied in reference to "global" variables, which means the United States in practice. In Asia, however, China potentially provides another anchor in addition to the US, given its size and its economic significance for the region. In 2017, China's GDP based on PPP valuation exceeded that of the US ($23.1 trillion vs. $19.4 trillion), and its share of global GDP had risen from 2.3% in 1980 to more than 18% in 2017. Moreover, the Chinese government bond market, which was virtually nonexistent in 1980, had grown beyond $7 trillion in outstanding bonds by 2019. Furthermore, as Farhi and Maggiori (2019) emphasize, China is emerging as a challenger to the dominance of the US dollar both in the International Monetary System and in the International Price System. Thus, we investigate the impact of macroeconomic and bond market developments of these two countries on local currency bond markets in Asia.

[1] This includes Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Venezuela. Source: BIS, Cbonds EM.

We address the questions raised above by studying the behavior of bond yields, both LC and USD, of four AP economies with active LC markets: Indonesia, Korea, Malaysia, and Thailand. We find strong evidence of the Twin Ds effect in periods of no distress via impulse responses: both AP and Chinese credit risk variables affect depreciation rates, and vice versa. The Twin Ds risk premiums are large even during expansions. They range between a half and two times the US bond risk premiums, depending on maturity. The risk-return trade-off from exposure to the Twin Ds risk is attractive, as investing in LC debt more than doubles the maximum Sharpe ratio compared to investment in global bonds only.

As the initial step of our analysis, we model the joint dynamics of macroeconomic and financial factors proposed as yield curve determinants in the literature. This approach serves two purposes. First, we can account for interactions between these factors as they affect sovereign credit spreads. Second, we can characterize not only the contemporaneous but also the multi-horizon role of these variables.

This approach is a departure from existing empirical frameworks. Most studies consider regressions of credit spreads on possible local and global drivers. That limits the analysis to contemporaneous relationships only. Furthermore, it does not account for interactions between variables. Last but not least, the choice of potential determinants is not tightly connected to a bond valuation framework.

Despite their relative macroeconomic stability, the selected countries do vary to some extent in terms of monetary policy, exchange rate stabilization policy, credit quality and tightness of capital controls. This is in contrast to the traditional analysis of bond markets in large advanced economies, which share a lot of similarities across all of these dimensions. Such cross-country differences among AP countries complicate the analysis of the factors driving bond yields in these countries. Another complication arises from the relatively short time span of data on most AP bond markets. This deprives researchers of rich time series evidence that is enjoyed by those who analyze bond markets in advanced economies, such as the United States.

We address both complications in one unifying framework by modeling state variables via a panel vector autoregression (VAR) with country-specific fixed effects. That is, we assume identical responses of local state variables to themselves or to US/Chinese counterparts. That same commonality allows us to establish time series, or predictability patterns, while exploiting cross-sectional information.

As a next step of our analysis, we take advantage of the rich term structure of yields in our dataset to study risk premiums associated with sovereign credit risk. To that end, we complement the VAR with a stochastic discount factor (SDF) thereby delivering pricing implications for bonds. In turn, this allows us to estimate market prices of various types of risk. One implication of this analysis is that we can infer the maximum Sharpe ratios associated with trading global bonds only, or trading global and local bonds. The difference between the two allows us to gauge the economic significance of AP bond markets for global investors.

We find that China plays a role similar to the United States quantitatively in terms of the variance decomposition of AP states and AP bond risk premiums. However, different types of variables from these two countries affect the dynamics and risk premiums in the AP region. Given our focus on the Twin Ds effects in AP countries, it is difficult for us to say more, but our findings suggest studying interactions between the United States, China and the rest of the world as a fruitful research avenue.

As regards local variables, they play an important quantitative role for the state dynamics. Variance decompositions show that local shocks explain from 36 to 95% of the variation in local state variables, depending on the variable and the country. In particular, local variables explain 88%, 83%, 44%, and 48% of the variation in sovereign credit spreads for Indonesia, Korea, Malaysia, and Thailand, respectively. Local output, however, has no significant impact on credit spreads at any horizon. The Twin Ds effect is manifested by the significant impact of the local credit factor on the depreciation rate of the local currency and vice versa.

Investment in AP LC bonds primarily offers exposure to regional undiversifiable credit risk. In contrast to USD bonds, the additional credit exposure is via the Twin Ds effect. This exposure is richly compensated even outside of default episodes. In particular, maximum Sharpe ratios, which capture the risk-return trade-off, more than double if one invests in AP bonds in addition to the global ones. The local variables contribute substantially to the variation in local bond risk premiums, with contributions ranging between 36% and 86%, depending on maturity and country. That these risk premiums depend on local variables is a manifestation of local time-varying risk bearing capacity.

Related literature

We build on the work of Du and Schreger (2016), who propose a methodology to construct LC credit spreads from foreign currency debt in emerging market economies and apply it to a single maturity (five years). Our analysis is also closely related to Augustin, Chernov, and Song (2018), who study the Twin Ds using LC- and USD-denominated sovereign credit default swaps (CDS) in the euro area. Their analysis is limited to one exchange rate across the different countries and a short sample starting in 2010.

Hofmann, Shim, and Shin (2017) document that movements in the dollar exchange rate of emerging market economies affect local currency government bond yields through changes in the sovereign credit risk. While they emphasize this interaction between currency and credit risk, they do not formally jointly model bond yields, exchange rates and credit risk or their drivers. Della-Corte, Sarno, Schmeling, and Wagner (2016) empirically show that the common component in sovereign credit risk correlates with currency depreciations and predicts currency risk premia. Buraschi, Sener, and Menguetuerk (2014) suggest that geographical funding frictions may be responsible for persistent mispricing of emerging market bonds denominated in EUR and USD. Mano (2013) proposes a descriptive segmented market model that is consistent with nominal and real exchange rate depreciation upon an exogenous default trigger.

Most of the literature has so far largely focused on the first D, that is, default or credit risk. In particular, researchers have investigated the role of global (US) and local variables (Hilscher and Nosebusch, 2010; Longstaff, Pan, Pedersen, and Singleton, 2011), or their relative importance (Augustin, 2018), in explaining credit spreads. As Borri and Verdelhan (2011) point out, much of the work either ignores the risk premiums (e.g., Uribe and Yue, 2006) or explicitly assumes risk-neutral investors. There are also data limitations: most empirical work uses the JP Morgan Emerging Market Bond Index (EMBI) that aggregates across different maturities of USD-denominated bonds (Borri and Verdelhan, 2011; Hilscher and Nosebusch, 2010). Longstaff, Pan, Pedersen, and Singleton (2011) study CDS contracts for one single maturity (five years). Very little work has been done on LC bonds of individual non-G7 countries. Augustin (2018) and Doshi, Jacobs, and Zurita (2017) exploit the entire term structure to identify the relative importance of global and local risks.

The literature on market segmentation implies that bonds could carry a risk premium for country-specific risk (e.g., Chaieb, Errunza, and Gibson, 2019). We do not address this possibility explicitly. Two considerations suggest that segmentation is, perhaps, not the main driver of our results. First, as we detail in the next section, all the AP markets we consider are essentially open to foreign investors. Second, the literature on market segmentation focuses on its effect on risk premiums of assets that have limited investors' access. That effort, in its turn, requires explicit modeling of the time-varying volatility of returns, but does not characterize the risk-return trade-off. Thus, the implication for maximal Sharpe ratios that we document is not available.

2 Institutional background on Asia-Pacific government bond markets

We study the behavior of LC bond yields of four AP economies: Indonesia, Korea, Malaysia, and Thailand. We view these four markets as in some sense representative of the Asia-Pacific region. These economies have all enjoyed relative macroeconomic stability, at least in the two decades following the Asian financial crisis. The bond markets of these four economies are also sufficiently developed and liquid to allow them to be meaningfully studied. Average bid-ask spreads are relatively narrow, typically below 5 basis points, although they tend to be somewhat higher in the Indonesian bond market. Moreover, these economies share the characteristic that their exchange rate arrangements are at least somewhat flexible[2]; a prerequisite for allowing us to examine issues related to the Twin Ds.

[2] According to the 2020 IMF Annual Report on Exchange Arrangements and Exchange Restrictions, the exchange rates of the four Asian countries in our sample are classified as de facto 'floating'.

As discussed above, the rapid growth in Asian local currency bond markets was spurred by Asian governments' desire to promote local currency bond issuance following the experience of the Asian financial crisis in 1997 (Figure 1). On the demand side, international investors came to view emerging market local currency bonds as a new investable asset class with potentially attractive returns and diversification benefits.

While the bond markets we study share a number of characteristics, including rapid growth rates, they also differ in important ways. One is the size of the markets. Among the four emerging Asian countries in our sample, Korea dominates in terms of size: its stock of Korean won-denominated bonds totaled around \$668 billion at the end of 2018. The Indonesian, Malaysian and Thai local currency government bond markets were around a quarter of the size of Korea, at around \$170 billion each in 2018. By contrast, the US and Chinese government bond markets totalled around \$16 trillion and \$7 trillion, respectively.

Another important way in which the four government bond markets differ is in terms of credit risk. At the end of 2018, Korea was rated AA by S&P, Malaysia A-, Thailand BBB+, and Indonesia BBB-. China had an A+ rating. While all these countries are now rated investment grade, this has not always been the case for some of the countries: Korea, for example, attained investment grade status in 1999, and Indonesia was rated speculative grade until mid-2017.

The countries in our sample also differ in terms of exchange rate regimes and monetary policy frameworks. With respect to exchange rate arrangements, while Indonesia, Korea and Thailand have floating exchange rates (with occasional interventions to manage excess volatility, and a 'managed-float' in the case of Thailand), Malaysia had a fixed exchange rate (to the USD) between 1998 and 2005, after which a managed float was introduced. As of mid-2005, China began implementing a managed exchange rate; first against the dollar, and since 2015 against a basket of currencies.

Finally, among the AP bond markets we consider there are differences in the degree of openness and accessibility to foreign investors. In particular, the Chinese bond market has in the past been highly regulated, although it has gradually opened up to foreign investors. Foreign investors used to be able to access Chinese capital markets - including its bond markets - only via the Qualified Foreign Institutional Investor (QFII) scheme (and its expanded counterpart, the RQFII). The scheme, which was launched in 2002, licensed major banks and institutional investors to invest in Chinese capital markets, subject to quotas. The total cap on overseas purchases of such assets reached \$300 billion by 2019, and was entirely scrapped the following year, thereby enabling foreign asset purchases without prior approval. Even before that, in 2016, authorities in China announced that foreign institutional investors would be given quota-free access to the Chinese Interbank Bond Market (CIBM), subject to certain registration requirements, via the CIBM Direct scheme. Similar access had been granted to some foreign central banks, sovereign wealth funds and a narrow set of long-term institutional investors as early as 2010. Moreover, in 2017, China launched Bond Connect, a bond market access scheme that allows foreign investors to trade in the Chinese bond markets through Hong Kong. As the Chinese bond market has opened up and developed, liquidity has improved over time (Bai, Fleming, and Horan, 2013).

As for the other AP bond markets we consider, they are essentially fully open to foreign investors, in some cases subject to reporting requirements.[3] In the case of the Korean market, foreign investors that do not have an office in Korea need to register with the Korean Financial Supervisory Service in order to invest in Korean securities. Foreign investors in the Indonesian bond market are, like domestic investors, subject to a requirement to obtain a so-called Single Investor Identification (which serves as a reporting device and investor identification) from the Indonesia Central Securities Depository (KSEI) in order to invest. Subject to this requirement, there are no limitations on foreign investors investing in Indonesian debt securities. Similarly, foreign investors may invest in Thai government bonds without any restrictions. Moreover, they do not face any market exit requirements, as long as they comply with certain limits on end-of-day balances of non-resident baht accounts. Finally, there are no entry or exit requirements for foreign investors in the Malaysian bond market.

3 Data and key variables

This section introduces the data we use in our analysis and basic relationships between the different types of bonds.

3.1 Data

For the purpose of our analysis, we need data on bond yields and macroeconomic factors for the Asian countries we study, as well as for the United States. We also require exchange rates between the currencies of the Asian countries in our sample and the dollar. Our sample ends in December 2019. The starting points are different depending on the variables and countries, as detailed below.

We use two types of macroeconomic data: inflation and a measure of economic activity. Inflation is measured as the monthly log-difference of the consumer price index for each country. For economic activity, we rely on the monthly growth rate of seasonally adjusted industrial production (IP). Figure 2 illustrates the macroeconomic data (in year-on-year growth terms). The source of this data is Bloomberg, except for Chinese industrial production data which comes from China's National Bureau of Statistics.[4]

[3]This is according to various editions of the ASEAN+3 Bond Market Guide by the Asian Development Bank.

[4]For China, authorities only release year-on-year IP growth rates only for the months March through December (and occasionally for February). This is due to the possibility that data for the first one or two months of the year can be affected significantly by the annual Spring Festival. Using the published y-o-y

The exchange rate data consists of end-of-month quotes of the Chinese yuan, the Indonesian rupiah, the Korean won, the Malaysian ringgit and the Thai baht, against the US dollar. All data is obtained from Bloomberg. Figure 3 displays the series (expressed as the number of US dollars per local currency).

For the renminbi and the ringgit, in particular, the management of the local currency against the US dollar up until 2015 and 2005, respectively, is quite evident in the figure. Ideally, one would want to incorporate these features into a model. Modeling currency management is a separate complicated task, so we choose to apply the same statistical model to all exchange rates.

We work with zero-coupon government bond yields. For the United States we rely on the standard dataset from the Federal Reserve, as constructed by Gürkaynak, Sack, and Wright (2007). For the five Asian economies we study (China, Indonesia, Korea, Malaysia and Thailand), data on local-currency zero-coupon yields is not readily available from official sources. We therefore rely on Bloomberg zero-coupon yields, which are estimated using a spline approach on available prices of individual bonds in domestic markets.[5]

In order to allow us to separately identify default and currency risks, we need USD-denominated bond yields for the Asian countries in our sample. In the case of Korea, we again rely on Bloomberg zero-coupon yields which are available for dollar-denominated bonds. For the remaining countries in our sample, the issuance of USD-denominated debt has in the past been infrequent and typically concentrated in very few maturities. This, in turn, has prevented the estimation of zero-coupon dollar curves for these countries.

To overcome this problem, we use CDS premiums across several maturities.[6] Specifically, we construct synthetic dollar yields by adding Chinese, Indonesian, Malaysian or Thai sovereign CDS premiums to US Treasury par yields across six different maturities. Using these six par dollar yields, we then construct zero-coupon dollar yields using the methodology of Nelson and Siegel (1987). We evaluate the accuracy of the procedure using the available Korean USD-bonds and CDS contracts. It recovers yields from CDS accurately.

In summary, we obtain 8 LC bonds per country: 1, 6, 12, 24, 36, 48, 60, and 120 months to maturity, and 6 foreign USD denominated bonds with maturities between 12 and 120 months. As a result, we have 14 bond yields for each AP country, and 8 for the United

growth rates, we back out a monthly Chinese IP level series for March-December each year. We then fill in the values for each January and February by assuming that monthly IP growth from December to March is constant. Finally, we seasonally adjust the resulting IP index series using the X13 method and proceed to calculate month-on-month growth rates.

[5]Indonesia and Malaysia issue Islamic bonds as well, but they are dropped from the Bloomberg data.

[6]With the exception of China, the reference obligation for the CDS contract is a USD-denominated issue of a sovereign. While Chinese CDS trading enjoys large net notional amounts, no information is available about the reference obligation. According to Longstaff et al. (2011), "... the contract ... explicitly references Chinese government international debt, and the only current Chinese international bond issues for the five-year horizon are US dollar-denominated issues." The issuance of USD debt at various maturity points has picked up in China since 2017.

States. In subsequent estimation we use yields of these maturities and one month prior to construct bond returns as described in Section 5.

The zero-coupon yield series available for our AP countries are relatively short compared to US data. Specifically, yield data is available as of September 2003 for China, from February 2003 for Indonesia, from September 1999 for Korea and Malaysia, and as of January 1998 for Thailand; see Figure 4.

Table 1 reports summary statistics for our data. Yields famously exhibit a high degree of persistence. This is also the case for the Asian countries we study, as evident from Figure 4. Interestingly, the slopes of the respective yield curves (defined as the difference between the 10-year and 1-month yields) show that only the US slope has turned negative over the sample period.

The volatility of yields varies between the countries, with the highest (Indonesia) being about four to five times higher than the lowest (Malaysia and China). Table 1 also shows that the volatility of the Chinese currency is substantially lower than for the other Asian currencies, and that the Indonesian currency is the most volatile one, largely due to very high volatility in the first few years of the sample (see Figure 3).

Based on the dollar yields described above, we construct a credit factor for each AP country, which we define as the spread between the 1-year USD-denominated yield for each country, minus the 1-year US Treasury yield. This credit factor is used in the estimation of our term structure model, as described below.

Figure 5 displays the credit factor. We note the coincident spike during the Global Financial Crisis. The different levels of credit risk are evident as well. Indonesia and Korea appear to be more risky than Malaysia and Thailand, with the latter two being similar to China. The summary statistics are reported in Table 1.

As a prima-facie evidence of bond risk premiums, we also report log excess returns computed from LC bonds. We see that overall Indonesia exhibits higher and more volatile returns than the rest of the countries. China's term structure of risk premiums is flat as compared to the United States. In contrast, the other AP countries exhibit steeper term structures.

3.2 Notation and basic relations

Suppose $M_{t,t+i}$ is a USD-denominated i-period nominal SDF. The prices of zero-coupon US bonds with maturity n are given by the standard pricing condition:

$$Q_t^n = E_t \left(M_{t,t+1} \cdot Q_{t+1}^{n-1} \right) \tag{1}$$

with corresponding yields $y_t^n = -n^{-1} \log Q_t^n \equiv -n^{-1} q_t^n$. The one-period return on an n-period bond is $r_{t+1}^{n,n-1} = q_{t+1}^{n-1} - q_t^n$.

Denote all the available information at time t, with the exception of credit events, by $\mathcal{F}_t$. The random time τ indicates an incidence of a credit event. We use the indicator variable z_t to denote the event, $z_{t+1} = 1$ if $\tau = t + 1$, and zero otherwise. The process

$$H_t \equiv Prob\,(\tau = t + 1 \mid \tau > t, \mathcal{F}_t)$$

denotes the conditional default probability of a given reference entity at day t, i.e., the hazard rate. Models of Poisson default arrival often associate the hazard rate with default intensity h_t via:

$$H_t = 1 - e^{-h_t}.$$

This is because the Poisson probability of no defaults is e^{-h_t}.

We assume that an AP bond may default with a fractional recovery of market value (RMV) equal to $1 - L$. The USD-denominated bond price is:

$$\widetilde{Q}_t^n \;=\; E_t\left(M_{t,t+1} \cdot \widetilde{Q}_{t+1}^{n-1}[(1 - z_{t+1}) + (1 - L)z_{t+1}]\right). \tag{2}$$

The corresponding yields are $\widetilde{y}_t^n = -n^{-1}\widetilde{q}_t^n$. The one-period return on an n-period bond is $\widetilde{r}_{t+1}^{n,n-1} = \widetilde{q}_{t+1}^{n-1} - \widetilde{q}_t^n$. The credit factor is the difference in yields between a one-year AP USD-denominated bond and a one-year US government bond, $\widetilde{c}_t^n = \widetilde{y}_t^n - y_t^n$.

Suppose S_t represents the value of one unit of local currency in terms of USD. The depreciation rate is $\Delta s_{t+1} = \log S_{t+1}/S_t$. An increase in the depreciation rate indicates a decrease in the value of USD. The LC bond price is:

$$\widehat{Q}_t^n \;=\; E_t\left(M_{t,t+1} \cdot S_{t+1}/S_t \cdot \widehat{Q}_{t+1}^{n-1}[(1 - z_{t+1}) + (1 - L)z_{t+1}]\right) \tag{3}$$

with corresponding yields $\widehat{y}_t^n = -n^{-1}\widehat{q}_t^n$. The bond return, in local currency, is $\widehat{r}_{t+1}^{n,n-1} = \widehat{q}_{t+1}^{n-1} - \widehat{q}_t^n$, and, in USD, is $\check{r}_{t+1}^{n,n-1} = \widehat{q}_{t+1}^{n-1} - \widehat{q}_t^n + \Delta s_{t+1}$.

By using the same hazard rate to value both USD and LC bonds, we are implicitly assuming no selective default. We do so because we cannot identify different hazard rates in our empirical work due to the lack of any defaults in our sample. We therefore have to be careful with the interpretation of our results.

At first blush, our focus on the Twin Ds would suggest studying the joint behavior of the depreciation rate Δs_{t+1} and the credit factor $\widetilde{c}_t$. The expressions above highlight that risk premiums associated with the two variables are determined by the properties of the SDF $M_{t,t+1}$. Indeed, consider the relative difference between the LC and USD AP bond prices

$$\begin{aligned}\frac{\widehat{Q}_t^n}{\widetilde{Q}_t^n} &= \frac{E_t\left(M_{t,t+\tau} \cdot \prod_{j=1}^{\tau}(1 - Lz_{t+j}) \cdot S_{t+\tau}/S_t\right)}{E_t\left(M_{t,t+\tau} \cdot \prod_{j=1}^{\tau}(1 - Lz_{t+j})\right)} \\ &= E_t(S_{t+\tau}/S_t) + (\widetilde{Q}_t^n)^{-1} cov_t\left(M_{t,t+\tau}\prod_{j=1}^{\tau}(1 - Lz_{t+j}), S_{t+\tau}/S_t\right).\end{aligned} \tag{4}$$

The first term in the second row reflects the expected currency depreciation. The second term reflects the pricing of the Twin Ds risk. Kremens and Martin (2019) refer to the covariance term as the quanto-implied FX risk premium.

In order to connect this expression to the Twin Ds premiums, consider the ratio of the USD excess returns on LC and USD AP bonds

$$\frac{\check{R}_{t+1}}{\widetilde{R}_{t+1}} = \frac{\widehat{Q}_{t+1}^{n-1}}{\widetilde{Q}_{t+1}^{n-1}} \cdot \left(\frac{\widehat{Q}_t^n}{\widetilde{Q}_t^n}\right)^{-1} \cdot \frac{S_{t+1}}{S_t}$$

This difference should be reflecting exposure to the Twin Ds. Substituting in Equation (4), taking conditional expectations, and using log-linearization, we get a measure of the Twin Ds risk premium:

$$\begin{aligned} \log E_t\left(\frac{\check{R}_{t+1}}{\widetilde{R}_{t+1}}\right) \quad &\approx \quad E_t\left[cov_{t+1}\left(\frac{M_{t+1,t+\tau}\prod_{j=2}^{\tau}(1-Lz_{t+j})}{\widetilde{Q}_{t+1}^{n-1}}, \frac{S_{t+\tau}/S_t}{E_t(S_{t+\tau}/S_t)}\right)\right] \\ &- \quad cov_t\left(\frac{M_{t,t+\tau}\prod_{j=1}^{\tau}(1-Lz_{t+j})}{\widetilde{Q}_t^n}, \frac{S_{t+\tau}/S_t}{E_t(S_{t+\tau}/S_t)}\right). \end{aligned} \tag{5}$$

The expressions inside of the covariance terms are innovations. The first term is the innovation in the "default-adjusted" SDF, $M_{t,t+\tau}\prod_{j=1}^{\tau}(1-Lz_{t+j})$, which reflects pricing of credit risk. The second term reflects the innovation in the depreciation rate. The Twin Ds premium reflects the expected change in the covariance of these two innovations.

Thus, we should study the SDF dynamics to understand the pricing of the Twin Ds. This conclusion leads us to look to bond prices as the relevant source of information about factors driving the SDF. As a result, we complement the two variables of primary interest (credit, exchange rate) with variables that are traditionally used in bond valuation models. Specifically, we consider principal components of bond returns and major macroeconomic variables.

3.3 State variables

Our dataset features bond prices of seven different maturities for each country. Thus, we would like to reduce the dimension of the bond data by selecting a lower-dimensional state vector. To this end, Table 2 reports a principal component (PC) analysis of yields. First, it shows that 72% of the joint variation in the 68 yield series from 6 different countries can be explained by the first PC. This degree of commonality is striking. A similar observation was made by Longstaff, Pan, Pedersen, and Singleton (2011) in the context of CDS of single maturity but a larger cross-section of countries.

Second, we see that the first 12 PCs explain 99.6% of the variation in returns. However, in contrast to the traditional single-country PCs, these components are difficult to interpret. Therefore, we construct a different set of 12 bond-return factors based on levels and slopes.

The US or local level, ℓ_t, or $\widehat{\ell}_{it}$, respectively, is selected to be equal to a yield on a single-horizon bond y_t^1, or $\widehat{y}_t^1$. The slope τ_t, or $\widehat{\tau}_{it}$ is the difference between yields on long- and short-horizon bonds: $\tau_t = y_t^{120} - y_t^1$ and $\widehat{\tau}_t = \widehat{y}_t^{120} - \widehat{y}_t^1$ (with superscripts denoting maturities measured in months). Further, because there is a lot of commonality between the US variables and their local counterparts, our final set of state variables uses the differences $\Delta_c\widehat{\ell}_{it} \equiv \ell_t - \widehat{\ell}_{it}$, and $\Delta_c\widehat{\tau}_{it} \equiv \tau_t - \widehat{\tau}_{it}$. Credit risk is captured by the credit spread $\tilde{c}_{it} \equiv \tilde{c}_{it}^{12}$, where the horizon is the shortest available with quality data.

We demonstrate the ability of these state variables to span the original PCs by regressing each of the 12 PCs on these variables and on the depreciation rates. We report the R^2 and adjusted R^2 from these regressions in Table 2. We see that each of the first seven PCs that jointly explain 95% of the variation in all returns are replicated by the state variables with a high degree of accuracy. The fit deteriorates afterwards with particular weakness in the last two PCs. Still, most of the variation in yields can be captured by our proposed variables. Also, we view clarity of interpretation of the factors as worthy of some deterioration in fit. Ultimately, what matters is how a model with the chosen state variables matches the observables.

As a result, we have the following set of state variables:

$$v_t = \begin{pmatrix} x_t \\ \widehat{x}_{1t} \\ \vdots \\ \widehat{x}_{Nt} \end{pmatrix} = \begin{pmatrix} \pi_{Ut} \\ g_{Ut} \\ \ell_{Ut} \\ \tau_{Ut} \\ \Delta s_{Ct} \\ \pi_{Ct} \\ g_{Ct} \\ \Delta_c \ell_{Ct} \\ \Delta_c \tau_{Ct} \\ c_{Ct} \\ \Delta s_{1t} \\ \widehat{\pi}_{1t} \\ \widehat{g}_{1t} \\ \Delta_c \widehat{\ell}_{1t} \\ \Delta_c \widehat{\tau}_{1t} \\ \widetilde{c}_{1t} \\ \vdots \\ \Delta s_{Nt} \\ \widehat{\pi}_{Nt} \\ \widehat{g}_{Nt} \\ \Delta_c \widehat{\ell}_{Nt} \\ \Delta_c \widehat{\tau}_{Nt} \\ \widetilde{c}_{Nt} \end{pmatrix} = \begin{pmatrix} \text{US inflation} \\ \text{US industrial production} \\ \text{US level} \\ \text{US slope} \\ \text{Chinese nominal depreciation rate} \\ \text{Chinese inflation} \\ \text{Chinese industrial production} \\ \text{US minus Chinese level} \\ \text{US minus Chinese slope} \\ \text{Chinese credit factor} \\ \text{nominal depreciation rate 1} \\ \text{local inflation 1} \\ \text{local industrial production 1} \\ \text{US minus local level 1} \\ \text{US minus local slope 1} \\ \text{local credit factor 1} \\ \vdots \\ \text{nominal depreciation rate } N \\ \text{local inflation } N \\ \text{local industrial production } N \\ \text{US minus local level } N \\ \text{US minus local slope } N \\ \text{local credit factor } N \end{pmatrix}. \quad (6)$$

4 No-arbitrage term structure model

A no-arbitrage term structure model has two major ingredients. First, we specify the dynamics of the state variables, v_t, via a VAR. As a particular case of a seemingly unrelated regression, the VAR delivers a relationship between credit spreads and other variables in the state vector in the style similar to the existing literature. The specifics of the VAR allows to study in-depth the dynamic interrelations of these variables.

Second, we specify the SDF that captures the behavior of prices of risk associated with v_t. That allows us to risk-adjust expectations implied by the VAR of the state. Subsequently, we can study asset-pricing implications using this framework.

4.1 VAR of the state

We model the joint dynamics via a first-order vector autoregression, VAR(1). We introduce a vector ϵ_t of $\mathrm{N}(0, \mathrm{I})$ innovations. This vector can be subdivided into global (US and Chinese) and AP innovations: $\epsilon_t = \left(\varepsilon_t^\top, \varepsilon_{1t}^\top, \ldots, \varepsilon_{Nt}^\top\right)^\top$. We specify the dynamics of v_t as

$$v_{t+1} \quad = \quad \mu_v + \Phi_v v_t + \Sigma_v \epsilon_{t+1}. \tag{7}$$

There is well-established evidence in the literature that bond prices co-move internationally (Driessen, Melenberg, and Nijman, 2003; Dahlquist and Hasseltoft, 2013; Jotikasthira, Le, and Lundblad, 2015), so our use of level and slope factors in deviation from their US counterparts provides us with a way of accommodating the local yield drivers while accounting for such international co-movement.

In the case of the four AP countries ($N = 4$) together with the US and China, the dimension of the state vector is 34. That implies $34 + 34^2 + 34 \times 35/2 = 1,785$ identified parameters that need to be estimated in a VAR(1). This is a daunting challenge in any situation. Our setting is complicated by relatively short samples and by the aforementioned cross-sectional variation in the countries' characteristics.

To estimate the parameters of such a dynamic model, we pool data from different countries (Ang and Chen, 2010; Bansal and Dahlquist, 2000 are early examples of this approach). This approach imposes the constraint that the underlying structure is the same for each country. Specifically, we assume that local macroeconomic and yield factors respond in the same way to local and global factors across all four AP markets. The US and Chinese variables are an exception. These variables respond to themselves and to each other only (they proxy for autonomous global variables), and their responses are different from responses of local variables to local variables of the same type. For example, US inflation is allowed to respond to US industrial production differently than Indonesian inflation responds to Indonesian industrial production. The cross-sectional differences between the countries are reflected in country-specific fixed effects, that is, country-specific means of the state variables.

To express this in algebraic terms,

$$E_t\left(x_{t+1}\right) = \mu_x + \Phi_x x_t, \tag{8}$$
$$E_t\left(\widehat{x}_{i,t+1}\right) = \mu_i + \Phi_{\widehat{x}} x_t + \Psi_{\widehat{x}} \widehat{x}_{it}, \quad i = 1, \ldots, N. \tag{9}$$

where the matrices Φ_x, $\Psi_{\widehat{x}}$, and $\Phi_{\widehat{x}}$ combine into Φ_v. The second equation is a simple version of a panel VAR with fixed effects. Here we treat the size of the cross-section N as fixed, and the time span as growing.

Lastly, a country's currency may devalue in the case of default – the aforementioned Twin Ds effect. We do not have such an event in our sample, so we account for this possibility in a largely symbolic fashion. First, we assume the currency jumps down in the case of default with a known jump size k. Second, because depreciation rates are part of our state we have to complement conditionally normal innovations with innovations of the form $-kz_{it}$ in each equation that correspond to a depreciation rate. Given that there are no default events in the sample, $z_{it} = 0$ for every country i and time t. This potential presence of jumps in currencies has two effects: it introduces a bias in the conditional mean of the depreciation rate, and it is reflected in the valuation of LC bonds, as per equation (3). We can identify k, but not the jump intensity, from the latter as we explain below.

We use the LDL representation for the covariance matrix of the state vector:

$$\Sigma_v \Sigma_v^\top = L_v D_v L_v^\top.$$

where D_v is a diagonal matrix with positive elements and L_v is a lower-triangular matrix with ones on the diagonal. We impose a block structure on L_v to reduce the number of parameters. For US and Chinese shocks, the corresponding rows of L_v consist of a 10×10 lower-triangular matrix L_x. For shocks from each country i the corresponding rows of L_v consist of two non-zero blocks: a 6×6 lower-triangular matrix $P_{\widehat{x}}$ corresponding to AP shocks from country i and a 6×10 matrix $L_{\widehat{x}}$ corresponding to US and Chinese shocks:

$$L_v = \begin{pmatrix} L_x & 0 & 0 & 0 & 0 \\ L_{\widehat{x}} & P_{\widehat{x}} & 0 & 0 & 0 \\ L_{\widehat{x}} & 0 & P_{\widehat{x}} & 0 & 0 \\ L_{\widehat{x}} & 0 & 0 & P_{\widehat{x}} & 0 \\ L_{\widehat{x}} & 0 & 0 & 0 & P_{\widehat{x}} \end{pmatrix},$$

and $\mathrm{diag}(L_v)$ is a vector of ones. Given the wide range of possible standard deviations for the diverse set of state variables, we do not constrain elements of the diagonal matrix D_v to be the same across AP countries.

The conditional covariance matrix allows us to implement a variance decomposition analysis. A variance decomposition measures the percentage contribution of a shock to variable j to the variation in variable i. Thus, the VAR enables us to move beyond inspection of the significance of regression coefficients, a standard tool in the extant literature, and to fully account for the state dynamics.

As is usually the case, the variance decomposition could be sensitive to the order of state variables. Therefore, instead of focusing on the impact of individual variables, we distinguish between the US, Chinese, and local variables only. We stack the odds against the local variables by ordering them last.

4.2 Stochastic discount factor

There is an important conceptual value to estimating the SDF. First, as emphasized by Duffee (2010), a no-arbitrage model can be used to determine Sharpe ratios for any dynamic trading strategy in fixed income, including all strategies that attain the maximum Sharpe ratio (MSR). Second, following Harvey (1995), we can use the model to characterize whether AP markets enhance an investor's opportunity set by increasing the MSR. Lastly, we can investigate the drivers behind the changing opportunity set.

We model the dynamics of the (log) SDF following a long tradition of affine models:

$$\begin{aligned} m_{t,t+1} &= -\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t - \lambda_t^\top \epsilon_{t+1} \\ &- \sum_{i=1}^{N} \left[\gamma_{it} z_{it+1} + h_{it}(e^{-\gamma_{it}} - 1)\right], \end{aligned} \tag{10}$$

with market prices of factor risk

$$\lambda_t = \Sigma_v^{-1} (\lambda_0 + \lambda_v v_t),$$

and market prices of non-US sovereign default risk, γ_{it}, which could have a flexible functional form. The variables z_{it} are the Poisson default arrival processes with intensity h_{it}.

Given that we have imposed a fixed-effects structure on the risk premium regressions, we do the same with λ_t. The elements of the vector λ_0 are country-specific. We assume that λ_v has a block structure. For US and Chinese factors the corresponding rows of λ_v consist of a 10×10 matrix λ_x that multiplies the US and Chinese factors x_t. For factors from each country i the corresponding rows of λ_v consist of two non-zero blocks: a 6×6 matrix $\chi_{\widehat{x}}$ that multiplies AP factors from country i and a 6×10 matrix $\lambda_{\widehat{x}}$ that multiplies the US and Chinese factors x_t.

$$\lambda_v = \begin{pmatrix} \lambda_x & 0 & 0 & 0 & 0 \\ \lambda_{\widehat{x}} & \chi_{\widehat{x}} & 0 & 0 & 0 \\ \lambda_{\widehat{x}} & 0 & \chi_{\widehat{x}} & 0 & 0 \\ \lambda_{\widehat{x}} & 0 & 0 & \chi_{\widehat{x}} & 0 \\ \lambda_{\widehat{x}} & 0 & 0 & 0 & \chi_{\widehat{x}} \end{pmatrix}.$$

The market price of default risk is $\gamma_{it} = -\log(h_{it}^*/h_{it})$, where h_{it}^* is a risk-adjusted default intensity. Without observing actual default events, it is impossible to identify the true default intensity h_{it}. Therefore, one cannot identify γ_{it} either. Thus, we focus on modeling h^*_{it}. We assume

$$h^*_{it} = \delta_{ih,0} + \delta_{h,\widehat{x}}^\top \widehat{x}_{it} + \delta_{h,x}^\top x_t \equiv \delta_{h,0} + \delta_{h,v}^\top v_t.$$

The results of Duffie and Singleton (1999) applied to the credit-risky bond valuation in Equation (2) imply that $\tilde{c}^1_{it} = Lh^*_{it}$. Thus, if the one-month credit spread was part of the state vector v_t, we would have $\delta_{h,0} = 0$, $\delta_{h,v} = L^{-1} \cdot \mathrm{e}_c$. Here e_c is a unit vector with zeros in all entries except for the one corresponding to the location of the ith credit spread in the overall vector v_t.

The one-month credit spread data are sporadic and of low quality. That is why $\tilde{c}^{12}_{it}$ is our choice to be an element of v_t. To streamline estimation and analytics, we impose the same restrictions on factor loadings appearing in h^*_{it}. Thus, in our model, the values of these loadings is an assumption rather than a result. The cost of any extra assumptions is a potentially less flexible model, which should not be an issue here given the large number of free parameters.

Because we cannot identify the default hazard premium, we consider the MSR that corresponds to normal innovations, $\mathrm{Vol}_t(m_{t,t+1}) = (\lambda_t^\top \lambda_t)^{1/2}$ (for log-returns). As the US-China block is autonomous in our model, we can compute the MSR implied by these two countries alone, and then, to gauge the improvement in the investment opportunity set, we can compute the MSR for the full model.

5 Estimation

Affine term structure models lend themselves naturally to some form of state-space formulation, where the state follows a VAR such as (7) and asset prices form a vector of observables. Traditionally, term structure models use bond yields as such observables. As we explain below, we follow Adrian, Crump, and Moench (2013) and use bond returns as observables. This leads to computational advantages in high-dimensional settings like ours. This section explains how our approach extends their framework.

5.1 Observable variables

Bond pricing equations (1) - (3) combined with the assumptions about the state v_t, (log) SDF $m_{t,t+1}$, and depreciation rate Δs_{t+1} imply bond valuation equations. The affine structure of the state variables and the SDF implies that log bond prices are linear functions of the state

$$\begin{aligned} q^n_t &= \bar{a}_n + \bar{b}_n^\top v_t, \\ \widetilde{q}^n_t &= \widetilde{a}_n + \widetilde{b}_n^\top v_t, \\ \widehat{q}^n_t &= \widehat{a}_n + \widehat{b}_n^\top v_t, \end{aligned}$$

where the intercepts and slopes are non-linear functions of the parameters in the dynamics of $m_{t,t+1}$, which control risk premiums.

The non-linearity of the parameters poses a significant challenge in estimation (e.g., Hamilton and Wu, 2012, Kim, 2008). Consequently, we use bond returns rather than yields as an observational input during estimation. To see why the change in the type of observations helps, consider returns on US bonds:

$$\begin{aligned}
r_{t+1}^{n,n-1} &= q_{t+1}^{n-1} - q_t^n \\
&= \bar{a}_{n-1} + \bar{b}_{n-1}^\top (\mu_v + \Phi_v v_t + \Sigma_v \epsilon_{t+1}) - \bar{a}_n - \bar{b}_n^\top v_t \\
&= \bar{a}_{n-1} + \bar{b}_{n-1}^\top \mu_v - \bar{a}_n + \left(\bar{b}_{n-1}^\top \Phi_v - \bar{b}_n^\top\right) v_t + \bar{b}_{n-1}^\top \Sigma_v \epsilon_{t+1} \\
&= \bar{c}_{n-1} + \left(\delta_{y,v}^\top + \bar{b}_{n-1}^\top \lambda_v\right) v_t + \bar{b}_{n-1}^\top \Sigma_v \epsilon_{t+1},
\end{aligned}$$

with $\bar{c}_{n-1} = \delta_{y,0} + \bar{b}_{n-1}^\top \lambda_0 - \bar{b}_{n-1}^\top \Sigma_v \Sigma_v^\top \bar{b}_{n-1}/2$. The last equality follows from applying the explicit expressions for $\bar{a}_n$ and $\bar{b}_n$. See Appendix A.1.

The advantage of this expression is that the risk premium parameters λ_0 and λ_v appear linearly in the intercept and slope. Thus, these parameters can be estimated using a cross-sectional (in maturity) regression of returns on the state. As a result, one does not need to get under the hood of $\bar{b}_n$ to estimate λ_v.

In the case of AP bonds, returns are affected not only by changes in bond prices, but by default as well. One can show that

$$\begin{aligned}
\widetilde{r}_{t+1}^{n,n-1} &= \widetilde{q}_{t+1}^{n-1} + \log[(1 - z_{t+1}) + (1 - L) z_{t+1}] - \widetilde{q}_t^n \\
&= \widetilde{c}_{n-1} + \left(\delta_{y,v}^\top + L\delta_{h,v}^\top + \widetilde{b}_{n-1}^\top \lambda_v\right) v_t + \widetilde{b}_{n-1}^\top \Sigma_v \epsilon_{t+1} - l z_{t+1}, \\
\widehat{r}_{t+1}^{n,n-1} &= \widehat{q}_{t+1}^{n-1} + \log[(1 - z_{t+1}) + (1 - L) z_{t+1}] - \widehat{q}_t^n \\
&= \widehat{c}_{n-1} + \left(\delta_{y,v}^\top + \widehat{K}\delta_{h,v}^\top - \delta_{s,v}^\top(\Phi_v - \lambda_v) + \widehat{b}_{n-1}^\top \lambda_v\right) v_t \\
&+ \widehat{b}_{n-1}^\top \Sigma_v \epsilon_{t+1} - l z_{t+1},
\end{aligned}$$

with $\widehat{K} = 1 - (1 - K)(1 - L)$, $K = 1 - e^{-k}$, and $L = 1 - e^{-l}$. We use the notation $\Delta s_t = \delta_{s,0} + \delta_{s,v}^\top v_t$ to emphasize the role of the depreciation rate in $\widehat{r}_{t+1}^{n,n-1}$. Given that Δs_t is an element of v_t, $\delta_{s,0} = 0$ and $\delta_{s,v} = \mathrm{e}_s$. Here e_s is a unit vector with zeros in all entries except for the one corresponding to the location of the i-th depreciation rate in the overall vector v_t. See Appendix A for a derivation. These expressions extend Adrian, Crump, and Moench (2013) to defaultable and foreign LC bonds.

Adrian, Crump, and Moench (2013) advocate a two-stage estimation strategy on the basis of the equations above. In the first stage the state dynamics (7) are estimated via a time-series regression. In the second stage, the risk premium parameters are estimated via a cross-sectional regression. We instead use Bayesian MCMC to estimate everything jointly.

Here "jointly" pertains not to the time-series and cross-section only. We also estimate multiple countries jointly – a key ingredient for studying spillovers and imposing constraints associated with fixed effects. See Appendix B for estimation details.

There are two potential drawbacks to the methodology. First, there is no way to impose internal consistency on yield-based factors if the same bonds are used as factors and as observables. That is in contrast to the yield-based estimation (e.g., Joslin, Singleton, and Zhu, 2011). We mitigate this issue by excluding returns on 1- and 120-month bonds from the set of observables.

Note that the one-month depreciation rate satisfies the no-arbitrage condition $\log E_t(M_{t,t+1}S_{t+1}/S_t) = y_t^1 - \widehat{y}_t^1$, a.k.a. Covered Interest Parity (CIP). Because one-month returns are omitted from the observation equation, CIP does not hold automatically. Thus, one has to take extra care by imposing restrictions associated with CIP on risk premium parameters.

Second, this approach treats the loadings b_n as (nuisance) free parameters thereby ignoring the structure imposed by no-arbitrage. That does not appear to be a concern in practice. Indeed, results in Duffee (2011) and Joslin, Le, and Singleton (2013) indicate that if the factor structure of an affine model is correct, then estimation recovers correct values of b_n even without imposing the no-arbitrage restrictions.

Another problem common to all estimation methods when using data with rare or no defaults is that $z_t = 0$ almost all the time. Thus, one cannot identify loss given default L. This parameter appears as a product with the default intensity loadings in bond returns. Thus, bonds are not informative about L either. Consequently, we calibrate $L = 0.4$. This is based on evidence provided by Cruces and Trebesch (2013), who report that the average loss across 180 sovereign defaults between 1970 and 2010 was between 0.37 and 0.40 (with a cross-sectional standard deviation of 0.28), depending on the formula applied to estimate the loss.

We do not have currency devaluation events in our sample either. Given our simple assumption about devaluation and assumed value of L, we can identify K from the combination of USD/LC bond returns. The estimated value of $k = \log(1 + K)$ is 0.02 (with a standard deviation of 0.01).

5.2 Unspanned macro variables

Unspanned variables (Duffee, 2011) occupy a special place in the bond risk premium literature. Conceptually, these are variables that do not appear in bond pricing expressions, but forecast bond excess returns nonetheless. This is possible if risk premiums for these variables line up in a particular way with the persistence matrix of the state. Nobody views this concept literally, but there are a lot of candidates proposed in the literature that are approximately unspanned. Whether the proposed variables are unspanned and whether they help in forecasting bond excess returns is a subject of current lively debate in the literature.

We are interested in unspanned variables for a different reason. When macro variables are unspanned, like in Joslin, Priebsch, and Singleton (2014), certain parameters in the matrix of risk premiums, λ_v, are not identified. Therefore, we can restrict them. That leads to a more parsimonious model, an important feature in a multi-country setting, and to a tremendous simplification in model estimation. We follow Joslin, Priebsch, and Singleton (2014) by assuming that inflation and industrial production are unspanned. We follow Chernov and Creal (2018) by assuming that depreciation rates are unspanned.

Because of these assumptions we can restrict the corresponding columns of b_n, $\widetilde{b}_n$, and $\widehat{b}_n$ to be equal to zero. Further, given that columns of b_n, $\widetilde{b}_n$, and $\widehat{b}_n$ are zero, the coefficients in the corresponding rows in the risk-premium matrix λ_v are not identified. Thus, we set them equal to zero as well. The depreciation rate is an exception because the corresponding risk premiums are identified from the combination of USD and LC bonds.

6 Results

Our joint estimation of the multi-country term structure model recovers both the dynamics of the state variables and the pricing of risk associated with those dynamics. As mentioned above, we impose the constraint that the underlying structure is identical across all smaller AP countries, but we allow it to be different for the United States and China. All the reported results are obtained by imposing zeros on all parameters whose probabilistic distance from 0 was less than 90% in an initial unrestricted estimation, and then re-estimating the model.

6.1 State dynamics

Estimated VAR

Table 3 reports unconditional means of elements of the state vector v_t. Tables 4 and 5 report various blocks of the state's persistence matrix Φ_v. Table 6 reports estimates of the covariance matrix $\Sigma_v \Sigma_v^\top$.

The results in Table 4 indicate that the global variables in our system depend on own-country variables, i.e. US variables depend on US lags, and similarly for Chinese variables. An exception is inflation: both US and Chinese inflation exhibit cross-country dynamics.

The last rows of $\Phi_{\widehat{x}}$ and $\Psi_{\widehat{x}}$ in Table 5 correspond to a regression of the AP credit spread $\tilde{c}_{it+1}$ on the entire state v_t. In spirit, this is closest to how the previous studies in the literature approach the impact of global versus local variables on sovereign credit risk. There is one global (Chinese IP growth) and one local variable (depreciation rate) that is significant, excluding the credit spread itself. Note that neither local nor US IP growth rates have any impact on credit spreads.

Of course, a discussion of one equation separately does not account for interactions between the variables controlled by matrices the Φ_v and Σ_v. Furthermore, the relative importance of the variables for the credit spread may change with the horizon. A strength of our framework is that we can move beyond inspection of the significance of regression coefficients (estimates of Φ_v) and to fully account for the state dynamics via impulse responses and variance decompositions.

Impulse responses

Figures 6 - 8 display statistically significant impulse responses organized around two themes. First is the traditional question in the literature, that is, the impact of global versus local variables on credit spreads (Figures 6 and 7). Second is the Twin Ds effect (Figures 7 and 8).

To identify structural shocks, the local variables are ordered by their "speed" rather than country. We start with "slow" macro variables and finish with the "fast" financial variables in this order: g_{it}, π_{it}, $\Delta_c \ell_{it}$, $\Delta_c \tau_{it}$, $\tilde{c}_{it}$, and Δs_{it}. Within each variable, we sort countries by their credit rating, starting with the highest: Korea, Malaysia, Thailand, Indonesia. We apply the recursive identification scheme to this order of variables. We display responses for Korea only as responses of other countries are the same, up to scale.

Starting with the responses of $\tilde{c}_{it}$ to shocks in financial variables, Figure 6 shows that the local term structure has significant effects. The relatively lower local level leads to a near-term decline in the credit spread consistent with an easing in borrowing conditions. The subsequent increase over longer horizons may be due to concerns about the risk of overheating from such easing down the line. The local slope also matters, with a relative flattening of the local yield curve - a form of long-term easing in borrowing conditions - leading to a decline in the credit spread that gradually reverts to zero.

Global yield factors play a role in local AP credit spreads as well. A higher US interest rate implies a tightening of global financial conditions, which leads to a persistent rise in the credit spread after an initial small dip. Likewise, a higher interest rate level in the United States relative to China is associated with an increase in the credit spread. An increase in the US slope, meanwhile, signals expectations of higher growth, and therefore leads to a gradual decline in the credit spread. This effect is economically large too, with the credit spread falling by as much as 12 basis points following a one standard deviation shock to the US slope.

With regard to responses to macro shocks, the response of $\tilde{c}_{it}$ to a shock in local output is missing from Figure 6 because the effect is insignificant. That is in contradiction to theories of strategic sovereign default. The only macro variable that affects the local credit spread is Chinese industrial production. An increase in Chinese output leads, counterintuitively, to an initial increase in the credit spread followed by a subsequent gradual decline. However, this effect is tiny: at its peak the credit spread rises by less than one basis point.

Figure 7 continues with shocks to the variables from the Twin Ds paradigm and their impact on the credit spread. An appreciation of the local currency or of the renminbi leads to a decline in the credit spread, which is intuitive. The effect of the local currency is particularly pronounced, with a one standard deviation appreciation resulting in a spread decline of around 7 basis points. From the perspective of the Twin Ds paradigm, we therefore see evidence of a strong impact of the exchange rate on the credit spread. Moreover, an increase in the Chinese credit spread leads to a very large increase in the AP spread, as it jumps by around 15 basis points immediately following a typical shock.

Figure 8 shows the other side of the Twin Ds by measuring the impact of credit and depreciation rates on the AP depreciation rate. The documented response to all shocks is transient. Exchange rates tend to be close to random walks. Thus, it is not a surprise that the depreciation rate is close to a serially uncorrelated variable.

A widening of the Chinese credit spread leads to an immediate sharp depreciation of the AP currency. A widening of the local spread leads to a depreciation as well. That is consistent with the response of c_{it} to Δs_{it} in Figure 7 and with the overall Twin Ds paradigm. Finally, an appreciation of the renminbi leads to a short-lived appreciation of the AP currency.

Variance decomposition

Variance decompositions measure the percentage contribution of a shock to variable j to the variation in variable i. As is usually the case, the variance decomposition could be sensitive to the order of state variables. Therefore, instead of focusing on the impact of individual variables, we distinguish between the US, Chinese and local variables only. We stack the odds in favor of the US variables by ordering them first. Chinese variables are ordered after US ones, but before local AP variables. The first part of Table 7, labeled “Factors” displays the results.

The headline result is that local variables are an important ingredient in the overall variation in AP factors. In particular, the contribution of local shocks to the variation in the local credit spread is 44% (Malaysia), 48% (Thailand), 83% (Korea), and 88% (Indonesia). Korea, Malaysia, and Thailand were studied by Longstaff, Pan, Pedersen, and Singleton (2011) as well. Although variables and methods differ, their metric termed “local ratio”, a ratio of regression R^2 with and without global variables, is the closest to the variance decomposition. The local ratios for the overlapping countries are similar to ours.

The contributions of local shocks to the variation in depreciation rates feature similar magnitudes: 62% (Malyasia) to 92% (Indonesia). The flipside of these numbers is that global variables have a relatively small effect. In the case of the US only 4 to 17% contribute to the credit spread, and, similarly, 4 to 19% contribute to the depreciation rate. Thus, the Chinese variables occupy a middle ground between the US and local ones. The cross-country differences are driven by the variation in conditional volatilities of credit spreads or depreciation rates in Table 6B, which is the only margin of cross-country differences in our model.

6.2 Prices of risk

Estimated model of the SDF

Tables 8 and 9 report estimates of λ_v. We don't report estimates of λ_0 because its non-zero elements are insignificant. The overall matrix is sparse, but still leaves room for rich differences in how the different factors affect risk premiums. To aid the interpretation, recall that the risk premium for factor v_{jt} is $-cov_t(m_{t,t+1}, \mathrm{e}_j^\top v_{t+1}) = (\lambda_0 + \lambda_v v_t)^\top \mathrm{e}_j$, where e_j is a vector with a one in location j and zeros in all other entries. As discussed earlier, prices of risks corresponding to macro variables, which are modeled as unspanned factors, are restricted to zero for identification purposes.

The US level's price of risk depends on all US variables but the level itself. The US slope's price depends on US IP growth only. This evidence is consistent with earlier studies on US data (Duffee, 2013; Haddad, Kozak, and Santosh, 2017).

For China, premiums for level, slope, and currency are significant. Premiums for Chinese yield variables depend on US output growth. The premium for the Chinese level depends on its own slope as well, which is consistent with evidence for advanced economies (Dahlquist and Hasseltoft, 2013). The currency premium depends on currency itself, the interest rate differential and Chinese credit spread.

Moving onto local AP variables, we have a situation similar to China's in that level, slope, currency, and credit risks have significant prices of risk. US output continues driving the yield risk premiums. In contrast to China, the local currency is significant for the credit risk variable (the Twin Ds effect).

Risk premiums

As highlighted, the estimated λ_t could be difficult to interpret. Thus, we study bond risk premiums corresponding to different horizons n. To highlight the risk premiums earned in excess of those for US bonds, we consider a USD-based investor who is engaging in long-short strategies. We compute risk premiums outside of the default episodes.

We introduce a new information set, $\mathcal{D}_t = \{\mathcal{F}_t, z_t = 0, z_{t+1} = 0\}$. It reflects a situation where the relevant sovereign defaults neither today nor tomorrow. That is the only conditional expectation we can compute because of lack of defaults in our sample. See also the discussion of this issue in Bansal and Dahlquist (2002).

Thus, expected excess (log) returns are:

$$
\begin{aligned}
\widehat{rx} - rx &\equiv E(\widehat{r}^{\,n,n-1}_{t+1} + \Delta s_{t+1} - r^{n,n-1}_{t+1} \mid \mathcal{D}_t), && \text{(long LC AP/ short US)} \\
\widetilde{rx} - rx &\equiv E(\widetilde{r}^{\,n,n-1}_{t+1} - r^{n,n-1}_{t+1} \mid \mathcal{D}_t), && \text{(long USD AP / short US)} \\
\widehat{rx} - \widetilde{rx} &\equiv E(\widehat{r}^{\,n,n-1}_{t+1} + \Delta s_{t+1} - \widetilde{r}^{\,n,n-1}_{t+1} \mid \mathcal{D}_t). && \text{(long LC AP/ short USD AP)}
\end{aligned}
$$

We refer to the first premium as the LC bond risk premium. One can think of the last two risk premiums as its decomposition. Long USD AP, short US reflect the impact of the credit risk premium. As Equation (5) indicates, the last risk premium is related to the Twin Ds premium.

Table 10 reports information about LC risk premiums and their components for horizons $n = 24$ and 60 months. One immediate observation is that premiums in excess of those in the US are quite sizeable. They are also volatile, indicating that the usual risk-return trade-off is at play. We will revisit this trade-off in a subsequent section. Finally, the term structure effects are modest.

The LC bond risk premium ranges between 2% and 9% with time-series volatility reaching 25% (annualized). Its decomposition into $\widetilde{rx} - rx$ and $\widehat{rx} - \widetilde{rx}$ indicates that between 30% and 60%, depending on a country and horizon, is coming from the latter. Time-series variation in the credit risk premium, $\widetilde{rx} - rx$, is relatively small. Malaysia is an exception in that most of its LC risk premium is coming from credit risk. The average Twin Ds premium is slightly negative.

Figure 9 displays time-series of the same risk premiums combined with local recessions marked by gray bars. The credit risk premium does not dip below zero (statistically and economically), which is intuitive. Moreover, it tends to rise during recessions, in particular severe ones. The Twin Ds premium occasionally goes below zero. Equation (5) helps to interpret this effect as it implies that occasionally the covariance between the "default-adjusted" SDF and the depreciation rate is expected to decline. The link between the two should be weak during expansions, which is consistent with the Figure. Further, in the case of Malaysia, the extended period of negative Twin Ds premium coincides with the peg to the dollar, which obviously weakens the covariance. That contributes to the negative average premium.

All these different risk premiums correspond to trading strategies. It would be interesting to think about optimal bond portfolios given that risk premiums are quite high. Constructing such portfolios in a dynamic setting is not an easy task. We can shortcut it and take advantage of our no-arbitrage setting to quantify the maximal benefits of investing in AP bonds.

Investment opportunities afforded by Asian bonds

We characterize the impact of adding AP bonds to the investment opportunity set. If they are redundant then the MSR corresponding to the global bonds would only be the same as the one obtained from a joint estimation using all bonds. The autonomous structure of the global variables x_t allows us to disentangle the global and local effects.

Figure 10 compares the global MSR, $[(\lambda_0 + \lambda_x x_t)^\top(\lambda_0 + \lambda_x x_t)]^{1/2}$ (top left panel) to the total MSR, $[(\lambda_0 + \lambda_v v_t)^\top(\lambda_0 + \lambda_v v_t)]^{1/2}$ (top right panel). The spike on the global MSR corresponds to the global financial crisis (GFC). One of the two spikes in the total MSR corresponds to GFC as well. Another one is the Asian financial crisis of 1998.

We see that the total MSR is always higher than the global one, averaging a monthly value of 1.89 vs 0.72 for the global MSR. The bottom left panel establishes significance of the difference by plotting the 5% upper confidence band for the global MSR and the 5% lower confidence band for the total MSR. This evidence shows that there is a statistically and economically significant gain to investing in AP bonds compared to limiting investments to US and Chinese bonds.

Finally, we exploit additivity of the squared MSR to characterize factors driving the improvement in the investment opportunity set. Specifically, note that $\lambda_t^\top \lambda_t = \sum_{j=1}^N (\lambda_t^\top \mathrm{e}_j)^2$, that is each element in the sum represents (squared) risk premium for risk j. We subtract the squared global MSR from the squared total MSR, and compute the percentage contribution of each of the local factors to that difference. The lower right panel of Figure 10 shows that a large component of the difference comes from compensation for credit and currency risks. The joint (credit/currency) contribution is 48% (19%/29%), on average, and varies between 25% and 80% (3% and 61% / 3% and 74%).

Variance decomposition

We quantify the percentage contribution of a shock to the state variables to the various risk premiums discussed above. The parts of Table 7, labeled with various premiums display the results for horizons $n = 24$ and 60. Local variables account for 9 to 51% of these for LC premiums. The ranges are $36 - 86\%$ for credit and the Twin Ds premiums. China's contribution flip: $22-41\%$ for the LC premium, and $5-22\%$ for the credit and the Twin Ds premiums. These contributions are smaller than those of the US variables but in a similar ballpark. Thus, local variables play an important role in risk premium variation as well as for the state dynamics.

Discussion

We have two main sets of results regarding the risk premiums. First, there are non-zero local risk premiums. Second, these premiums depend on local variables. These conclusions are surprising if one takes a view that "local" means "diversifiable".

To understand this evidence better we drill down to the shock structure of the variables that is reported in Table 6. As discussed earlier, four local variables, $\Delta_c \widehat{\ell}_{it}$, $\Delta_c \widehat{\tau}_{it}$, Δs_{it}, and $\widetilde{c}_{it}$ command risk premiums. Out of these, besides their own shocks, $\widetilde{c}_{it}$ and Δs_{it} are the two variables that have significant exposure to global shocks. Exposures to local variables primarily arise via exposure to local currency risk.

The interesting question is whether the local shocks affecting the credit and currency factors are priced. The answer would be affirmative if the elements of λ_t corresponding to the respective local shocks are non-zero. Our analysis of the MSR offers an answer. The difference in the squared MSRs reflects compensation for exposure to local shocks, by construction. As noted earlier, it is the compensation for local shocks to the credit and currency factors that drives a large part of the difference.

This evidence is suggestive of undiversifiable local credit and currency risk. To investigate this further, we perform PC analysis of the local credit and currency factors. The first PC of AP credit spreads explains 92% of their variation, while that of depreciation rates explains only 67%. Thus, indeed it appears that one cannot diversify the credit risk within the region. This conclusion is consistent with the evidence in Longstaff, Pan, Pedersen, and Singleton (2011) who find that the regional credit spread, an average of the CDS premium in a region, is significant in contemporaneous regressions of local credit spreads. The same is true albeit to a weaker extent for currency risk.

As regards the functional dependence of risk premiums on local factors, it is difficult to provide a definitive answer in the context of our no-arbitrage model. We could speculate that local variables could be proxying for local willingness to bear risk.

7 Conclusion

We contribute to the literature on determinants of sovereign credit spreads. We focus on the interaction between credit and currency risks, the Twin Ds, reflected in local currency (LC) bonds issued by sovereigns in Asia-Pacific (AP). LC bonds enable us to contribute to the existing literature by measuring this risk outside of default episodes and to examine how the risk is priced. We find strong interaction between credit and currency risks. The risk premium for this interaction is economically comparable to interest rate and credit risks.

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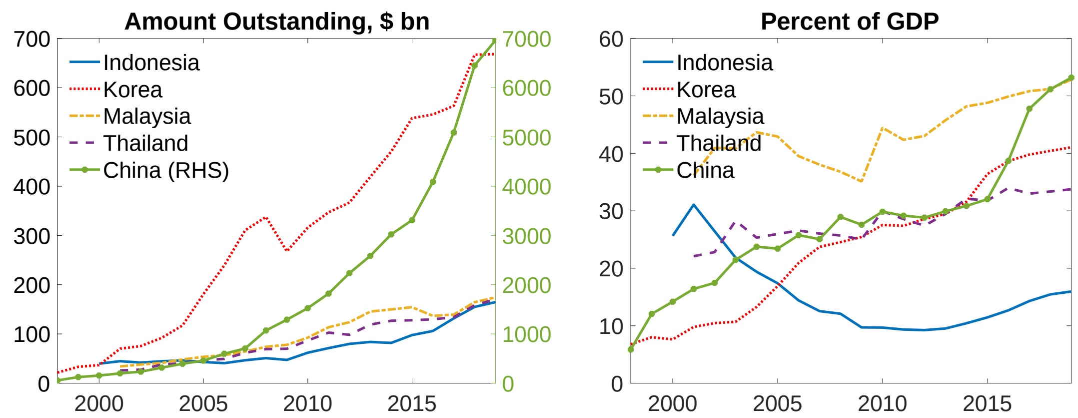
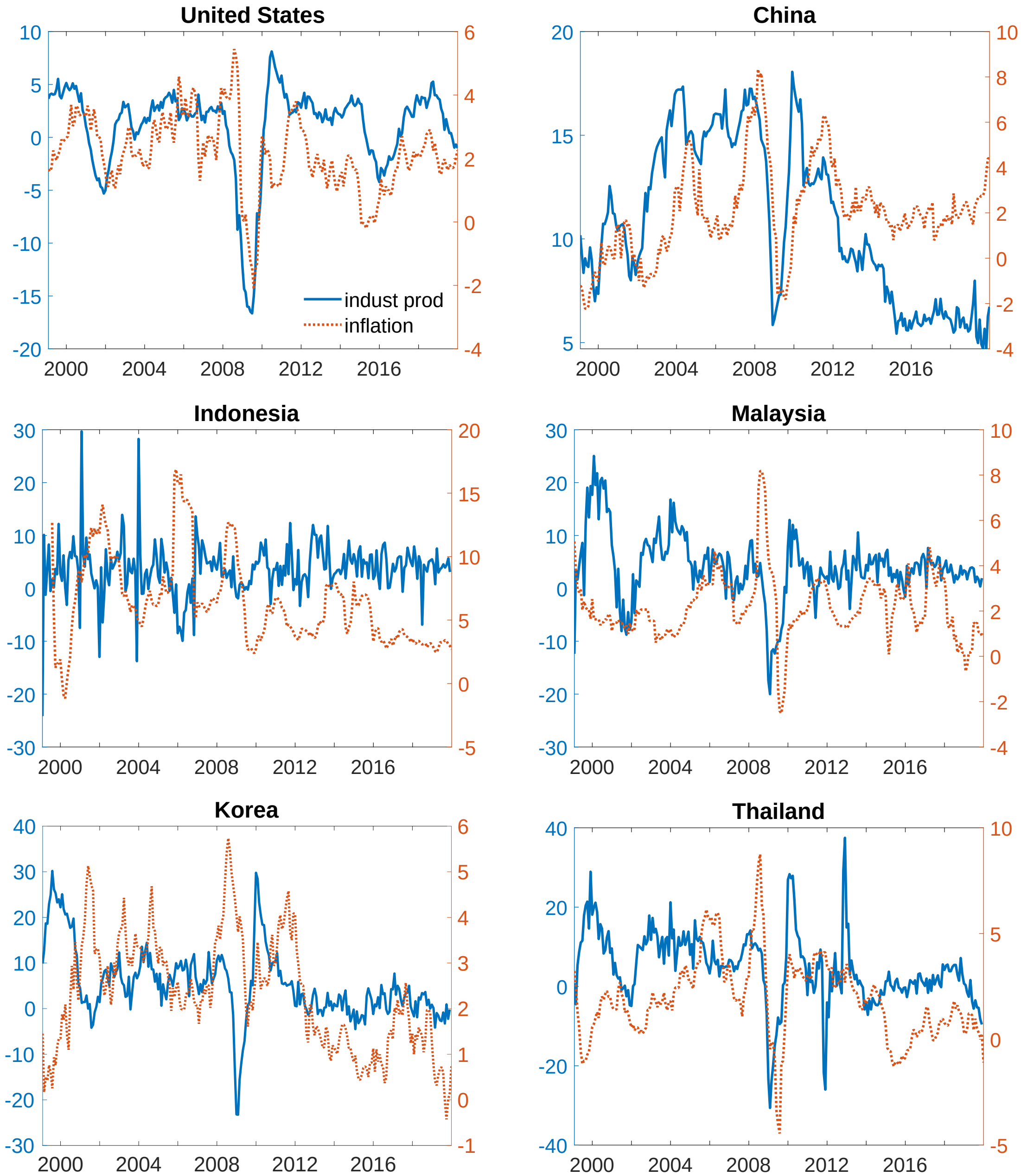
Figure 1
Local currency government bond markets



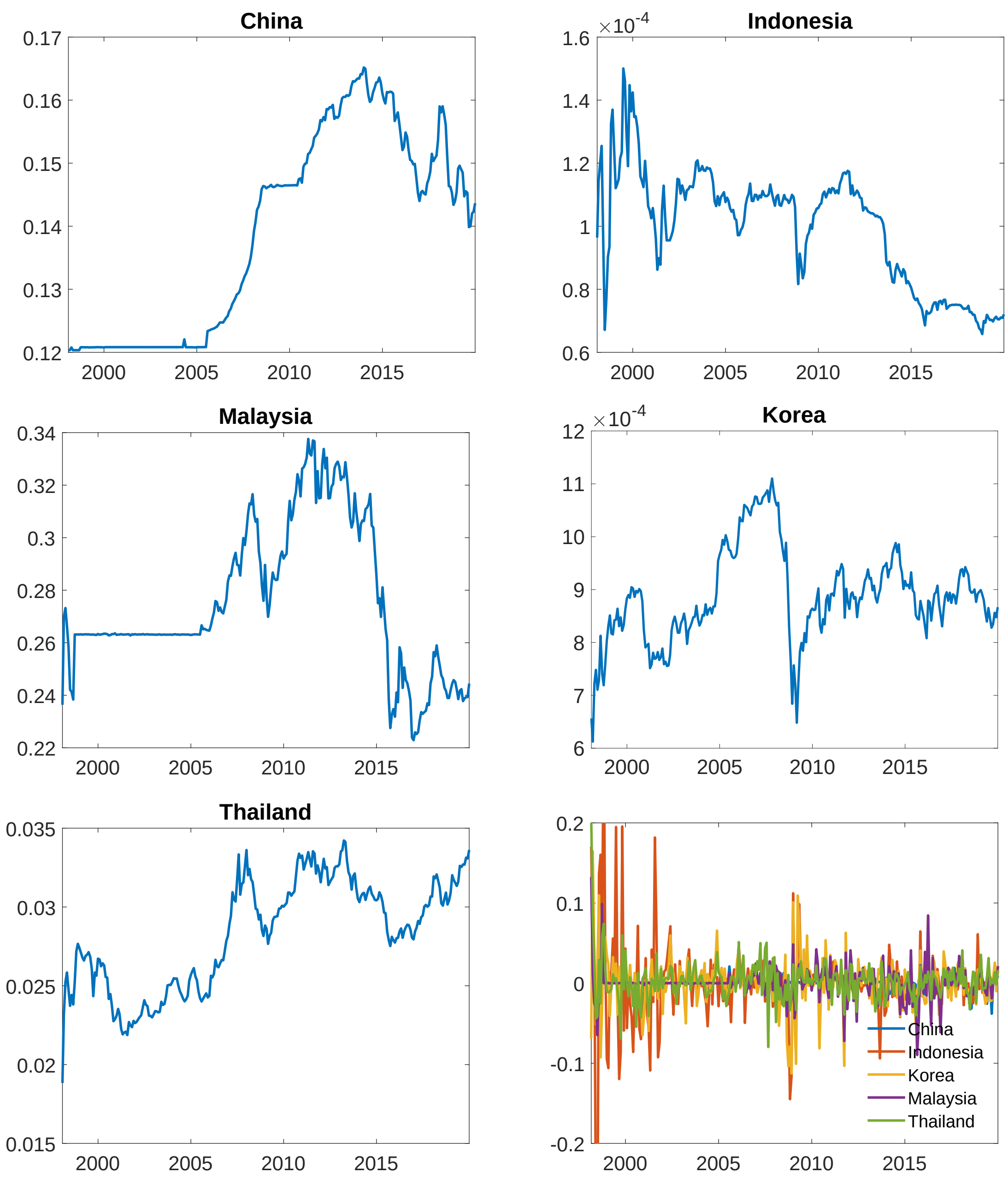
Source: Asian Development Bank

Figure 2
Macroeconomic variables



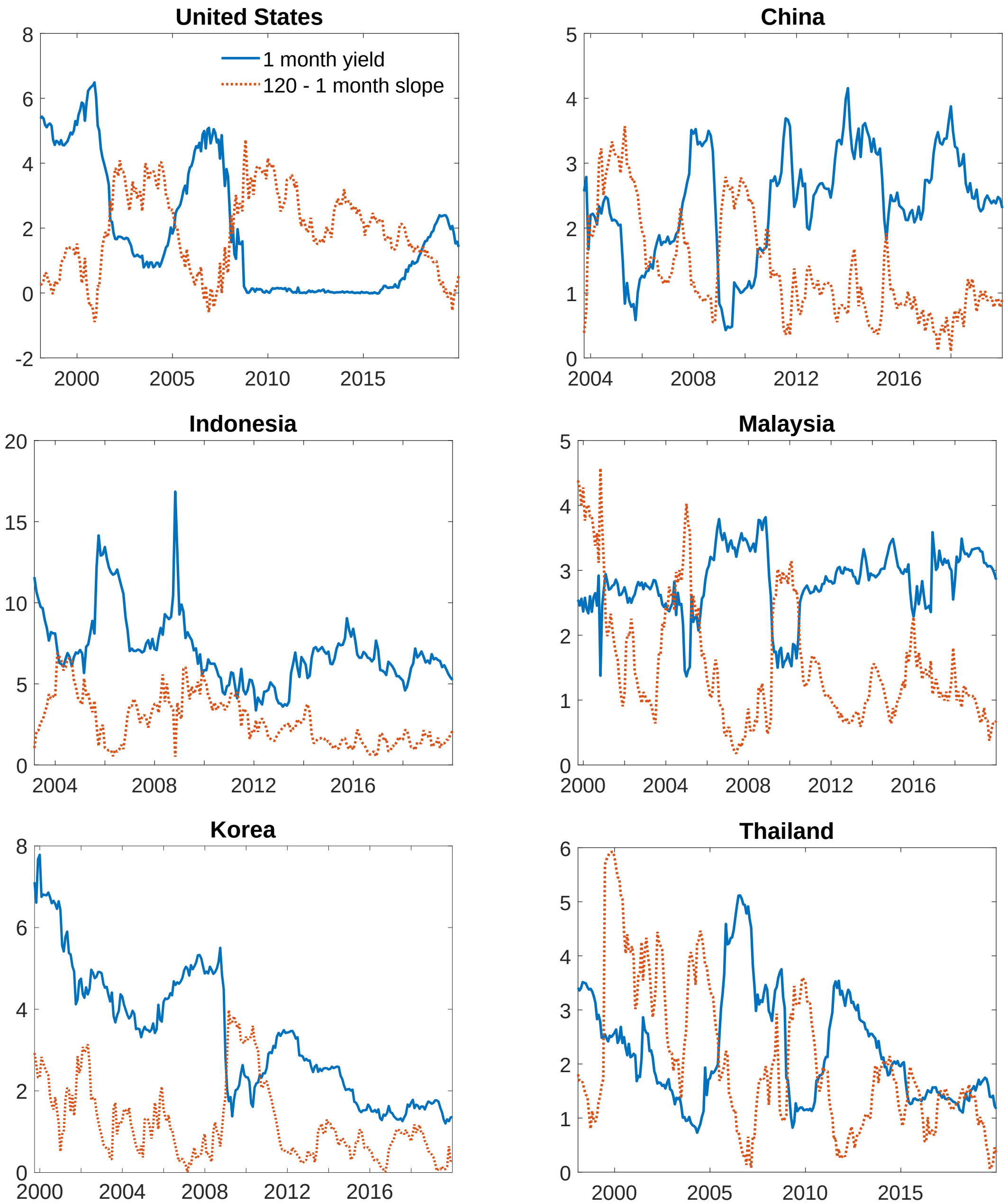
Year-on-year log-changes in industrial production and CPI. Source: Bloomberg.

Figure 3
Exchange rates



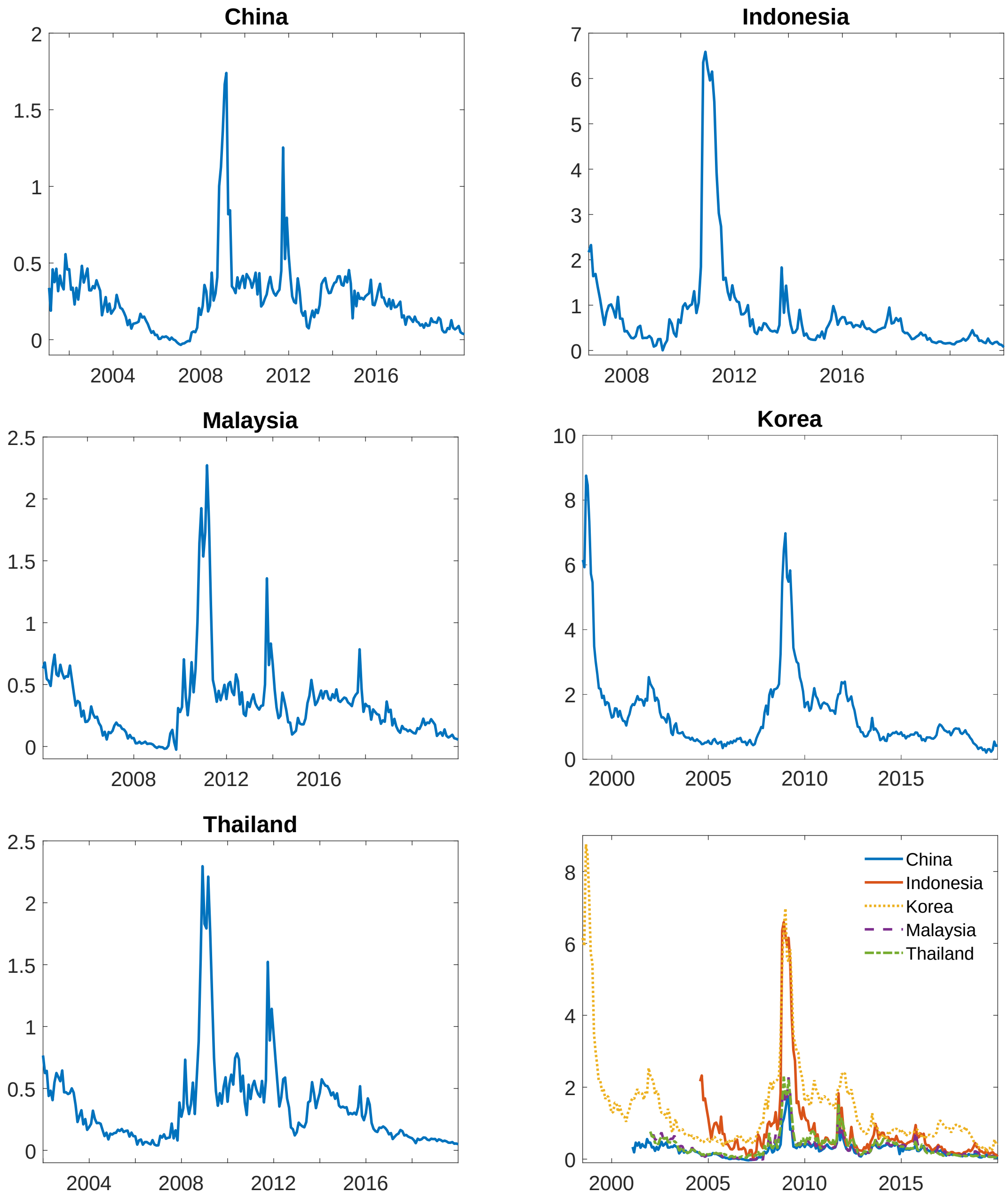
Nominal exchange rates S_t, expressed as the number of US dollars per unit of foreign currency. Log depreciation rates Δs_t are in the bottom right. Source: Bloomberg.

Figure 4
Local currency bond yields: Level and slope



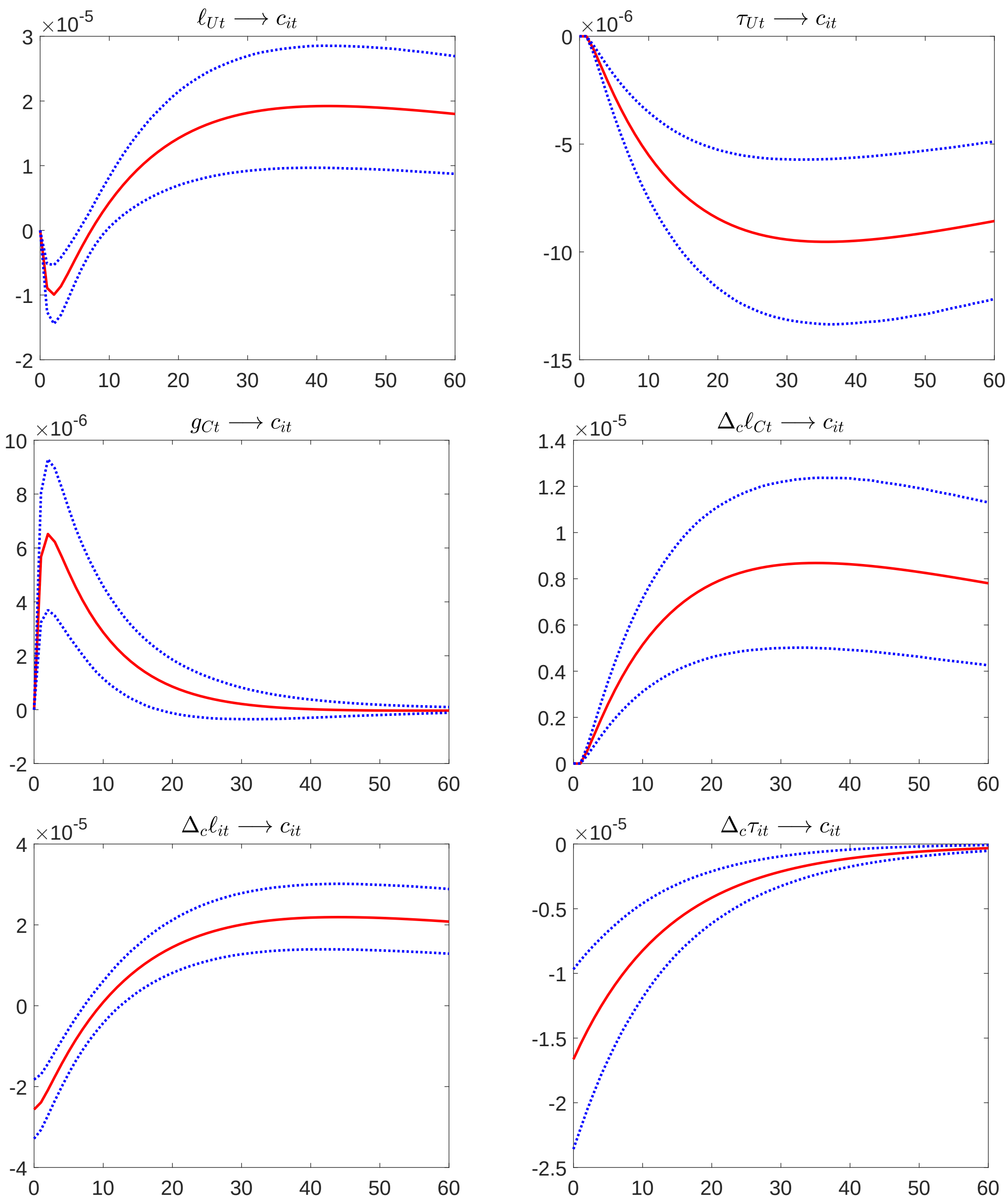
Level and slope of domestic, continuously compounded zero-coupon yield curves. Source: Bloomberg.

Figure 5
Sovereign credit factor



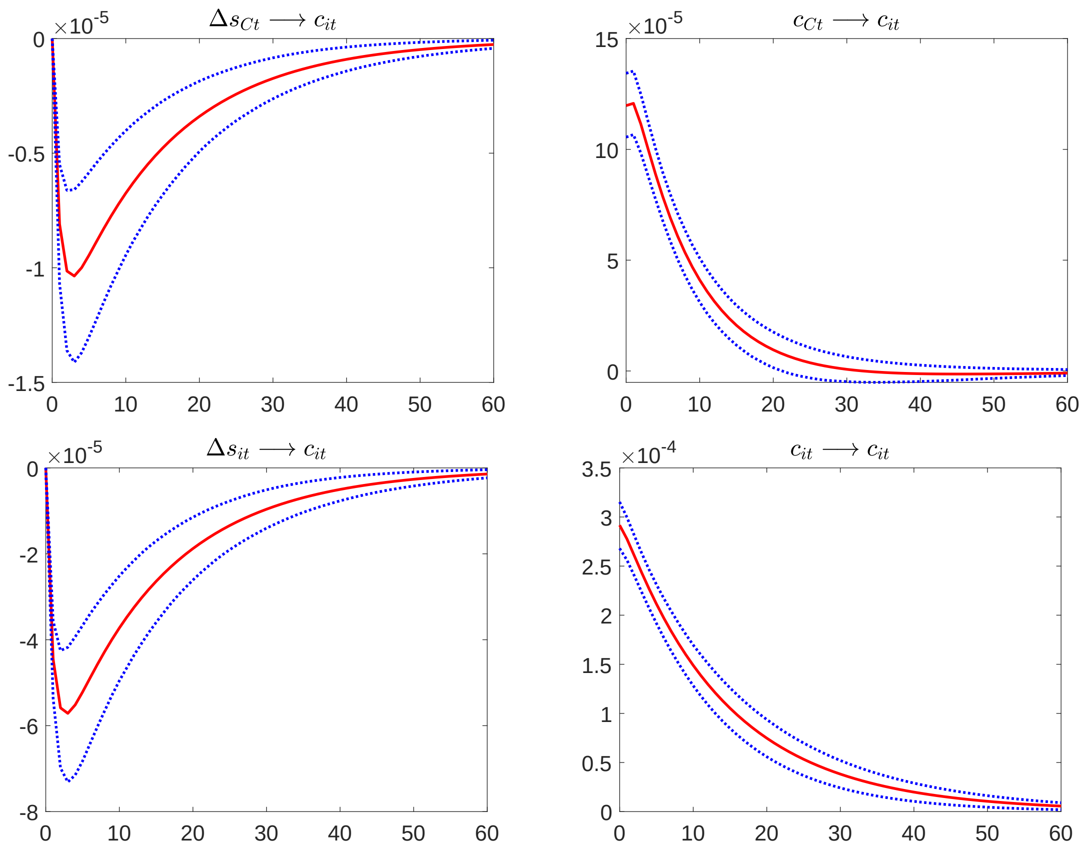
One-year spread between USD-denominated Asian bond yields and US Treasury bond yield.

Figure 6
Impulse responses: global and local shocks to credit spread



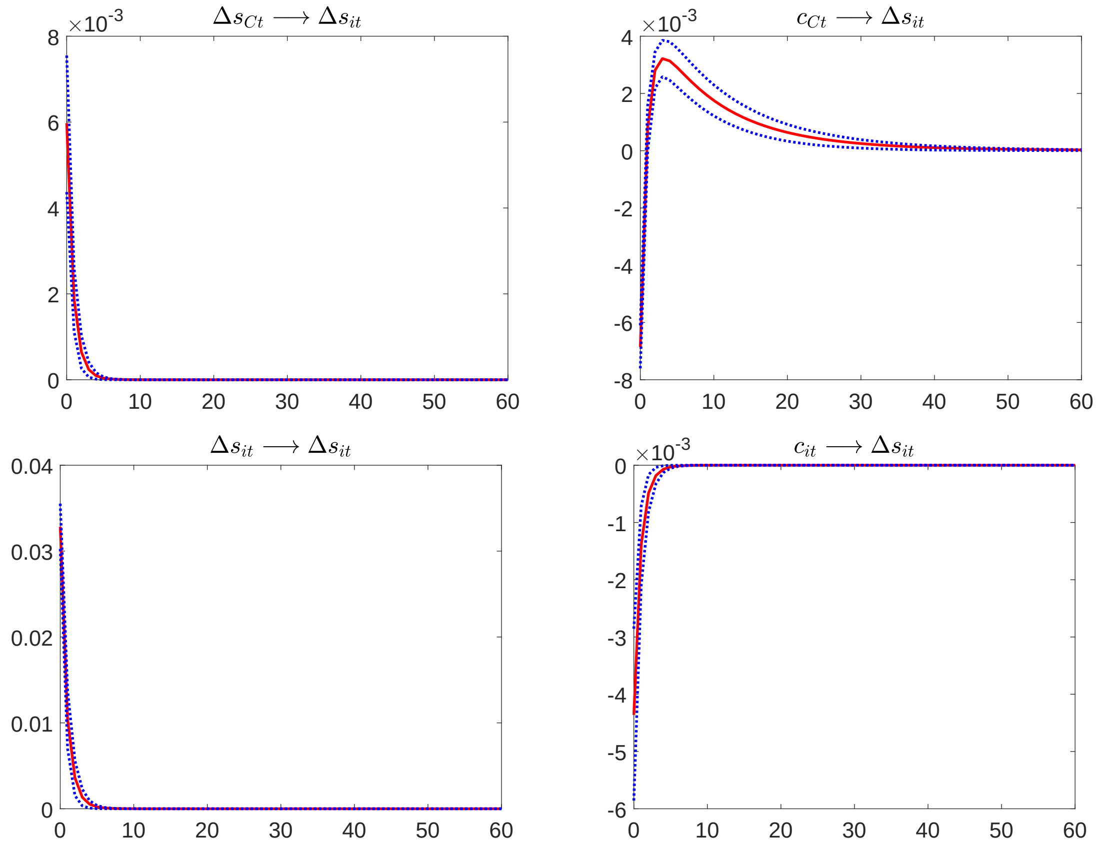
Response to a one standard deviation shock, expressed in decimals per month.

Figure 7
Impulse responses: credit spread responds to credit and currency shocks



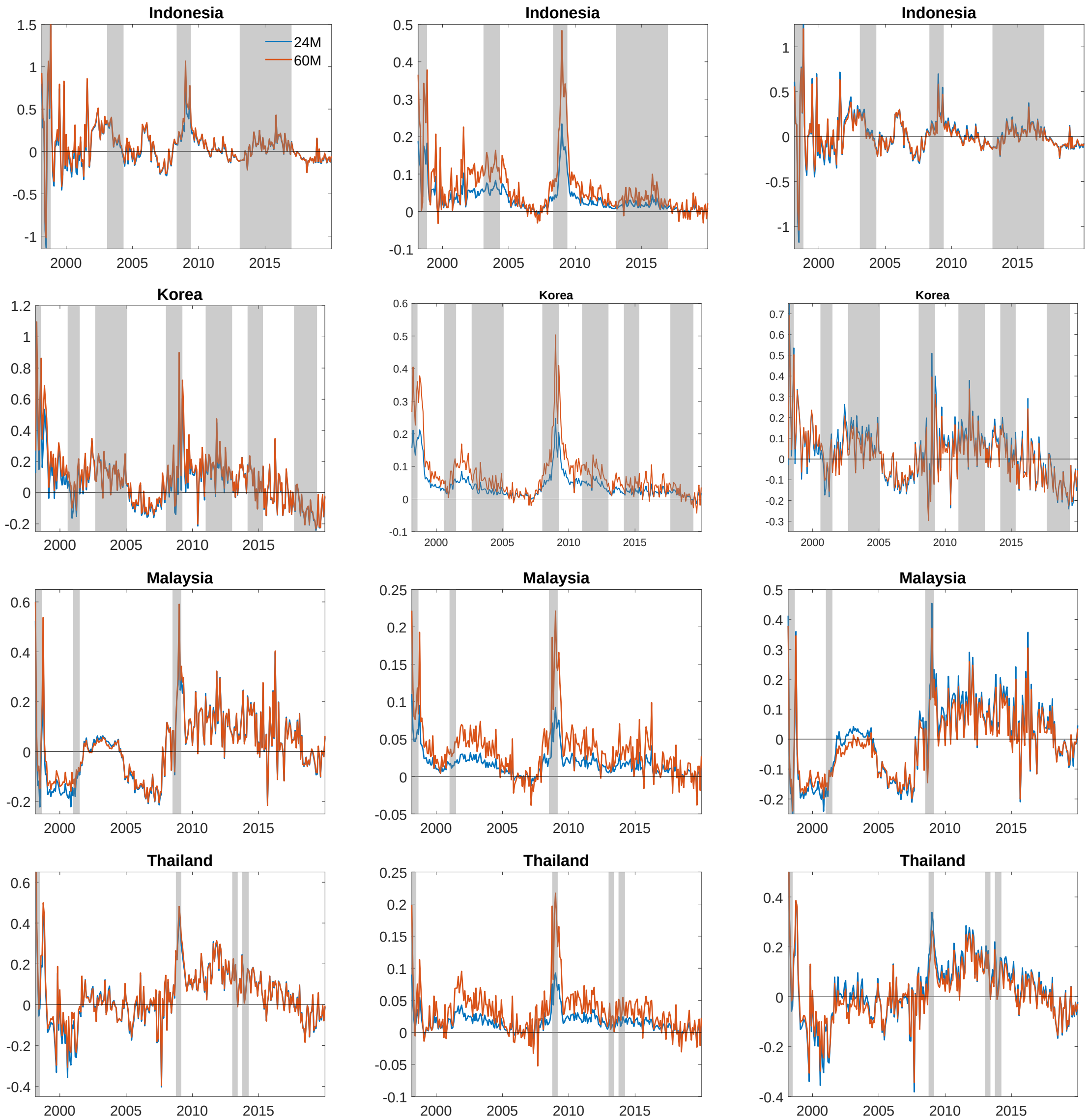
Response to a one standard deviation shock, expressed in decimals per month.

Figure 8
Impulse responses: local currency depreciation rate responds to credit and currency shocks



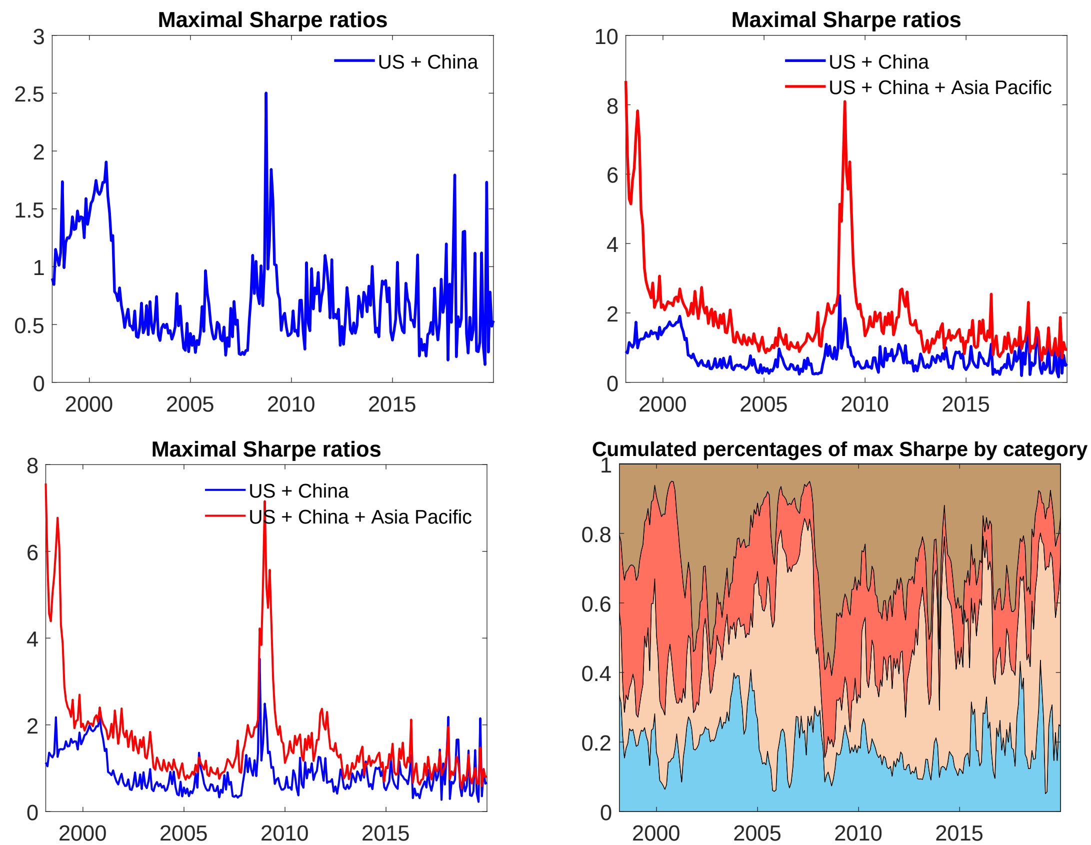
Response to a one standard deviation shock, expressed in decimals per month.

Figure 9
Bond risk premiums



The left column shows the LC bond premium, $\breve{rx} - rx$*; the next two are related to the credit premium,* $\widetilde{rx} - rx$*; and the Twin Ds premium,* $\breve{rx} - \widetilde{rx}$*. The definitions are in the main text. The second and third columns add up to the first. All numbers are annualized. Gray bars indicate local recessions. Source: OECD for Indonesia and Korea, the Bry and Boschan (1971) algorithm for Malaysia, and Thailand's Office of the National Economic and Social Development Council.*

Figure 10
Maximal Sharpe ratios



We plot maximal Sharpe ratios for log returns, $\sqrt{\lambda_t^\top \lambda_t}$, for US and Chinese assets only (blue line) and for all assets combined (red line) in the top right panel. The top left panel provides more detail on US and China combination. The bottom left panel quantifies significance of the difference between the two by plotting the 95th percentile for US and China and 5th percentiles for all assets combined. The bottom right panel illustrates the contribution of the different types of risk premiums, in per cent, to the difference between the squares of the two MSRs (squared MSRs are additive in the squared risk premiums). These values are cummulated in order from the bottom to top as: Credit, Currency, Slope, Level. For expositional purposes we smooth the contribution amount by computing a three-month centered, equal-weighted moving average.

Table 1: Summary statistics

					Country		
Variable	Statistic	US	China	Indonesia	Korea	Malaysia	Thailand
Macro							
Depr. rate	mean	–	0.008	-0.013	0.013	0.002	0.026
	st.dev.	–	0.090	0.658	0.397	0.256	0.296
	serial corr.	–	0.304	0.149	-0.039	0.032	0.147
Inflation	mean	0.021	0.020	0.086	0.024	0.023	0.019
	st.dev.	0.043	0.079	0.164	0.049	0.049	0.059
	serial corr.	0.482	0.165	0.631	0.279	0.291	0.357
IP growth	mean	0.011	0.107	0.026	0.049	0.034	0.040
	st.dev.	0.079	0.078	0.756	0.274	0.321	0.425
	serial corr.	0.203	0.239	-0.459	-0.027	-0.388	-0.126
Yields							
1y	mean	0.022	0.026	0.077	0.037	0.031	0.025
	st.dev.	0.020	0.007	0.024	0.017	0.004	0.010
	serial corr.	0.990	0.952	0.934	0.978	0.900	0.971
3y	mean	0.026	0.030	0.085	0.041	0.035	0.031
	st.dev.	0.018	0.006	0.025	0.019	0.004	0.011
	serial corr.	0.985	0.945	0.951	0.974	0.898	0.957
10y	mean	0.038	0.037	0.098	0.046	0.044	0.043
	st.dev.	0.014	0.006	0.025	0.020	0.007	0.014
	serial corr.	0.976	0.939	0.959	0.973	0.927	0.953
Credit	mean	–	0.002	0.002	0.003	0.002	0.002
	st.dev.	–	0.001	0.001	0.002	0.001	0.001
	serial corr.	–	0.981	0.970	0.969	0.989	0.990
Excess returns							
1y	mean	0.006	0.005	0.015	0.009	0.005	0.007
	st.dev.	0.021	0.026	0.090	0.026	0.019	0.023
3y	mean	0.018	0.012	0.035	0.023	0.015	0.019
	st.dev.	0.088	0.073	0.268	0.094	0.058	0.100
10y	mean	0.047	0.017	0.067	0.056	0.040	0.050
	st.dev.	0.310	0.249	0.846	0.344	0.239	0.483

Sample statistics of macroeconomic factors, selected yields, and one-month bond returns. 'Depr. rate' refers to the deprecation rate of the local currency relative to the US dollar. The depreciation rate, inflation and industrial production growth are measured as monthly log-differences and scaled by 12. The yields and the credit factor are zero-coupon based and expressed in annual continuously compounded terms. Excess returns are in logs, monthly, scaled by 12, in relation to one-month LC bond return. All series end Dec. 2018; all FX and macro series start in January 1998. Yield series for the US and Thailand start Jan. 1998, for China Sep. 2003, for Indonesia Feb. 2003, and for Korea and Malaysia Sep. 1999.

Table 2: Principal components

	PCs	R^2	R^2_{adj}
1	71.50	99.36	99.27
2	81.80	98.26	98.02
3	89.57	98.87	98.71
4	93.82	97.34	96.98
5	95.99	98.90	98.75
6	97.48	99.12	98.99
7	98.21	96.03	95.49
8	98.85	88.80	87.29
9	99.09	91.70	90.58
10	99.32	86.67	84.87
11	99.47	80.07	77.39
12	99.60	78.69	75.81

In the first column, we report per cent of variation in international yields explained by first 12 principal components constructed from the US, Chinese, and AP yields. The second and third column displays how much variation in a given principal component can be explained by depreciation rates and the bond-yield-related elements of the state vector v_t : ℓ_{Ut} and τ_{Ut} for the US; $\Delta_c \ell_{Ct}$ and $\Delta_c \tau_{it} \Delta_c \widehat{\ell}_{Ct}$ for China, and $\Delta_c \widehat{\tau}_{it}$ for AP.

Table 3: VAR estimation, $\bar{\mu}_x$

x_t	US	China	Indonesia	Korea	Malaysia	Thailand
Δs_t	—	4.24e-05	0.002	-0.012	-6.84e-04	-8.16e-04
	(—)	(0.001)	(0.008)	(0.008)	(0.007)	(0.007)
π_t	0.002	0.002	0.005	0.002	0.002	0.002
	(3.50e-04)	(5.31e-04)	(5.40e-04)	(3.79e-04)	(4.00e-04)	(4.17e-04)
g_t	9.23e-04	0.009	0.007	0.007	0.010	0.010
	(4.86e-04)	(0.002)	(0.006)	(0.005)	(0.005)	(0.005)
g_t	-0.003	2.40e-04	-0.006	2.94e-04	-0.001	-4.90e-04
	(0.008)	(0.005)	(0.001)	(0.001)	(0.001)	(0.001)
ℓ_t	0.001	-3.65e-04	-3.25e-04	2.17e-04	5.59e-04	-1.55e-05
	(6.02e-04)	(5.72e-04)	(0.002)	(0.001)	(0.001)	(0.001)
τ_t	—	1.23e-04	3.64e-04	4.82e-04	1.19e-04	1.34e-04
	(—)	(7.88e-05)	(2.55e-04)	(2.71e-04)	(1.93e-04)	(1.95e-04)

Posterior mean and standard deviation of the unconditional mean $\bar{\mu}_x$. Depreciation rate Δs_t, Slope $\tau_t = y_t^{120} - y_t^1$. Level is $\ell_t = y_t^1$. Foreign credit factor c_t. Inflation: π_t. Industrial production g_t

Table 4: VAR estimation, Φ_x

x	π_{Ut}	g_{Ut}	ℓ_{Ut}	τ_{Ut}	Δs_{Ct}	π_{Ct}	g_{Ct}	$\Delta_c \ell_{Ct}$	$\Delta_c \tau_{Ct}$	c_{Ct}
π_{Ut+1}	0.342	0	0	0	0	0.071	0	0	-0.190	0
	(0.045)	(—)	(—)	(—)	(—)	(0.025)	(—)	(—)	(0.097)	(—)
g_{Ut+1}	0.331	0.151	0	0	0	0	0	0	0	0
	(0.109)	(0.051)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
ℓ_{Ut+1}	0	0	0.998	0	0	0	0	0	0	0
	(—)	(—)	(0.002)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
τ_{Ut+1}	0	0	0	0.973	0	0	0	0	0	0
	(—)	(—)	(—)	(0.017)	(—)	(—)	(—)	(—)	(—)	(—)
Δs_{Ct+1}	0	0	0	0	0.318	0	0	-0.138	0	3.866
	(—)	(—)	(—)	(—)	(0.058)	(—)	(—)	(0.085)	(—)	(1.564)
π_{Ct+1}	-0.397	0	0	0	0	0.165	0.135	0	0	0
	(0.111)	(—)	(—)	(—)	(—)	(0.059)	(0.053)	(—)	(—)	(—)
g_{Ct+1}	0	0	0	0	0	0	0.219	0.152	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(0.062)	(0.161)	(—)	(—)
$\Delta_c \ell_{Ct+1}$	0	0	0	0	0	0	0	0.995	0	0.113
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(0.003)	(—)	(0.016)
$\Delta_c \tau_{Ct+1}$	0	0	0	0	0	0	0	0	0.961	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(0.011)	(—)
c_{Ct+1}	0	0	0	0	0	0	6.98e-04	0	0	0.896
	(—)	(—)	(—)	(—)	(—)	(—)	(4.41e-04)	(—)	(—)	(0.023)

Posterior mean and standard deviation of elements of the persistence matrix for the US and Chinese variables. Depreciation rate Δs_t, Slope $\tau_t = y_t^{120} - y_t^1$. Level is $\ell_t = y_t^1$. Foreign credit factor c_t. Inflation: π_t. Industrial production g_t. Cross-country difference Δ_c.

Table 5: Persistence of AP countries

Panel A. VAR estimation, $\Phi_{\widehat{x}}$

$\widehat{x}_i$ / x	π_{Ut}	g_{Ut}	ℓ_{Ut}	τ_{Ut}	Δs_{Ct}	π_{Ct}	g_{Ct}	$\Delta_c \ell_{Ct}$	$\Delta_c \tau_{Ct}$	c_{Ct}
Δs_{it+1}	0	0.013	0	0	0	0	0	0	0	23.332
	(—)	(0.029)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(3.436)
$\widehat{\pi}_{it+1}$	0	0	0	0	0.033	0	0.051	0	0	0
	(—)	(—)	(—)	(—)	(0.017)	(—)	(0.020)	(—)	(—)	(—)
$\widehat{g}_{it+1}$	0.776	0	0	12.845	0	0	0.014	-0.090	-7.669	-54.640
	(0.265)	(—)	(—)	(1.588)	(—)	(—)	(0.027)	(0.099)	(1.348)	(1.915)
$\Delta_c \widehat{\ell}_{it+1}$	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c \widehat{\tau}_{it+1}$	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widetilde{c}_{it+1}$	0	0	0	0	0	0	9.08e-04	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(3.83e-04)	(—)	(—)	(—)

Panel B. VAR estimation, $\Psi_{\widehat{x}}$

$\widehat{x}_i$	Δs_{it}	$\widehat{\pi}_{it}$	$\widehat{g}_{it}$	$\Delta_c \widehat{\ell}_{it}$	$\Delta_c \widehat{\tau}_{it}$	$\widetilde{c}_{it}$
Δs_{it+1}	0.314	0	0	-4.630	0	0
	(0.097)	(—)	(—)	(0.535)	(—)	(—)
$\widehat{\pi}_{it+1}$	-0.015	0.283	0	0	0	0
	(0.004)	(0.030)	(—)	(—)	(—)	(—)
$\widehat{g}_{it+1}$	0	0	-0.150	0	0	9.619
	(—)	(—)	(0.022)	(—)	(—)	(1.349)
$\Delta_c \widehat{\ell}_{it+1}$	0	0	0	0.991	0	0
	(—)	(—)	(—)	(0.003)	(—)	(—)
$\Delta_c \widehat{\tau}_{it+1}$	7.94e-04	0	0	0	0.977	0
	(2.09e-04)	(—)	(—)	(—)	(0.008)	(—)
$\widetilde{c}_{it+1}$	-0.001	0	0	0	0	0.931
	(2.41e-04)	(—)	(—)	(—)	(—)	(0.012)

Posterior mean and standard deviation of the AP countries. Depreciation rate Δs_t*, Slope* $\tau_t = y_t^{120} - y_t^1$*. Level is* $\ell_t = y_t^1$*. Foreign credit factor* c_t*. Inflation:* π_t*. Industrial production* g_t*. Cross-country difference* Δ_c*.*

Table 6: VAR estimate, Σ_v

Panel A. L_v

	π_{Ut}	g_{Ut}	ℓ_{Ut}	τ_{Ut}	Δs_{Ct}	$\widehat{\pi}_{Ct}$	$\widehat{g}_{Ct}$	$\Delta_c\widehat{\ell}_{Ct}$	$\Delta_c\widehat{\tau}_{Ct}$	$\widetilde{c}_{Ct}$	Δs_{it}	$\widehat{\pi}_{it}$	$\widehat{g}_{it}$	$\Delta_c\widehat{\ell}_{it}$	$\Delta_c\widehat{\tau}_{it}$	$\widetilde{c}_{it}$
π_{Ut}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
g_{Ut}	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
ℓ_{Ut}	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
τ_{Ut}	0	0	-0.207	1	0	0	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(0.073)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
Δs_{Ct}	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widehat{\pi}_{Ct}$	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widehat{g}_{Ct}$	0.260	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	(0.211)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c\widehat{\ell}_{Ct}$	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c\widehat{\tau}_{Ct}$	0	0	0	0.641	0	0	0	-0.633	1	0	0	0	0	0	0	0
	(—)	(—)	(—)	(0.149)	(—)	(—)	(—)	(0.151)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widetilde{c}_{Ct}$	0	0	0	0	0	0	0	0	-0.011	1	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(0.034)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
Δs_{it}	0	0	12.026	0	0.821	0	0	0	0	-54.533	1	0	0	0	0	0
	(—)	(—)	(3.300)	(—)	(0.214)	(—)	(—)	(—)	(—)	(2.752)	(—)	(—)	(—)	(—)	(—)	(—)
$\widehat{\pi}_{it}$	0.351	0	0	0	0	0.050	0	0	0	0	0	1	0	0	0	0
	(0.073)	(—)	(—)	(—)	(—)	(0.037)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widehat{g}_{it}$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c\widehat{\ell}_{it}$	0	0	0.366	-0.332	0	0	0	0.133	0	0	0	0	0	1	0	0
	(—)	(—)	(0.116)	(0.070)	(—)	(—)	(—)	(0.041)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c\widehat{\tau}_{it}$	0	0	-0.294	0.811	0	0	0	0	0	0	0	0	0	-0.542	1	0
	(—)	(—)	(0.100)	(0.067)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(0.051)	(—)	(—)
$\widetilde{c}_{it}$	0	0	0	0	0	0	0	0	0	0.953	-0.001	0	0	-0.112	-0.081	1
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(0.067)	(3.57e-04)	(—)	(—)	(0.031)	(0.035)	(—)

Panel B. D_v

x_t	US	China	Indonesia	Korea	Malaysia	Thailand
Δs_t	—	0.007	0.054	0.033	0.021	0.023
	(—)	(5.38e-04)	(0.004)	(0.003)	(0.002)	(0.002)
$\widehat{\pi}_{it}(\pi_t)$	0.003	0.006	0.006	0.003	0.004	0.004
	(2.44e-04)	(4.48e-04)	(4.32e-04)	(2.46e-04)	(2.83e-04)	(3.03e-04)
$\widehat{g}_{it}$ (g_t)	0.006	0.006	0.059	0.023	0.026	0.034
	(4.56e-04)	(5.04e-04)	(0.005)	(0.002)	(0.002)	(0.003)
$\Delta_c\widehat{\ell}_{it}$ (ℓ_t)	5.61e-04	5.86e-04	7.02e-04	2.29e-04	2.03e-04	1.97e-04
	(2.20e-04)	(1.43e-04)	(5.75e-05)	(1.93e-05)	(1.71e-05)	(2.16e-05)
$\Delta_c\widehat{\tau}_{it}$ (τ_t)	2.80e-04	2.65e-04	4.55e-04	2.06e-04	1.72e-04	3.04e-04
	(2.56e-05)	(4.99e-05)	(3.75e-05)	(1.71e-05)	(1.51e-05)	(2.76e-05)
$\widetilde{c}_{it}$	—	1.26e-04	2.43e-04	2.89e-04	7.60e-05	8.64e-05
	(—)	(1.29e-05)	(2.26e-05)	(2.43e-05)	(6.96e-06)	(7.78e-06)

Estimates of dynamics of v_t. $\Sigma_v\Sigma_v^\top = L_v D_v L_v^\top$.

Table 7: Variance decomposition

	Indonesia			Korea			Malaysia			Thailand		
	US	CHN	local	US	CHN	local	US	CHN	local	US	CHN	local
Factors												
Δs_{it}	0.04	0.04	0.92	0.11	0.11	0.78	0.19	0.19	0.62	0.18	0.18	0.64
$\widehat{\pi}_{it}$	0.04	0.01	0.95	0.11	0.03	0.86	0.08	0.02	0.89	0.08	0.02	0.90
$\widehat{g}_{it}$	0.04	0.07	0.89	0.15	0.27	0.59	0.13	0.24	0.63	0.10	0.17	0.73
$\Delta_c\widehat{\ell}_{it}$	0.09	0.01	0.89	0.44	0.06	0.50	0.48	0.07	0.45	0.50	0.07	0.43
$\Delta_c\widehat{\tau}_{it}$	0.17	0.00	0.83	0.56	0.01	0.43	0.63	0.01	0.36	0.46	0.01	0.53
$\widetilde{c}_{it}$	0.04	0.08	0.88	0.06	0.11	0.83	0.17	0.38	0.44	0.16	0.36	0.48
LC credit premium, $\widehat{rx} - rx$												
24	0.31	0.19	0.50	0.30	0.19	0.51	0.47	0.41	0.12	0.46	0.39	0.15
60	0.37	0.22	0.42	0.36	0.21	0.42	0.50	0.41	0.09	0.49	0.39	0.11
Credit premium, $\widetilde{rx} - rx$												
24	0.08	0.05	0.87	0.29	0.19	0.52	0.36	0.23	0.41	0.35	0.23	0.42
60	0.11	0.05	0.84	0.34	0.18	0.48	0.40	0.22	0.38	0.35	0.18	0.46
Twin Ds premium, $\widehat{rx} - \widetilde{rx}$												
24	0.09	0.05	0.86	0.31	0.18	0.52	0.38	0.22	0.39	0.38	0.22	0.40
60	0.14	0.07	0.79	0.35	0.19	0.46	0.44	0.25	0.30	0.41	0.23	0.37

We report contribution, in per cent, of shocks to the state variables v_t to the unconditional variance of the states themselves or to the currency and credit components of bond risk premiums. Our factor order is such that US variables are given a first chance to explain the relevant variation with a residual attributed to the local factors. We report cumulative contribution of US vs local variables and do not distinguish between the shocks to individual elements of v_t. Because many elements of v_t are persistent we exclude the impact of its own shock on a variable in the first part of the table. That is why the US and local columns do not add up to 100%.

Table 8: SDF estimation, λ_x

x	π_{Ut}	g_{Ut}	ℓ_{Ut}	τ_{Ut}	Δs_{Ct}	π_{Ct}	g_{Ct}	$\Delta_c \ell_{Ct}$	$\Delta_c \tau_{Ct}$	c_{Ct}
π_{Ut}	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
g_{Ut}	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
ℓ_{Ut}	0.003	0.013	0	-0.043	0	0	0	0	0	0
	(5.20e-04)	(0.004)	(—)	(0.017)	(—)	(—)	(—)	(—)	(—)	(—)
τ_{Ut}	0	-0.010	0	0	0	0	0	0	0	0
	(—)	(0.004)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
Δs_{Ct}	0	0	0	0	0.318	0	0	-1.138	0	3.866
	(—)	(—)	(—)	(—)	(0.058)	(—)	(—)	(0.085)	(—)	(1.564)
π_{Ct}	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
g_{Ct}	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c \ell_{Ct}$	0	0.015	0	0	0	0	0	0	-0.041	0
	(—)	(0.004)	(—)	(—)	(—)	(—)	(—)	(—)	(0.011)	(—)
$\Delta_c \tau_{Ct}$	0	-0.014	0	0	0	0	0	0	0	0
	(—)	(0.004)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
c_{Ct}	0	0	0	0	0	0	0	0	0	-0.021
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(0.026)

Posterior mean and standard deviation of the risk premium matrix λ_x (US and China). Depreciation rate Δs_t, Slope $\tau_t = y_t^{120} - y_t^1$. Level is $\ell_t = y_t^1$. Foreign credit factor c_t. Inflation: π_t. Industrial production g_t. Cross-country difference Δ_c.

Table 9: Risk premiums of AP countries

Panel A. SDF estimation, $\lambda_{\widehat{x}}$

$\widehat{x}_i$ / x	π_{Ut}	g_{Ut}	ℓ_{Ut}	τ_{Ut}	Δs_{Ct}	π_{Ct}	g_{Ct}	$\Delta_c \ell_{Ct}$	$\Delta_c \tau_{Ct}$	c_{Ct}
Δs_{it}	0	0.013	0	0	0	0	0	0	0	23.332
	(—)	(0.029)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(3.436)
$\widehat{\pi}_{it}$	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widehat{g}_{it}$	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c \widehat{\ell}_{it}$	0	0.014	0	0	0	0	0	0	-0.026	0
	(—)	(0.004)	(—)	(—)	(—)	(—)	(—)	(—)	(0.007)	(—)
$\Delta_c \widehat{\tau}_{it}$	0	-0.013	0	0	0	0	0	0	0	0
	(—)	(0.004)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\widetilde{c}_{it}$	0	0	0	0	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)	(—)

Panel B. SDF estimation, $\chi_{\widehat{x}}$

$\widehat{x}_i$	Δs_{it}	$\widehat{\pi}_{it}$	$\widehat{g}_{it}$	$\Delta_c \widehat{\ell}_{it}$	$\Delta_c \widehat{\tau}_{it}$	$\widetilde{c}_{it}$
Δs_{it}	0.314	0	0	-5.630	0	0
	(0.097)	(—)	(—)	(0.535)	(—)	(—)
$\widehat{\pi}_{it}$	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)
$\widehat{g}_{it}$	0	0	0	0	0	0
	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta_c \widehat{\ell}_{it}$	0	0	0	0	0	-0.140
	(—)	(—)	(—)	(—)	(—)	(0.022)
$\Delta_c \widehat{\tau}_{it}$	0	0	0	0	0	0.180
	(—)	(—)	(—)	(—)	(—)	(0.031)
$\widetilde{c}_{it}$	-0.001	0	0	0	0	-0.081
	(2.66e-04)	(—)	(—)	(—)	(—)	(0.013)

Posterior mean and standard deviation of the AP countries. Depreciation rate Δs_t, Slope $\tau_t = y_t^{120} - y_t^1$. Level is $\ell_t = y_t^1$. Foreign credit factor c_t. Inflation: π_t. Industrial production g_t. Cross-country difference Δ_c.

Table 10: Bond risk premiums

Panel A. 24 months

	US	Indonesia	Korea	Malaysia	Thailand
rx	0.0059	—	—	—	—
	(0.0048)	(—)	(—)	(—)	(—)
$\widehat{rx} - rx$	—	0.0553	0.0672	0.0166	0.0390
	(—)	(0.2499)	(0.1666)	(0.1356)	(0.1433)
$\widetilde{rx} - rx$	—	0.0336	0.0409	0.0167	0.0160
	(—)	(0.0383)	(0.0424)	(0.0169)	(0.0157)
$\widehat{rx} - \widetilde{rx}$	—	0.0216	0.0263	-0.0001	0.0229
	(—)	(0.2245)	(0.1384)	(0.1268)	(0.1325)

Panel B. 60 months

	US	Indonesia	Korea	Malaysia	Thailand
rx	0.0213	—	—	—	—
	(0.0152)	(—)	(—)	(—)	(—)
$\widehat{rx} - rx$	—	0.0780	0.0911	0.0240	0.0469
	(—)	(0.2596)	(0.1854)	(0.1322)	(0.1433)
$\widetilde{rx} - rx$	—	0.0637	0.0751	0.0345	0.0335
	(—)	(0.0773)	(0.0812)	(0.0373)	(0.0364)
$\widehat{rx} - \widetilde{rx}$	—	0.0143	0.0160	-0.0104	0.0134
	(—)	(0.2020)	(0.1254)	(0.1106)	(0.1196)

The table offers summary statistics – mean (volatility) – of bond risk premiums with maturities $n = 24$ and 60 months earned over a single month. As a reference, we report the expected bond excess returns in the US, rx. The LC bond premium is $\widehat{rx} - rx$; the credit premium is $\widetilde{rx} - rx$; the Twin Ds premium is $\widehat{rx} - \widetilde{rx}$. The definitions are in the main text. The last two premiums add up to the second. All numbers are annualized.

Appendix A Bond prices

Appendix A.1 US bonds

The US short rate is

$$y_t^1 = \delta_{y,0} + \delta_{y,v}^\top v_t$$

We conjecture that the price of a US bond is $Q_t^n = \exp\left(\bar{a}_n + \bar{b}_n^\top v_t\right)$. The price of a 1 period bond is

$$\begin{aligned}
Q_t^1 &= \mathrm{E}_t\left(M_{t,t+1}\right) \\
&= \mathrm{E}_t\left[\exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t - \lambda_t^\top\epsilon_{t+1} - \sum_{i=1}^N\left[\gamma_{i,t}z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\right] \\
&= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right)\mathrm{E}_t\left[\exp\left(-\lambda_t^\top\epsilon_{t+1} - \sum_{i=1}^N\left[\gamma_{i,t}z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\right] \\
&= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \sum_{i=1}^N h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right)\mathrm{E}_t\left[\exp\left(-\sum_{i=1}^N\gamma_{i,t}z_{i,t+1}\right)\right] \\
&= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t\right)
\end{aligned}$$

This implies the initial conditions $\bar{a}_1 = -\delta_{y,0}$ and $\bar{b}_1 = -\delta_{y,v}$.

The price of an n-period US bond is

$$\begin{aligned}
Q_t^n &= \mathrm{E}_t\left[\exp\left(m_{t,t+1}\right)Q_{t+1}^{n-1}\right] \\
&= \mathrm{E}_t\left[\exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t - \lambda_t^\top\epsilon_{t+1} - \sum_{i=1}^N\left[\gamma_{i,t}z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right] + \bar{a}_{n-1} + \bar{b}_{n-1}^\top v_{t+1}\right)\right] \\
&= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t + \bar{a}_{n-1} + \bar{b}_{n-1}^\top\left[\mu_v + \Phi_v v_t\right]\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\sum_{i=1}^N\left[\gamma_{i,t}z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right] + \left[\bar{b}_{n-1}^\top\Sigma_v - \lambda_t^\top\right]\epsilon_{t+1}\right)\right] \\
&= \exp\left(\bar{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t + \bar{b}_{n-1}^\top\left[\mu_v + \Phi_v v_t\right] - \lambda_t^\top\Sigma_v^\top\bar{b}_{n-1} + \frac{1}{2}\bar{b}_{n-1}^\top\Sigma_v\Sigma_v^\top\bar{b}_{n-1}\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\sum_{i=1}^N\left[\gamma_{i,t}z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\right] \\
&= \exp\left(\bar{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t + \bar{b}_{n-1}^\top\left[\mu_v + \Phi_v v_t\right] - \lambda_t^\top\Sigma_v^\top\bar{b}_{n-1} + \frac{1}{2}\bar{b}_{n-1}^\top\Sigma_v\Sigma_v^\top\bar{b}_{n-1}\right)
\end{aligned}$$

where the loadings are

$$\bar{a}_n = \bar{a}_{n-1} - \delta_{y,0} + \bar{b}_{n-1}^\top\left(\mu_v - \lambda_0\right) + \frac{1}{2}\bar{b}_{n-1}^\top\Sigma_v\Sigma_v^\top\bar{b}_{n-1} \tag{A.1}$$

$$\bar{b}_n = \left(\Phi_v - \lambda_v\right)^\top\bar{b}_{n-1} - \delta_{y,v} \tag{A.2}$$

US yields are $y_t = a_n + b_n^\top v_t$ with $a_n = -n^{-1}\bar{a}_n$ and $b_n = -n^{-1}\bar{b}_n$.

Appendix A.2 USD foreign bonds subject to default risk

We conjecture that the price of a foreign bond denominated in US dollars can be written as $\tilde{Q}_t^n = \exp\left(\tilde{a}_n + \tilde{b}_n^\top v_t\right)$ for unknown coefficients $\tilde{a}_n$ and $\tilde{b}_n$. An $n = 1$ period foreign bond is

$$\begin{aligned}
\widetilde{Q}_{jt}^1 &= \mathrm{E}_t\left[M_{t,t+1}\left[(1 - z_{j,t+1}) + (1 - L)\, z_{j,t+1}\right]\right] \\
\widetilde{Q}_{jt}^1 &= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\lambda_t^\top \epsilon_{t+1} - \sum_{i=1}^N \left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widetilde{Q}_{jt}^1 &= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t\right) \mathrm{E}_t\left[\exp\left(-\lambda_t^\top \epsilon_{t+1}\right)\right] \\
&\quad \left[1 - L\exp\left(-\sum_{i=1}^N h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right) \mathrm{E}_t\left[z_{j,t+1}\exp\left(-\sum_{i=1}^N \gamma_{i,t} z_{i,t+1}\right)\right]\right] \\
\widetilde{Q}_{jt}^1 &= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t\right)\left[1 - L h_{jt}\exp\left(-\gamma_{jt}\right)\right] \mathrm{E}\left[\exp\left(-\lambda_t^\top \epsilon_{t+1}\right)\right] \\
\widetilde{Q}_{jt}^1 &= \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t\right)\exp\left(\log\left[1 - L h_{jt}^*\right]\right) \\
\widetilde{Q}_{jt}^1 &\approx \exp\left(-\delta_{y,0} - \delta_{y,v}^\top v_t\right)\exp\left(-L h_{jt}^*\right)
\end{aligned}$$

where the last line follows from a Taylor series approximation $\log(1 + x) \approx x$ for small x. The initial loadings are

$$\begin{aligned}
\widetilde{a}_1 &= -\delta_{y,0} - L\delta_{h,0} - L\delta_{h,v}^\top \\
\widetilde{b}_1 &= -L\delta_{h,v} - \delta_{y,v}
\end{aligned}$$

Assuming the pricing equation holds for an $n - 1$ period bond, the price of an n-period bond is

$$\begin{aligned}
\widetilde{Q}_{jt}^n &= \mathrm{E}_t\left[M_{t,t+1}\widetilde{Q}_{j,t+1}^{n-1}\left[(1 - z_{j,t+1}) + (1 - L)\, z_{j,t+1}\right]\right] \\
\widetilde{Q}_t^n &= \mathrm{E}_t\left[M_{t,t+1}\exp\left(\widetilde{a}_{n-1} + \widetilde{b}_{n-1}^\top v_{t+1}\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widetilde{Q}_{jt}^n &= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(\widetilde{b}_{n-1}^\top v_{t+1} - \lambda_t^\top \epsilon_{t+1} - \sum_{i=1}^N \left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widetilde{Q}_{jt}^n &= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t + \widetilde{b}_{n-1}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n-1}\right)^\top \epsilon_{t+1} - \sum_{i=1}^N \left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widetilde{Q}_{jt}^n &= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t + \widetilde{b}_{n-1}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n-1}\right)^\top \epsilon_{t+1} - \sum_{i=1}^N \left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widetilde{Q}_{jt}^n &= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t + \widetilde{b}_{n,x}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n,x}\right)^\top \epsilon_{t+1} + \log\left[1 - L h_{j,t}^*\right]\right)\right]
\end{aligned}$$

Next, we take a first-order Taylor series expansion of the log-function $\log\left(-Lh^*_{j,t}\right) \approx 1 - Lh^*_{j,t}$. We get an approximation

$$
\begin{aligned}
\widetilde{Q}^n_t &\approx \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t + \widetilde{b}_{n,x}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n,x}\right)^\top \epsilon_{t+1} - Lh^*_{j,t}\right)\right] \\
&= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - Lh^*_{j,t} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t + \widetilde{b}_{n-1}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n,x}\right)^\top \epsilon_{t+1}\right)\right] \\
&= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - Lh^*_{j,t} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top \lambda_t + \widetilde{b}_{n-1}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t\right) \\
&\quad \exp\left(\frac{1}{2}\left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n,x}\right)^\top \left(\lambda_t - \Sigma_v^\top \widetilde{b}_{n,x}\right)\right) \\
&= \exp\left(\widetilde{a}_{n-1} - \delta_{y,0} - Lh^*_{j,t} - \delta_{y,v}^\top v_t + \widetilde{b}_{n-1}^\top \mu_v + \widetilde{b}_{n-1}^\top \Phi_v v_t - \lambda_t^\top \Sigma_v^\top \widetilde{b}_{n-1} + \frac{1}{2}\widetilde{b}_{n-1}^\top \Sigma_v \Sigma_v^\top \widetilde{b}_{n-1}\right)
\end{aligned}
$$

This implies the bond loadings are

$$
\begin{aligned}
\widetilde{a}_n &= \widetilde{a}_{n-1} - \delta_{y,0} - L\delta_{h,0} + \widetilde{b}_{n-1}^\top \left(\mu_v - \lambda_0\right) + \frac{1}{2}\widetilde{b}_{n-1}^\top \Sigma_v \Sigma_v^\top \widetilde{b}_{n-1} \\
\widetilde{b}_n &= \left(\Phi_v - \lambda_v\right)^\top \widetilde{b}_{n-1} - \delta_{y,v} - L\delta_{h,v}
\end{aligned}
$$

Appendix A.3 LC foreign bonds subject to default

We conjecture that the price of a foreign bond denominated in US dollars can be written as $\widehat{Q}_t^n = \exp\left(\widehat{a}_n + \widehat{b}_n^\top v_t\right)$ for unknown coefficients $\widehat{a}_n$ and $\widehat{b}_n$. For an $n = 1$ period bond, we find

$$\begin{aligned}
\widehat{Q}_{jt}^1 &= \mathrm{E}_t\left[M_{t,t+1}\frac{S_{t+1}}{S_t}\left[(1-z_{t+1}) + (1-L)\, z_{j,t+1}\right]\right] \\
\widehat{Q}_{jt}^1 &= \mathrm{E}_t\left[M_{t,t+1}\exp\left(\Delta s_{t+1}\right)\left[1 - L z_{t+1}\right]\right] \\
\widehat{Q}_{jt}^1 &= \exp\left(\delta_{s,0} - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(\delta_{s,v}^\top v_{t+1} - k z_{j,t+1} - \lambda_t^\top\epsilon_{t+1} - \sum_{i=1}^N\left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widehat{Q}_{jt}^1 &= \exp\left(\delta_{s,0} + \delta_{s,v}^\top\mu_v + \delta_{s,v}^\top\Phi_v v_t - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)^\top\epsilon_{t+1} - k z_{j,t+1} - \sum_{i=1}^N\left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widehat{Q}_{jt}^1 &= \exp\left(\delta_{s,0} + \delta_{s,v}^\top\mu_v + \delta_{s,v}^\top\Phi_v v_t - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)^\top\epsilon_{t+1}\right)\exp\left(-k z_{j,t+1} - \sum_{i=1}^N\left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right] \\
\widehat{Q}_{jt}^1 &= \exp\left(\delta_{s,0} + \delta_{s,v}^\top\mu_v + \delta_{s,v}^\top\Phi_v v_t - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) \\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)^\top\epsilon_{t+1}\right)\right] \\
&\quad \mathrm{E}_t\left[\exp\left(-k z_{j,t+1} - \sum_{i=1}^N\left[\gamma_{i,t} z_{i,t+1} + h_{i,t}\left(e^{-\gamma_{i,t}} - 1\right)\right]\right)\left[1 - L z_{j,t+1}\right]\right]
\end{aligned}$$

Next, we integrate out $z_{i,t+1}$ to get

$$\begin{aligned}
\widehat{Q}_{jt}^1 &= \exp\left(\delta_{s,0} + \delta_{s,v}^\top\mu_v + \delta_{s,v}^\top\Phi_v v_t - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) \\
&\quad \exp\left(-h_{j,t}^* K\right)\left[1 - L h_{jt}^*\exp\left(-k\right)\right] \\
&\quad \exp\left(\frac{1}{2}\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)^\top\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)\right)
\end{aligned}$$

Then, we take a Taylor series expansion

$$\begin{aligned}
\widehat{Q}_{jt}^1 &\approx \exp\left(\delta_{s,0} + \delta_{s,v}^\top\mu_v + \delta_{s,v}^\top\Phi_v v_t - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) \\
&\quad \exp\left(-\left[K + L - LK\right] h_{j,t}^*\right) \\
&\quad \exp\left(\frac{1}{2}\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)^\top\left(\lambda_t - \Sigma_v^\top\delta_{s,v}\right)\right)
\end{aligned}$$

where we define $K = \left(1 - e^{-k}\right)$ and $\widehat{K} = K + L - LK$. This gives

$$\begin{aligned}
\widehat{Q}_{jt}^1 &= \exp\left(\delta_{s,0} + \delta_{s,v}^\top\mu_v + \delta_{s,v}^\top\Phi_v v_t - \delta_{y,0} - \delta_{y,v}^\top v_t - \frac{1}{2}\lambda_t^\top\lambda_t - \widehat{K} h_{j,t}^*\right) \\
&\quad \exp\left(\frac{1}{2}\lambda_t^\top\lambda_t - \lambda_t^\top\Sigma_v^\top\delta_{s,v} + \frac{1}{2}\delta_{s,v}^\top\Sigma_v\Sigma_v^\top\delta_{s,v}\right)
\end{aligned}$$

The bond loadings at $n=1$ are

$$\begin{aligned}\widehat{a}_1 &= \delta_{s,0}+\delta_{s,v}^{\top}(\mu_v-\lambda_0)-\delta_{y,0}-\widehat{K}\delta_{h,0}+\frac{1}{2}\delta_{s,v}^{\top}\Sigma_v\Sigma_v^{\top}\delta_{s,v}\\ \widehat{b}_1 &= (\Phi_v-\lambda_v)^{\top}\delta_{s,v}-\delta_{y,v}-\widehat{K}\delta_{h,v}\end{aligned}$$

For a general n period bond, we find

$$\begin{aligned}
\widehat{Q}_{jt}^{n} &= \mathrm{E}_t\left[M_{t,t+1}\widehat{Q}_{t+1}^{n-1}\frac{S_{t+1}}{S_t}\left[(1-z_{j,t+1})+(1-L)\,z_{j,t+1}\right]\right]\\
\widehat{Q}_{jt}^{n} &= \mathrm{E}_t\left[M_{t,t+1}\widehat{Q}_{t+1}^{n-1}\exp(\Delta s_{t+1})\left[1-Lz_{j,t+1}\right]\right]\\
\widehat{Q}_{jt}^{n} &= \exp\left(\widehat{a}_{n-1}+\delta_{s,0}-\delta_{y,0}-\delta_{y,v}^{\top}v_t-\frac{1}{2}\lambda_t^{\top}\lambda_t\right)\\
&\quad \mathrm{E}_t\left[\exp\left(\left(\bar{b}_{n-1}+\delta_{s,v}\right)^{\top}v_{t+1}-\lambda_t^{\top}\epsilon_{t+1}-kz_{j,t+1}-\sum_{i=1}^{N}\left[\gamma_{i,t}z_{i,t+1}+h_{i,t}\left(e^{-\gamma_{i,t}}-1\right)\right]\right)\left[1-Lz_{j,t+1}\right]\right]\\
\widehat{Q}_{jt}^{n} &= \exp\left(\widehat{a}_{n-1}+\delta_{s,0}+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\mu_v+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\Phi_v v_t-\delta_{y,0}-\delta_{y,v}^{\top}v_t-\frac{1}{2}\lambda_t^{\top}\lambda_t\right)\\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)^{\top}\epsilon_{t+1}-kz_{j,t+1}-\sum_{i=1}^{N}\left[\gamma_{i,t}z_{i,t+1}+h_{i,t}\left(e^{-\gamma_{i,t}}-1\right)\right]\right)\left[1-Lz_{j,t+1}\right]\right]\\
\widehat{Q}_{jt}^{n} &= \exp\left(\widehat{a}_{n-1}+\delta_{s,0}+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\mu_v+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\Phi_v v_t-\delta_{y,0}-\delta_{y,v}^{\top}v_t-\frac{1}{2}\lambda_t^{\top}\lambda_t\right)\\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)^{\top}\epsilon_{t+1}\right)\exp\left(-kz_{j,t+1}-\sum_{i=1}^{N}\left[\gamma_{i,t}z_{i,t+1}+h_{i,t}\left(e^{-\gamma_{i,t}}-1\right)\right]\right)\left[1-Lz_{j,t+1}\right]\right]\\
\widehat{Q}_{jt}^{n} &= \exp\left(\widehat{a}_{n-1}+\delta_{s,0}+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\mu_v+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\Phi_v v_t-\delta_{y,0}-\delta_{y,v}^{\top}v_t-\frac{1}{2}\lambda_t^{\top}\lambda_t\right)\\
&\quad \mathrm{E}_t\left[\exp\left(-\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)^{\top}\epsilon_{t+1}\right)\right]\\
&\quad \mathrm{E}_t\left[\exp\left(-kz_{j,t+1}-\sum_{i=1}^{N}\left[\gamma_{i,t}z_{i,t+1}+h_{i,t}\left(e^{-\gamma_{i,t}}-1\right)\right]\right)\left[1-Lz_{j,t+1}\right]\right]
\end{aligned}$$

Calculate the integral

$$\begin{aligned}
\widehat{Q}_{jt}^{n} &= \exp\left(\widehat{a}_{n-1}+\delta_{s,0}+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\mu_v+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\Phi_v v_t-\delta_{y,0}-\delta_{y,v}^{\top}v_t-\frac{1}{2}\lambda_t^{\top}\lambda_t\right)\\
&\quad \exp\left(\frac{1}{2}\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)^{\top}\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)\right)\\
&\quad \exp\left(-h_{j,t}^{*}K\right)\left[1-Lh_{jt}^{*}\exp(-k)\right]
\end{aligned}$$

Next, we use a Taylor series expansion

$$\begin{aligned}
\widehat{Q}_{jt}^{n} &\approx \exp\left(\widehat{a}_{n-1}+\delta_{s,0}+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\mu_v+\left(\delta_{s,v}+\widehat{b}_{n-1}\right)^{\top}\Phi_v v_t-\delta_{y,0}-\delta_{y,v}^{\top}v_t-\frac{1}{2}\lambda_t^{\top}\lambda_t\right)\\
&\quad \exp\left(\frac{1}{2}\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)^{\top}\left(\lambda_t-\Sigma_v^{\top}\left(\widehat{b}_{n-1}+\delta_{s,v}\right)\right)\right)\\
&\quad \exp\left(-h_{j,t}^{*}\left[K+L-LK\right]\right)
\end{aligned}$$

This implies that

$$
\begin{aligned}
\widehat{a}_n &= \widehat{a}_{n-1} + \delta_{s,0} - \delta_{y,0} - \widehat{K}\delta_{h,0} + \left(\widehat{b}_{n-1} + \delta_{s,v}\right)^\top (\mu_v - \lambda_0) \\
&\quad + \frac{1}{2}\left(\widehat{b}_{n-1} + \delta_{s,v}\right)^\top \Sigma_v \Sigma_v^\top \left(\widehat{b}_{n-1} + \delta_{s,v}\right) \\
\widehat{b}_n &= (\Phi_v - \lambda_v)^\top \left(\widehat{b}_{n-1} + \delta_{s,v}\right) - \delta_{y,v} - \widehat{K}\delta_{h,v}
\end{aligned}
$$

where $\widehat{K} = K + L - LK$.

Appendix B Estimation

Appendix B.1 Observables

We observe a vector of exchange rates Δs_t, macro economic variables, and yield factors. We place these in the state vector v_t as in (6). In addition to the state variables, we observe one-period returns on US zero coupon bonds $r_t^{n-1,n}$ for monthly maturities $n = (6, 12, 24, 36, 48, 60)$. We also observe returns on foreign bonds denominated in foreign currency $\widehat{r}_t^{n-1,n}$ for maturities $n = (6, 12, 24, 36, 48, 60)$. Finally, we also observe returns on foreign bonds denominated in US dollars $\tilde{r}_t^{n-1,n}$ for maturities $n = (24, 36, 48, 60, 120)$. We stack all these returns in a vector r_t. We write these returns as

$$
\begin{aligned}
r_{t+1}^{n,n-1} &= \bar{c}_{n-1} + \bar{\psi}_{n-1}^\top v_t + \bar{b}_{n-1}^\top v_{t+1} && \text{(B.3)} \\
\tilde{r}_{t+1}^{n,n-1} &= \tilde{c}_{n-1} + \tilde{\psi}_{n-1}^\top v_t + \tilde{b}_{n-1}^\top v_{t+1} && \text{(B.4)} \\
\widehat{r}_{t+1}^{n,n-1} &= \widehat{c}_{n-1} + \widehat{\psi}_{n-1}^\top v_t + \widehat{b}_{n-1}^\top v_{t+1} && \text{(B.5)}
\end{aligned}
$$

where the loadings are taken from the expressions for bond prices

$$
\begin{aligned}
\bar{c}_{n-1} &= \delta_{y,0} - \bar{b}_{n-1}^\top (\mu_v - \lambda_0) - \frac{1}{2}\bar{b}_{n-1}^\top \Sigma_v \Sigma_v^\top \bar{b}_{n-1} && \text{(B.6)} \\
\widetilde{c}_{n-1} &= \delta_{y,0} + L\delta_{h,0}^* - \widetilde{b}_{n-1}^\top (\mu_v - \lambda_0) - \frac{1}{2}\widetilde{b}_{n-1}^\top \Sigma_v \Sigma_v^\top \widetilde{b}_{n-1} && \text{(B.7)} \\
\widehat{c}_{n-1} &= \delta_{y,0} + \widehat{K}\delta_{h,0}^* - \delta_{s,0} - \left(\widehat{b}_{n-1} + \delta_{s,v}\right)^\top (\mu_v - \lambda_0) && \text{(B.8)} \\
&\quad - \frac{1}{2}\left(\widehat{b}_{n-1} + \delta_{s,v}\right)^\top \Sigma_v \Sigma_v^\top \left(\widehat{b}_{n-1} + \delta_{s,v}\right) && \text{(B.9)} \\
\bar{\psi}_{n-1}^\top &= -\bar{b}_{n-1}^\top (\Phi_v - \lambda_v)^\top + \delta_{y,v}^\top && \text{(B.10)} \\
\widetilde{\psi}_{n-1}^\top &= -\widetilde{b}_{n-1}^\top (\Phi_v - \lambda_v) + \delta_{y,v}^\top + L\delta_{h,v}^{*,\top} && \text{(B.11)} \\
\widehat{\psi}_{n-1}^\top &= -\left(\widehat{b}_{n-1} + \delta_{s,v}\right)^\top (\Phi_v - \lambda_v) + \delta_{y,v}^\top + \widehat{K}\delta_{h,v}^{*,\top} && \text{(B.12)}
\end{aligned}
$$

Collecting all countries together, the vector of returns r_t has dimension $n_r \times 1$ with $n_r = 61$.

We write the model as

$$
\begin{aligned}
v_t &= \mu_v + \Phi_v v_{t-1} + \Sigma_v \varepsilon_t \qquad \varepsilon_t \sim \mathrm{N}(0, \mathbf{I}) && \text{(B.13)} \\
r_t &= C + \Psi v_{t-1} + B v_t + \eta_t \qquad \eta_t \sim \mathrm{N}(0, \Omega) && \text{(B.14)}
\end{aligned}
$$

where η_t are a vector of *i.i.d.* measurement errors. We assume the matrix Ω is diagonal with asset specific variances $\omega_{i,i}^2$ for $i = 1, \ldots, n_r$. The vector C and matrices Ψ and B contain the bond loadings (B.6)-(B.12) stacked in order of ascending maturities. We note that the parameters λ_0 and $\bar{\mu}_v$ enter the vector C while Ψ is a function of the matrices Φ_v and λ_v. For future reference below, we can write these as

$$C = C_c + C_{\bar{\mu}}\bar{\mu}_x + C_\lambda \lambda_0 \tag{B.15}$$

$$\Psi \cdot v_{t-1} = \Psi_c + \Psi_{\phi,t-1}\text{vec}\left(\Phi_v\right) + \Psi_{\lambda,t-1}\text{vec}\left(\lambda_v\right) \tag{B.16}$$

where $\Psi_{\phi,t-1}$ and $\Psi_{\lambda,t-1}$ are functions of v_{t-1}.

In practice, both the state vector v_t and the observation vector r_t contain missing values. During the MCMC algorithm, we impute the missing values in the state vector v_t using the Kalman filter. We do not impute the missing values in returns r_t.

Appendix B.2 Identification and restrictions

We model the macroeconomic factors for inflation and industrial production as unspanned. This implies that the columns of the factor loadings B associated with inflation or industrial production are a-priori equal to zero. Under this assumption, we cannot identify the rows of λ_0 and λ_v associated with these factors because they always enter the model multiplicatively as $B\lambda_0$ and $B\lambda_v$. We set these rows of λ_0 and λ_v equal to zero.

We also impose the restrictions on λ_0 and λ_v associated with covered interest parity (CIP). This restriction would be automatically imposed in a no-arbitrage model. The CIP restriction implies that rows of the risk neutral drift μ_v^* and rows of the risk neutral autocovariance matrix Φ_v^* associated with any depreciation rates must satisfy the conditions

$$\begin{aligned} \mathbf{e}_{s,i}^\top \mu_v^* &= -\frac{1}{2}\mathbf{e}_{s,i}^\top \Sigma_v \Sigma_v^\top \mathbf{e}_{s,i} \\ \mathbf{e}_{s,i}^\top \Phi_v^* &= \mathbf{e}_{\Delta_c \ell,i}^\top \end{aligned}$$

We use $\mathbf{e}_{s,i}$ to denote a unit vector that selects out the depreciation rate of the i-th country. Similarly, we use $\mathbf{e}_{\Delta_c \ell,i}$ to denote a unit vector that selects out of the state vector the interest rate differential of the i-th country.

Since we parameterize the model in terms of λ_0 and λ_v, the CIP restrictions imply that

$$\begin{aligned} \mathbf{e}_{s,i}^\top \lambda_0 &= \mathbf{e}_{s,i}^\top \mu_v + \frac{1}{2}\mathbf{e}_{s,i}^\top \Sigma_v \Sigma_v^\top \mathbf{e}_{s,i} \\ \mathbf{e}_{s,i}^\top \lambda_v &= \mathbf{e}_{s,i}^\top \Phi_v - \mathbf{e}_{\Delta_c \ell,i}^\top \end{aligned}$$

These state that rows of λ_0 and λ_v associated with the depreciation rate must be restricted.

Appendix B.3 Prior distributions

Given the large number of parameters in the model, we use informative priors for all the parameters of the model.

- Let Ω be a diagonal matrix with dimension $n_r \times n_r$. Note that r_t has dimension $n_r \times 1$. We assume that each element ω_{ii}^2 has prior distribution $\omega_{ii}^2 \sim \text{IG}\left(\underline{\nu}_{r,i}, \underline{w}_{r,i}\right)$. We calculate the unconditional sample variance of each return $\bar{v}_{r,i} = V\left(r_{i,t}\right)$. For each return i, we calculate the shape and scale parameters of the prior distribution as $\underline{w}_{r,i} = 2 + \bar{v}_{r,i}^2/(0.95 \cdot \bar{v}_{r,i})$ and $\underline{\nu}_{r,i} = \left(\underline{w}_{r,i} - 1\right)\bar{v}_{r,i}$.

- We place a prior distribution on the individual elements of the matrix $\Sigma_v \Sigma_v^\top = L_v D_v L_v^\top$. For the diagonal elements of D_v, we place inverse Gamma priors $d_{v,i} \sim \text{IG}\left(\underline{\nu}_{x,i}, \underline{w}_{x,i}\right)$ on each of the diagonal elements. We calculate the unconditional sample variance of each of the state variables $\bar{v}_{x,i} = V\left(x_{i,t}\right)$ and set $E\left(d_{v,i}\right) = \bar{v}_{x,i}$. For each state variable i, we calculate the shape and scale parameters of the prior distribution as $\underline{w}_{x,i} = 2 + \bar{v}_{x,i}^2/(0.5 \cdot \bar{v}_{x,i})$ and $\underline{\nu}_{x,i} = \left(\underline{w}_{x,i} - 1\right)\bar{v}_{x,i}$. The matrix L_v is lower triangular with individual values $\ell_{v,ij}$. We place an independent prior $\ell_{v,ij} \sim N\left(0,1\right)$ on these values.
- We place a prior on the unconditional mean of the state vector $\bar{\mu}_v \sim \text{N}\left(\underline{\mu}_v, \underline{V}_\mu\right)$. First, we calculate the unconditional sample mean of the factors $\bar{x}$ and set $\underline{\mu}_v = \bar{x}$. We set $\underline{V}_\mu$ to be a diagonal matrix and we choose the variances to be large enough to cover the support of the data.
- We place a joint prior on the matrix Φ_x by considering the individual elements $\phi_{x,ij}$. We place a normal prior on the diagonal elements $\phi_{x,ii} \sim \text{N}\left(0.8, 0.025\right)$. For the off-diagonal elements, we use a normal prior with a larger variance $\phi_{x,ij} \sim \text{N}\left(0,1\right)$. The "joint" prior comes from truncating the distribution over Φ_x to only accept draws in the stationarity region.
- The vector λ_0 needs restricted for identification as discussed in Appendix B.2. Let λ_0^r denote the vector of unrestricted parameters. These values can be written as

$$\lambda_0 \quad = \quad S_c + S_\mu \mu_v + S_\lambda \lambda_0^r$$

where S_c, S_μ, and S_λ are selection matrices that impose the identifying and CIP restrictions. The vector λ_0^r has prior $\lambda_0^r \sim \text{N}\left(\underline{\mu}_\lambda, \underline{V}_\lambda\right)$. We set $\underline{\mu}_\lambda = 0$. The covariance matrix $\underline{V}_\lambda$ is diagonal with variances chosen so that the unconditional annual Sharpe Ratio's has a high probability of being less than 1.
- The matrix λ_v needs restricted for identification as discussed in Appendix B.2. The matrix λ_v has individual elements $\lambda_{v,ij}$. We use a normal prior distribution on each element $\lambda_{v,ij} \sim \text{N}\left(0, 0.001\right)$. Again, the variance of this distribution is chosen so that the conditional annual Sharpe Ratio's has a high probability of being less than 1.
- The matrix of factor loadings B in equation B.3 has many free parameters. For returns of different maturities but of the same kind of bond (e.g. holding period returns on US bonds), we note that the bond loadings of different maturities $\bar{b}_n$ and $\bar{b}_{n-1}$ should be related through the recursion (A.2). This implies that the columns of the matrix B in (B.14) should be smooth functions of maturity. To impose this structure, we assign a conditional prior to the entries $b_{i,j}$ of the matrix B. As long as the return belongs to the same type of asset class (e.g. holding period returns on US bonds), we set $b_{i,j} \sim \text{N}\left(0.9 b_{i-1,j}, 0.01\right)$. This implies that bond loadings for each factor j on returns of the same asset class are correlated. The bond loadings are independent across different factors j. Furthermore, the loadings $b_{i,j}$ on returns across asset classes are uncorrelated, e.g. loadings for US bonds are unrelated to Chinese bonds.

In practice, some data points are missing which implies that some of the state variables v_t are missing. We use $(v_{1:T}^o, r_{1:T}^o)$ to denote the observed data and $v_{1:T}^m$ to denote the missing data, respectively. The log-likelihood function is

$$\mathcal{L} \quad = \quad \log p\left(v_1^o, \ldots, v_T^o, r_1^o, \ldots, r_T^o | \theta\right) \; = \; \sum_{t=1}^{T} \log p\left(v_t^o | v_{t-1}^o, \theta\right) + \sum_{t=1}^{T} \log p\left(r_t^o | v_t^o; \theta\right)$$

where v_0 are equal to the unconditional mean. We note that the dimensions of v_t and r_t change through time. The likelihood with missing observations can still be computed using the Kalman filter.

Appendix B.4 Estimation

Let θ denote all the parameters of the model and define $v_{1:T} = (v_1, \ldots, v_T)$ and $r_{1:T} = (r_1, \ldots, r_T)$. The joint posterior distribution over the parameters and missing data is given by

$$p\left(\theta, v_{1:T}^m | v_{1:T}^o, r_{1:T}^o\right) \quad \propto \quad p\left(v_{1:T}^o, r_{1:T}^o | \theta\right) p\left(\theta\right),$$

where $p\left(v^o_{1:T}, r^o_{1:T}|\theta\right)$ is the likelihood of the observed data and $p\left(\theta\right)$ is the prior distribution. We use Markov-chain Monte Carlo to draw from the posterior.

Appendix B.4.1 MCMC algorithm

We provide a brief description of the MCMC algorithm. We use a Gibbs sampler that iterates between drawing from each of the full conditional distributions.

- Place the model in linear, Gaussian state space form as described in Appendix B.4.2. Draw the state variables and unconditional means $(v^o_{1:T}, \bar{\mu}_v, \lambda^r_0)$ from their full conditional distribution using the Kalman filter and simulation smoothing algorithm.
- Draw the factor loadings in the matrix B of the returns

$$r_t \quad = \quad C + \Psi v_{t-1} + B v_t + \eta_t$$

We draw the free parameters in each row of B, row-by-row. These are the vectors $\bar{b}_{n-1}, \tilde{b}_{n-1}$ and $\widehat{b}_{n-1}$ in the bond loadings (B.3)-(B.5). For the current discussion, we denote each row of B as b_i. The vector b_i also enters each row of the vector Γ and the matrix Ψ in the expressions (B.6)-(B.12). With a Gaussian prior on the parameters in each row, the full conditional posterior is unknown because b_i enters each row of Γ as a quadratic form, e.g. the Jensen's inequality term $b_i^\top \Sigma_v \Sigma_v^\top b_i$. Consequently, the full conditional distribution is unknown and we use a Metropolis-Hastings algorithm. We note that if this term were linear in the loadings b_i, then the full-conditional would be a standard draw from a linear regression model. Therefore, our proposal distribution approximates the quadratic term by setting the quadratic term to a constant $b_i^\top \Sigma_v \Sigma_v^\top \mathbf{b}_i$ where $\mathbf{b}_i$ is fixed from previous runs of the algorithm. We then use the linear regression model as our proposal inside the Metropolis-Hastings algorithm. The acceptance rates for each row of B are roughly 70%.
- Draw the free elements of $\Sigma_v \Sigma_v^\top = L_v D_v L_v^\top$ from their full conditional using a random-walk Metropolis algorithm. We tuned the covariance matrix of the random-walk based on previous estimates from the algorithm with a targeted acceptance rate of roughly 25-30%. In practice, we propose the diagonal elements of the matrix D_v using a logarithmic transformation $\log\left(d_{v,i}\right)$ to guarantee that the proposal for $d_{v,i}$ is positive. In this step, we avoid conditioning on the missing values $v^m_{1:T}$ by using the Kalman filter to calculate the log-likelihood within the Metropolis-Hastings acceptance ratio.
- Let $\bar{v}_t = v_t - \bar{\mu}_v$ denote the demeaned factors. We draw the free elements of Φ_v and λ_v from their full conditional distribution using standard results for Bayesian multiple regression. We write the model as a regression model

$$Y_t = X_t \beta + \varepsilon_t$$

where $Y_t = \left(\bar{v}_t^\top \ d_t^\top\right)^\top$ and $\beta = \left(\mathrm{vec}\left(\Phi_x\right)^\top \ \mathrm{vec}\left(\lambda_v\right)^\top\right)^\top$. The regressors X_t contain lagged and contemporaneous values of $\bar{v}_{t-1}, v_{t-1}$ and v_t. The left-hand side variables d_{it} for $i = 1, \ldots, n_r$ are equal to the returns minus any known constants. For example, for a return on US bonds we write

$$d_{it} = r_{it} - c_i - \psi_{i,c} - b_i^\top v_t$$

where c_i and $\psi_{i,c}$ are the coefficients in the representation (B.15)-(B.16). The missing returns r_{it} are omitted in the vector Y_t by selecting out only those equations that are observed at each relevant date. Otherwise, draws from this model are standard for Bayesian multiple regression.
- Let T_i denote the number of observed values for the i-th return r_{it}. The full conditional posterior distribution of the diagonal elements of Ω are $\omega^2_{ii} \sim \text{Inv-Gamma}\left(\bar{\nu}_{r,i}, \bar{\kappa}_{r,i}\right)$ where $\bar{\nu}_{r,i} = \underline{w}_{r,i} + T_{r,i}$ and $\bar{\kappa}_{r,i} = \underline{w}_{r,i} + \frac{1}{2}\sum_{t=1}^{T_i}\left(r_{it} - c_i - \psi_i' v_{t-1} - b_i' v_t\right)^2$ where c_i, ψ_i, and b_i denote the relevant entries of return i in C, Ψ and B of equation (B.14).

Appendix B.4.2 State space form

We place this model in the following linear, Gaussian state space form

$$Y_t = Z\alpha_t + d + u_t \qquad u_t \sim \mathrm{N}\left(0, H\right), \tag{B.17}$$

$$\alpha_{t+1} = T\alpha_t + c + Rv_t \qquad v_t \sim \mathrm{N}\left(0, Q\right). \tag{B.18}$$

where the initial condition is $\alpha_1 \sim \mathrm{N}\left(a_{1|0}, P_{1|0}\right)$.

We draw the vectors $\bar{\mu}_x$ and λ_0^r jointly with the missing state variables by including them in the state vector. We define the system matrices from (B.17)-(B.18) as

$$d = \begin{pmatrix} 0 \\ C_c + C_\lambda S_c \end{pmatrix} \qquad Z = \begin{pmatrix} \mathrm{I} & 0 & 0 & 0 \\ B & \Psi & C_{\bar{\mu}_x} + C_\lambda S_\mu & C_\lambda S_\lambda \end{pmatrix} \qquad H = \begin{pmatrix} 0 & 0 \\ 0 & \Omega \end{pmatrix} \qquad Q = \mathrm{I}$$

$$\alpha_t = \begin{pmatrix} v_t \\ v_{t-1} \\ \bar{\mu}_v \\ \lambda_0^r \end{pmatrix} \qquad T = \begin{pmatrix} \Phi_v & 0 & (\mathrm{I} - \Phi_v) & 0 \\ \mathrm{I} & 0 & 0 & 0 \\ 0 & 0 & \mathrm{I} & 0 \\ 0 & 0 & 0 & \mathrm{I} \end{pmatrix} \qquad c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad R = \begin{pmatrix} \Sigma_v \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$a_{1|0} = \begin{pmatrix} \underline{\mu}_v \\ \underline{\mu}_v \\ \underline{\mu}_v \\ \underline{\lambda}^r \end{pmatrix} \qquad P_{1|0} = \begin{pmatrix} \Sigma_v \Sigma_v^\top + \underline{V}_\mu & \underline{V}_\mu & \underline{V}_\mu & 0 \\ \underline{V}_\mu & \underline{V}_\mu & \underline{V}_\mu & 0 \\ \underline{V}_\mu & \underline{V}_\mu & \underline{V}_\mu & 0 \\ 0 & 0 & 0 & \underline{V}_\lambda \end{pmatrix}$$

We use the Kalman filter and simulation smoothing algorithm to draw the missing values and the unconditional means jointly.

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