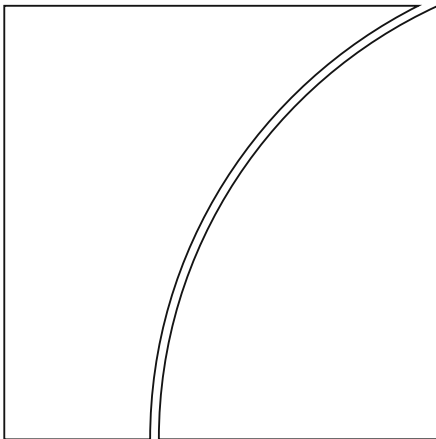




BANK FOR INTERNATIONAL SETTLEMENTS



BIS Working Papers

No 991

Monetary policy and endogenous financial crises

by Frederic Boissay, Fabrice Collard, Jordi Galí and
Cristina Manea

Monetary and Economic Department

January 2022 (revised July 2024)

JEL classification: E1, E3, E6, G01.

Keywords: Inflation targeting, low-rate-for-long,
search for yield, financial instability, backstops.

BIS Working Papers are written by members of the Monetary and Economic Department of the Bank for International Settlements, and from time to time by other economists, and are published by the Bank. The papers are on subjects of topical interest and are technical in character. The views expressed in them are those of their authors and not necessarily the views of the BIS.

This publication is available on the BIS website (www.bis.org).

© *Bank for International Settlements 2022. All rights reserved. Brief excerpts may be reproduced or translated provided the source is stated.*

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

Monetary Policy and Endogenous Financial Crises

F. Boissay* F. Collard† J. Galí‡ C. Manea§¶

July 30, 2024

Abstract

What are the channels through which monetary policy affects financial stability? Can (and should) central banks prevent financial crises by tolerating higher price volatility? To what extent may monetary policy itself brew financial fragility? We study these questions through the lens of a textbook New Keynesian model augmented with capital accumulation and endogenous financial crises due to adverse selection in credit markets. Our main findings are threefold. First, monetary policy affects the probability of a crisis not only in the short-term (through its usual effects on aggregate demand) but also over the medium-term (through its effects on capital accumulation). Second, the central bank can significantly reduce the incidence of financial crises in the medium-term by tolerating higher price volatility in the short-term. Third, financial crises may occur after a long period of loose monetary policy, as the central bank abruptly reverses course and hikes its policy rate.

Keywords: Inflation targeting, low-for-long policy rate, adverse selection, financial crises

JEL classification: E1, E3, E6, G01.

*Bank for International Settlements. Email: frederic.boissay@bis.org.

†Toulouse School of Economics (CNRS). Email: fabrice.collard@gmail.com.

‡CREI, Universitat Pompeu Fabra and Barcelona School of Economics. Email: jgali@crei.cat.

§Bank for International Settlements. Email: cristina.manea@bis.org.

¶The views expressed in this paper are our own and should not be interpreted as reflecting the views of the Bank for International Settlements. We thank our discussants H. Ai, I. Correia, K. Dogra, F. Gourio, G. Nuno, M. Piazzesi, C. Rogers and F. Romei, as well as F. Alvarez, G. Barlevy, P. Beaudry, C. Borio, A. Fabiani, F. De Fiore, M. Del Negro, P. Disyatat, G. Galati, A. Gutiérrez-Daza, M. Hoffmann, T. Holden, E. Mendoza, B. Mojon, C. Pflueger, F. Ravenna, P. Rungcharoentkitkul, S. Schmitt-Grohé, J. Sim, L. Svensson, O. Tristani, H. Uhlig, I. Vetlov, R. Wouters, E. Zakrajšek, J. Weber and seminar and conference participants at the BdF-EUI conference, NYU Conference in celebration of Mark Gertler’s Research, NBER Summer Institute, NBER ME Spring meeting, EEA-ESEM Annual Congress, Ridge Conference, National Bank of Romania, National Bank of Belgium, ECB, Danmarks Nationalbank–Bundesbank–Norges Bank Conference, Bank of Japan, Bank of Korea, HKMA, Deutsche Bundesbank, Atlanta Fed, Kansas City Fed, San Francisco Fed, New York Fed, International Monetary Fund, Vimacro meeting, Bank for International Settlements, Universitat Pompeu Fabra, CREST, CICM2024, Toulouse School of Economics, Trinity College, Bank of Canada Workshop on Monetary Policy, Cattolica Sacro Cuore, T2M, WE ARE IN conference, Padova Macro Talks, Konstanz Monetary Theory Seminar, Oxford Saïd – ETH Zürich macrofinance conference, DNB research seminar, Bank of Japan, Hong-Kong EA Conference, LSE, Banca d’Italia–Bocconi–CEPR Conference on “Financial Stability and Regulation”, Banco de México, Midwest Macro Meetings, University of Warwick MIMA Conference, China International Conference in Macro and SED Annual Meeting for helpful discussions. Galí acknowledges financial support from the European Research Council under the European Union’s Horizon 2020 research and innovation program, Advanced Grant (882332–HEMPEF), from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019–000915–S), and from the Generalitat de Catalunya, through CERCA and SGR Programme (2017–SGR-1393). Collard acknowledges financial support from the Agence Nationale de la Recherche under grant ANR-17-EURE-0010 (Investissements d’Avenir program).

“Credit markets are characterized by imperfect and asymmetric information. These informational frictions can interact with other economic forces to produce periods of credit-market stress (...). A high level of credit-market stress, as in a severe financial crisis, may in turn produce a deep and prolonged recession.”

Bernanke (2023), Nobel Prize Lecture, p. 1

“A prolonged period of low interest rates can create incentives for agents to take on greater credit risks in an effort to reach for yield.”

Stein (2013), p. 6

1 Introduction

The impact of monetary policy on financial stability remains a controversial topic. On the one hand, loose monetary policy can help stave off financial crises. In response to the 9/11 terrorist attacks and Covid-19 pandemic, for example, central banks swiftly lowered interest rates and acted as a backstop to the financial sector. These moves likely prevented a financial collapse that would otherwise have exacerbated the damage to the economy. On the other hand, keeping policy rates low-for-long may fuel a credit and asset price boom and contribute to the build-up of financial vulnerabilities, if not pose risks to financial stability. Recent empirical studies indeed suggest that too loose a monetary policy significantly increases the probability of a financial crisis in the medium-term (Grimm et al. (2023), Jiménez et al. (2023)).¹

This ambivalence prompts the question of the adequate monetary policy in an environment where credit markets are fragile and crises may have varied causes. *What are the channels through which monetary policy affects financial stability? Can (and should) central banks prevent financial crises by tolerating higher price volatility? To what extent may monetary policy itself brew financial vulnerabilities?*

We study these questions through the lens of a novel New Keynesian (NK) model that features a credit market subject to an adverse selection/moral hazard problem. As in Mankiw (1986), Bernanke and Gertler (1990) and Azariadis and Smith (1998), these financial frictions surface when the real returns on investments are low: low capital returns prompt some borrowers to invest in alternative (“below-the-radar”) projects that are privately beneficial but raise the probability of credit default to the detriment of lenders—a behaviour sometimes dubbed “search for yield” (Martinez-Miera and Repullo (2017)).² In our model, low returns may have

¹A case in point is the 2007–8 Great Financial Crisis. Taylor (2011) refers to the period 2003–2005 in the US as the “Great Deviation”, which he characterises as one when monetary policy became less rule-based, less predictable, and excessively loose. Other notable examples of financial crises preceded by loose monetary policy include the Japanese (Ito and Mishkin (2006)) and Swedish (Englund (1999)) crises in the early 1990s.

²In practice, search-for-yield behaviour may come in different guise like excessive/reckless risk-taking, false information disclosure, scams, outright embezzlement (Mishkin (1991), Piskorski et al. (2015), Garmaise (2015), Mian and Sufi (2017)). Garmaise (2015) and Mian and Sufi (2017), for example, provide evidence that sub-prime

varied causes, such as a large adverse non-financial shock or a protracted investment boom driven by low-for-long monetary policy rates. In the latter case, the longer the period of low policy rates, the longer the boom is likely to last and the bigger the capital stock in the economy. Because of decreasing marginal returns, the accumulation of capital eventually exhausts profitable investment opportunities and erodes capital returns, prompting borrowers to search for yield. The consequent rise in moral hazard and credit default risk may then induce prospective lenders to panic and refuse to lend, triggering a sudden collapse of the credit market—a “financial crisis”. Even though default risk is limited to some specific (sub-prime) borrowers, the adverse selection problem and resulting uncertainty as to where the exposure to such borrowers resides suffices to undermine lenders’ confidence. In effect, the root cause of financial fragility in our model is not borrowers’ default *per se*—which is an out-of-equilibrium outcome—but rather lenders’ fear of being defaulted upon.

Our model departs from the textbook three-equation NK model (Galí (2015)) in a few and straightforward ways.

First, we introduce a credit market. Our approach focuses on the central role played by credit markets in the reallocation of capital across heterogeneous firms. In the spirit of Bernanke and Gertler (1990) and Khan and Thomas (2013), we assume that firms are subject to transitory idiosyncratic productivity shocks—in addition to the usual persistent aggregate ones. This heterogeneity induces productive firms to borrow funds on a credit market in order to buy capital from unproductive firms, and unproductive firms to lend the proceeds of the sales of their capital goods.³ A well-functioning credit market thus supports the efficient reallocation of capital from unproductive to productive firms and augments aggregate productivity. In contrast, a dysfunctional credit market entails capital mis-allocation and a fall in aggregate productivity. As Oulton and Sebastiá-Barriel (2016) and Gopinath et al. (2017) show, capital mis-allocation and the resulting fall in total factor productivity are indeed salient empirical features of financial crises.⁴

Second, we introduce frictions in this credit market. Our approach emphasises the central role played by moral hazard and asymmetric information in the fragility of credit markets (Bernanke (2023)). As in other models featuring both frictions (Bernanke and Gertler (1990), Gertler and Rogoff (1990) and Azariadis and Smith (1998)), we assume that borrowers have

borrowers fraudulently overstated their assets and income in order to obtain loans during the 2002–2006 mortgage credit boom in the US, which eventually ended up with the financial panic of 2007 (Gorton (2008), Gorton (2009)). Such practices are well-documented causes of distrust and financial stress (Aliber and Kindleberger (2015), Griffin (2021)).

³Our narrative in terms of inter-firm lending should not be taken at face value but rather interpreted as broadly capturing the role of credit markets in reallocating initially mis-allocated resources. For example, Bernanke and Gertler (1990) note (page 94) that “one may think of this [inter-firm] borrowing as being channeled through competitive financial intermediaries, which use no resources in the process of intermediation and earn no profits in equilibrium”. In Section B.1 in the online appendix, we present a version of our model with banks, in which productive firms borrow from banks to buy capital goods from unproductive firms and the latter deposit the proceeds of the sales in the banks. One may alternatively think of the inter-firm credit market as a rental market for capital, as in Moll (2014).

⁴Additional empirical evidence on capital mis-allocation during financial crises can be found in, e.g., Foster et al. (2016), Duval et al. (2019).

private information about their productivity and that unproductive firms may borrow, engage in below-the-radar activities, abscond, and default —a form of moral hazard. To get unproductive firms (the natural lenders) to sell their capital stock and lend the proceeds of the sale —rather than borrow and abscond— the equilibrium loan rate must be above a minimum threshold. At the same time, for productive firms (the natural borrowers) to afford a loan, their return on capital must be above the loan rate. The upshot is that, when the return on capital falls below the minimum loan rate threshold, productive firms cannot (afford to) get the unproductive ones to lend. Since *all* firms may want to borrow in that case, lenders’ information about prospective borrowers deteriorates and the adverse selection problem kicks in, causing a “panic” —in the sense of [Gorton \(1988\)](#).⁵ This modelling approach is motivated by two complementary sets of historical studies: those that emphasise the panic-like aspect of financial crises and ascribe panics to asymmetric information and moral hazard, such as [Gorton \(1988\)](#), [Mishkin \(1991\)](#) and [Gorton \(2009\)](#); and those that explain the rise in moral hazard by a change in macro-economic fundamentals and, notably, loose-for-long monetary policy, such as [Corsetti et al. \(1999\)](#) and [Jiménez et al. \(2014\)](#).⁶

The third departure from the textbook NK model is that we allow for endogenous capital accumulation and persistent deviations from the steady state. All else equal, the credit market is fragile when the capital stock is far above its steady state and the return on capital is low. One important implication is that financial crises may occur on the back of a protracted credit/investment boom (as documented by, *e.g.*, [Schularick and Taylor \(2012\)](#), [Gorton and Ordoñez \(2020\)](#)), and may therefore be predictable (as shown by [Greenwood et al. \(2022\)](#)). Finally, we solve the model globally to capture the non-linearities embedded in the endogenous booms and busts of the credit market.⁷

The baseline version of our model features both aggregate supply and demand shocks and assumes that the central bank follows a standard Taylor rule. We set the non-financial parameters at their standard values and the financial ones so that, in the simulated stochastic steady state, the economy spends 10% of the time in a financial crisis and aggregate productivity falls by 1.8% due to financial frictions in a crisis —as observed in OECD countries. Despite its stylised nature, our model does a fair job in capturing salient facts about historical financial crises. On the one hand, it is able to reproduce the median evolution of several relevant macro-financial

⁵In line with [Gorton \(1988\)](#), the financial panics modelled in this paper are fundamental-based. [Gorton \(1988\)](#) indeed distinguishes fundamental-based from non-fundamental-based panics. The latter are panics due to self-confirming equilibria, caused by shifts in the beliefs of agents which are unrelated to the real economy (“sunspots”). Fundamental-based panics, in contrast, are due to an asymmetry of information between borrowers and lenders and are systematically related to macro-economic events that change lenders’ perception of borrowers’ riskiness —as in our model. [Gorton \(1988\)](#) shows empirically that financial panics during the US National Banking Era (1863–1914) were fundamental-based.

⁶Related works include [Bernanke \(1995\)](#), [Mishkin \(1999\)](#), [Brunnermeier \(2009\)](#), [Gorton and Metrick \(2012\)](#), [Dang et al. \(2019\)](#) who also consider the combination of asymmetric information and moral hazard as the root cause of financial panics; and [Stein \(2013\)](#), [Grimm et al. \(2023\)](#) and [Jiménez et al. \(2023\)](#), who also highlight the causal link between low-for-long policy rates and excessive risk-taking/moral hazard.

⁷As we show later, the transition from “normal times” to “crisis times” induces a sudden and discrete drop in aggregate productivity. This non-linearity requires that our model be solved numerically and globally.

variables (*e.g.* productivity, credit/capital, capital return, output, inflation, monetary policy rate) around a broad range of past episodes. On the other hand, our model also acknowledges the differences across those episodes and, more particularly, the fact that many of them, but not all, were predictable. For example, 45% of the crises in our model are predictable — a proportion broadly similar to that reported in [Greenwood et al. \(2022\)](#).

We use our model to study whether monetary policy can tame endogenous booms and busts; whether a central bank can (and should) depart from its objective of price stability to prevent financial crises; and whether monetary policy can by itself brew financial vulnerabilities. In the process, we compare the performance of the economy under Taylor-type rules and regime-contingent rules, and also study the effect of discretionary monetary policy on financial stability.

Our main findings are threefold.

First, monetary policy affects the probability of a crisis not only in the short-term but also over the medium-term. A policy that prevents the excess fall in firms’ capital returns, *i.e.* that “cuts the left tail” of firms’ return distribution can help stem crises. Such policy requires a two-pronged approach that consists in stimulating aggregate demand in the face of adverse shocks —the short-term effect— while preemptively slowing down capital accumulation during booms —the medium-term effect. One example of such policy consists in responding to the deviation of the aggregate real return on capital from its steady state, in addition to output and inflation —an “augmented” Taylor rule. We find that, with this type of rule, the central bank can halve the time spent in crisis compared to a case where it follows a standard Taylor rule or a *strict* inflation targeting policy (henceforth, SIT).

Second, implementing an augmented Taylor rule requires from the central bank to tolerate a higher price volatility and, therefore, entails a trade-off between price and financial stability. On balance, our preferred augmented Taylor rule increases welfare significantly compared to a standard Taylor rule —but marginally compared to SIT. One way to alleviate the price/financial stability trade-off is to follow a more flexible (regime-contingent) policy rule, whereby the central bank commits to price stability in normal times *and* to doing whatever needed to forestall a crisis in stress times. We find that such “backstop” policy significantly improves welfare upon our preferred augmented Taylor rule as well as upon SIT. We also discuss the central bank’s balancing act between returning to its normal-times price stability objective quickly —at the risk of a resurgence of financial stress— *versus* slowly —at the risk of keeping inflation unnecessarily high. We find that monetary policy should return to normal faster when the cause of financial stress is a short-lived exogenous negative shock than when it is a protracted credit/investment boom.⁸

⁸One novel feature of our model is that it accounts for the role of monetary policy not only as a tool to achieve price stability but also as a potential tool to restore credit markets’ functionality ([Bank for International Settlements \(2022\)](#), [Duffie and Keane \(2023\)](#)). Relatedly, it also captures the potential tensions and trade-off between a central bank’s price and financial stability objectives. Examples of such tensions include, for example, the Savings & Loans crisis in the 1980s, the GFC, the May 2013 “taper tantrum” episode and, more recently, the Bank of England’s purchases of government bonds to address the November 2022 gilt market turmoil ([Hauser \(2023\)](#)), and the run on Silicon Valley Bank in March 2023.

Third, we study the effects of discretionary monetary policy interventions, *i.e.* random deviations from a Taylor-type rule. This last piece of analysis emphasises the opposite effects on financial stability of lowering the policy rate *versus* keeping it low-for-long. On the one hand, and all else equal, rate cuts boost aggregate demand and raise the returns on capital, dissuading search-for-yield behaviour. Hence, a one-off temporary rate cut helps lower the probability of a crisis in the short run while, on the flip side, a one-off hike may trigger a crisis. On the other hand, keeping the policy rate low for a long time stimulates the accumulation of capital and gradually erodes capital returns over time, eventually prompting investors to search for yield. The upshot is that a crisis is more likely when the central bank discretionarily hikes its policy rate after having kept it low-for-long, *i.e.* when it implements a so-called “U-shaped” monetary policy (Jiménez et al. (2023)).

We consider our model as a first step toward more complex models that would feature a richer set of policies, amplification mechanisms, and financial imbalances.

For one, we purposely leave out two potential ingredients that, while relevant, would not bring much additional insights into the role of monetary policy in the genesis of financial crises. Since we are interested in the effects of *monetary* policy, we abstract from other policies, such as macro-prudential or fiscal policies. Our intention is not to argue that these policies are not effective or should not be used as a first line of defence against the build-up of financial vulnerabilities. Rather, it is to understand better how monetary policy can by itself create, amplify, or mitigate risks to the financial system.⁹ In the same vein, since our goal is to study *endogenous* financial crises and how financial vulnerabilities build up, we deliberately leave out financial amplification mechanisms such as costly state verification (as in Bernanke and Gertler (1989), Bernanke et al. (1999)), collateral constraints (as in Kiyotaki and Moore (1997), Iacoviello (2005)) or leverage constraints (as in Gertler and Kiyotaki (2011), Gertler and Karadi (2011)). Such mechanisms are useful to explain how exogenous adverse shocks may set in motion deleveraging spirals and ultimately have big *consequences*. But they are not meant to capture the *causes* of financial crises, *e.g.* to explain their panic-like aspect and why they tend to occur on the back of protracted credit/investment booms. Adding amplification mechanisms in our model would enrich its description of how financial crises unfold but would not add to its explanation of how crisis risk builds up.¹⁰

Finally, while our model can account for several of the main characteristics of past crises, it is admittedly not meant to capture them all. For example, it does not speak to other elements common to many financial crises like rational bubbles (Galí (2014)) or to financial imbalances arising from agents’ “irrational exuberance” (Greenspan (1996)) and, more generally,

⁹Macro-prudential policies are generally still perceived as not offering full protection against financial stability risks, not least due to the rise of market finance and non-bank financial intermediation (Woodford (2012), Stein (2013, 2021), Schnabel (2021), Bernanke (2022)). For a discussion of macro-prudential policies in a static model featuring the same financial frictions as in our model, see Garcia-Macia and Villacorta (2022).

¹⁰For a version of the adverse selection/moral hazard problem considered here with collateral and endogenous collateral creation, see Boissay and Cooper (2020). In this version of the problem, a credit market collapse entails a shortage of collateral that amplifies the collapse.

from psychological biases and other deviations from fully rational expectations (Gennaioli et al. (2012), Gertler et al. (2020), Fontanier (2022)).¹¹

The paper proceeds as follows. Section 2 reviews key stylised facts of financial crises. Section 3 describes our theoretical framework, with a focus on the micro-foundations of endogenous financial crises, and discusses the channels through which monetary policy affects financial stability. Section 4 presents the parametrisation of the model as well as the average simulated dynamics around financial crises. Section 5 revisits the “divine coincidence” result and analyses whether the central bank should depart from its objective of price stability to prevent financial crises. Section 6 studies the effect of monetary policy surprises on financial stability and shows how discretionary monetary policy can breed financial vulnerabilities. Section 7 sets our work in the literature. A last section concludes.

2 Salient Facts about Financial Crises

The empirical literature has identified several salient facts about financial crises that are common across a broad range of historical episodes. Figure 1 recapitulates these facts. Later (in Section 4.2), we will briefly review these facts in the light of our model. For now, we illustrate them by reporting the median dynamics of key macro-financial variables relating to credit, productivity, and monetary policy around financial crises in 18 advanced economies since 1945.

Most of the data reported in Figure 1 are from the latest release of the Jordà–Schularick–Taylor [Macrohistory Database](#).¹² The only exceptions are asset prices—which we take from [Global Financial Data](#) (as in Greenwood et al. (2022)), the capital stock and the output-to-capital ratio—from the [International Monetary Fund \(2021\)](#)’s [Capital Stock Dataset](#), and total factor productivity adjusted for labour utilisation—from Jordà et al. (2023).¹³

Credit and Financial Vulnerabilities. Financial crises tend to follow (unusually large) booms in credit and asset prices (*e.g.* Borio and Lowe (2002), Schularick and Taylor (2012); Figure 1, panels (a) and (b), plain *versus* dashed line) which in turn fuel an investment boom and capital accumulation (panel (c)). Greenwood et al. (2022) document that, on average, a significant proportion (around 40–64%) of financial crises are preceded by such booms and, in that sense, are predictable.¹⁴ Other empirical studies show that the longer the boom lasts, the more likely it is to stoke financial vulnerabilities (Dell’Ariccia et al. (2016), Jiménez et al. (2023)).

¹¹Stein (2013) makes a distinction between two complementary views of financial crises: the “preferences and beliefs” one, which assumes that agents have psychological biases (*e.g.* optimism, overconfidence); and the “agency and incentives” one, which assumes that agents have conflicts of interest and private information. Ours pertains to the latter view.

¹²This database combines annual macro-financial data with a chronology of a banking crises (see Jordà et al. (2017)).

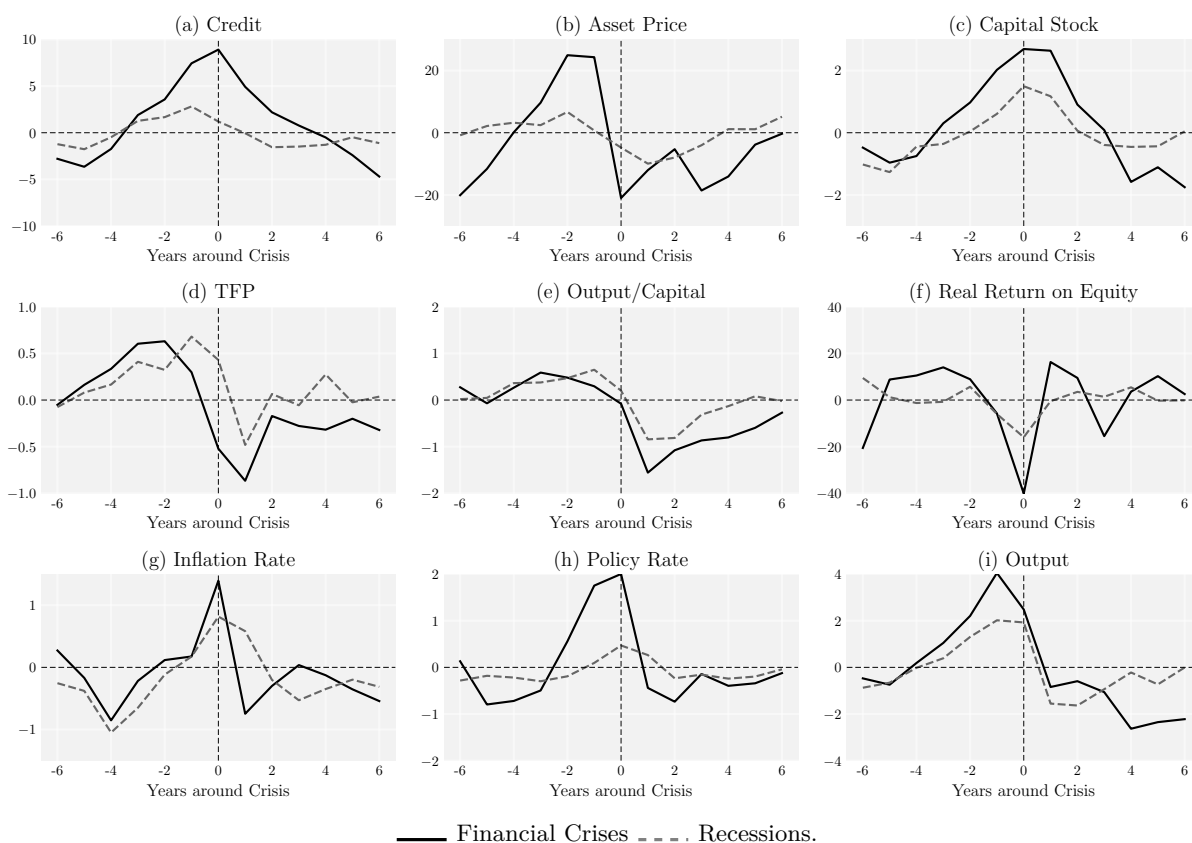
¹³We thank Òscar Jordà, Sanjay Singh, and Alan Taylor for sharing the utilisation-adjusted productivity data.

¹⁴This finding, which relies on a sample of 50 crises in 42 countries over the period 1950–2016, challenges the view that all financial crises would be (in Greenwood et al. (2022)’s language) “bolts from the sky”, *i.e.* due to large and exogenous adverse financial shocks.

When these vulnerabilities eventually surface, credit markets may freeze and the economy may tank, causing a particularly deep and long recession (Cerra and Saxena (2008), Jordà et al. (2013); panel (i), plain *versus* dashed line). Such “financial recessions” tend to be characterised by an unusually large and prolonged fall in total factor productivity compared to normal recessions (panel (d), plain *versus* dashed line) as the tightening in credit conditions inhibits factor reallocation across businesses and impairs the usual “cleansing effect” of recessions (Foster et al. (2016), Oulton and Sebastiá-Barriel (2016), Gopinath et al. (2017), Duval et al. (2019)).

That said, not all credit booms go bust. Dell’Ariccia et al. (2016) and Greenwood et al. (2022) document that roughly one in three credit booms are “bad booms” followed by a financial crisis —and hence that two thirds are “good booms”. Recent empirical literature further shows that bad booms have specific characteristics and, more particularly, that they often follow a rise-and-fall in aggregate productivity and a U-shaped path of monetary policy rates.

Figure 1: Median Dynamics Around Financial Crises



Notes: Median dynamics of key macro-financial variables around financial crises in the post-WW2 period. All variables are annual and de-trended using Hodrick and Prescott (1997) with $\lambda = 100$. The recession dates are taken from Jordà et al. (2013). Bar the inflation rate (panel (g)), one obtains similar dynamics when the sample period is extended back to 1870. A detailed description of all data series and crisis dynamics over the full sample period (1870–2020) are reported in Section A.1 of the online appendix. For all variables, the *median* crisis dynamics are essentially the same as the *average* ones. We chose to report the former only to emphasise that the results are immune to potential outliers and not driven by specific crisis episodes.

Productivity. The typical credit boom preceding financial crises generally starts with a protracted increase in aggregate productivity (Gorton and Ordoñez (2020); Figure 1, panel (d)).¹⁵ The latter at first raises the real return on capital (measured here by the output-to-capital ratio (panel (e)) and real equity returns (f)) above trend, which boosts investment and credit demand; hence the boom in credit (panel (a)) and asset prices (panel (b)). Economic and financial vulnerabilities generally surface as the productivity gains peter out. Gorton and Ordoñez (2020) indeed document that bad credit booms display relatively large productivity declines (panel (d)) and Paul (2022) shows that such declines are robust predictors of financial crises. As productivity falls, firms reduce their utilisation of their (possibly oversized) capital stock, which weighs on capital returns (panels (e) and (f)). The fall in productivity and capital returns starts about two years before the crisis.

Monetary policy. Another characteristic of bad credit booms is their association with a U-shaped path of monetary policy rates —defined as a prolonged period of relatively low rates followed by rapid hikes (Figure 1, panel (h)). Grimm et al. (2023) provide empirical evidence that discretionarily keeping monetary policy loose for an extended period of time can cause a boom in credit and beget financial instability down the road. Schularick et al. (2021) show that discretionary rate hikes during a credit boom may trigger a financial crisis. Together, these findings dovetail with those of Jiménez et al. (2023), who show that a U-shaped monetary policy is conducive to a crisis. The latter study also notes that rate hikes may only trigger a crisis if rates were kept low for long beforehand. Using loan-level data, it further establishes a *causal chain* that links the low-for-long rates with search-for-yield behaviour and the subsequent crisis. At first, the rate cuts boost the supply of credit; but the longer the period of low rates, the scarcer the profitable lending opportunities, and the more likely it is that credit flows toward riskier or less productive investments. When the central bank eventually hikes its policy rate, these vulnerabilities come to the fore and a crisis breaks out.

To our knowledge, there is no empirical analysis of the reasons that may prompt a central bank to implement a U-shaped monetary policy in the first place. But the fact that such policy tends to go hand in hand with a U-shaped inflation rate (compare panels (g) and (h)) suggests that it may stem, at least partially, from a *systematic* response of the central bank to inflation—and not necessarily from discretionary interventions. This will be the case if the central bank follows an inflation-targeting rule whereby it cuts (raises) its policy rate whenever inflation falls below (increases above) target.¹⁶

¹⁵The GFC in the United States, for example, was preceded by the boom-bust in information technologies of the early 2000s (Fernald (2015)). Similarly, the 1990-91 Japanese crisis was preceded by the boom-bust in electronics in the 1980s; and the Great depression followed the end of the second industrial revolution in the early 1920s (Cao and L’Huillier (2018) and references therein).

¹⁶The periods that preceded the 1990-91 Japanese crisis and the GFC in the United States are often cited as examples of periods of dis-inflationary credit booms with relatively low policy rates. Anecdotal evidence suggests that, in 1987, the Bank of Japan considered the possibility of tightening monetary policy in order to slow down the overheating economy but, with inflation close to 0%, lacked the arguments to justify a rate hike and eventually raised its policy rate only after inflation rose to above 2%, in 1989 (Okina et al. (2001), Ito and Mishkin (2006)). Similarly, Greenspan (2003)’s testimony before the US Congress in July 2003 emphasises how

3 Model

Our model is a variation of the textbook NK model (Galí (2015)), with sticky prices *à la* Rotemberg (1982), capital accumulation, and financial frictions.

3.1 Agents

The economy is populated with a central bank, a continuum of identical households, a continuum of monopolistically competitive retailers $i \in [0, 1]$, as well as with a continuum of competitive intermediate goods producers $j \in [0, 1]$ (henceforth, “firms”). The only non-standard agents are the firms, which experience idiosyncratic productivity shocks that prompt them to resize their capital stock and participate in a credit market.

3.1.1 Central Bank

The central bank sets the policy rate i_t according to the following simple policy rule:¹⁷

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \quad (1)$$

where $1/\beta$ is the gross natural rate of interest in the deterministic steady state —with $\beta \in (0, 1)$ the household’s discount factor, π_t and Y_t are aggregate inflation and output in period t , and Y is aggregate output in the deterministic steady state. As baseline, we consider Taylor (1993)’s original rule (henceforth, TR93) with parameters $\phi_\pi = 1.5$ and $\phi_y = 0.125$ (for quarterly data). In the analysis, we also experiment with different types of rule, including SIT, Taylor-type rules, and regime-contingent rules (Section 5).¹⁸

3.1.2 Households

The representative household is infinitely-lived. In period t , the household supplies N_t hours of work at nominal wage rate W_t , consumes a Dixit–Stiglitz consumption basket of differentiated goods $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$, with $C_t(i)$ the consumption of good i purchased at price $P_t(i)$, and invests their savings in a private nominal bond B_{t+1} in zero net supply and in equity $P_t K_{t+1}(j)$ issued by newborn firm j , with $j \in [0, 1]$. The household can thus be seen as a venture capitalist providing startup equity funding to firms.¹⁹

Since the new capital goods can be produced instantly and one-for-one with final goods and are homogeneous to the old ones (net of depreciation and maintenance costs), all capital goods are purchased at price P_t . And since firms are identical at the time they raise funding,

challenging it can be for a central bank to hike rates in the midst of a dis-inflationary boom.

¹⁷Given that there is no growth trend in our model, the term Y_t/Y corresponds to the GDP gap with respect to its long-run trend (or de-trended GDP) as defined in Taylor (1993)’s seminal paper.

¹⁸In Section A.6 of the online appendix, we show that our analysis and results are robust to considering an alternative Taylor rule whereby the central bank reacts to expected —as opposed to current— inflation.

¹⁹In the baseline model, we assume for simplicity that firms finance their startup capital stock by issuing equity but they could of course also issue debt. In Section B.3.1 of the online appendix, we show that a version of the model where firms issue riskfree debt to households (*i.e.* debt that is not subject to moral hazard) is strictly isomorphic to our baseline model.

we anticipate (to economise on notations) that all firms obtain the same funding and purchase the same quantity of capital goods K_{t+1} at the end of period t for use in period $t + 1$.

The household maximises their expected lifetime utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right]$$

subject to the sequence of budget constraints

$$\int_0^1 P_t(i)C_t(i)di + B_{t+1} + P_tK_{t+1} \leq W_tN_t + (1 + i_{t-1}^b)B_t + P_tK_t \int_0^1 (1 + r_t^q(j))dj + \Upsilon_t$$

for $t = 0, 1, \dots, +\infty$. In the above, $\mathbb{E}_t(\cdot)$ denotes the expectation conditional on the information set available at the end of period t , $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ is the price of the consumption basket, Υ_t is a lump-sum component of nominal income (which includes retailers' dividends and lump-sum taxes), $r_t^q(j)$ is firm j 's *ex post* real rate of return on equity, and i_t^b is the private nominal bond yield defined by

$$i_t^b \equiv \frac{1 + i_t}{Z_t} - 1 \quad (2)$$

where Z_t corresponds to a wedge between the private bond yield i_t^b and the policy rate i_t (as in [Smets and Wouters \(2007\)](#)), and follows an exogenous AR(1) process $\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \varepsilon_t^z$ with $\rho_z \in (0, 1)$, where $\varepsilon_t^z \sim N(0, \sigma_z^2)$ is realised at the beginning of period t . Following the literature, we interpret Z_t as an aggregate demand shock.²⁰

The first order conditions describing the household's optimal behaviour are standard and given by (in addition to a transversality condition):

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad \forall i \in [0, 1] \quad (3)$$

$$\frac{\chi N_t^\varphi}{C_t^{1-\sigma}} = \frac{W_t}{P_t} \quad (4)$$

$$1 = \beta(1 + i_t^b)\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{1 + \pi_{t+1}} \right] \quad (5)$$

$$1 = \beta\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (1 + r_{t+1}^q(j)) \right] \quad \forall j \in [0, 1] \quad (6)$$

where $\pi_t \equiv P_t/P_{t-1} - 1$. Equation (3) determines the optimal composition of the household's consumption basket. Equation (4) states that optimal labour supply behaviour requires that the marginal rate of substitution between consumption and leisure be equal to the real wage. The no-arbitrage conditions (5) and (6) determine the optimal demands for bonds and equity.

²⁰See, *e.g.*, [Galí et al. \(2012\)](#), [Barsky et al. \(2014\)](#), and [Fisher \(2015\)](#). This shock has the opposite effect of a risk-premium shock. All else equal, a higher Z_t ($\varepsilon_t^z > 0$) lowers the return i_t^b on private bonds (from (2)) and, therefore, increases current consumption (from (5)) and induces a portfolio re-balancing from bonds toward firm equity (from (6)), which stimulates investment. In a model with endogenous capital accumulation and no capital adjustment costs, like ours, this type of demand shock generates a positive correlation between consumption and investment —unlike a discount factor shock.

3.1.3 Retailers

Retailers are infinitely-lived and endowed with a linear production technology

$$Y_t(i) = X_t(i) \quad (7)$$

that transforms $X_t(i)$ units of the (single) intermediate good into $Y_t(i)$ units of a differentiated final good $i \in [0, 1]$.

Retailers sell their output in a monopolistically competitive environment subject to their individual downward sloping demand schedules and to nominal price rigidities à la [Rotemberg \(1982\)](#). Each retailer i sets its price $P_t(i)$ subject to adjustment costs $\frac{\varrho}{2} P_t Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2$, where $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ denotes aggregate output. The demand for final goods emanates from households (who consume), firms (which invest), and retailers (which incur menu costs). Capital investment goods and menu costs take the form of a basket of final goods similar to that of consumption goods. Accordingly, retailer i faces the demand schedule

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \quad \forall i \in [0, 1] \quad (8)$$

where $Y_t = C_t + I_t + \frac{\varrho}{2} Y_t \pi_t^2$, with I_t the aggregate basket of investment goods defined by $I_t \equiv \left(\int_0^1 I_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$, and $\frac{\varrho}{2} Y_t \pi_t^2$ the real value of aggregate menu costs in the symmetric equilibrium.

At the beginning of period t , retailer i chooses the price $P_t(i)$ that maximises the market value of its current and future profits

$$\max_{\{P_t(i)\}_{t=0, \dots, +\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t} \left[\frac{P_t(i)}{P_t} Y_t(i) - \frac{(1-\tau)p_t}{P_t} Y_t(i) - \frac{\varrho}{2} Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 \right] \right\},$$

subject to the sequence of demand schedules (8) for $t = 0, \dots, +\infty$, where $\Lambda_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)^{-\sigma}$ is the stochastic discount factor between period t and $t+k$, p_t is the unit price of intermediate goods used as inputs, which are subsidised at rate $\tau = 1/\epsilon$.²¹

In the symmetric equilibrium, where $Y_t(i) = Y_t$ and $P_t(i) = P_t$, the optimal price setting behaviour satisfies

$$(1 + \pi_t)\pi_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} (1 + \pi_{t+1})\pi_{t+1} \right) - \frac{\epsilon-1}{\varrho} \left(\frac{\mathcal{M}_t - \mathcal{M}}{\mathcal{M}_t} \right) \quad (9)$$

where \mathcal{M}_t is retailers' average markup given by

$$\mathcal{M}_t \equiv \frac{P_t}{(1-\tau)p_t} > 0 \quad (10)$$

and $\mathcal{M} \equiv \epsilon/(\epsilon-1)$ is the desired markup level which would prevail in the absence of nominal rigidities. According to (9), inflation will be positive when markups are below their desired level (i.e. when $\mathcal{M}_t - \mathcal{M} < 0$), for in that case retailers will increase prices in order to realign markups closer to their desired level.

²¹This subsidy corrects for monopolistic market power distortions in the deterministic steady state of the model.

3.1.4 Intermediate Goods Producers (“Firms”)

The intermediate goods sector consists of overlapping generations of firms that live one period, are born at the end of period $t-1$ and die at the end of period t . Firms are perfectly competitive, and produce a homogeneous good, whose price p_t they take as given. They are identical *ex ante* but face idiosyncratic productivity shocks *ex post*, which they cushion by borrowing or lending on a short term (intra-period) credit market. As in [Bernanke and Gertler \(1989\)](#), [Fuerst \(1995\)](#), [Bernanke et al. \(1999\)](#), “generations” in our model should be thought of as representing the entry and exit of firms from such credit markets, rather than as literal generations; a “period” in our model may therefore be interpreted as the length of a financial contract.²²

Consider a generic firm $j \in [0, 1]$ born at the end of period $t-1$.

At birth, this firm receives $P_{t-1}K_t$ startup equity funding, which it uses to buy K_t units of capital goods. Among the latter, $(1-\delta)K_{t-1}$ are old capital goods that they purchase from the previous generation of firms, where δ is the rate of depreciation of capital, and I_{t-1} are newly produced capital goods, with $K_t = (1-\delta)K_{t-1} + I_{t-1}$.²³

At the beginning of period t , firm j experiences an aggregate shock, A_t , as well as an idiosyncratic productivity shock, $\omega_t(j)$, and has access to a constant-return-to-scale technology represented by the production function

$$X_t(j) = A_t(\omega_t(j)K_t(j))^\alpha N_t(j)^{1-\alpha} \quad (11)$$

where $K_t(j)$ and $N_t(j)$ denote the levels of capital and labour that firm j uses as inputs and $X_t(j)$ is the associated output. The idiosyncratic shock $\omega_t(j) \in \{0, 1\}$ takes the value 0 for a fraction μ of the firms (“unproductive firms”) and 1 for a fraction $1-\mu$ of the firms (“productive firms”).²⁴ We denote the set of unproductive firms by $\Omega_t^u \equiv \{j \mid \omega_t(j) = 0\}$ and that of productive firms by $\Omega_t^p \equiv \{j \mid \omega_t(j) = 1\}$. Aggregate productivity A_t evolves randomly according to a stationary AR(1) process $\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a$ with $\rho_a \in (0, 1)$ and $\varepsilon_t^a \sim N(0, \sigma_a^2)$, where the innovation ε_t^a is realised at the beginning of period t .

Upon observing $\omega_t(j)$, firm j may resize its capital stock by purchasing or selling capital goods on a secondary capital goods market. Of course, firms also always have the option to stay put and keep their capital stock idle throughout the period. In that case, we assume that

²²The overlapping generation approach is standard in macroeconomic models because it provides a tractable framework for dynamic general equilibrium analysis with firm heterogeneity. In the presence of agency costs, this approach is a way to ignore multi-period financial contracts contingent on past debt repayments (see *e.g.* [Gertler \(1992\)](#) for an example of multi-period contracts in a three-period model). In Sections B.2 and B.4 of the online appendix, we discuss the robustness of our analysis when firms live infinitely or are heterogeneous *ex ante* (*i.e.* before they incur the idiosyncratic productivity shocks).

²³Given that firms live only one period, the inter-temporal decisions regarding capital accumulation within the intermediate good sector are, in effect, taken by the households—their shareholders.

²⁴As will become clear later, one advantage of the Bernoulli (as opposed to a continuous) distribution is that financial frictions only surface during financial crises, not in normal times—where the entire capital stock is used productively. This property is appealing because it allows us to isolate the effects of agents’ anticipation of a crisis and to illustrate the presence of financial externalities (see Section 4.3). In earlier versions of the model, where we considered a continuous distribution of $\omega_t(j)$, financial frictions also affected capital allocation in normal times but only marginally so.

capital depreciates at the same rate (or must be maintained at the same cost) δ as when it is used productively.²⁵

To fill any gap between its desired capital stock $K_t(j)$ and its initial (predetermined) one, K_t , firm j may borrow or lend on a credit market at real interest rate r_t^c . This market operates in lockstep with the secondary capital goods market. If $K_t(j) > K_t$, firm j borrows and uses the funds to buy capital goods. If $K_t(j) < K_t$, it instead sells capital goods and lends the proceeds of the sale to other firms. Firm j therefore buys $K_t(j) - K_t$ (if $K_t(j) > K_t$) or sells $K_t - K_t(j)$ (if $K_t(j) < K_t$) capital goods, hires labour $N_t(j)$, and produces intermediate goods $X_t(j)$.

At the end of period t , the firm sells its production to retailers, pays workers, sells its undepreciated capital $(1 - \delta)K_t(j)$, and repays $P_t(1 + r_t^c)(K_t(j) - K_t)$ to lenders if $K_t(j) > K_t$ or receives $P_t(1 + r_t^c)(K_t - K_t(j))$ from borrowers if $K_t(j) < K_t$. Let $D_t(j)$ denote firm j 's dividend payout, expressed in final goods. Then, one obtains

$$P_t D_t(j) = p_t X_t(j) - W_t N_t(j) + P_t(1 - \delta)K_t(j) - P_t(1 + r_t^c)(K_t(j) - K_t) \quad (12)$$

for $j \in [0, 1]$. Using relations (10), (11) and (12), firm j 's real rate of return on equity can be expressed as

$$r_t^q(j) \equiv \frac{D_t(j)}{K_t} - 1 = \frac{X_t(j)}{(1 - \tau)\mathcal{M}_t K_t} - \frac{W_t N_t(j)}{P_t K_t} - (r_t^c + \delta) \frac{K_t(j) - K_t}{K_t} - \delta \quad \forall j \in [0, 1] \quad (13)$$

The objective of firm j is to maximise $r_t^q(j)$ with respect to $N_t(j)$ and $K_t(j)$. We present the maximization problem of unproductive and productive firms in turn.

Choices of an Unproductive Firm. It is easy to see that unproductive firms all take the same decisions and choose $N_t(j) = 0$, $X_t(j) = 0$, and $K_t(j) = K_t^u$, for all $j \in \Omega_t^u$. The adjusted capital stock K_t^u will be determined later, as we solve the equilibrium of the credit market (see Section 3.2). Using (13), firm j 's maximization problem can therefore be written as

$$\max_{K_t^u} r_t^q(j) = r_t^c - (r_t^c + \delta) \frac{K_t^u}{K_t} \quad \forall j \in \Omega_t^u \quad (14)$$

In the above expression, the first term is the return from selling capital and lending the proceeds, while the second term is the opportunity cost of keeping capital idle.

Choices of a Productive Firm. Productive firms all take the same decisions, and choose $N_t(j) = N_t^p$, $X_t(j) = X_t^p$, and $K_t(j) = K_t^p$ for all $j \in \Omega_t^p$, where the optimal labour demand N_t^p satisfies the first order condition

$$\frac{W_t}{P_t} = \frac{1 - \alpha}{1 - \tau} \frac{X_t^p}{\mathcal{M}_t N_t^p} = \frac{1 - \alpha}{1 - \tau} \frac{A_t}{\mathcal{M}_t} \left(\frac{K_t^p}{N_t^p} \right)^\alpha \quad (15)$$

²⁵This assumption simply implies that the marginal return on capital is always higher for a productive firm that produces than for a firm that does not produce and keeps its capital idle, as relation (18) below shows. Hence, the option to keep capital idle is only sensible for an unproductive firm.

and will be determined later, along with the adjusted capital stock K_t^p . Let

$$\Phi_t \equiv \alpha \frac{X_t^p}{K_t^p} = \alpha \left(\frac{N_t^p}{K_t^p} \right)^{1-\alpha} \quad (16)$$

denote the marginal product of capital for a productive firm. Using the optimal labour demand in (15), one obtains that Φ_t is a function of the real wage W_t/P_t and retailer's markup \mathcal{M}_t ,

$$\Phi_t = \alpha A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{(1-\tau)\mathcal{M}_t \frac{W_t}{P_t}} \right)^{\frac{1-\alpha}{\alpha}}$$

which firm j takes as given. Using (13), (15), and (16), the maximization problem of a productive firm j can be written as

$$\max_{K_t^p} r_t^q(j) = r_t^c + (r_t^k - r_t^c) \frac{K_t^p}{K_t} \quad \forall j \in \Omega_t^p \quad (17)$$

where

$$r_t^k \equiv \frac{\Phi_t}{(1-\tau)\mathcal{M}_t} - \delta > -\delta \quad (18)$$

denotes the marginal return on capital (after depreciation) for a productive firm, and is taken as given by firm j .

3.2 Market Clearing

We first consider the benchmark case of a frictionless credit market, where the idiosyncratic productivity shocks can be observed by all potential investors, and where financial contracts are fully enforceable, with no constraint on the amount that a firm can borrow. Then, we introduce financial frictions.

3.2.1 Frictionless Credit Market

Let $L^D(r_t^c)$ and $L^S(r_t^c)$ denote the aggregate demand and supply of credit, and assume that there is no friction on the credit market. The mass μ of unproductive firms are the natural lenders. Given relation (14), these firms sell their entire capital stock K_t and lend the proceeds of the sale when $r_t^c > -\delta$, implying $K_t^u = 0$ and $L^S(r_t^c) = \mu K_t$ in that case. When $r_t^c = -\delta$, they are indifferent between lending, keeping their capital idle, and borrowing: $L^S(r_t^c) \in (-\infty, \mu K_t]$. When $r_t^c < -\delta$, they borrow as much as possible in order to buy capital goods and keep them idle: $K_t^u = +\infty$ and $L^S(r_t^c) = -\infty$. The aggregate credit supply (by unproductive firms) is therefore given by

$$L^S(r_t^c) = \mu (K_t - K_t^u) = \begin{cases} \mu K_t & \text{for } r_t^c > -\delta \\ (-\infty, \mu K_t] & \text{for } r_t^c = -\delta \\ -\infty & \text{for } r_t^c < -\delta \end{cases}$$

and is represented by the black line in Figure 2.

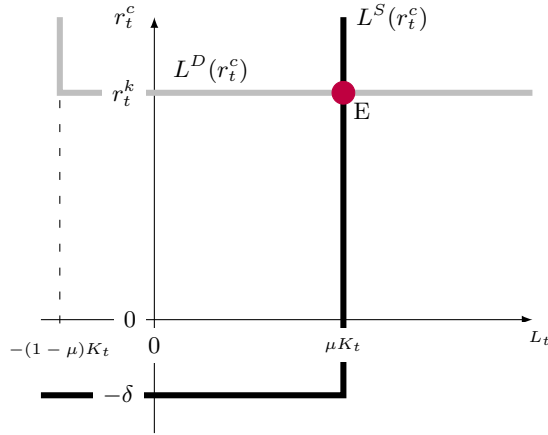
The mass $1 - \mu$ of productive firms are the natural borrowers. Given relation (17), these firms borrow as much as possible when $r_t^c < r_t^k$, implying $K_t^p = +\infty$ and $L^D(r_t^c) = +\infty$ in

that case. When $r_t^c = r_t^k$, they are indifferent between borrowing, staying put, and lending: $L^D(r_t^c) \in [-(1-\mu)K_t, +\infty)$. When $r_t^c > r_t^k$, they sell their entire capital stock K_t and lend the proceeds of the sale: $K_t^p = 0$ and $L^D(r_t^c) = -(1-\mu)K_t$. The aggregate credit demand (from productive firms) is therefore given by

$$L^D(r_t^c) = (1-\mu)(K_t^p - K_t) = \begin{cases} -(1-\mu)K_t & \text{for } r_t^c > r_t^k \\ [-(1-\mu)K_t, +\infty) & \text{for } r_t^c = r_t^k \\ +\infty & \text{for } r_t^c < r_t^k \end{cases}$$

and is represented by the gray line in Figure 2.

Figure 2: Frictionless Credit Market Equilibrium



Notes: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate credit demand (gray) curves, in the absence of financial frictions. This figure is drawn for $r_t^k > -\delta$, as implied by relation ((18)).

In equilibrium E , $r_t^c = r_t^k > -\delta$ and $K_t^u = 0$, implying that $r_t^q(j) = r_t^k = r_t^c$ for all $j \in [0, 1]$. As the mass μ of unproductive firms lend their entire capital stock K_t to the mass $1-\mu$ of productive firms, the equilibrium is also characterised by

$$K_t^p = \frac{K_t}{1-\mu} \quad (19)$$

In this economy, capital goods are perfectly reallocated and used productively. Our model then boils down to the textbook NK model with endogenous capital accumulation and a representative intermediate goods firm.

3.2.2 Frictional Credit Market

Consider now the case with financial frictions. We assume that borrowers have limited liability and the possibility to hide idle capital from lenders, sell it at the end of the period, “go underground” with the proceeds of the sale, and default. This feature, which allows borrowers to divert funds from their intended use, is common in models of investment under moral hazard (*e.g.* Gertler and Rogoff (1990), Azariadis and Smith (1998), Boissay et al. (2016)). Since idle capital depreciates at rate δ , the unit gross return of a borrower that defaults is $1-\delta$. We assume that the borrower incurs a cost equal to a fraction θ of the funds borrowed in that case,

with θ capturing the cost of hiding from lenders or the fraction of the purchased capital goods that the borrower cannot abscond with. We further assume that this cost is lower than the unit return of the purchased capital goods, *i.e.*

$$\theta < 1 - \delta$$

so that default can be worthwhile. In contrast, we assume that the firms that use their capital to produce intermediate goods always repay their debt entirely at the end of the period. One can think of these firms, which utilise capital and labour as inputs, as firms operating transparently and whose cashflow cannot be concealed.

Borrowers' limited liability opens the door to moral hazard: to boost its profit, a firm may borrow, purchase more capital, keep its capital stock idle, abscond, and default. Note that we also allow productive firms to keep capital idle and default. But since their return on capital when they produce is higher than that of keeping capital idle (see relation (18)), it should be clear that, for these firms, the opportunity cost of absconding and defaulting is higher than for unproductive ones.

In addition, we assume that lenders do not observe a given firm j 's productivity $\omega_t(j)$, and hence cannot assess individual firms' incentives to borrow and default.²⁶ As a result, lenders face an adverse selection problem, whereby unproductive firms may pretend they are productive, borrow, and default. To deter unproductive firms from borrowing, lenders must limit the amount that any individual firm can borrow.²⁷ Proposition 1 follows.

Proposition 1. (*Firms' Incentive-Compatible Borrowing Limit*) *A firm cannot borrow and purchase more than a fraction ψ_t of its initial capital stock:*

$$\frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \max \left\{ \frac{r_t^c + \delta}{1 - \delta - \theta}, 0 \right\} \quad (20)$$

Proof. Suppose that an unproductive firm were to mimic a productive firm by borrowing and purchasing $K_t^p - K_t \geq 0$ capital goods, keep its capital stock K_t^p idle, resell it at the end of the period, and then default. In this case, the firm's net payoff from defaulting would be $P_t(1 - \delta)K_t^p - \theta P_t(K_t^p - K_t)$, where the term $\theta P_t(K_t^p - K_t)$ is the cost of defaulting. That firm will not default as long as this payoff is smaller than the return $P_t(1 + r_t^c)K_t$ from selling its entire capital stock and lending the proceeds of the sale —its only viable alternative option. The incentive compatibility constraint that ensures that no unproductive firm defaults thus reads $(1 - \delta)K_t^p - \theta(K_t^p - K_t) \leq (1 + r_t^c)K_t$. Let ψ_t denote the firm's borrowing limit, with $\psi_t \geq 0$. Then the inequality in Proposition 1 follows from re-arranging this incentive compatibility constraint and from the non-negativity of ψ_t . \square

²⁶As shown in Section B.6 of the online appendix, both moral hazard and asymmetric information are necessary for the equilibrium outcome to depart from the first best one.

²⁷The contractual problem considered here is similar to that in [Bernanke and Gertler \(1990\)](#), [Gertler and Rogoff \(1990\)](#), [Azariadis and Smith \(1998\)](#) and [Boissay et al. \(2016\)](#).

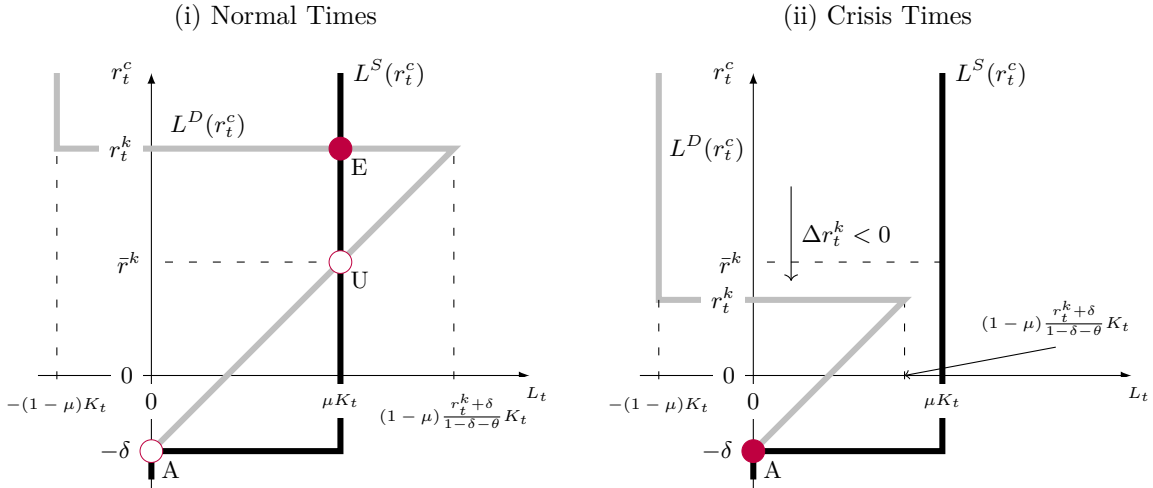
As long as condition (20) is satisfied, unproductive firms (and *a fortiori* the productive ones) will refrain from borrowing and defaulting. The ratio on the right-hand side of (20) reflects the choice of an unproductive firm between lending, with excess return of $r_t^c + \delta$ (numerator), or borrowing and defaulting, with return of $1 - \delta - \theta$ (denominator). Condition (20) shows that the higher the loan rate, the higher the unproductive firm's opportunity cost of defaulting, and the higher borrowers' incentive-compatible borrowing limit. By contrast, a fall in the loan rate induces lenders to lower the borrowing limit and retreat from the loan market. As we show next, lenders' retreat may lead to a collapse of the loan market.

We are now in the position to construct the credit supply and demand schedules (see Figure 3). Given relation (14) and Proposition 1, the aggregate credit supply, $L^S(r_t^c)$, represented by the black lines in Figure 3, reads:

$$L^S(r_t^c) = \mu(K_t - K_t^u) = \begin{cases} \mu K_t & \text{for } r_t^c > -\delta \\ [0, \mu K_t] & \text{for } r_t^c = -\delta \\ 0 & \text{for } r_t^c < -\delta \end{cases} \quad (21)$$

When $r_t^c > -\delta$, the mass μ of unproductive firms sell their capital stock K_t and lend the proceeds on the credit market, implying $L^S(r_t^c) = \mu K_t$. When $r_t^c = -\delta$, they are indifferent between lending and keeping their capital stock K_t idle, implying $L^S(r_t^c) \in [0, \mu K_t]$. When $r_t^c < -\delta$, they keep their capital idle: $L^S(r_t^c) = 0$.

Figure 3: Credit Market Equilibrium



Notes: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' incentive-compatible aggregate credit demand (gray) curves. In panel (i), the demand curve is associated with a value of r_t^k strictly above \bar{r}^k and multiple equilibria A, E, and U. In this case, U and A are ruled out on the ground that they are unstable (for U) and Pareto-dominated (for A). In panel (ii), the demand curve is associated with a value of r_t^k strictly below \bar{r}^k and A as unique equilibrium. The threshold for the loan rate, \bar{r}^k , is constant and corresponds to the minimum incentive-compatible loan rate that is required to ensure that every unproductive firm sells its entire capital stock and lends the proceeds of the sale—rather than borrows and absconds.

Taking into account borrowers' incentive compatibility constraint, the aggregate credit de-

mand, $L^D(r_t^c)$, is given by (using (17) and Proposition 1):

$$L^D(r_t^c) = (1 - \mu)(K_t^p - K_t) = \begin{cases} -(1 - \mu)K_t & \text{for } r_t^c > r_t^k \\ [-(1 - \mu)K_t, (1 - \mu)\psi_t K_t] & \text{for } r_t^c = r_t^k \\ (1 - \mu)\psi_t K_t & \text{for } r_t^c < r_t^k \end{cases} \quad (22)$$

and is represented by the gray lines in Figure 3. When $r_t^c > r_t^k$, productive firms prefer to sell their capital and lend the proceeds rather than borrow: $L^D(r_t^c) = -(1 - \mu)K_t$. When $r_t^c = r_t^k$, they are indifferent but may borrow up to $\psi_t K_t$, where ψ_t the borrowing limit defined in (20), implying $L^D(r_t^c) \in [-(1 - \mu)K_t, (1 - \mu)\psi_t K_t]$. When $r_t^c < r_t^k$, they borrow up to the limit, implying $L^D(r_t^c) = (1 - \mu)\psi_t K_t$.

Proposition 2. (Existence of an Active Credit Market) *An equilibrium with trade exists if and only if*

$$r_t^k \geq \bar{r}^k \equiv \frac{(1 - \theta)\mu - \delta}{1 - \mu} \quad (23)$$

Proof. From panel (i) in Figure 3, it is clear that an equilibrium with trade exists if and only if there is a range of interest rates for which demand (in gray) intersects supply (in black) for a strictly positive amount of credit, *i.e.* if and only if $\lim_{r_t^c \nearrow r_t^k} L^D(r_t^c) \geq L^S(r_t^k)$. Using relations (20), (21) and (22), this condition can be re-written as $(1 - \mu)(r_t^k + \delta)/(1 - \delta - \theta) \geq \mu \Leftrightarrow r_t^k \geq ((1 - \theta)\mu - \delta)/(1 - \mu)$. \square

The interest rate threshold \bar{r}^k is the minimum return on capital that guarantees the existence of an equilibrium with trade. Perhaps more intuitively, \bar{r}^k can also be seen as the minimum loan rate that unproductive firms require in order to lend on the credit market rather than borrow funds and abscond in search for yield. To see this, notice that borrowers' incentive compatibility constraint underpinning Proposition 1, $(1 - \delta)K_t^p - \theta(K_t^p - K_t) \leq (1 + r_t^c)K_t$, can be re-written as a condition on the loan rate: $r_t^c \geq (1 - \delta - \theta)(K_t^p - K_t)/K_t - \delta$, which simply means that an unproductive firm has an incentive to lend only if the loan rate is high enough. For this condition to be satisfied in an equilibrium with trade, *i.e.* when $\mu K_t = (1 - \mu)(K_t^p - K_t)$, one must therefore have $r_t^c \geq \bar{r}^k \equiv ((1 - \theta)\mu - \delta)/(1 - \mu)$, which corresponds to the right-hand side of relation (23). Further, notice that productive firms only borrow funds if their return on capital is higher than the cost of funds, *i.e.* if $r_t^k \geq r_t^c$ (see (17)). When $r_t^k < \bar{r}^k$, productive firms cannot afford to pay the minimum loan rate that unproductive firms require in order to lend, and the credit market collapses.²⁸

When condition (23) holds, there exist multiple equilibria, denoted by E , U , and A in panel (i) of Figure 3. In what follows, we focus on equilibria A and E which, unlike U , are stable under tatônnement.²⁹ When condition (23) does not hold, A (for ‘‘Autarky’’) is the only possible equilibrium. We describe equilibria A and E in turn.

²⁸In Section B.5 of the online appendix we show that, in that case, no lender can make a positive profit by charging a risk premium that would compensate for the losses due to the default of unproductive firms. In other words, there exists no pooling equilibrium with trade.

²⁹We rule out equilibrium U because it is not tatônnement-stable. An equilibrium rate r_t^c is tatônnement-stable if, following any small perturbation to r_t^c , a standard adjustment process —whereby the loan rate goes up (down)

Consider equilibrium A , where $r_t^c = -\delta$. At that rate, unproductive firms are indifferent between keeping their capital idle or selling it and lending the proceeds. Hence, any supply of funds within the interval $[0, \mu K_t]$ is consistent with optimal firm behaviour. However, the incentive compatible amount of funds that can be borrowed at that rate is zero ($\psi_t = 0$). As a result, $L^D(-\delta) = L^S(-\delta) = 0$ and there is no trade and no capital reallocation: $K_t^u = K_t^p = K_t$. In what follows, we refer to this autarkic equilibrium as a “financial crisis”.

Equilibrium E , in contrast, features a loan rate $r_t^c = r_t^k \geq \bar{r}^k > -\delta$, at which every unproductive firm sells capital to productive firms, as if there were no financial frictions (*i.e.* equilibrium E in panel (i) of Figure 3 is the same as equilibrium E in Figure 2). In that case, there is perfect capital reallocation, with $K_t^u = 0$ and $K_t^p = K_t/(1 - \mu)$. We refer to this equilibrium as “normal times”.

Finally, when productive firms’ return on capital r_t^k falls below the threshold \bar{r}^k , so that condition (23) is not satisfied anymore (panel (ii) of Figure 3), the range of loan rates for which $L^D(r_t^c) \geq L^S(r_t^c) > 0$ vanishes altogether. In that case, only the autarkic equilibrium A survives.

Equilibrium construction. Solving the general equilibrium is not trivial because the condition of existence of an active credit market (23) involves productive firms’ return on capital r_t^k , which depends on whether the credit market is active in the first place.

To solve the equilibrium we proceed in two stages. We first conjecture that equilibrium E exists —and therefore co-exists with the autarkic equilibrium A — and assume that, in that case, market participants coordinate on equilibrium E , the most efficient of the two equilibria.³⁰ We then solve the general equilibrium under this conjecture, determine r_t^k , and verify *ex post* that $r_t^k \geq \bar{r}^k$. If this condition is satisfied, we conclude that E is the equilibrium. Otherwise, we conclude that an active credit market equilibrium cannot emerge and therefore that the autarkic equilibrium A is the only possible equilibrium.³¹

Sudden switches from normal to crisis times in our model are akin to the fundamental-based panics described in Gorton (1988). When the return on capital falls below the minimum loan rate, productive firms cannot (afford to) get the unproductive ones to lend. As unproductive firms too may want to borrow and are indistinguishable from productive ones, lenders’ information about prospective borrowers deteriorates and the adverse selection problem surfaces, causing lenders to panic and to refuse to lend.

whenever there is excess demand (supply) of credit — pulls r_t^c back to its equilibrium value (see Mas-Colell et al. (1995), Chapter 17). Note that U and E yield the same aggregate outcome and overall rate of return on equity $\int_0^1 r_t^q(j) dj$, and only differ in terms of the distribution of individual returns $r_t^q(j)$ across firms.

³⁰There are of course several —but less parsimonious— ways to select the equilibrium. For example, one could introduce a sunspot, *e.g.* assume that firms coordinate on equilibrium E (*i.e.* are “optimistic”) with some constant and exogenous probability whenever this equilibrium exists. It should be clear, however, that the central element of our analysis is Proposition 2 for the existence of E , not the selection of E conditional on its existence. In other terms, our analysis does not hinge on the assumed equilibrium selection mechanism.

³¹In effect, the way we solve the equilibrium amounts to ruling out coordination failures: a crisis breaks out if and only if A is the only possible equilibrium.

A Negative Yield Gap Presages Search for Yield. For what follows, it is useful to derive the relationship between firms' average return on equity and productive firms' return on capital. Let

$$r_t^q \equiv \int_0^1 r_t^q(j) dj \quad (24)$$

be firms' average return on equity. In equilibrium, this return is equal to (see Section A.3 in the online appendix)

$$r_t^q = \begin{cases} r_t^k & \text{if } r_t^k \geq \bar{r}^k \\ -\mu\delta + (1-\mu)r_t^k & \text{otherwise} \end{cases} \quad (25)$$

which in turn implies that $r_t^q = r_t^k$ in normal times and therefore that condition (23) can be re-written in terms of return on equity r_t^q as $r_t^q \geq \bar{r}^k$.

Definition 1. (Yield Gap) *The yield gap $(1+r_t^q)/(1+r^q)$ is the gap between firms' average gross return on equity $1+r_t^q$ and its deterministic steady state value $1+r^q$.*

Given that financial crises have a low frequency, a realistic parametrisation of the model (see Section 4.1) requires that there is no crisis in the deterministic steady state, *i.e.* that $r^q > \bar{r}^k$. Since in normal times $r_t^q = r_t^k$ (see (25)), a positive yield gap ($r_t^q > r^q$) indicates that the economy is well above the crisis threshold and, in that sense, resilient to adverse aggregate shocks. When the yield gap is negative ($r_t^q < r^q$), in contrast, unproductive firms may be inclined to search for yield and the credit market is vulnerable: even a small adverse shock and change in fundamental can tip the economy into a crisis. Later, in Section 5.1, we study the possibility that the central bank systematically responds to the yield gap—in addition to inflation and output—to enhance the resilience of the credit market.

3.2.3 Other Markets

As only productive firms hire labour and produce, the labour and intermediate goods markets clear when

$$N_t = \int_{j \in \Omega_t^p} N_t(j) dj = (1-\mu)N_t^p \quad (26)$$

$$Y_t = \int_{j \in \Omega_t^p} X_t(j) dj = (1-\mu)X_t^p \quad (27)$$

and the final goods market clears when

$$Y_t = C_t + I_t + \frac{\theta}{2} Y_t \pi_t^2 \quad (28)$$

3.3 General Equilibrium Outcome

The level of aggregate output depends on the equilibrium of the credit market. In normal times, the entire capital stock of the economy is used productively and, given K_t and N_t , aggregate output is the same as in an economy without financial frictions:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad (29)$$

In crisis times, in contrast, unproductive firms keep their capital idle, only a fraction $1 - \mu$ of the economy's aggregate capital stock is used productively, and aggregate productivity falls.³² For the same K_t and N_t , output is therefore lower than in normal times:

$$Y_t = A_t ((1 - \mu)K_t)^\alpha N_t^{1-\alpha} \quad (30)$$

All else equal, the aggregate productivity loss caused by the financial crisis amounts to a fraction $1 - (1 - \mu)^\alpha$ of aggregate output.

Corollary 1. (*Monetary Policy and Financial Stability*) *A crisis breaks out in period t if and only if*

$$\frac{Y_t}{\mathcal{M}_t K_t} < \frac{1 - \tau}{\alpha} \left(\frac{(1 - \theta)\mu - \delta}{1 - \mu} + \delta \right)$$

Proof. This inequality follows from combining relations (16), (18), (23), (27), and the result that $K_t^p = K_t/(1 - \mu)$ in normal times. \square

What are the channels through which monetary policy affects financial stability? Corollary 1 makes clear that crises may emerge through a fall in aggregate output (the “Y-channel”), a rise in retailers’ markup (the “M-channel”), or excess capital accumulation (the “K-channel”). Given a (predetermined) capital stock K_t , a crisis is more likely to break out following a shock that lowers output or increases the markup, as productive firms’ return on capital r_t^k may fall below the crisis threshold \bar{r}^k in either case. Such a shock does not need to be large to trigger a crisis, if the economy has accumulated a large enough capital stock. Indeed, when K_t is high, all else equal, productive firms’ average return on equity tends to be relatively low and the credit market fragile. As we show in the next section, the capital stock may be especially high towards the end of an unusually long economic boom. In this case, even a modest change in Y_t or \mathcal{M}_t may trigger a crisis.

The upshot is that the central bank may affect the probability of a crisis both in the short and in the medium run. In the short run, it may do so through the effect of contemporaneous changes in its policy rate on output and inflation (the Y- and M-channels). To see this, consider the effects of an unexpected rate hike. On impact, the hike works to reduce aggregate demand for final and intermediate goods, weighing on retailers’ prices and costs. In the presence of menu costs, prices adjust more slowly than costs and retailers’ markups rise. The concomitant fall in aggregate demand and rise in the markup, in turn, weighs on firms’ average return on equity, bringing the economy closer to a crisis. In the medium run, in contrast, monetary policy affects financial stability through its impact on the household’s saving behaviour and capital accumulation (the K-channel). For example, a central bank that commits itself to systematically and forcefully responding to fluctuations in output will —all else equal— tend to slow down capital accumulation during booms, thereby enhancing the resilience of the credit market.

³²Even though in normal times the aggregate production function is the same as in an economy with a frictionless credit market, N_t and K_t (and therefore output) will in general be higher in our model than in the frictionless case. The reason is that households tend to accumulate precautionary savings and work more to compensate for the fall in consumption should a crisis break out. All else equal, the mere anticipation of a crisis induces the economy to accumulate more capital in normal times compared to a frictionless economy.

4 Anatomy of a Financial Crisis

The aim of this section is to describe the “average” dynamics around financial crises under a sensible parametrisation of the model. This section also briefly links the model-based crisis dynamics with those observed in the data, with a focus on the role of monetary policy and the predictability of financial crises.

4.1 Parametrisation of the Model

We parameterise our model based on quarterly data under Taylor (1993)’s original monetary policy rule (*i.e.* with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$). The standard parameters of the model take the usual values (see Table 1).

Table 1: Parametrisation

Parameter	Target	Value
<i>Preferences</i>		
β	4% annual real interest rate	0.989
σ	Logarithmic utility on consumption	1
φ	Inverse Frish elasticity equals 2	0.5
χ	Steady state hours equal 1	0.81
<i>Technology and price setting</i>		
α	64% labour share	0.36
δ	6% annual capital depreciation rate	0.015
ϱ	Same slope of the Phillips curve as with Calvo price setting	58.22
ϵ	20% markup rate	6
<i>Aggregate TFP (supply) shocks</i>		
ρ_a	Standard persistence	0.95
σ_a	Volatility of inflation and output in normal times (in %)	0.81
<i>Aggregate Demand shocks</i>		
ρ_z	Standard persistence	0.95
σ_z	Volatility of inflation and output in normal times (in %)	0.16
<i>Interest rate rule</i>		
ϕ_π	Response to inflation under TR93	1.5
ϕ_y	Response to output under TR93	0.125
<i>Financial Frictions</i>		
μ	Productivity falls by 1.8% due to financial frictions during a crisis	0.05
θ	The economy spends 10% of the time in a crisis	0.522

The utility function is logarithmic with respect to consumption ($\sigma = 1$). The parameters of labour dis-utility are set to $\chi = 0.814$ and $\varphi = 0.5$ so as to normalise hours to one in the deterministic steady state and to obtain an inverse Frish labour elasticity of 2 —this is in the ballpark of the calibrated values used in the literature. We set the discount factor to $\beta = 0.989$, which corresponds to an annualised average return on financial assets of about 4%. The elasticity of substitution between intermediate goods ϵ is set to 6, which generates a markup of 20% in the deterministic steady state. Given this, we set the capital elasticity parameter α to 0.36

to obtain a labour income share of 64%. We assume that capital depreciates by 6% per year ($\delta = 0.015$). We set the price adjustment cost parameter to $\varrho = 58.2$, so that the model generates the same slope of the Phillips curve as in a Calvo pricing model with an average duration of prices of 4 quarters. The persistence of the technology and demand shocks is standard and set to $\rho_a = \rho_z = 0.95$. Their standard deviations are set so as to replicate the volatility of inflation and output in normal times: $\sigma_a = 0.008$ and $\sigma_z = 0.001$.

Compared to the textbook NK model, there are two additional parameters: the share of unproductive firms μ and the default cost θ . Parameter μ directly affects the cost of financial crises in terms of productivity and output loss (see relation (30)). Given $\alpha = 0.36$, we set $\mu = 5\%$ so that capital mis-allocation entails a further 1.8% ($= 1 - (1 - 0.05)^{0.36}$) fall in aggregate productivity during a financial crisis.³³ This (momentary) productivity loss comes *on the top of* that due to the adverse TFP shock that may trigger the crisis. Parameter θ governs the degree of moral hazard and, given μ , the incidence of financial crises. Proposition 2 shows that the lower θ , the higher the minimum marginal return on capital required for an active credit market to exist and, as a result, the higher the probability of a crisis. We set $\theta = 52.2\%$ so that the economy spends 10% of the time in a crisis in the stochastic steady state.³⁴

4.2 Simulated Dynamics Around Financial Crises and Link to Facts

The aim of this section is to describe the macroeconomic dynamics around financial crises. We compute these dynamics in two steps. First, we solve our non-linear model numerically using a global solution method.³⁵ Second, we feed the model with aggregate productivity and demand shocks and simulate it over 10,000,000 periods. Second, we identify the starting dates of financial crises and compute the average of macro-financial variables in the 24 quarters around these dates. To filter out the potential noise due to the aftershocks of past crises, we only report averages for “new” crises, *i.e.* crises that follow at least 24 quarters of normal times.

The average crisis dynamics, reported in Figure 4, are the outcome of both the two exogenous non-financial shocks (panel (a)) and the endogenous response of the economy to these shocks (other panels). The results suggest that these dynamics can be broadly decomposed into three phases: a boom, a slowdown, and a bust.

The Boom. The average crisis dynamics begin with a protracted sequence of small positive productivity and demand shocks, 8 to 24 quarters before the start of the crisis (Figure 4, panel

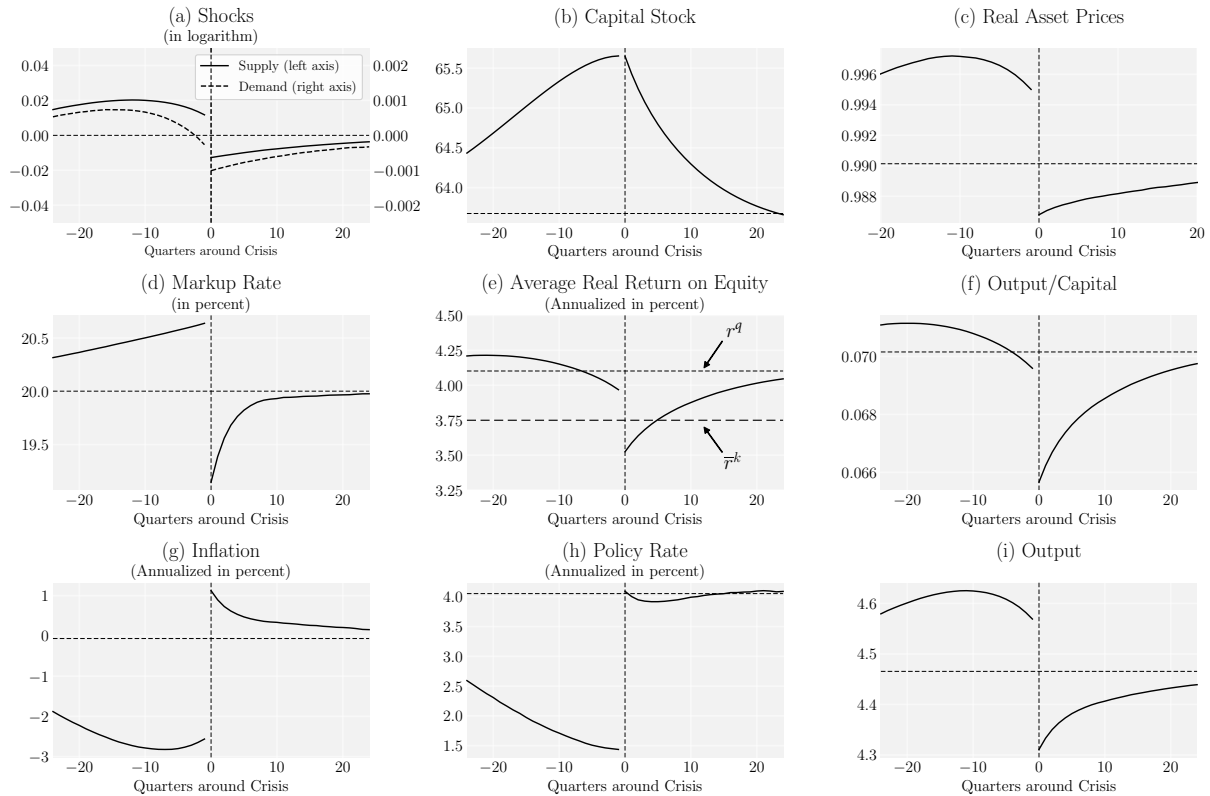
³³While there is a general agreement that financial frictions impair the re-allocation of capital across firms, the resultant aggregate productivity loss is hard to measure. Estimates vary depending on the data and methodology: *e.g.* about 1% of total factor productivity in Oulton and Sebastián-Barriol (2016), 1–2% in Gilchrist et al. (2013), 2.4% in Duval et al. (2019), up to 5% in Fernald (2015). We opt for an intermediate value of 1.8%.

³⁴Romer and Romer (2017) and Romer and Romer (2019) construct a semiannual financial distress index for 31 OECD countries and rank the level of distress between 0 (“no stress”) to 14 (“extreme crisis”). Using their data, we compute the fraction of the time these countries spent in financial distress at or above level 4 (“minor crisis” or worse) over the period 1980–2017, and obtain 10.57%.

³⁵Our model cannot be solved linearly because panic-like crises induce discontinuities in the optimal decision rules. Details on the numerical solution method are provided in Section A.10 of the online appendix.

(a)). These positive shocks entail an economic boom (panel (i)), a rise in the real price of the safe asset (panel (c)) and high capital returns, as measured by the return on equity (panel (e)) and the output to capital ratio (panel (f)). This early phase is also characterised by a positive yield gap: $r_t^q - r^q > 0$ (panel (e)), and sustained capital accumulation (panel (b)).³⁶

Figure 4: Simulated Dynamics Around Crises



Notes: Average dynamics of the economy around the beginning of a crisis (in quarter 0) in the stochastic steady state of the TR93 economy. To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 24 quarters of normal times. The horizontal dotted lines correspond to the average values in the stochastic steady state. In panel (e), the upper horizontal dashed line corresponds to the deterministic steady state value r^q , the lower one to the crisis threshold \bar{r}^k . Since the capital stock is financed externally through equity issuance, its dynamics (panel (b)) correspond to those of equity funding (not reported). In Section B.3.1 of the online appendix, we show that, to finance their startup capital, firms are indifferent between issuing equity or riskless debt, provided that they can issue such debt. In that case, the dynamics of the capital stock in panel (b) can also be interpreted as those of riskless debt. The asset price reported in panel (c) corresponds to the real price of an asset that returns one unit of consumption good next period, *i.e.* $\beta \mathbb{E}_t [(C_{t+1}/C_t)^{-\sigma}]$ (see, *e.g.* Cochrane (2001)). In the stochastic steady state, the *average* crisis dynamics are essentially the same as the *median* ones for all variables except the aggregate shocks (panel (a)), whose values we discretised for the purpose of the numerical resolution of the model (see Section A.10 in the online appendix). The median dynamics of the shocks are nothing but a “stepwise version” of the average ones.

As the positive productivity and demand shocks have opposite effects on prices, the evolution of inflation indicates which shock has the biggest footprint. The prolonged fall in inflation rate and rise in markups (panels (g) and (d)) suggests that, on balance, the boom is mainly driven by

³⁶In the baseline version of the model considered here, investment is entirely financed externally through equity issuance, implying that the investment boom goes hand in hand with an equity issuance boom. As noted in Section B.3 in the online appendix, however, the baseline model is isomorphic to a version of the model where firms can finance their entire startup capital stock with riskless debt. Under this condition, the investment boom in panel (b) can be interpreted as a credit boom, and financial crises are preceded by concomitant booms in credit and in asset prices.

the productivity shocks.³⁷ Under TR93 (our baseline), the persistent dis-inflationary pressures induce the central bank to cut its policy rate and to keep it low-for-long (panel (h)). As a result, the run-up to the average crisis is characterised by both low inflation and monetary easing. Monetary easing, in turn, further boosts investment and capital accumulation.

The Slowdown. In the 8 quarters that precede the crisis, productivity gains subside and output falls toward its steady state (Figure 4, panels ((a) and (i)). As long as productivity remains above its steady state, households nonetheless continue to accumulate savings and accumulate capital, albeit at a slower pace (panel (b)). Similarly, even though the fall in productivity works to raise prices, inflation still remains below its steady state (panel (g)) and markups continue to rise (panel (d)). In turn, the combination of lower productivity, higher markups and rising capital stock weighs on firms’ real equity returns, which fall below their steady state during the slowdown (panels (e) and (f)). The yield gap, $r_t^q - r^q$, turns negative 4 to 8 quarters before the crisis.

The Bust. The negative yield gap marks the entry of the economy into a region of financial fragility. Lower capital returns entail a lower equilibrium loan rate. And the lower loan rate gives unproductive firms more incentives to search for yield, stoking lenders’ fear of being defaulted upon. The credit market eventually breaks down as a relatively modest adverse productivity shock (and the endogenous response of the economy thereto) pulls equity returns further down and below the crisis threshold (Figure 4, panels (a), (e) and (f)). The average crisis is characterised by a severe recession (panel (i)) and asset price correction (panel (c)). On average, output falls by 6.6% during a crisis (Table 2, row (1), column “Output Loss”).

Role of Monetary Policy. Systematic monetary policy plays an important role in the dynamics that precede crises. Figure 5 depicts the transmission chain for the average crisis dynamics³⁸. At first, keeping the policy rate low-for-long during a dis-inflationary boom (Figure 4, panel (h)) boosts aggregate demand, raises capital returns, and stimulates capital accumulation (panel (b)).³⁹ Over time, however, a prolonged accumulation of capital gradually erodes capital returns, exposing the credit market to potential adverse shocks. When adverse produc-

³⁷In Section A.5 of the online appendix, we report the dynamics of crises in a version of the model with either supply or demand shocks. The comparison of these dynamics makes clear that demand-driven booms are inflationary while supply-driven ones are dis-inflationary.

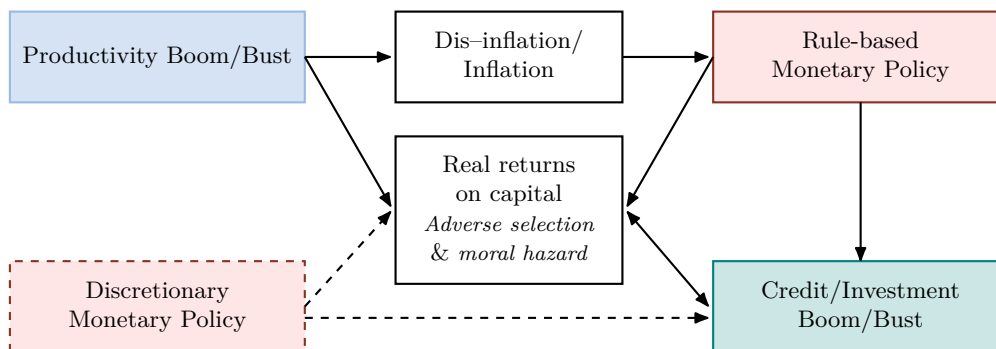
³⁸This transmission chain applies to the crises that follow a productivity-driven boom (a large share of the crises in our model). It does not apply to the small share of crises that follow demand-driven booms (Section A.5, Figure A.7), or to those that do not follow a boom. Relatedly, note that the effects of monetary policy on financial stability depend on the type of shocks hitting the economy. When demand shocks prevail, capital accumulation goes hand in hand with inflationary (instead of dis-inflationary) pressures. In that case, raising the policy rate to tame inflation helps to slow down capital accumulation and reduce the risk of financial stress in the medium run in our model, as described in Boissay et al. (2023).

³⁹Brunnermeier and Julliard (2008) and Piazzesi and Schneider (2008) argue that the dis-inflationary pressures in the first half of the 2000s in the US may have fueled the credit and asset price boom that preceded the GFC, due to people suffering from money or inflation illusion. Our model does not capture this mechanism, as we assume that all agents are fully rational. But it nonetheless captures the historical regularity that financial crises are preceded by a dis-inflationary boom (Figures 1 and 4, panels (g) and (i)).

tivity shocks hit and productivity reverts back to its steady state, inflation picks up, prompting the central bank to raise its policy rate (panel (h)) and eventually leading to the bust. At that stage, though, the rate hike acts more as a catalyst than as the root cause of the crisis, to the extent that the same hike may not have led to a crisis had monetary policy not been so-loose-for-so-long and the capital stock so high in the first place.⁴⁰

Link to Facts. Despite its stylised nature, our model does a fair job in capturing the most salient facts on financial crises (compare Figures 1 and 4). In particular, it is able to account for the rise in capital stock and output, the boom-bust in productivity and capital returns, as well as for the U-shaped dynamics of the inflation and policy rates observed in the run-up to historical financial crises. To our knowledge, our model thus provides a first theoretical explanation for why financial crises tend to follow a U-shaped monetary policy (Schularick et al. (2021), Grimm et al. (2023), Jiménez et al. (2023)).

Figure 5: Boom-Bust Episodes: the Role of Productivity and Monetary Policy



Notes: This diagram summarises the interactions between the main macro-financial factors (credit/investment, productivity, and monetary policy) that have been shown to contribute to the build-up of financial risks ahead of financial crises both in the data (Figure 1) and in the model (Figure 4).

That said, the average dynamics in Figure 4 mask some differences across crises. In particular, 45% of the crises in the stochastic steady state are “predictable”, in the sense that they are associated to a one-quarter-ahead crisis probability above 0.8 in the quarter that precedes them.⁴¹ In turn, predictable and unpredictable crises have distinct characteristics (see Figure A.4 in the online appendix). The former tend to be preceded by an investment boom and break out despite aggregate productivity being above its steady state value. By contrast, unpredictable crises break out in the wake of a large drop in productivity below its steady state value without the economy having experienced an investment boom. This latter feature is consistent with the notion that large exogenous shocks are hard to predict. Noticeably — but somewhat fortuitously, the proportion of predictable crises in our model (45%) is broadly in line with that

⁴⁰As Corollary 1 shows, abundant capital — a capital “glut” — is indeed a pre-condition for a financial crisis to break out in the absence of a large adverse shock.

⁴¹Figure A.3 (left-hand panel) of the online appendix shows that the distribution of the probability of a crisis (conditional on a crisis happening next quarter) in the stochastic steady state is bi-modal, with one set of crises associated to a crisis probability below 0.2 (“unpredictable” crises) and another associated to a crisis probability above 0.8 (“predictable” crises).

(between 40% and 64%) reported in Greenwood et al. (2022).

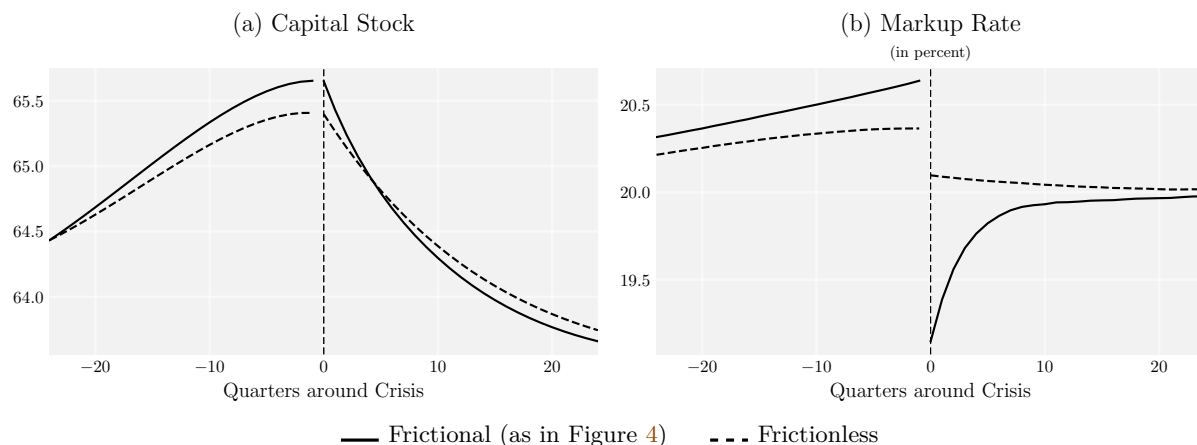
4.3 Crisis Anticipations and Externalities

The above discussion prompts the question why boom–driven crises take place even though agents anticipate them. The reason is that neither households nor retailers internalise the effects of their individual choices on financial fragility and that, somewhat paradoxically, their anticipation of a crisis induces them to precipitate (rather than avert) it.

When a crisis is looming, households seek to hedge against the future recession and smooth their consumption by accumulating precautionary savings, which contributes to increasing capital even further. Boissay et al. (2016) refer to this phenomenon as a “savings glut” externality.

Similar financial externalities stem from retailers. All else equal, the collapse of the credit market during a crisis induces a fall in aggregate productivity (term $(1 - \mu)^\alpha \in (0, 1)$ in relation (30)), and hence less dis–inflationary (or more inflationary) pressures compared to an economy with a frictionless credit market.⁴² To smooth their menu costs over time, retailers typically reduce their prices by less (or increase them by more) ahead of a crisis, thus raising their markup above the level that would otherwise prevail absent financial frictions. Since higher markups reduce firms’ return on equity, retailers’ response to financial fragility makes the financial sector even more fragile.⁴³

Figure 6: Crisis Anticipations, Saving/Capital Glut and Markup Externalities



Notes: Comparison of two economies under TR93 with a frictional *versus* frictionless credit market around the beginning of a crisis (in quarter 0). For the frictional credit market economy: same average dynamics as in Figure 4. For the frictionless credit market economy: counterfactual average dynamics, when the economy starts with the same capital stock in quarter -24 and is fed with the same aggregate shocks as the frictional credit market economy.

As Figure 6 illustrates, agents’ anticipation of a crisis underpins these externalities. The focus here is on the run–up phase to the average financial crisis, from quarters -24 to -1. The

⁴²This feature tallies with the “missing dis–inflation” during the GFC (Gilchrist et al. (2017)).

⁴³These “markup externalities” due to the presence of financial frictions come on the top of the usual aggregate demand externalities (Blanchard and Kiyotaki (1987)).

experiment consists in comparing the dynamics of capital and markups before a crisis with their dynamics in a counter-factual economy without financial frictions that is fed with the very same shocks. Since the credit market functions equally well in the two economies before the crisis, the difference reflects agents’ anticipations of a crisis and response thereto. It also informs us about how capital and markups would have evolved absent financial frictions. Figure 6 shows that both the capital stock and markup are higher when households and retailers anticipate a crisis, illustrating the presence of a capital glut. All else equal, the relatively high capital stock and markup make the crisis more likely. These externalities call for policy intervention, which we study next.

5 The “Divine Coincidence” Revisited

In the absence of financial frictions, SIT simultaneously eliminates inefficient fluctuations in prices and output gap and achieves the first best allocation —the so-called “divine coincidence” (Blanchard and Galí (2007)), as shown in Table 2 (row (6), column “Frictionless”). In the presence of financial frictions, in contrast, SIT does not deliver the first best allocation. In our model, in particular, the welfare loss under SIT is strictly positive, and amounts to 0.23% in terms of consumption equivalent variation (Table 2, row (6), column “Welfare Loss”).⁴⁴

This finding prompts the question: *Can (and should) central banks prevent financial crises by tolerating higher price volatility?* To answer this question, we study the trade-off between price and financial stability and compare welfare under SIT *versus* alternative monetary policy rules. We consider three types of rule: standard Taylor-type rules, Taylor-type rules augmented with the yield gap, and regime-contingent rules. Throughout this section, we focus on the systematic component of monetary policy and abstract from unexpected discretionary deviations from the pre-announced policy rule.

5.1 Price *versus* Financial Stability Trade-off

Comparing the effects of varied Taylor-type rules reveals a trade-off between price and financial stability. We find that the central bank can reduce the incidence and severity of crises by deviating from price stability, and reacting to output and the yield gap in addition to inflation. More precisely, Table 2 shows that, all else equal, raising ϕ_y from 0.125 to 0.375 in the Taylor-type rule (1) reduces the percentage of the time spent in crisis from 10% to 4.1% (Table 2, rows (1) *versus* (3), column “Time in Crisis”) as well as the output loss due to a crisis from 6.6% to 4.4% (column “Output Loss”). However, these financial stability gains come at the cost of higher inflation volatility (2.5% compared to 1.2%, column “Std(π_t)”).

To some extent, price instability can also contribute to financial fragility through markups (M-channel, see Section 3.3). All else equal, raising ϕ_π from 1.5 to 2.5 in the Taylor-type rule (1) reduces both the volatility of inflation from 1.2% to 0.5% (rows (1) *versus* (5), column “Std(π_t)”).

⁴⁴Since the distortions due to sticky prices are fully neutralised under SIT, this welfare loss is entirely due to the cost of financial crises.

and the time spent in crisis from 10% to 9.6% (column “Time in Crisis”). Improvements in financial stability *via* the M-channel are however limited, with a hard lower bound of crisis incidence at 9.4% under SIT. Further reducing the time spent in crisis requires departing from SIT at the cost of inflation volatility (rows (2) and (3)). Therefore, the central bank faces a trade-off between price and financial stability in our model.

Table 2: Economic Performance and Welfare Under Alternative Policy Rules

Rule	Model with Financial Frictions						Frictionless		
	parameters			Time in	Length	Output	Std(π_t)	Welfare	Welfare
	ϕ_π	ϕ_y	ϕ_r	Crisis/Stress (in %)	(quarters)	Loss (in %)	(in pp)	Loss (in %)	Loss (in %)
Taylor-type Rules									
(1)	1.5	0.125	–	[10]	4.8	6.6	1.2	0.82	0.56
(2)	1.5	0.250	–	7.2	4.0	5.4	1.8	1.48	1.21
(3)	1.5	0.375	–	4.1	3.1	4.4	2.5	3.10	2.07
(4)	2.0	0.125	–	9.7	5.0	7.2	0.6	0.41	0.17
(5)	2.5	0.125	–	9.6	5.1	7.5	0.5	0.31	0.08
SIT									
(6)	$+\infty$	–	–	9.4	5.1	8.1	–	0.23	0.00
Augmented Taylor-type Rules									
(7)	1.5	0.125	5.0	5.1	3.8	5.5	1.2	0.70	–
(8)	5.0	0.125	5.0	8.7	4.9	7.4	0.2	0.23	–
(9)	10.0	0.125	25.0	8.0	4.8	7.2	0.3	0.21	–
Backstop Rules									
(10)	1.5	0.125	–	15.5	–	–	1.2	0.56	–
(11)	$+\infty$	–	–	17.1	–	–	0.5	0.10	–

Notes: Statistics of the stochastic steady state ergodic distribution. “Time in Crisis/Stress” is the percentage of the time that the economy spends in a crisis in the case of the log-linear rule, or in stress in the case of the backstop rules. “Length” is the average duration of a crisis/stress period (in quarters). “Output Loss” is the percentage fall in output from one quarter before the crisis until the trough of the crisis (in %). “Std(π_t)” is the standard deviation of inflation in the stochastic steady state (in %). “Welfare Loss” is the loss of welfare relative to the first best economy, expressed in terms of consumption equivalent variation (in percentage points), and corresponds to the percentage of permanent consumption the household should be deprived of in the first best economy to reach the same level of welfare as in our economy with nominal and financial frictions. In the case of the frictionless credit market economy (column “Frictionless”), the SIT economy reaches the first best and there is no welfare loss in this case. In the case of the frictional credit market and the TR93 rule (case with $\phi_\pi = 1.5$, $\phi_y = 0.125$, and $\phi_r = 0$), the economy spends by construction 10% of the time in a crisis (square brackets; see Section 4.1).

Which leg of the trade-off dominates in terms of welfare is a quantitative question. We find that, on balance, the welfare loss due to price instability more than offsets the gain from enhanced financial stability under *standard* Taylor-type rules (rows (2)-(3) *versus* (6), column “Welfare Loss”). Hence, even though it is associated with a strictly positive (0.23%) welfare loss due to a relatively high incidence and severity of financial crises, SIT improves welfare upon standard Taylor-type rules.

Next we ask whether following other —more informed— Taylor-type rules could improve welfare. The evolution of the yield gap in Figure 4 (panel (f)) suggests that a potential candidate

is a rule whereby the central bank responds positively to this gap on the top of inflation and output. To study this possibility, we consider the following *augmented* Taylor rule (A–TR),

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \left(\frac{1 + r_t^q}{1 + r^q}\right)^{\phi_r} \quad (31)$$

with $\phi_r > 0$.

There are good reasons why this type of rule may improve welfare. On the one hand, and all else equal, it requires setting the policy rate above that of the corresponding standard Taylor rule during economic booms, when the yield gap is positive ($r_t^q > r^q$).⁴⁵ A higher policy rate helps to slow down capital accumulation and keep financial imbalances from building up during such booms. On the other hand, the augmented Taylor rule also requires from the central bank to set the policy rate below that of a standard rule when the economy approaches a crisis and the yield gap turns negative ($r_t^q < r^q$). At that point, lowering the policy rate helps to boost aggregate demand and steer the economy away from the financial fragility region.

And indeed Table 2 shows that responding to the yield gap fosters financial stability and increases welfare compared to standard Taylor–type rules. For example, the economy spends only 5.1% of the time in a crisis under the augmented TR93 rule (A–TR93) with $\phi_r = 5$, against 10% under TR93 (row (7) *versus* (1), column “Time in Crisis”). Setting $\phi_r > 0$ does not materially affect inflation volatility compared to TR93, implying a positive net effect on welfare: the welfare loss falls from 0.82% under TR93 to 0.70% under the A–TR93 rule (row (1) *versus* (7), column “Welfare Loss”). In turn, responding more aggressively to inflation helps to lower the overall welfare loss significantly down to 0.23%, *i.e.* to the same level as under SIT (row (6) *versus* (8)). Trying several values for ϕ_π , ϕ_y , and ϕ_r , we could reduce the welfare loss further down to 0.21% (row (9)) but not much beyond that. This latter result suggests that, compared to SIT, the cost of experiencing higher inflation volatility in normal times under augmented Taylor rules broadly balances the benefit of experiencing fewer financial crises.⁴⁶

5.2 “Backstop” Rules

We now consider more complex, “regime–contingent” monetary policy rules, whereby the central bank commits itself to following TR93 or SIT in normal times but also to doing whatever needed whenever necessary—and therefore exceptionally deviating from these rules—to forestall a crisis. In those instances, we assume that the central bank deviates “just enough” to avert the crisis, *i.e.* sets its policy rate so that $r_t^k = \bar{r}^k$ (see Proposition 2).⁴⁷ We refer to such contingent rule as a “backstop” rule.

⁴⁵Since the yield gap tends to be positive during the dis–inflationary booms that precede financial crises (see Figure 4), one implication of following an augmented Taylor rule is that the central bank will tend to set higher rates during such booms. Figure A.10 in Section A.7 of the online appendix illustrates this point. To some extent, following an augmented Taylor rule can be seen as akin to “leaning against the wind” (Svensson (2017a,b)), with the difference that it additionally requires from the central bank to boost the economy during recessions.

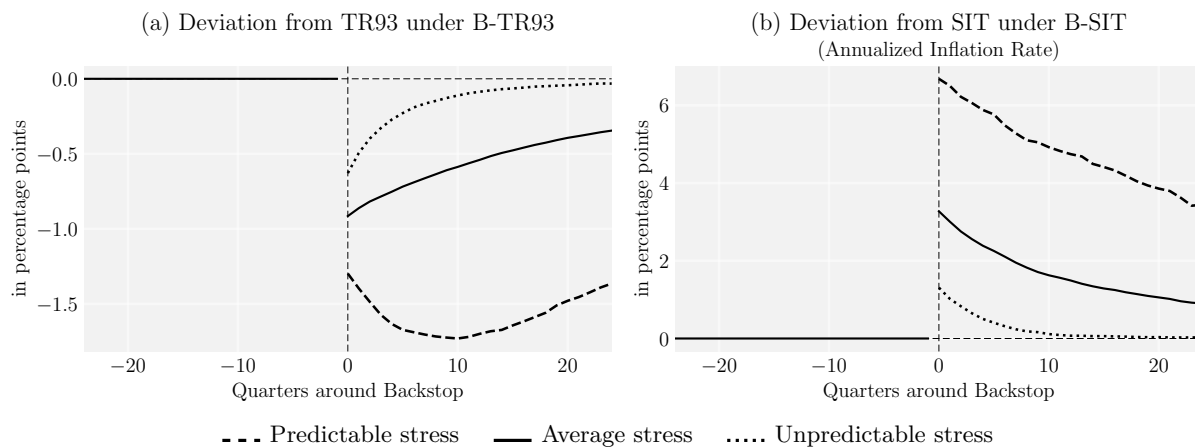
⁴⁶Section A.8 in the online appendix provides more details on the mechanisms underpinning these results.

⁴⁷In the case of a Taylor–type rule $1 + i_t = (1 + \pi_t)^{1.5} (Y_t/Y)^{0.125} \varsigma_t/\beta$, for example, this consists in setting the term ς_t equal to 1 if $r_t^k \geq \bar{r}^k$ and such that $r_t^k = \bar{r}^k$ whenever (and only then) r_t^k would otherwise be lower than \bar{r}^k . Likewise, under SIT, the central bank tolerates just enough deviations from inflation target so that $r_t^k = \bar{r}^k$.

There are two good reasons for considering this type of rule. The first is conceptual. As a financial crisis corresponds to a regime shift, a monetary policy rule followed in —and designed for— normal times is unlikely to be adequate during periods of financial stress. In effect, regime switches call for a regime–contingent strategy. Our contention is that, by giving the central bank more flexibility in its policy response, such strategy can alleviate the trade–off between price and financial stability discussed in the previous section. The second reason is practical: our backstop rule speaks to the “backstop principle” that most central banks in advanced economies have *de facto* been following since the GFC and that consists in deviating from conventional (“normal times”) monetary policy when necessary to restore financial market functionality.⁴⁸ Our analysis can therefore be seen as an attempt to assess the costs and benefits of post–GFC monetary policy strategies.

We show below that backstop rules can significantly improve welfare compared to both SIT and Taylor–type rules. We start by reporting in Figure 7 the average systematic deviations from TR93 and SIT that are needed in stress times to ward off crises (plain lines) and refer to these backstop policies as B–TR93 (panel (a)) and B–SIT (panel (b)), respectively.⁴⁹ The deviations are reported in terms of the policy rate for B–TR93 and the annualised inflation rate for B–SIT.

Figure 7: Backstop Necessary to Stave off a Crisis and Normalisation Path



Notes: Average deviations from the normal times’ policy rule that the central bank must commit itself to in order to forestall a financial crisis (quarter 0) and normalisation path (after quarter 0). Panel (a): deviation of the nominal policy rate, in percentage points, when the central bank otherwise follows TR93. Panel (b): deviation of the inflation target from zero, in percentage points, when the central bank otherwise follows SIT. For the purpose of the exercise, financial stress is defined as a situation where there would have been a crisis absent the monetary policy backstop. A stress episode is classified as “predictable” if the crisis probability in the quarter that precedes it (quarter –1) was in the *top* decile of its ergodic distribution. This type of episode typically follows an investment boom. In contrast, an “unpredictable” stress episode refers to a situation where the crisis probability in the quarter that precedes it was in the *bottom* decile of its ergodic distribution. This type of episode is typically due to adverse aggregate shocks. For a more detailed discussion on predictable and unpredictable episodes, see Section A.4 and Figure A.3 (panel (a)) in the online appendix.

Figure 7 shows that the central bank must loosen its policy compared to normal times,

⁴⁸For recent discussions on the backstop principle, see [Bank for International Settlements \(2022\)](#), [Hauser \(2023\)](#), and [Duffie and Keane \(2023\)](#).

⁴⁹Such deviation of the policy rate is akin to what [Akinci et al. \(2020\)](#) call “ R^{**} ”.

i.e. cut the policy rate by almost 1 percentage point below TR93 or temporarily tolerate a 3 percentage point higher inflation rate under SIT.⁵⁰ It also shows that the backstop policy must be unwound gradually, reflecting the time it takes for financial vulnerabilities to dissipate. In our model, the adequate normalisation path is narrow. Tightening monetary policy more slowly would lead to unnecessary high inflation and costs due to nominal rigidities. Tightening too quickly would result in a financial crisis and a “hard landing”.

One important determinant of the speed of normalisation is the type of financial vulnerabilities that are being addressed. When the stress is due to an exogenous adverse shock (“Unpredictable stress”), the central bank can set its policy rate (roughly) in line with the TR93 rule already after 10 quarters (panel (a), dotted line). When it is due to an excessive investment boom (“Predictable stress”), in contrast, the normalisation takes much longer and is still far from over after 24 quarters (dashed line). The reason is clear. As the central bank intervenes to stem stress, it concomitantly slows down the adjustment that would be necessary to eliminate the capital glut that causes stress in the first place. As a result, monetary policy must remain accommodative for longer to prevent a crisis.⁵¹

Finally, we study the net welfare gain of following a backstop rule. The results are reported at the bottom of Table 2. Two results stand out. First, backstopping the economy unambiguously improves welfare. In the case of B–TR93, the welfare loss is reduced from 0.82% (absent a backstop) to 0.56% (row (1) *versus* (10), column “Welfare Loss”), which is essentially the same as in the economy with no financial frictions (row (1), column “Frictionless”). In the case of B–SIT, the welfare loss falls by more than half, from 0.23% without backstop to 0.1% with backstop (row (6) *versus* (11), column “Welfare Loss”). Second, the financial sector is *more* fragile when the central bank commits itself to backstopping the economy. Under B–SIT, for instance, the central bank has to backstop the economy and deviate from its normal times policy rule more than 17% of the time, whereas under SIT the economy would spend only 9.4% of the time in a crisis (row (11) *versus* (6), column “Time in Crisis/Stress”). The reason is that, as they forestall financial crises, backstop policies also delay the downward adjustment of the capital stock that would be necessary to sustain high capital returns and deter search–for–yield behaviour throughout the business cycle. On average, the capital stock is therefore higher under B–SIT than under SIT, which in turn raises the economy’s exposure to adverse shocks.⁵² The credit market is therefore more prone to financial stress when the central bank provides a backstop.⁵³

⁵⁰Throughout, we assume that the central bank is not constrained by a zero lower bound on the nominal policy rate.

⁵¹This result echoes with that of models where crises are driven by an exogenous financial shock to borrowers’ collateral or net worth (*e.g.* Andrés et al. (2013), Manea (2020)). In these models, the optimal policy indeed also consists in lowering the policy rate during a crisis.

⁵²In Section A.9 of the online appendix, we show that the accumulation of capital is however slower in the run–up to periods of financial stress with a backstop than without. As households factor in the central bank’s commitment to backstop the economy, they indeed accumulate less precautionary savings before periods of financial stress than they otherwise would, which in turn slows down capital accumulation.

⁵³As Hauser (2021) puts it, [monetary policy backstops] “*are an appropriate response to a truly unprecedented situation —just as powerful anti-inflammatory medicines are the right solution to a sudden and massive flare*”

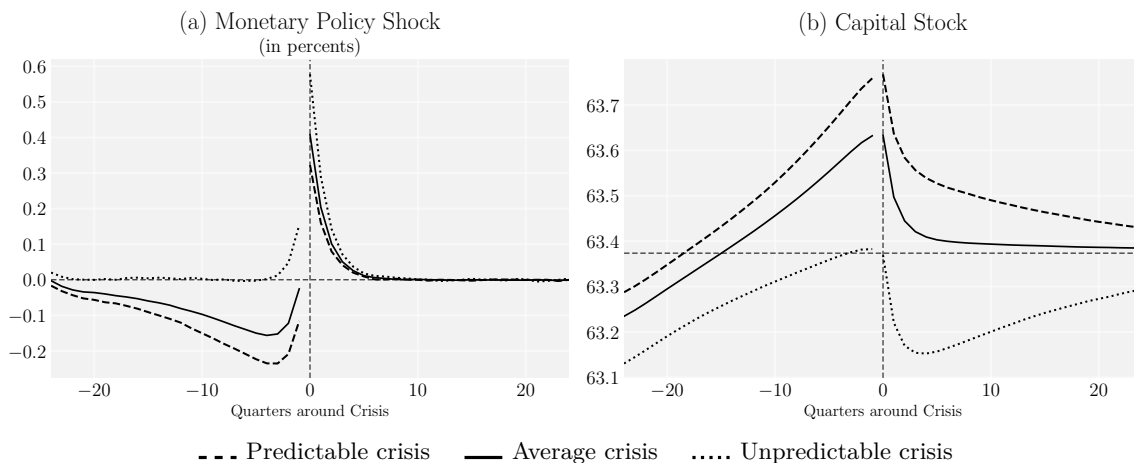
6 Discretion as a Source of Financial Instability

To what extent may monetary policy itself brew financial vulnerabilities? We now turn to the effects on financial stability of *unexpected and discretionary* deviations from the monetary policy rule. In his narrative of the GFC, Taylor (2011) argues that discretionary loose monetary policy may have exposed the economy to financial stability risks—the “Great Deviation” view. This section revisits this narrative and assesses the potential detrimental effects of monetary policy surprises. For the purpose of pinpointing these effects, we consider a TR93 economy that experiences monetary policy shocks and where these shocks are the only source of aggregate uncertainty—*i.e.* we discard the supply and demand shocks. More specifically, we consider a monetary policy rule of the form

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{1.5} \left(\frac{Y_t}{Y}\right)^{0.125} \varsigma_t$$

where the monetary policy shock ς_t follows an AR(1) process $\ln(\varsigma_t) = \rho_\varsigma \ln(\varsigma_{t-1}) + \epsilon_t^\varsigma$, with $\rho_\varsigma = 0.5$ and $\sigma_\varsigma = 0.0025$, as in Galí (2015). We are interested in the dynamics of monetary policy shocks around crises in this new environment.

Figure 8: Rates too Low for too Long May Lead to a Crisis



Notes: Average discretionary deviations from TR93 (panel (a)) and evolution of the capital stock (panel (b)) around the beginning of a crisis (quarter 0), in an economy with only monetary policy shocks. A financial crisis is classified as “predictable” if the crisis probability in the quarter that precedes it (quarter -1) was in the *top* decile of its ergodic distribution. A crisis is classified as “unpredictable” if the crisis probability in the quarter that precedes it was in the *bottom* decile of its ergodic distribution. For a more detailed discussion on predictable and unpredictable stress, see Section A.4 and Figure A.3 (panel (a)) in the online appendix.

The results, reported in Figure 8 (panel (a)), show that the average crisis breaks out after a long period of unexpected monetary easing as the central bank reverses course (plain line). Keeping the policy rate *too low for too long* stimulates capital accumulation (panel (b)), which in turn undermines the resilience of the credit market to shocks via the K-channel. The crisis is then triggered by three consecutive, unexpected, and abrupt interest rate hikes toward the end

up. But such drugs are less well suited to treating long-term conditions— and there is every reason to believe that, absent further action, we will see more frequent periods of dysfunction in markets (...) if business model vulnerabilities persist.”

of the boom. The comparison of the dynamics of predictable (dashed line) and unpredictable (dotted line) crises further shows that the looser—for longer the monetary policy, the smaller the hikes “needed” to trigger a crisis.

These findings are consistent with the recent empirical evidence that an unanticipated protracted discretionary loose monetary policy followed by rapid rate hikes—or U-shaped policy rate path—is conducive to financial instability (Schularick et al. (2021), Grimm et al. (2023), Jiménez et al. (2023)). More generally, our analysis highlights that discretionary monetary policy may also be, on its own, a source of financial instability (as depicted in Figure 5, dashed arrows).

7 Contribution and Relation to the Literature

One contribution of this paper is to propose a new monetary model in which adverse selection/moral hazard in credit markets gives rise to endogenous financial panics. Another contribution is to use this model to parse the link between systematic and discretionary monetary policies and financial fragility. In the process, we show that our model can account for several features common to a broad range of historical crisis episodes. We also compare the performances of a set of monetary policy rules in terms of price stability, financial stability, and welfare. This comparison yields novel insights into the adequate monetary policy strategy when credit markets are fragile.

The present paper straddles several strands of the literature.

Banking Models with Adverse Selection and Moral Hazard. The financial frictions considered here are similar to those in classical principal–agent banking models such as Stiglitz and Weiss (1981), Mankiw (1986) and more particularly Bernanke and Gertler (1990), Gertler and Rogoff (1990), and Azariadis and Smith (1998). In these models, entrepreneurs typically differ in terms of the riskiness/productivity of their projects and have an informational advantage over lenders. This asymmetry of information creates an agency problem between lenders and entrepreneurs–borrowers. While the nature of this agency problem and attendant frictions (*e.g.* the type of adverse selection and moral hazard) may vary across models, a general result is that, ultimately, the aggregate outcome ought to improve with the “creditworthiness” of borrowers, reflected in their net worth, payoffs, or capital returns.⁵⁴ When these payoffs decrease, incentives deteriorate, informational frictions become more prevalent and the credit market may

⁵⁴In these models, higher pledgeable payoffs invariably improve borrowers’ incentives and prompt lenders to raise their supply of credit and borrowers’ borrowing limit (*i.e.* they induce an outward shift of the loan supply curve), ultimately improving the equilibrium outcome. By contrast, and all else equal, a rise in the (off-equilibrium) loan market rate may affect incentives either way depending on the friction considered. In Stiglitz and Weiss (1981) and Mankiw (1986), for example, a rise in the loan rate along the credit supply curve crowds out the *safest* borrowers, whereas in Gertler and Rogoff (1990), Azariadis and Smith (1998) and the present model, it instead crowds out the *riskiest* ones. The difference is due to the fact that, in the latter models, firms also hold internal funds and have the option to lend these funds on the credit market. When inefficient firms have internal funds, a rise in the (off-equilibrium) loan rate *increases* their opportunity cost of investing in risky projects to the detriment of lenders.

collapse, as in [Mankiw \(1986\)](#), [Azariadis and Smith \(1998\)](#), and [Boissay et al. \(2016\)](#).⁵⁵ Our contribution is to embed this classical information view of financial crises into an otherwise standard; dynamic; stochastic; general equilibrium; fully rational expectations NK framework in which there is a role for monetary policy.

Macro–models with Financial Crises. Following [Kiyotaki and Moore \(1997\)](#), [Gertler and Kiyotaki \(2011\)](#), [Gertler and Karadi \(2011\)](#), [Jermann and Quadrini \(2012\)](#), a large body of the macro–financial literature models crises as situations where borrowers’ financial constraint (*e.g.* a leverage or collateral constraint) tightens after an exogenous adverse financial shock (typically a “capital quality” or capital pledgeability shock). In more recent papers (*e.g.* [Gertler and Kiyotaki \(2015\)](#), [Boissay et al. \(2016\)](#), [Gertler et al. \(2020\)](#)) financial crises take the form of micro–founded endogenous panics, as in our case. Ours complements previous work in that it studies the role of monetary policy in the genesis of crises.

Monetary Policy and Financial Stability. Our work is related to the literature on whether central banks should “lean against the wind” to avert financial crises ([Woodford \(2012\)](#), [Svensson \(2017a\)](#), [Gourio et al. \(2018\)](#)), which introduces endogenous crises in otherwise standard NK frameworks.⁵⁶ These works assume specific and reduced form relationships to describe how macro–financial variables (*e.g.* credit gap, credit growth, leverage) affect the likelihood of a crisis. In our model, in contrast, crises—including their probability and size—are micro–founded and derived from first principles. Our approach has important advantages for policy analysis. One, of course, is that our policy experiments are immune to the Lucas critique. Another is that monetary policy influences not only the crisis probability but also the size of the recessions that typically follow crises, and therefore the associated welfare cost. Yet another advantage of our approach is that, even though crises can be seen as credit booms “gone bust”, not all booms are equally conducive to crises—a key element to determine how hard to lean against booms. More generally, our findings do not hinge on any postulated reduced functional form for the probability or size of a crisis. In this sense, ours can be seen as a fairly general framework that provides micro–foundations to the setups in [Woodford \(2012\)](#), [Svensson \(2017a\)](#) and [Gourio et al. \(2018\)](#).

Financial Crises and Capital Mis–allocation. The “Great Productivity Slowdown” ([Duvall et al. \(2019\)](#)) that followed the GFC prompted the question why dysfunctional credit markets can cause large disruptions in the reallocation of capital within the economy and, therethrough, large reductions in aggregate total factor productivity.⁵⁷ To answer this question, [Khan and](#)

⁵⁵In that respect, our approach is also related to [Dang et al. \(2019\)](#), who propose a model where financial panics may occur when the collateral or the pledgeable future returns on capital backing debts have lost enough value to make the private acquisition of information worthwhile, thus giving rise to adverse selection.

⁵⁶Other contributions include [Bernanke and Gertler \(2000\)](#), [Galí \(2014\)](#), [Filardo and Rungcharoenkitkul \(2016\)](#), [Cairó and Sim \(2018\)](#), [Ajello et al. \(2019\)](#), [Fontanier \(2022\)](#), [Coimbra and Rey \(2023\)](#).

⁵⁷Other related papers seek to explain *business cycle* fluctuations in aggregate productivity by financial frictions and capital mis–allocation, *e.g.* [Eisfeldt and Rampini \(2006\)](#), [Ai et al. \(2020\)](#), [Cooper and Schott \(2023\)](#).

Thomas (2013) develop a model where investment is irreversible and firms face idiosyncratic productivity shocks as well as exogenous financial shocks to their borrowing constraint (*e.g.* net-worth or capital-quality shocks). Moll (2014) shows that such a constraint is especially relevant when the idiosyncratic productivity shocks are transitory because, in this case, productive firms do not have enough time to accumulate enough cash out of their cash flow to self-finance their investments. Midrigan and Xu (2014) distinguish two types of capital re-allocation: from unproductive to productive incumbents, and from incumbents to new entrants. Ottonello (2021) considers a form of mis-allocation whereby a fraction of the capital stock is kept idle during financial crises (as in our model) — a situation he refers to as “capital unemployment”. One common feature of these contributions is that financial crises are modelled as exogenous, large adverse *financial* shocks. In our model, in contrast, financial crises stem from the economy experiencing standard adverse *non-financial* shocks on the back of financial imbalances. Our approach can therefore be seen as endogenising the financial shocks typically considered in the existing macro-financial literature (Galí (2018)).

In the process, we bring this literature into line with the recent empirical literature that shows that a large share of financial crises are predictable, as opposed to “bolts from the sky” (Schularick and Taylor (2012), Greenwood et al. (2022)). This feature is important because the “bolts from the sky” *versus* “endogenous” views of financial crises have very different implications for the conduct of monetary policy. In the former view, monetary policy can only *react* in the short-term to adverse financial shocks. In the latter, in contrast, monetary policy also has a bearing on the build-up of financial imbalances and can *act preemptively* to lower the probability of a financial crisis in the medium-term (as we discussed in Section 5).

Heterogeneous Agents NK Models. Our paper also belongs to the literature on the transmission of monetary policy in heterogeneous agent New Keynesian (HANK) models. Most existing HANK models focus on household heterogeneity and study the channels through which this heterogeneity shapes the effects of monetary policy on aggregate demand (Guerrieri and Lorenzoni (2017), Kaplan et al. (2018), Auclert (2019), Debortoli and Galí (2021)). In contrast, our model considers firm heterogeneity (as in Adam and Weber (2019), Manea (2020), Ottonello and Winberry (2020)) and the role of credit markets in channelling resources to the most productive firms.

8 Concluding Remarks

In this paper, we have proposed an extension of the textbook NK model that allows for the possibility of endogenous financial crises. This extension features capital accumulation, heterogeneous firms, and a credit market that permits an efficient reallocation of capital across firms. Absent frictions in the credit market, the equilibrium outcome boils down to that of the standard model with a representative firm. With financial frictions, in contrast, an incentive-compatibility constraint may at times prevent capital from being fully reallocated to the most

efficient firms. When the marginal return on capital falls (*e.g.* due to a protracted investment boom), notably, borrowers have more incentives to invest in alternative (“below the radar”) projects, stoking fears of default and possibly causing prospective lenders to panic and refuse to lend. In such an environment, monetary policy affects the probability of a crisis not only in the short run —through its usual effects on aggregate demand— but also in the medium run —through its effect on capital accumulation.

We use the model to conduct several monetary policy experiments. We show that a policy that consists in rapidly tightening monetary policy after having kept it loose-for-long can lead to financial crises. This is the case regardless of the motivation underpinning such “U-shaped” policy, that is: whether it is due to a rule-based response to inflation or to discretionary deviations from the policy rule. We also show that a central bank can increase welfare by following a backstop rule whereby it commits to doing whatever needed whenever necessary to forestall crises. Once backstops are activated, the speed at which monetary policy can be normalised without inducing a crisis depends on the source of financial vulnerabilities, *i.e.* a boom or an unusually large adverse shock.

Our model can be seen as a first step toward more complex models featuring a richer set of frictions and policies. One straightforward extension could consist in studying the effects of a zero lower bound (ZLB) on the policy rate. The implications of such a constraint are not clear-cut: while a ZLB may constrain the use of backstop policies, it could also reduce the need for such interventions by capping rate cuts during dis-inflationary booms —thereby fostering financial stability in the medium run. It would also be interesting to introduce macro-prudential policies that help rein in the accumulation of capital during economic booms, and to study the policy mix in that case. These extensions are left to future research.

References

- K. ADAM AND H. WEBER (2019): “Optimal Trend Inflation,” *American Economic Review*, 109(2), 702–37.
- H. AI, K. LI, AND F. YANG (2020): “Financial intermediation and capital reallocation,” *Journal of Financial Economics*, 138(3), 663–686.
- A. AJELLO, T. LAUBACH, D. LÓPEZ-SALIDO, AND R. NAKATA (2019): “Financial stability and optimal interest rate policy,” *International Journal of Central Banking*, 15(1), 279–326.
- O. AKINCI, G. BENIGNO, M. DEL NEGRO, AND A. QUERALTO (2020): “The financial (in)stability real interest rate, R^{**} ,” Federal Reserve Bank of New York Staff Report No. 946.
- R. ALIBER AND C. KINDLEBERGER (2015): *Mania, panics, and crashes: a history of financial crises*. Wiley Investment Classics, 7th edition.
- J. ANDRÉS, O. ARCE, AND C. THOMAS (2013): “Banking competition, collateral constraints, and optimal monetary policy,” *Journal of Money, Credit and Banking*, 45(s2), 87–125.
- A. AUCLERT (2019): “Monetary Policy and the Redistribution Channel,” *American Economic Review*, 109(6), 2333–67.

- C. AZARIADIS AND B. SMITH (1998): “Financial intermediation and regime switching in business cycles,” *American Economic Review*, 88(3), 516–536.
- BANK FOR INTERNATIONAL SETTLEMENTS (2022): “Market dysfunction and central bank tools,” Insights from the Markets Committee Working Group on Market Dysfunction and Central Banks, https://www.bis.org/publ/mc_insights.htm.
- R. BARSKY, A. JUSTINIANO, AND L. MELOSI (2014): “The natural rate of interest and its usefulness for monetary policy,” *The American Economic Review P&P*, 104(5), 37–43.
- B. BERNANKE (1995): “The Macroeconomics of the Great Depression: A Comparative Approach,” *Journal of Money, Credit, and Banking*, 27, 1–28.
- B. BERNANKE (2022): *21st Century Monetary Policy: The Federal Reserve from the Great Inflation to COVID-19*. W. W. Norton & Company.
- (2023): “Nobel Lecture: Banking, Credit, and Economic Fluctuations,” *American Economic Review*, 113(5), 1143–69.
- B. BERNANKE AND M. GERTLER (1989): “Agency costs, net worth, and business fluctuations,” *American Economic Review*, 79, 14–31.
- (1990): “Financial fragility and economic performance,” *The Quarterly Journal of Economics*, 105(1), 87–114.
- (2000): “Monetary policy and asset price volatility,” NBER Working Paper No 7559.
- B. BERNANKE, M. GERTLER, AND S. GILCHRIST (1999): *The financial accelerator in a quantitative business cycle framework*, vol. 1. Handbook of Macroeconomics.
- O. BLANCHARD AND J. GALÍ (2007): “Real wage rigidities and the New Keynesian model,” *Journal of money, credit and banking*, 39, 35–65.
- O. BLANCHARD AND N. KIYOTAKI (1987): “Monopolistic competition and the effects of aggregate demand,” *American Economic Review*, 77(4), 647–666.
- F. BOISSAY, F. COLLARD, C. MANEA, AND A. SHAPIRO (2023): “Monetary Tightening and Financial Stress during Supply- versus Demand-driven Inflation,” BIS Working Paper, number 1155.
- F. BOISSAY, F. COLLARD, AND F. SMETS (2016): “Booms and Banking Crises,” *Journal of Political Economy*, 124(2), 489–538.
- F. BOISSAY AND R. COOPER (2020): “The Collateral Composition Channel,” *American Economic Journal: Macroeconomics*, 12(1), 41–75.
- C. BORIO AND P. LOWE (2002): “Asset prices, financial and monetary stability: exploring the nexus,” BIS Working Paper number 114.
- M. BRUNNERMEIER (2009): “Deciphering the liquidity and credit crunch 2007–2008,” *Journal of Economic Perspectives*, 23(1), 77–100.
- M. BRUNNERMEIER AND C. JULLIARD (2008): “Money Illusion and Housing Frenzies,” *The Review of Financial Studies*, 21(1), 135–180.
- I. CAIRÓ AND J. SIM (2018): “Income inequality, financial crises, and monetary policy,” Federal Reserve Board, Finance and Economics Discussion Series, No 2018-048.
- D. CAO AND J-P. L’HUILIER (2018): “Technological revolutions and the three great slumps: A medium-run analysis,” *Journal of Monetary Economics*, 96, 93–108.

- V. CERRA AND S. SAXENA (2008): “Growth Dynamics: The Myth of Economic Recovery,” *American Economic Review*, 98(1), 439–57.
- J. COCHRANE (2001): *Asset Pricing*. Princeton University Press.
- N. COIMBRA AND H. REY (2023): “Financial Cycles with Heterogeneous Intermediaries,” *The Review of Economic Studies*, 91(2), 817–857.
- R. COOPER AND I. SCHOTT (2023): “Capital Reallocation and the Cyclicalities of Aggregate Productivity,” *Quantitative Economics*, 14, 1337–1365.
- G. CORSETTI, P. PESENTI, AND N. ROUBINI (1999): “What caused the Asian currency and financial crisis?,” *Japan and the World Economy*, 11, 305–373.
- T. DANG, G. GORTON, AND B. HOLMSTROM (2019): “The Information View of Financial Crises,” NBER Working Papers 26074, National Bureau of Economic Research, Inc.
- D. DEBORTOLI AND J. GALÍ (2021): “Monetary policy with heterogeneous agents: insights from TANK models,” Universitat Pompeu Fabra, Economics Working Papers, No 1686.
- G. DELL’ARICCIA, D. IGAN, L. LAEVEN, AND H. TONG (2016): “Credit booms and macro-financial stability,” *Economic Policy*, 31(86), 299–355.
- D. DUFFIE AND F. KEANE (2023): “Market-function asset purchases,” Federal Reserve Bank of New York Staff Report No. 1054.
- R. DUVAL, G. HONG, AND Y. TIMMER (2019): “Financial frictions and the Great Productivity Slowdown,” *Review of Financial Studies*, 33(2), 475–503.
- A. EISFELDT AND A. RAMPINI (2006): “Capital reallocation and liquidity,” *Journal of Monetary Economics*, 53(3), 369–399.
- P. ENGLUND (1999): “The Swedish banking crisis: roots and consequences,” *Oxford Review of Economic Policy*, 15(3), 80–97.
- J. FERNALD (2015): *Productivity and potential output before, during, and after the Great Recession* vol. 29 of *NBER Macroeconomics Annual*, pp. 1–51–214. University of Chicago Press.
- A. FILARDO AND P. RUNGCHAROENKITKUL (2016): “A quantitative case for leaning against the wind,” BIS Working Paper No 594.
- J. FISHER (2015): “On the structural interpretation of the Smets—Wouters “risk premium” shock,” *Journal of Money, Credit and Banking*, 47, 511–516.
- P. FONTANIER (2022): “Optimal policy for behavioral financial crises,” Unpublished manuscript.
- L. FOSTER, C. GRIM, AND J. HALTIWANGER (2016): “Reallocation in the Great Recession: cleansing or not?,” *Journal of Labor Economics*, 34(S1), S296–S331.
- T. FUERST (1995): “Monetary and Financial Interactions in the Business Cycle,” *Journal of Money, Credit and Banking*, 27, 1321–1338.
- J. GALÍ (2015): *Monetary policy, inflation, and the business cycle: an introduction to the New Keynesian framework and its applications*. Princeton University Press.
- J. GALÍ, F. SMETS, AND R. WOUTERS (2012): “Unemployment in an estimated New Keynesian model,” *NBER Macroeconomics Annual*, 26, 329–360.

- J. GALÍ (2014): “Monetary policy and rational asset price bubbles,” *American Economic Review*, 104(3), 721–52.
- (2018): “The State of New Keynesian Economics: A Partial Assessment,” *Journal of Economic Perspectives*, 32(3), 87–112.
- D. GARCIA-MACIA AND A. VILLACORTA (2022): “Macroprudential Policy with Liquidity Panics,” *The Review of Financial Studies*, 36(5), 2046–2090.
- M. GARMAISE (2015): “Borrower Misreporting and Loan Performance,” *The Journal of Finance*, 70(1), 449–484.
- N. GENNAIOLI, A. SHLEIFER, AND R. VISHNY (2012): “Neglected risks, financial innovation, and financial fragility,” *Journal of Financial Economics*, 104(3), 452–468.
- M. GERTLER (1992): “Financial capacity and output fluctuations in an economy with multi-period financial relationships,” *Review of Economic Studies*, 59, 455–472.
- M. GERTLER AND P. KARADI (2011): “A model of unconventional monetary policy,” *Journal of Monetary Economics*, 58(1), 17–34.
- M. GERTLER AND N. KIYOTAKI (2011): “Financial intermediation and credit policy in business cycle analysis,” in *Handbook of Monetary Economics*, vol. 3A, chap. 11, pp. 547–599. Amsterdam: Elsevier Science.
- (2015): “Banking, liquidity, and bank runs in an infinite horizon economy,” *American Economic Review*, 105(7), 2011–2043.
- M. GERTLER, N. KIYOTAKI, AND A. PRESTIPINO (2020): “A macroeconomic model with financial panics,” *Review of Economic Studies*, 87(1), 240–288.
- M. GERTLER AND K. ROGOFF (1990): “North-South lending and endogenous domestic capital market inefficiencies,” *Journal of Monetary Economics*, 26, 245–66.
- S. GILCHRIST, R. SCHOENLE, J. SIM, AND E. ZAKRAJŠEK (2017): “Inflation dynamics during the financial crisis,” *American Economic Review*, 107(3), 785–823.
- S. GILCHRIST, J. SIM, AND E. ZAKRAJŠEK (2013): “Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs,” *Review of Economic Dynamics*, 16(1), 159–176.
- G. GOPINATH, S. KALEMLI-OZCAN, L. KARABARBOUNIS, AND C. VILLEGAS-SANCHEZ (2017): “Capital allocation and productivity in South Europe,” *Quarterly Journal of Economics*, 132(4), 1915–1967.
- G. GORTON (1988): “Banking Panics and Business Cycles,” *Oxford Economic Papers*, 40(4), 751–781.
- (2008): “The panic of 2007,” NBER Working Paper No 14358.
- (2009): “Information, liquidity, and the (ongoing) panic of 2007,” *American Economic Review: Papers and Proceedings*, 99(2), 567–572.
- G. GORTON AND A. METRICK (2012): “Securitized banking and the run on repo,” *Journal of Financial Economics*, 104(3), 425–451, Market Institutions, Financial Market Risks and Financial Crisis.
- G. GORTON AND G. ORDOÑEZ (2020): “Good booms, bad booms,” *Journal of the European Economic Association*, 18(2), 618–665.

- F. GOURIO, A. KASHYAP, AND J. SIM (2018): “The trade-offs in leaning against the wind,” *IMF Economic Review*, 66, 70–115.
- A. GREENSPAN (1996): “The Challenge of Central Banking in a Democratic Society,” Remarks by Chairman Alan Greenspan At the Annual Dinner and Francis Boyer Lecture of The American Enterprise Institute for Public Policy Research.
- (2003): “Testimony of Chairman Alan Greenspan,” Federal Reserve Board’s semiannual monetary policy report to the Congress before the Committee on Financial Services, U.S. House of Representatives.
- R. GREENWOOD, S. HANSON, A. SCHLEIFER, AND J. SØRENSEN (2022): “Predictable financial crises,” *Journal of Finance*, 77(2), 863–921.
- J. GRIFFIN (2021): “Ten years of evidence: was fraud a force in the financial crisis?,” *Journal of Economic Literature*, 59(4), 1293–1321.
- M. GRIMM, Ò. JORDÀ, M. SCHULARICK, AND A. TAYLOR (2023): “Loose Monetary Policy and Financial Instability,” NBER Working Paper No 30958.
- V. GUERRIERI AND G. LORENZONI (2017): “Credit crises, precautionary savings, and the liquidity trap,” *Quarterly Journal of Economics*, 132(3), 1427–1467.
- A. HAUSER (2021): “From lender of last resort to market maker of last resort *via* the dash for cash: why central banks need new tools for dealing with market dysfunction,” Speech given at Thomson Reuters Newsmaker, London 7 January 2021.
- (2023): “Looking through a glass onion: lessons from the 2022 LDI intervention,” Speech given at the “Initiative on Global Markets’ Workshop on Market Dysfunction”, University of Chicago Booth School of Business.
- R. HODRICK AND E. PRESCOTT (1997): “Postwar US business cycles: an empirical investigation,” *Journal of Money, credit, and Banking*, pp. 1–16.
- M. IACOVIELLO (2005): “House prices, borrowing constraints, and monetary policy in the business cycle,” *American economic review*, 95(3), 739–764.
- INTERNATIONAL MONETARY FUND (2021): *IMF Investment and capital stock dataset, 1960-2019*. Expenditure Policy (EP) Division, Fiscal Affairs Department (FAD).
- T. ITO AND F. MISHKIN (2006): *Two Decades of Japanese Monetary Policy and the Deflation Problem*. University of Chicago Press.
- U. JERMANN AND V. QUADRINI (2012): “Macroeconomic Effects of Financial Shocks,” *American Economic Review*, 102(1), 238–71.
- G. JIMÉNEZ, E. KUVSHINOV, J-P. PEYDRÓ, AND B. RICHTER (2023): “Monetary Policy, Inflation, and Crises: new Evidence from History and Administrative Data,” Available at SSRN: <https://ssrn.com/abstract=4304896>.
- G. JIMÉNEZ, J-L. ONGENA, S. AND PEYDRÓ, AND J. SAURINA (2014): “Hazardous times for monetary policy: what do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking?,” *Econometrica*, 82(2), 463–505.
- Ò. JORDÀ, M. SCHULARICK, AND A. TAYLOR (2013): “When credit bites back,” *Journal of money, credit and banking*, 45(s2), 3–28.
- (2017): “Macrofinancial history and the new business cycle facts,” *NBER macroeconomics annual*, 31(1), 213–263.

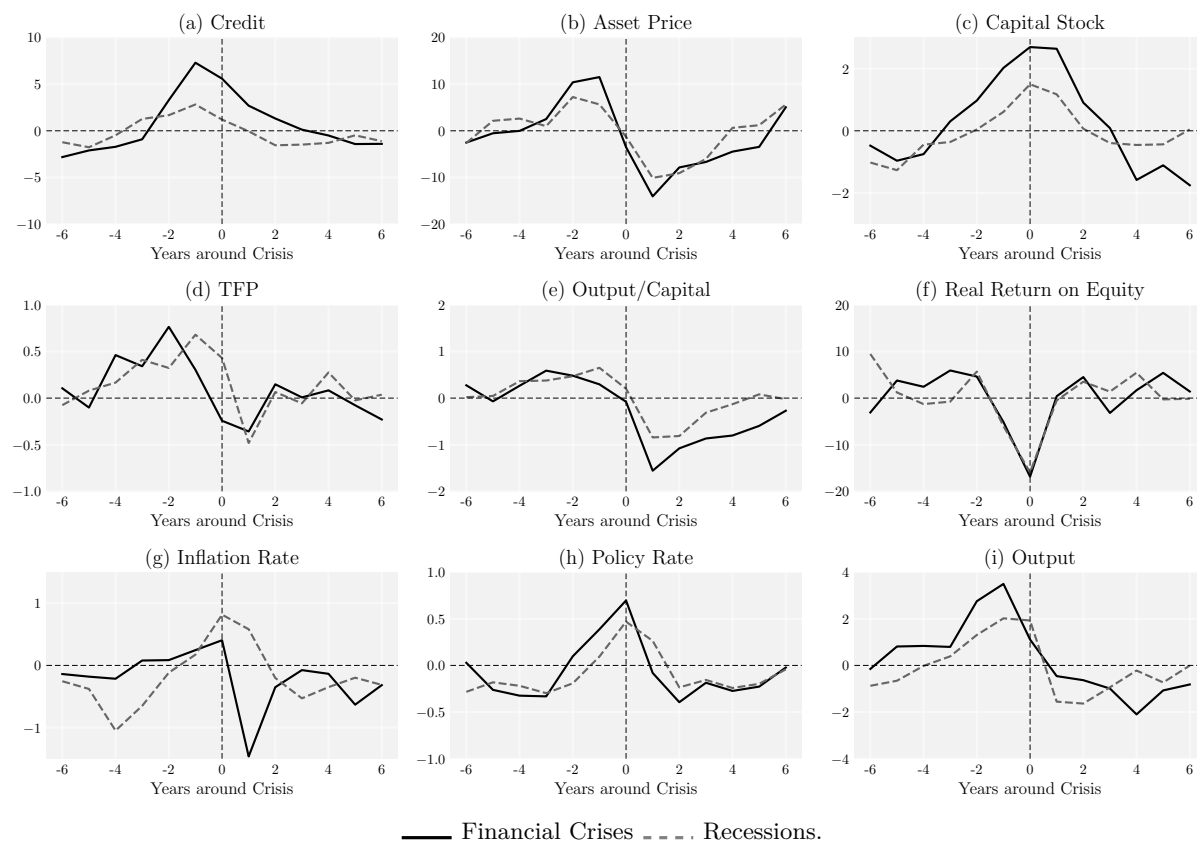
- Ò. JORDÀ, S. SINGH, AND A. TAYLOR (2023): “The long-run effects of monetary policy,” Federal Reserve Bank of San Francisco Working Paper No 2020-01.
- G. KAPLAN, B. MOLL, AND G. VIOLANTE (2018): “Monetary policy according to HANK,” *American Economic Review*, 108(3), 697–743.
- A. KHAN AND J. THOMAS (2013): “Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity,” *Journal of Political Economy*, 121(6), 1055–1107.
- N. KIYOTAKI AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105(2), 211–247.
- C. MANEA (2020): “Monetary policy with financially constrained and unconstrained firms,” Essays in monetary economics, phd thesis, Universitat Pompeu Fabra.
- G. MANKIW (1986): “The allocation of credit and financial collapse,” *Quarterly Journal of Economics*, 101(Aug), 455–470.
- D. MARTINEZ-MIERA AND R. REPULLO (2017): “Search for yield,” *Econometrica*, 85(2), 351–378.
- A. MAS-COLELL, M. WHINSTON, AND J. GREEN (1995): *Microeconomic Theory*. Oxford university Press.
- A. MIAN AND A. SUFI (2017): “Fraudulent income overstatement on mortgage applications during the credit expansion of 2002 to 2005,” *Review of Financial Studies*, 30(6), 1831–1864.
- V. MIDRIGAN AND D. Y. XU (2014): “Finance and Misallocation: Evidence from Plant-Level Data,” *American Economic Review*, 104(2), 422–58.
- F. MISHKIN (1991): “Asymmetric Information and Financial Crises: A Historical Perspective,” in *Financial Markets and Financial Crises*, pp. 69–108. University of Chicago Press.
- (1999): “Lessons from the Asian crisis,” *Journal of International Money and Finance*, 18(4), 709–723.
- B. MOLL (2014): “Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?,” *American Economic Review*, 104, 3186–3221.
- K. OKINA, M. SHIRAKAWA, AND S. SHIRATSUKA (2001): “The asset price bubble and monetary policy: Japan’s experience in the late 1980s and the lessons,” *Monetary and Economic Studies (special edition)*, 19(2), 395–450.
- P. OTTONELLO (2021): “Capital unemployment,” Mimeo, University of Michigan.
- P. OTTONELLO AND T. WINBERRY (2020): “Financial heterogeneity and the investment channel of monetary policy,” *Econometrica*, 88(6), 2473–2502.
- N. OULTON AND M. SEBASTIÁ-BARRIEL (2016): “Effects of financial crises on productivity, capital and employment,” *Review of Income and Wealth*, 63.
- P. PAUL (2022): “Historical Patterns of Inequality and Productivity around Financial Crises,” Federal Reserve Bank of San Francisco Working Paper Series No 2017-23.
- M. PIAZZESI AND M. SCHNEIDER (2008): “Inflation Illusion, Credit, and Asset Prices,” in *Asset Prices and Monetary Policy*. University of Chicago Press.
- T. PISKORSKI, A. SERU, AND J. WITKIN (2015): “Asset quality misrepresentation by financial intermediaries: evidence from the RMBS Market,” *Journal of Finance*, LXX(6), 2635–2678.

- C. ROMER AND D. ROMER (2017): “New evidence on the aftermath of financial crises in advanced countries,” *American Economic Review*, 107(10), 3072–3118.
- (2019): “Fiscal space and the aftermath of financial crises: how it matters and why,” *Brookings Papers on Economic Activity*.
- J. ROTEMBERG (1982): “Monopolistic price adjustment and aggregate output,” *Review of Economic Studies*, 49(4), 517–531.
- K. ROUWENHORST (1995): “Asset pricing implications of equilibrium business cycle models, ” in *Frontiers of Business Cycle Research*, ed. by T. Cooley, pp. 294–330. Princeton University Press.
- I. SCHNABEL (2021): “Monetary policy and financial stability,” Speech by Isabel Schnabel, Member of the Executive Board of the ECB, at the fifth annual conference of the European Systemic Risk Board.
- M. SCHULARICK AND A. TAYLOR (2012): “Credit booms gone bust: monetary policy, leverage cycles, and financial crises, 1870–2008,” *American Economic Review*, 102(2), 1029–1061.
- M. SCHULARICK, L. TER STEEGE, AND F. WARD (2021): “Leaning against the Wind and Crisis Risk,” *American Economic Review: Insights*, 3(2), 199–214.
- F. SMETS AND R. WOUTERS (2007): “Shocks and frictions in US business cycles: a Bayesian DSGE approach,” *American Economic Review*, 97(3), 586–606.
- J. STEIN (2013): “Overheating in credit markets: origins, measurement, and policy responses,” in *Speech given to the symposium on Restoring Household Financial Stability After the Great Recession*, Federal Reserve Bank of St. Louis, St. Louis, Missouri, February, vol. 7.
- (2021): “Can policy tame the credit cycle?,” *IMF Economic Review*, 63, 5–22.
- J. STIGLITZ AND A. WEISS (1981): “Credit Rationing in Markets with Imperfect Information,” *American Economic Review*, 71(Jun), 393–410.
- A. SUFI AND A. TAYLOR (2022): *Financial crises: a survey*, vol. 6. Handbook of International Economics.
- L. SVENSSON (2017a): “Cost-benefit analysis of leaning against the wind,” *Journal of Monetary Economics*, 90, 193–213.
- (2017b): “Leaning against the wind: the Role of different assumptions about the costs,” NBER Working Paper No. 23745.
- J. TAYLOR (1993): “Discretion versus policy rules in practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.
- (2011): “Macroeconomic lessons from the Great Deviation,” in *NBER Macroeconomics Annual*, ed. by D. Acemoglu, and M. Woodford, vol. 25, p. 387–95. University of Chicago Press.
- M. WOODFORD (2012): “Inflation targeting and financial stability,” *Sveriges Riksbank Economic Review*, 1, 1–32.

A Online Appendix

A.1 Stylised Facts of Financial Crises: Full JST Sample

Figure A.1: Median Dynamics Around Financial Crises



Notes: Same as Figure 1, for the full sample period (1870–2020).

A.2 Summary of the Model

Our model can be summarised by the following 13 equations:⁵⁸

1. $Z_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1} (1 + r_{t+1}) \right\}$
2. $1 = \mathbb{E}_t \left\{ \Lambda_{t,t+1} (1 + r_{t+1}^q) \right\}$
3. $\frac{W_t}{P_t} = \chi N_t^\varphi C_t^\sigma$
4. $Y_t = A_t (\omega_t K_t)^\alpha N_t^{1-\alpha}$
5. $\frac{W_t}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{(1 - \alpha) Y_t}{\mathcal{M}_t N_t}$
6. $r_t^q + \delta = \frac{\epsilon}{\epsilon - 1} \frac{\alpha Y_t}{\mathcal{M}_t K_t}$
7. $(1 + \pi_t) \pi_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} (1 + \pi_{t+1}) \pi_{t+1} \right) - \frac{\epsilon - 1}{\varrho} \left(\frac{\mathcal{M}_t - \frac{\epsilon}{\epsilon - 1}}{\mathcal{M}_t} \right)$
8. $1 + i_t = \frac{1}{\beta} (1 + \pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y}$
9. $Y_t = C_t + I_t + \frac{\varrho}{2} Y_t \pi_t^2$
10. $\Lambda_{t,t+1} \equiv \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$
11. $1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}$
12. $K_{t+1} = I_t + (1 - \delta) K_t$
13. $\omega_t = \begin{cases} 1 & \text{if } r_t^q \geq \frac{(1-\theta)\mu - \delta}{1-\mu} \\ 1 - \mu & \text{otherwise} \end{cases}$

A.3 Proof of Relation (25)

Using equations (14), (17) and (24), one can re-write r_t^q as $r_t^q = \mu (r_t^c - (r_t^c + \delta) K_t^u / K_t) + (1 - \mu) (r_t^c + (r_t^k - r_t^c) K_t^p / K_t)$. In normal times, $K_t^u = 0$ and $K_t^p = K_t / (1 - \mu)$, which implies that $r_t^q = r_t^k$. In crisis times, $r_t^c = -\delta$ and $K_t^u = K_t^p = K_t$, which implies that $r_t^q = -\mu\delta + (1 - \mu)r_t^k$.

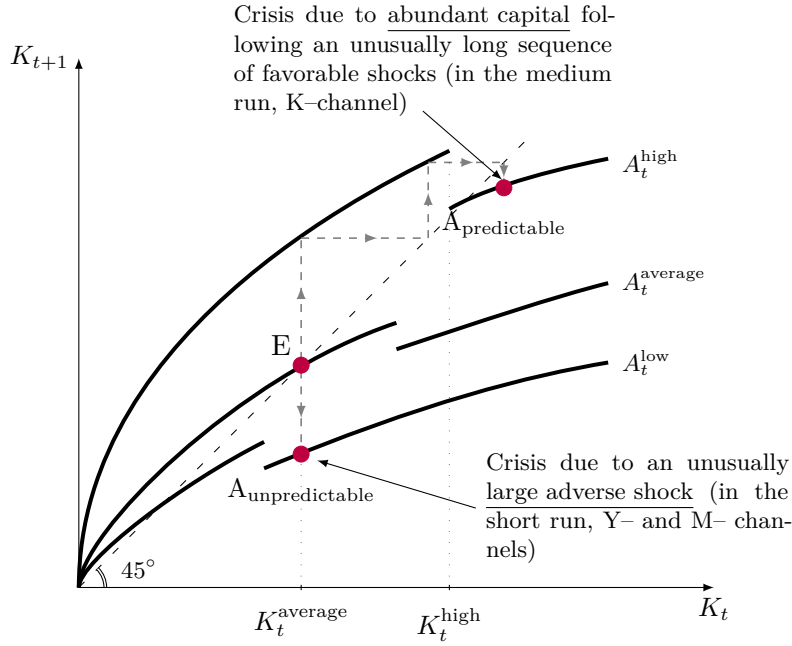
A.4 Financial Crises: Polar Types and Multiple Causes

Figure A.2 is a stylised representation of the optimal capital accumulation decision rule, which expresses K_{t+1} as a function of state variables K_t and A_t .⁵⁹ During a crisis, the household dis-saves to consume, which generates less investment and a fall in the capital stock, as captured by the discontinuous downward breaks in the decision rules.

⁵⁸In the list of equations below, relation 2 is obtained after noticing that, given the definition of the firm's average return on equity (24), $\mathbb{E}_t(\Lambda_{t,t+1}(1 + r_{t+1}^q)) = \mathbb{E}_t(\Lambda_{t,t+1}(1 + r_{t+1}^q(j))) \forall j \in [0, 1]$. Relation 6 is obtained as follows. In normal times: using (16), (18), (27) and $K_t^p = K_t / (1 - \mu)$, one obtains $r_t^k + \delta = \alpha Y_t / ((1 - \tau) \mathcal{M}_t K_t)$ which, given the first row in (25) and that $\tau = 1/\epsilon$, yields relation 6. In crisis times: using relations (16), (18), (27) and $K_t^p = K_t$, one obtains $r_t^k + \delta = \alpha Y_t / ((1 - \mu)(1 - \tau) \mathcal{M}_t K_t)$ which, given the second row in (25) and that $\tau = 1/\epsilon$, yields relation 6. All the other relations are straightforward and from the main text.

⁵⁹For the purpose of the illustration, we abstract from the demand shock Z_t .

Figure A.2: Optimal Decision Rules $K_{t+1}(K_t, A_t)$ and Two Polar Types of Crisis



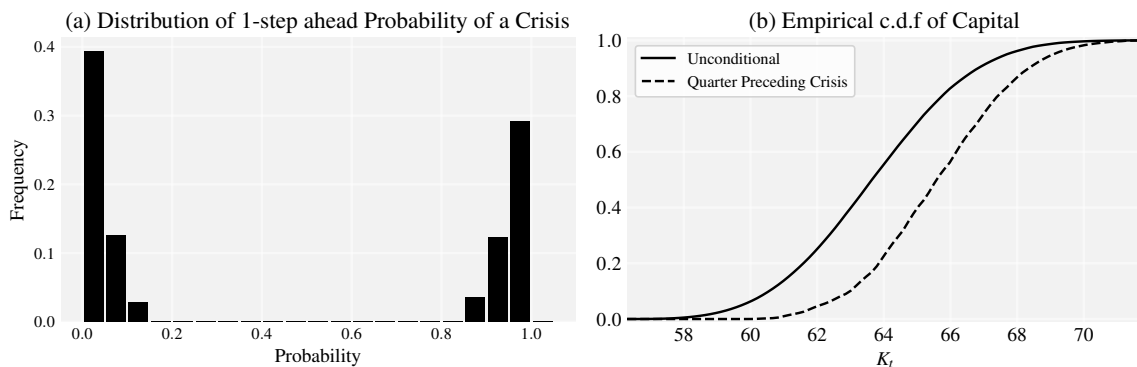
Notes: Stylised representation of the optimal decision rule for the capital stock.

There are two polar types of crises. The first one can be characterised as “unpredictable”: for an average level of capital stock K_t^{average} , a crisis breaks out when productive firms’ marginal return on capital, r_t^k , falls below the required incentive compatible loan rate, \bar{r}^k (see Proposition 2). In Figure A.2, this is the case in equilibrium $A_{\text{unpredictable}}$, where aggregate productivity A_t falls from A_t^{average} to A_t^{low} . This type of crisis is hard to predict, insofar as it is due to an unusually large adverse shock.

The other polar type of crisis can be characterised as “predictable”: following a long period of high productivity A_t^{high} , the household accumulates savings and feeds an investment boom that increases the stock of capital. All other things equal, the rise in the capital stock reduces productive firms’ marginal return on capital until $r_t^k < \bar{r}^k$. The crisis then breaks out as K_t exceeds K_t^{high} , without the economy experiencing any adverse shock, as in equilibrium $A_{\text{predictable}}$. This type of crisis is predictable to the extent that the protracted investment boom (and associated fall in firms’ average return on equity) that precede it can be used as early warning.

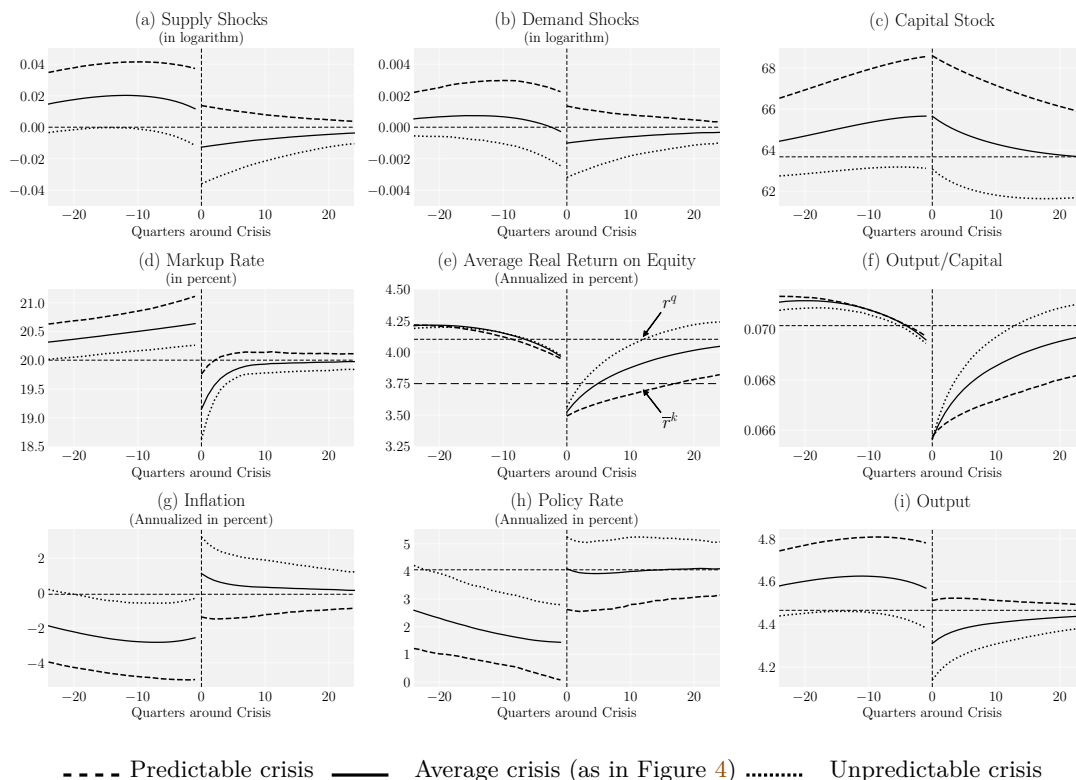
In the stochastic steady state of our model, crises can be seen as different blends of the two polar types. To document this heterogeneity, we report in Figure A.3 the distribution the crisis probability (panel (a)) in the quarter before a crisis (quarter -1). The distribution is clearly bimodal: about 55% of the crises are associated with a crisis probability of less than 20% in the quarter that preceded, *i.e.* were not predictable; and 45% are associated with a crisis probability above 80%, *i.e.* were predictable. Panel (b) also shows that the level of the capital stock in the quarter that precedes financial crises tends to be higher than that in the stochastic steady state. These results are consistent with recent empirical evidence that financial crises are, by and large, predictable and the byproducts of credit booms (see Greenwood et al. (2022),

Figure A.3: Predictable *versus* Unpredictable Crises



Notes: Panel (a): Ergodic distribution of the one-step ahead crisis probability in the quarter that precedes financial crises in the TR93 economy. The one-step ahead crisis probability is defined as $\mathbb{E}_{t-1} \left(\mathbb{1} \left(\frac{Y_t}{M_t K_t} < \frac{1-\tau}{\alpha} \left(\frac{(1-\theta)\mu-\delta}{1-\mu} + \delta \right) \right) \right)$, where $\mathbb{1} \{ \cdot \}$ is a dummy variable equal to one when the inequality inside the curly braces holds (*i.e.* there is a crisis) and to zero otherwise (see Corollary 1). Panel (b): Ergodic cumulative distribution of the capital stock in the TR93 economy, unconditional (plain line) or conditional on being in a crisis next quarter (dashed line).

Figure A.4: Dynamics of Predictable and Unpredictable Crises

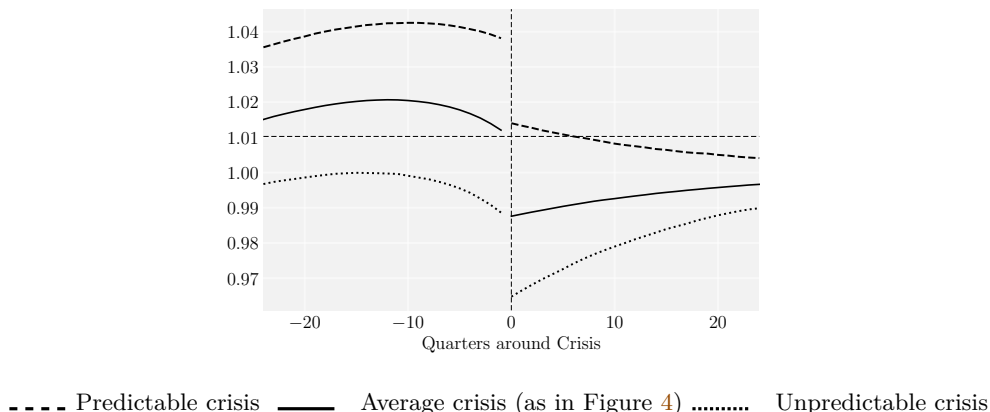


Notes: Simulations for the TR93 economy. Average dynamics of the economy around the beginning of all (black line, as in Figure 4), predictable (dashed) and unpredictable (gray) crises (in quarter 0). The subset of predictable (unpredictable) crises correspond to the crises whose one-step-ahead probability in quarter -1 is in the top (bottom) decile of its distribution (see Figure A.3, panel (a)). For the evolution of asset prices, see Figure A.5.

Figure A.4 further shows how the average dynamics around predictable (dashed line) and unpredictable (dotted line) crises differ. For the purpose of this exercise, we define a crisis as “predictable” (respectively “unpredictable”) if the crisis probability in the quarter that precedes

it (*i.e.* quarter -1) is in the top (respectively bottom) decile of its distribution (Figure A.3, panel (a)). In line with Figure A.2, we find that unpredictable crises occur when aggregate productivity and demand shocks are negative (panels (a) and (b), dotted line), as in the case of crisis $A_{\text{unpredictable}}$ in Figure A.2, whereas predictable crises occur despite shocks being positive, and follow an investment boom (panel (c), dashed line), as in the case of crisis $A_{\text{predictable}}$.

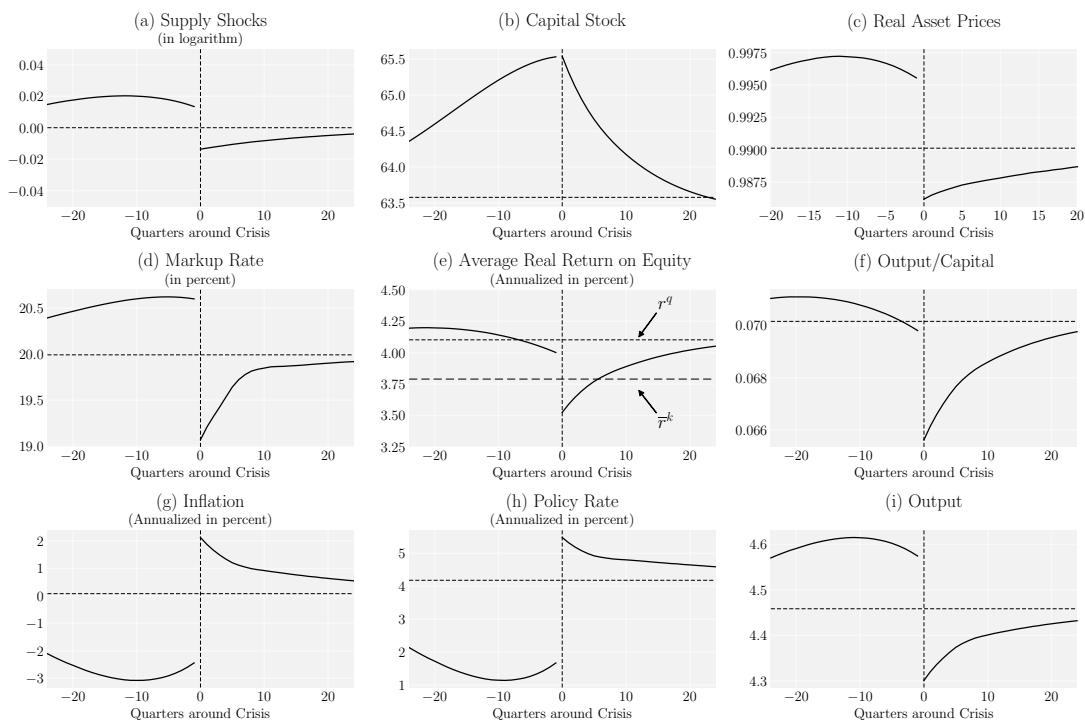
Figure A.5: Asset Price Dynamics around Predictable and Unpredictable Crises



Notes: Same experiment as in Figure A.4.

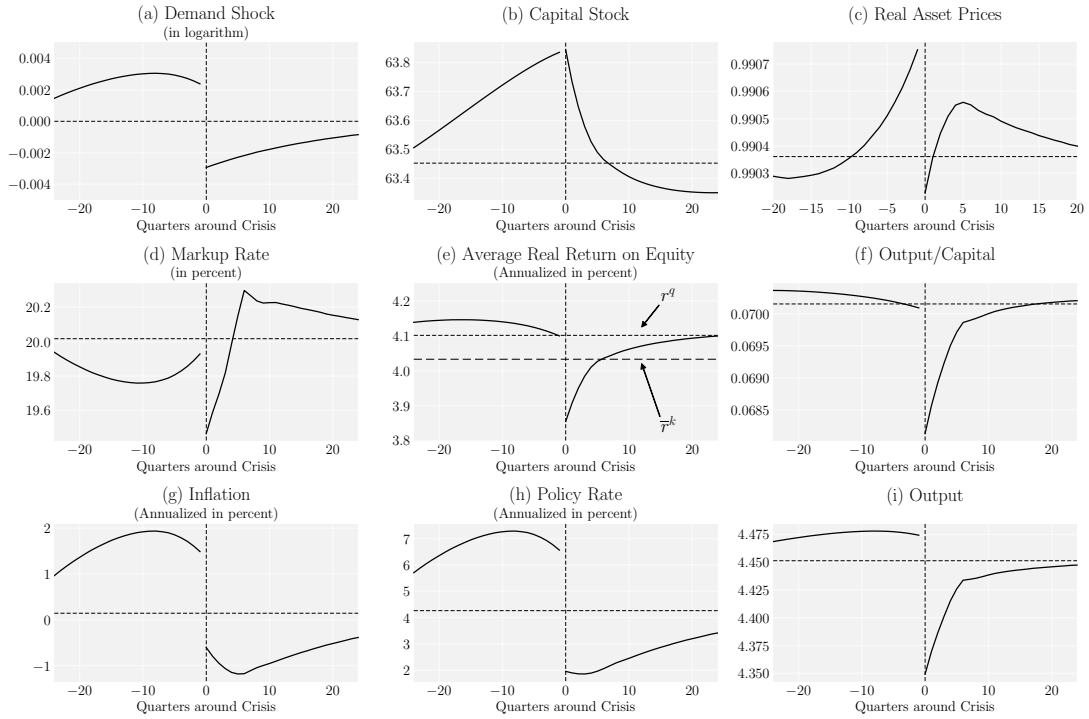
A.5 Crisis dynamics: supply or demand shocks only

Figure A.6: Simulated Dynamics Around Crises: Supply Shocks only



Notes: Same as Figure 4 in an economy subject to supply shocks only. The model is re-parameterised so that the economy spends 10% of the time in crisis.

Figure A.7: Simulated Dynamics Around Crises: Demand Shocks only



Notes: Same as Figure 4 in an economy subject to demand shocks only. The model is re-parameterised so that the economy spends 10% of the time in crisis.

A.6 Taylor Rule with Expected Inflation

This section presents the statistics and dynamics of financial crises in a model where the central bank would target *expected* inflation instead of *current* inflation, *i.e.* would follow the following Taylor rule:

$$1 + i_t = \frac{1}{\beta} (1 + \mathbb{E}_t[\pi_{t+1}])^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y}$$

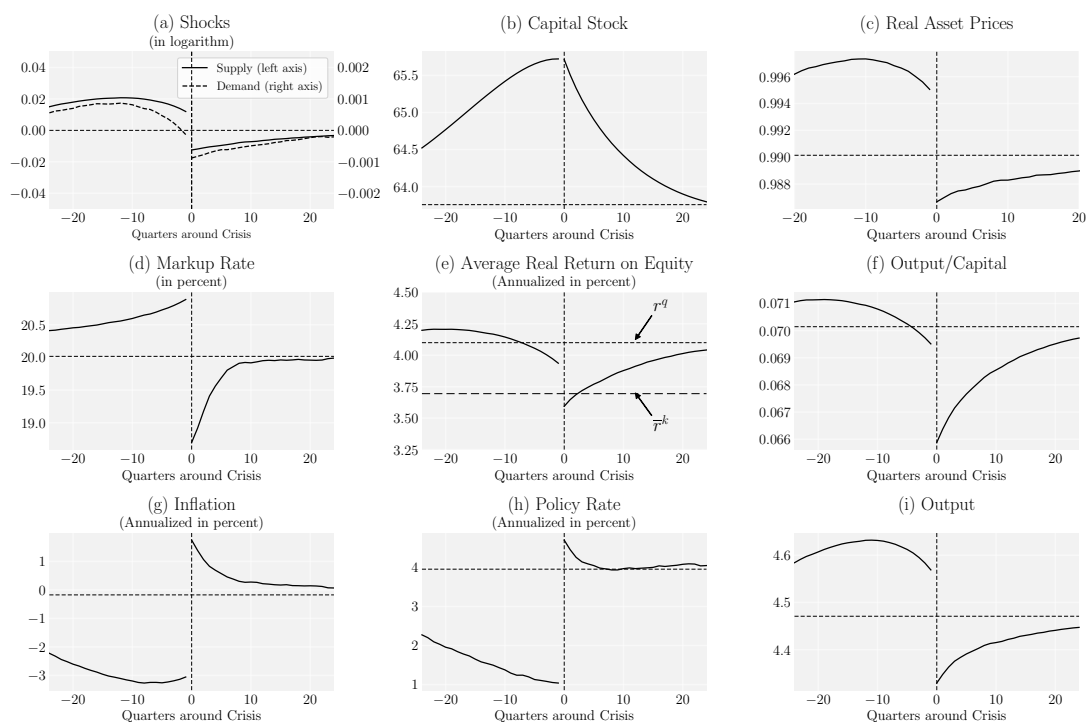
Table A.3 and Figures A.8 and A.9 show that our results are essentially the same as in our baseline model and therefore that our analysis is robust to considering the above alternative monetary policy rule.

Table A.3: Economic Performance and Welfare Under TR93

Rule	parameters			Model with Financial Frictions				Frictionless	
	ϕ_π	ϕ_y	ϕ_r	Time in Crisis (in %)	Length (quarters)	Output Loss (in %)	Std(π_t) (in pp)	Welfare Loss (in %)	Welfare Loss (in %)
Current Inf.	1.5	0.125	–	[10]	4.8	6.6	1.2	0.82	0.56
Expected Inf.	1.5	0.125	–	[10]	5.1	6.3	1.4	0.98	0.71

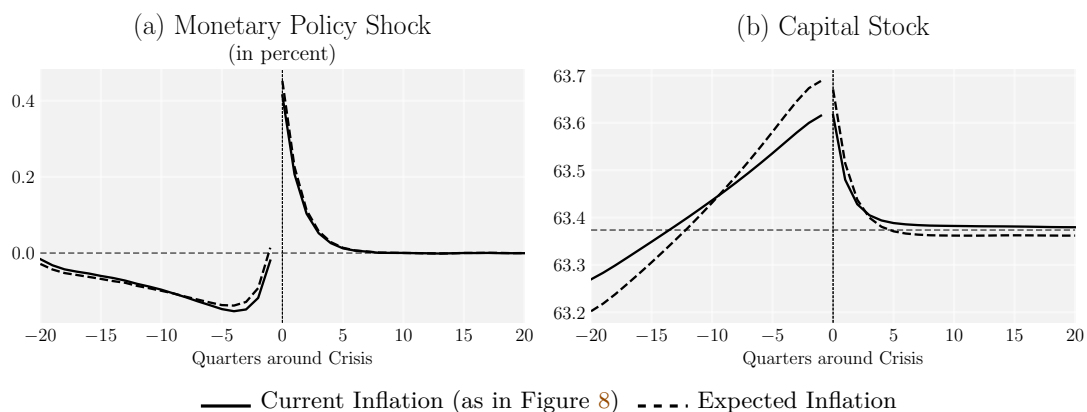
Notes: Same statistics as in Table 2. For the purpose of comparison, parameter θ of the model with expected inflation targeting was set so that the economy also spends 10% of the time in a crisis in that case.

Figure A.8: Average Crisis Dynamics Under Expected Inflation Targeting



Notes: Same as Figure 4 when the Taylor rule features expected inflation. The model is re-parameterised so that the economy spends 10% of the time in crisis.

Figure A.9: Discretionary Monetary Policy Shocks: Current vs Expected Inflation Targeting



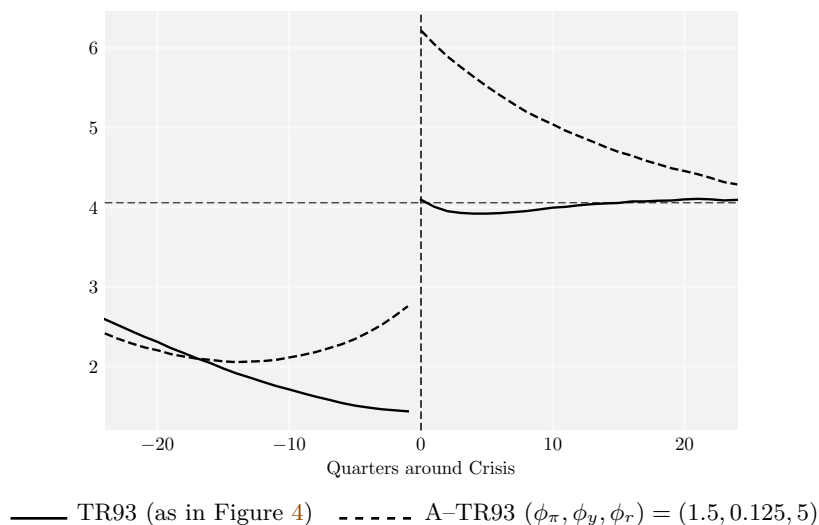
Notes: Same as Figure 8.

A.7 Augmented Taylor Rule in a Dis-Inflationary Boom

Figure A.10 below reports the evolution of the monetary policy rate under TR93 and an augmented Taylor rule during a dis-inflationary boom. To fix ideas, the dis-inflationary boom considered is the same as the one that precedes the average financial crisis in our baseline model, *i.e.* under TR93 (see Figure 4). The comparison of the two policy rate paths shows that, despite the dis-inflationary pressures (Figure 4, panel (g)), the central bank is more restrictive during the boom under the augmented Taylor rule up throughout the 16 quarters before the

beginning of the crisis.

Figure A.10: Policy Rate under a Standard *versus* Augmented Taylor Rule

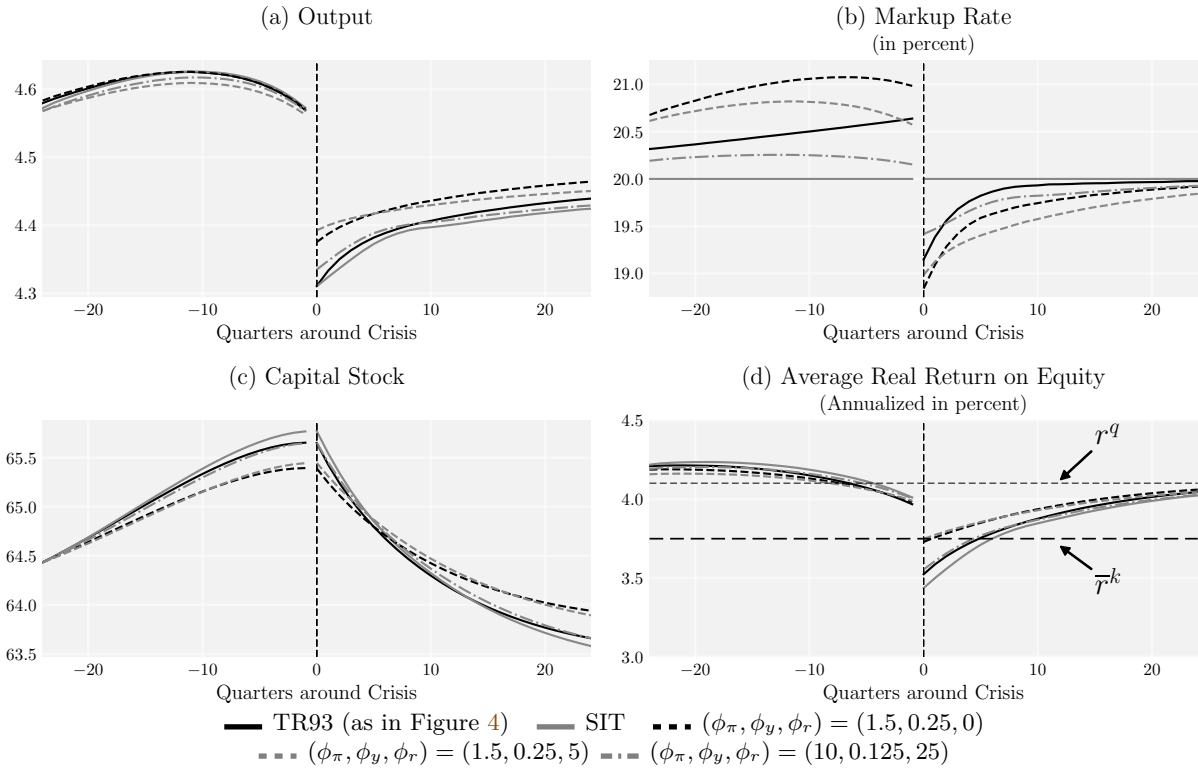


Notes: The model is solved under the assumption that the central bank follows either TR93 or A-TR93 (with parameters $(\phi_\pi, \phi_y, \phi_r) = (1.5, 0.125, 5)$). In the latter case, the counterfactual dynamics are derived by feeding the model with the same sequence of aggregate shocks as those that lead to a crisis under TR93 (Figure 4, panel (a)), starting with the same level of capital stock in quarter -24 .

A.8 Discussion on the Effects of Augmented Taylor Rules

Counter-factual analyses help gain intuition about the effects of A-TR discussed in Section 5.1. In Figure A.11 below, we compare the average dynamics of the economy under TR93 (black line) with counterfactual dynamics in economies under SIT (gray line), a Taylor-type rule with $\phi_\pi = 1.5$ and $\phi_y = 0.25$ (dashed black line), an augmented Taylor-type rule with $\phi_\pi = 1.5$, $\phi_y = 0.25$ and $\phi_r = 5$ (dashed gray line), and another one with $\phi_\pi = 10$, $\phi_y = 0.125$ and $\phi_r = 25$ as in row (9) of Table 2 (dash-dotted gray line). For the purpose of the comparison, these economies are fed with the very same sequences of shocks as those that lead to a crisis under TR93.

Figure A.11: Counterfactual Booms and Busts



Notes: For TR93: same average dynamics as in Figure 4. For the other rules: counterfactual average dynamics, when the economy starts with the same capital stock in quarter -24 and is fed with the same aggregate shocks as those that lead to a crisis under TR93 (Figure 4, panel (a)). In panel (d), the upper horizontal dashed line corresponds to the deterministic steady state value r^q and the lower one to the crisis threshold \bar{r}^k .

Consider first the dynamics of the economy from quarters -24 to -1 . These dynamics help understand how the different policies act on the savings glut and markup externalities in the boom phase. Panel (d) suggests that responding more aggressively to output or to the yield gap has overall a limited effect on the firms' average return on equity. This is due to offsetting effects on capital accumulation and markups (panels (b) and (c)). On the one hand, such policies mean that the central bank commits itself to boosting demand during recessions and curbing growth during booms. The former tends to reduce households' needs for precautionary savings while the latter lowers investors' expected returns during booms. Compared to TR93 or SIT, both effects contribute to slowing down capital accumulation and increase the resilience of credit markets through the K-channel (panel (c)). On the other hand, however, responding more aggressively to output or to the yield gap works to dampen inflationary pressures during booms, implying higher markups and less resilience through the M-channel (panel (b)). On balance, these opposite effects offset each other during the boom.

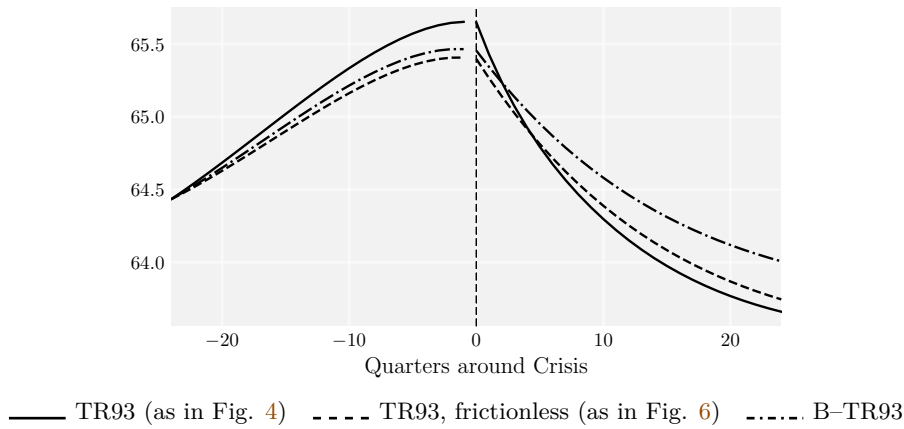
The main difference between the policy rules comes from the response of the economy at the time of the crisis, in quarter 0. While output and equity returns fall under all rules, they fall by less when ϕ_y or ϕ_r are higher —keeping all else equal. The reason is clear: following the adverse aggregate shocks (Figure 4, panels (a) and (b)), such rules imply a bigger fall in the policy rate, which boosts aggregate demand, lifts firms' average return on equity (Figure A.11,

panel (d)), and helps avoid a crisis. Responding more aggressively to output or to the yield gap thus helps to foster financial stability by cushioning the impact of the shocks (Y-channel).

A.9 Capital Accumulation Under Backstop

Figure A.12 shows that the accumulation of capital is slower under B-TR93 than under TR93 in the run-up to financial stress episodes and financial crises, respectively. The difference reflects the household's lesser need for accumulating precautionary savings when the central bank commits to backstop the economy which, in effect, amounts to providing households with an insurance against the fall in their revenues should financial stress emerge.

Figure A.12: Capital Accumulation under B-TR93 *versus* TR93



Notes: Comparison of three economies: under TR93 with a frictional credit market (baseline, as in Figure 4); under TR93 with a frictionless credit market (as in Figure 6); and under B-TR93. For the latter two economies: counterfactual average dynamics of the capital stock, when the economy starts with the same capital stock in quarter -24 and is fed with the same aggregate shocks as in the baseline.

A.10 Global Solution Method

We first discretise the distribution of the aggregate shocks using [Rouwenhorst \(1995\)](#)'s approach. The latter involves a Markov chain representation of the shock, s_t , with $s_t \in \{a_1, \dots, a_{n_a}\} \times \{z_1, \dots, z_{n_z}\}$ and transition matrix $\mathbb{T} = (\varpi_{ij})_{i,j=1}^{n_a n_z}$ where $\varpi_{ij} = \mathbb{P}(s_{t+1} = s_j | s_t = s_i)$. In what follows, we use $n_a = 5$ and $n_z = 5$. We look for an approximate representation of consumption, the marginal cost ($mc \equiv 1/\mathcal{M}$) and the gross nominal interest rate (\hat{i}) as a function of the endogenous state variables in each regime, *e.g.* normal times and crisis times. More specifically, we use the approximation⁶⁰

$$G_x(K_t; s) = \begin{cases} \sum_{j=0}^{p_x} \psi_j^x(n, s) T_j(\nu(K)) & \text{if } K \leq K^*(s) \\ \sum_{j=0}^{p_x} \psi_j^x(c, s) T_j(\nu(K)) & \text{if } K > K^*(s) \end{cases} \text{ for } x = \{c, mc, \hat{i}\}$$

where $T_j(\cdot)$ is the Chebychev polynomial of order j and $\nu(\cdot)$ maps $[\underline{K}; K^*(s)]$ in the normal regime (respectively $[K^*(s); \bar{K}]$ in the crisis regime) onto interval $[-1; 1]$.⁶¹ $\psi_j^x(r, s)$ denotes the

⁶⁰Throughout this section, we denote $\hat{\pi} = 1 + \pi$ and $\hat{i} = 1 + i$.

⁶¹More precisely, $\nu(K)$ takes the form $\nu(K) = 2 \frac{K - \underline{K}}{K^*(s) - \underline{K}} - 1$ in the normal regime and $\nu(K) = 2 \frac{K - K^*(a, z)}{K - K^*(s)} - 1$ in the crisis regime.

coefficient of the Chebychev polynomial of order j for the approximation of variable x when the economy is in regime r and the shocks are $s = (a, z)$. p_x denotes the order of Chebychev polynomial we use for approximating variable x .

$K^*(s)$ denotes the threshold in physical capital beyond which the economy falls in a crisis, defined as

$$r_t^k + \delta = \frac{\alpha Y_t}{(1 - \tau) \mathcal{M}_t K_t} = \frac{\mu(1 - \delta - \theta)}{1 - \mu} \quad (32)$$

This value is unknown at the beginning of the algorithmic iterations, insofar as it depends on the agents' decisions. We therefore also need to formulate a guess for this threshold.

A.10.1 Algorithm

The algorithm proceeds as follows.

1. Choose a domain $[K_m, K_s]$ of approximation for K_t and stopping criteria $\varepsilon > 0$ and $\varepsilon_k > 0$. The domain is chosen such that K_m and K_s are located 25% away from the deterministic steady state of the model (located in the normal regime). We chose $\varepsilon = \varepsilon_k = 1e^{-4}$.
2. Choose an order of approximation p_x (we pick $p_x = 9$) for $x = \{c, mc, \hat{i}\}$, compute the n_k roots of the Chebychev polynomial of order $n_k > p$ as

$$\zeta_\ell = \cos\left(\frac{(2\ell - 1)\pi}{2n_k}\right) \text{ for } \ell = 1, \dots, n_k$$

and formulate an initial guess⁶² for $\psi_j^x(n, s)$ for $x = \{c, mc, \hat{i}\}$ and $i = 1, \dots, n_a \times n_z$. Formulate a guess for the threshold $K^*(s)$.

3. Compute K_ℓ , $\ell = 1, \dots, 2n_k$ as

$$K_\ell = \begin{cases} (\zeta_\ell + 1) \frac{K^*(s) - K_m}{2} + K_m & \text{for } K \leq K^*(s) \\ (\zeta_\ell + 1) \frac{K_s - K^*(s)}{2} + K^*(s) & \text{for } K > K^*(s) \end{cases}$$

for $\ell = 1, \dots, 2n_k$.

4. Using a candidate solution $\Psi = \{\psi_j^x(r, s_i); x = \{c, \hat{\pi}, \hat{i}\}, r = \{n, c\}, i = 0 \dots p_x\}$, compute approximate solutions $G_c(K; s_i)$, $G_{\hat{\pi}}(K; s_i)$ and $G_{\hat{i}}(K; s_i)$ for each level of K_ℓ , $\ell = 1, \dots, 2n_k$ and each possible realization of the shock vector s_i , $i = 1, \dots, n_a \times n_z$ and the over quantities of the model using the definition of the general equilibrium of the economy (see below). In particular, compute the next period capital $K'_{\ell,i} = G_K(K_\ell; z_i)$ for each $\ell = 1, \dots, 2n_k$ and $i = 1 \dots n_a \times n_z$.
5. Using the next period capital and the candidate approximation, solve the general equilibrium to obtain next period quantities and prices entering households' and retailers'

⁶²The initial guess is obtained from a first order approximation of the model around the deterministic steady state.

expectations, and compute expectations

$$\tilde{\mathcal{E}}_{c,t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(G_c(K'_{\ell,i}, z'_s))(1 + r^{k'}(K'_{\ell,i}, z'_s)) \right] \quad (33)$$

$$\tilde{\mathcal{E}}_{\hat{i},t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[\frac{u'(G_c(K'_{\ell,i}, z'_s))}{G_{\hat{\pi}}(K'_{\ell,i}, z'_s)} \right] \quad (34)$$

$$\tilde{\mathcal{E}}_{\hat{\pi},t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(G_c(K'_{\ell,i}, z'_s))G_Y(K'_{\ell,i}, z'_s)G_{\hat{\pi}}(K'_{\ell,i}, z'_s)(G_{\hat{\pi}}(K'_{\ell,i}, z'_s) - 1) \right] \quad (35)$$

6. Use expectations to compute new candidate c , mc and \hat{i}

$$\tilde{c}_t = u'^{-1}(\tilde{\mathcal{E}}_{c,t}) \quad (36)$$

$$\tilde{u}_t = z \frac{u'(G_c(K_{\ell}, z_i))}{\tilde{\mathcal{E}}_{\hat{i},t}} \quad (37)$$

$$\tilde{m}\tilde{c}_t = (1 - \tau) + \frac{\varrho}{\epsilon} \left(G_{\hat{\pi}}(K_{\ell}, z_i)(G_{\hat{\pi}}(K_{\ell}, z_i) - 1) - \frac{\tilde{\mathcal{E}}_{\hat{\pi},t}}{u'(G_c(K_{\ell}, z_i))G_y(K_{\ell}, z_i)} \right) \quad (38)$$

7. Project \tilde{c}_t , \tilde{u}_t , $\tilde{m}\tilde{c}_t$ on the Chebychev polynomial $T_j(\cdot)$ to obtain a new candidate vector of approximation coefficients, $\tilde{\Psi}$. If $\|\tilde{\Psi} - \Psi\| < \varepsilon\xi$ then a solution was found and go to step 8, otherwise update the candidate solution as

$$\xi\tilde{\Psi} + (1 - \xi)\Psi$$

where $\xi \in (0, 1]$ can be interpreted as a learning rate, and go back to step 3.

8. Upon convergence of Ψ , compute $\tilde{K}^*(s)$ that solves (32). If $\|\tilde{K}^*(s) - K^*(s)\| < \varepsilon_k\xi_k$ then a solution was found, otherwise update the threshold as

$$\xi_k\tilde{K}^*(s) + (1 - \xi_k)K^*(s)$$

where $\xi_k \in (0, 1]$ can be interpreted as a learning rate on the threshold, and go back to step 3.

A.10.2 Computing the General Equilibrium

This section explains how the general equilibrium is solved. Given a candidate solution Ψ , we present the solution for a given level of the capital stock K , a particular realization of the shocks (a, z) . For convenience, and to save on notation, we drop the time index.

For a given guess on the threshold, $K^*(a, z)$, test the position of K . If $K \leq K^*(a, z)$, the economy is in normal times. Using the approximation guess, we obtain

$$C = G_c^n(K, s), \hat{\pi} = G_{\hat{\pi}}^n(K, s), mc = G_{mc}^n(K, s)$$

and $\omega = 1$. If $K > K^*(a, z)$, the economy is in crisis times. Using the approximation guess, we get immediately

$$C = G_c^c(K, s), \hat{\pi} = G_{\hat{\pi}}^c(K, s), mc = G_{mc}^c(K, s) = \frac{1}{\mathcal{M}}$$

and $\omega = 1 - \mu$.

From the production function and the definition of the marginal cost, we get

$$N = \left(\frac{1 - \alpha}{\chi(1 - \tau)\mathcal{M}} a(\omega K)^\alpha C^{-\sigma} \right)^{\frac{1}{\alpha + \varphi}}$$

Using the Taylor rule, we obtain gross inflation as

$$\hat{\pi} = \pi^* \left(\frac{\beta \hat{i}}{(Y/Y^*)^{\phi_y}} \right)^{\frac{1}{\phi_\pi}}$$

Output then directly obtains from the production function as

$$Y = a(\omega K)^\alpha N^{1 - \alpha}$$

The rate of return on capital follows as

$$r^k = \frac{\alpha}{1 - \tau} \frac{Y}{\mathcal{M}K} - \delta$$

The investment level obtains directly from the resource constraint as

$$X = Y - C - \frac{\varrho}{2} (\hat{\pi} - 1)^2 Y$$

implying a value for the next capital stock of

$$K' = X + (1 - \delta)K$$

A.10.3 Accuracy

In order to assess the accuracy of the approach, we compute the relative errors an agent would make if they used the approximate solution. In particular, we compute the quantities

$$\begin{aligned} \mathcal{R}_c(K, z) &= \frac{C_t - \left(\beta \mathbb{E}_t \left[C_{t+1}^{-\sigma} (1 + r_{t+1}^q) \right] \right)^{-\frac{1}{\sigma}}}{C_t} \\ \mathcal{R}_i(K, z) &= \frac{C_t - \left(\beta \frac{\hat{i}_t}{z_t} \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{\hat{\pi}_{t+1}} \right] \right)^{-1/\sigma}}{C_t} \\ \mathcal{R}_{\hat{\pi}}(K, z) &= \hat{\pi}_t (\hat{\pi}_t - 1) - \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \hat{\pi}_{t+1} (\hat{\pi}_{t+1} - 1) \right] + \frac{\epsilon - 1}{\varrho} \left(1 - \frac{\epsilon}{\epsilon - 1} \cdot \frac{1}{\mathcal{M}_t} \right) \end{aligned}$$

where $r_{t+1}^q \equiv \int_0^1 r_{t+1}^q(j) dj$, and $\mathcal{R}_c(K, z)$ and $\mathcal{R}_i(K, z)$ denote the relative errors in terms of consumption an agent would make by using the approximate expectation rather than the “true” rational expectation in the household’s Euler equation. $\mathcal{R}_{\hat{\pi}}(K, z)$ corresponds to the error on inflation. All these errors are evaluated for values for the capital stock that lie outside of the grid that was used to compute the solution. We used 1,000 values uniformly distributed between K_m and K_s . Table A.4 reports the average of absolute errors, $E^x = \log_{10} \left(\frac{1}{n_k \times n_a \times n_z} \sum |\mathcal{R}_x(K, s)| \right)$, for $x \in \{c, \hat{i}, \hat{\pi}\}$.

Concretely, $E^c = -5.23$ in the case $(\phi_\pi, \phi_y, \phi_r) = (1.5, 0.125, 0)$ means that the average error an agent makes in terms of consumption by using the approximated decision rule —rather

than the true one— under TR93 amounts to \$1 per \$171,000 spent. The largest approximation errors in the decision rules are made at the threshold values for the capital stock where the economy shifts from normal to crisis times (in the order of \$1 per \$2500 of consumption).

Table A.4: Accuracy Measures

ϕ_π	ϕ_y	ϕ_r	E^c	E^i	E^π
Taylor-type Rules					
1.5	0.125	–	-5.23	-5.00	-4.83
1.5	0.250	–	-5.13	-4.72	-4.67
1.5	0.375	–	-5.07	-4.61	-4.56
2.0	0.125	–	-5.15	-5.10	-4.84
2.5	0.125	–	-5.15	-5.16	-4.88
SIT					
$+\infty$	–	–	-5.31	–	–
Augmented Taylor-type Rules					
1.5	0.125	5.0	-5.37	-5.21	-5.04
5.0	0.125	5.0	-5.34	-5.56	-5.09
10.0	0.125	75.0	-5.35	-5.39	-5.09
Backstop Rules					
1.5	0.125	–	-5.80	-5.29	-5.39
$+\infty$	–	–	-5.74	–	-4.60

Notes: $E^x = \log_{10}(\frac{1}{n_k \times n_a \times n_z} \sum |\mathcal{R}_x(K, s)|)$ is the average of the absolute difference, in terms of the level of consumption, that is obtained if agents use the approximated expectation of variable x instead of its “true” rational expectation, for $x \in \{c, \hat{\lambda}, \hat{\pi}\}$.

B Model Robustness

The aim of this section is to illustrate the robustness of our results by showing that they hold in four alternative versions of our model: (B.1) with intermediated finance, (B.2) with infinitely-lived firms, (B.3) with *ex ante* debt financing, and (B.4) with *ex ante* heterogeneous firms. In addition, we show that (B.5) there cannot be a rationing equilibrium with trade on the credit market, and analyse the cases where (B.6) there is only one financial friction —either moral hazard or asymmetric information. The latter analysis allows us to show that both frictions are necessary for our model to feature credit market collapses.

B.1 Intermediated Finance

The aim of this section is to show that our baseline model with inter-firm credit is isomorphic to a model with bank credit.

We are interested in whether capital reallocation can also take place through banks, without banks making losses. For this, we consider a representative and competitive bank that purchases K_t capital goods on credit at rate r_t^d (“deposits”) from unproductive firms and sells $K_t^p - K_t > 0$

capital goods on credit (“loans”) at rate r_t^ℓ to productive firms. The bank faces the same financial frictions as the firms. It is not able to enforce contracts with borrowers and does not observe firms’ idiosyncratic productivities. But it is not a source of financial frictions itself, in the sense that it can credibly commit itself to paying back its deposits. The rest of the model is unchanged.

The bank’s profit is the sum of the gross returns on the loans (first term) minus the cost of deposits (last term):

$$\max_{K_t^p} (1 - \mu)(1 + r_t^\ell)(K_t^p - K_t) - \mu(1 + r_t^d)K_t \quad (39)$$

The bank’s objective is to maximise its profit with respect to K_t^p given r_t^ℓ and r_t^d , subject to its budget constraint

$$(1 - \mu)(K_t^p - K_t) = \mu K_t \quad (40)$$

as well to productive firms’ participation constraint

$$r_t^\ell \leq r_t^k \quad (41)$$

and unproductive firms’ incentive compatibility constraint

$$(1 - \delta)K_t^p - \theta(K_t^p - K_t) \leq (1 + r_t^d)K_t \quad (42)$$

The latter constraint means that unproductive firms must be better-off when they deposit their funds K_t with the bank (for a return r_t^d , on the right-hand side) than when they borrow $K_t^p - K_t$ and abscond (left-hand side).

Since the bank’s profit increases with r_t^ℓ and decreases with r_t^d , a necessary condition for the bank to be active is that its profit be positive when r_t^ℓ and r_t^d satisfy (41) and (42) with equality, respectively. Substituting relation (40) in the expression of the bank’s profit in (39) and in (42) respectively yields the non-negative profit condition

$$r_t^k \geq r_t^d$$

and the incentive compatibility condition

$$r_t^d \geq \bar{r}^k \equiv \frac{(1 - \theta)\mu - \delta}{1 - \mu} \quad (23a)$$

the combination of which corresponds to condition (23). It follows that, when $r_t^k < \bar{r}^k$ and the credit market has collapsed, there is no room for financial intermediation either. When $r_t^k \geq \bar{r}^k$, financial intermediation may arise. But as unproductive firms can lend to productive ones at rate $r_t^c = r_t^k$ directly *via* the credit market in that case (see equilibrium E in Figure 3), the bank must offer the same conditions, with $r_t^\ell = r_t^d = r_t^k$, in order to be competitive —and therefore makes zero profit.

A version of the model with banks is therefore isomorphic to our baseline model with dis-intermediated finance. This result is intuitive. As long as banks face the same agency problem

as other prospective lenders, whether financial transactions take place directly through a credit market, as in our baseline model, or indirectly through a loan market is irrelevant: these two markets rise and collapse in sync —and yield the same general equilibrium outcome.⁶³

B.2 Infinitely-lived Firms and Stigma Effects

The aim of this section is to show that our results would not change if firms lived infinitely.

Assume that firms live infinitely, the rest of the model being unchanged —*e.g.* firms' idiosyncratic productivities are still independently distributed every period. Since the household can freely re-balance their entire equity portfolio across firms, it is optimal for the household to perfectly diversify its portfolio and fund every firm with the same amount. Hence, all firms start afresh with the same startup equity funding and capital stock every period. In the absence of stigma associated to default, firms' borrowing limit remains the same as in Proposition 1, and whether firms live infinitely is immaterial.

Our model is robust to introducing stigma effects. Probably the simplest way to see this is to notice that parameter θ can be seen as capturing a —possibly non-pecuniary— reputational cost of default and therefore a stigma effect.

Another way to model stigmas would be to assume that a firm that defaults is banned from the credit market for, say, $K \geq 1$ periods. In this case, the penalty cost of a default θ_t and crisis threshold \bar{r}_t^k would become endogenous and vary over time with the present franchise (or continuation) value, say V_t , of having access to the credit market in the future. V_t would be equal to the discounted sum of the expected net future returns that a firm would forgo by being banned from the credit market and depend (among other things) on the expectation of the future rates of return r_{t+k}^k (with $k = 1, \dots, K$). While such extension would be particularly hard to solve numerically, a simple thought experiment helps to see why the model mechanisms and results would not change either in that case.

Consider a given crisis threshold \bar{r}_t^k and an adverse exogenous productivity shock that lowers r_t^k down to a level close to —but still above— \bar{r}_t^k . As the return on capital gets closer to the crisis threshold, firms would anticipate a higher risk of a crisis in the near future and factor in a lower franchise value V_t of credit market access. Following the decline in V_t , the crisis threshold \bar{r}_t^k would go up, further reducing the gap between r_t^k and \bar{r}_t^k . It follows that the mere expectation of a fragile credit market in the future would make the credit market *even more* likely, inducing households to accumulate yet more precautionary savings and intermediate firms to further raise their markups ahead of crises (see the discussion Section 4.3) compared to the baseline model. In turn, larger externalities would lead to more frequent booms and busts.⁶⁴ The upshot is that, in a version of the model where access to credit market carries a franchise value that

⁶³This equivalence result only emphasises that the key element of our model is the agency problem that lenders face, and not the financial infrastructure (financial markets or banks) considered.

⁶⁴Of course, crises being more frequent, one would have to re-parameterise this version of the model so that the economy still spends 10% of the time in a crisis, as in the baseline model.

acts as borrowers’ “skin in the game”, the mechanisms and trade-offs would be essentially the same as in the baseline model.

B.3 *Ex ante* Debt Financing

The aim of this section is to show that our results carry through if, at the end of period $t - 1$, firms finance their startup capital stock K_t with debt instead of equity.

Assume that, at the end of period $t - 1$, firms finance a share $1 - \gamma$ of their capital with equity and a share γ with debt, and that debt carries a real interest rate r_t^d . In that case, a firm may end up with two distinct types of debt at the beginning of period t : a “legacy”, *inter*-period debt $\gamma P_{t-1} K_t$, and a “new”, *intra*-period debt $\psi_t P_t K_t$. The implications of legacy debt issuance depends on whether firms can default on this debt or not. We study these two cases in turn.

B.3.1 Riskless Legacy Debt

A preliminary and straightforward observation is that our model would be unchanged if we assumed that firms cannot default on households —*i.e.* that they can issue pure riskless debt at the end of period $t - 1$. Such situation amounts to assuming a higher creditor protection for legacy debt holders (households) than for new debtholders (unproductive firms) and, more precisely, that the cost of hiding from debtholders, denoted θ^h , exceeds the gain from defaulting, *i.e.*

$$\theta^h \geq 1 + r_t^d \quad \forall t$$

In that case, defaulting on legacy debt is not worthwhile, firms always repay this type of debt, and firms’ incentives to default on inter-firm loans in period t are unchanged. In other terms, assuming that firms can issue riskless debt would not change anything compared to our baseline model. Moreover, the Modigliani–Miller theorem applying, firms would also be indifferent between financing their start-up capital with equity or with debt. In such version of the model, the capital stock K_t can therefore be seen as being entirely financed with riskless debt (case with $\gamma = 1$) as opposed to equity (case with $\gamma = 0$).

B.3.2 Risky Legacy Debt

Next, assume that firms may default on legacy debt at the end of period t in the similar way as they can default on their inter-firm debt, *i.e.*

$$\theta^h < 1 + r_t^d \quad \forall t$$

As firms enter period t , lenders on the credit market understand that their legacy debt increases borrowers’ incentives to default. Accordingly, they limit the amount that a borrower can borrow so that an unproductive firm does not have any incentive to borrow and abscond:

$$(1 - \delta)K_t^P - \theta(K_t^P - K_t) + \theta^h \gamma K_t \leq (1 + r_t^c)K_t - (1 + r_t^d)\gamma K_t$$

$$\Leftrightarrow \frac{K_t^p - K_t}{K_t} \leq \frac{r_t^c + \delta - \gamma(1 + r_t^d - \theta^h)}{1 - \delta - \theta} \quad (43)$$

with possibly $\theta^h = \theta$, and where $\theta(K_t^p - K_t)$ and $\theta^h \gamma K_t$ are the costs of defaulting on inter-firm and legacy debts, respectively. Relation (43) shows that market rates may have varied (and opposite) effects on incentives: a higher cost of *legacy debt* (r_t^d) deteriorates incentives (as in [Stiglitz and Weiss \(1981\)](#) and [Mankiw \(1986\)](#)), whereas a higher return on financial *assets* (r_t^c) improves incentives (as in [Bernanke and Gertler \(1990\)](#), [Gertler and Rogoff \(1990\)](#), [Azariadis and Smith \(1998\)](#)).

Given condition (43), the condition of existence of an active credit market becomes

$$r_t^k \geq \bar{r}_{t-1}^k \equiv \frac{(1 - \theta)\mu - \delta}{1 - \mu} + \gamma(1 + r_t^d - \theta^h) \quad (23b)$$

where only the last term differs from the baseline condition (23). Since r_t^d is predetermined, the presence of legacy debt essentially raises the crisis threshold but does not materially affect the condition of existence of an active credit market —which still rests on the level of capital returns r_t^k .

Condition (23b) illustrates and emphasises a general result of banking models with agency costs ([Bernanke and Gertler \(1990\)](#)): in the presence of agency costs, the equilibrium outcome is ultimately determined by the “creditworthiness” of borrowers, reflected here by the return on capital r_t^k — and not by the level of the loan rate as such. In our model, the higher the return on capital, the more room for manoeuvre lenders have to address the agency problem, and the more robust the credit market.

In this extension of the model, crisis dynamics ought to be similar to those in our baseline model. To see this, assume that households anticipate in $t - 1$ a crisis (and therefore more loan defaults) in t . Since households understand that debt is riskier, they will charge a higher nominal loan rate *ex ante*: all else equal, the real loan rate r_t^d will go up. Following the rise in their cost of debt, unproductive firms’ incentives to default will rise, making the crisis even more likely (condition (23b)). If anything, such inter-temporal complementarities will therefore work to amplify the dynamics in our model —rather than lessen them.

Finally, assume that firms can choose their funding mix γ at the end of period $t - 1$. Given that its legacy debt impedes its borrowing capacity in period t (compare (20) and (43)), it is clear that it is always optimal for any given firm to finance its startup capital stock entirely through equity. Hence, if firms are given the choice of their *ex ante* debt structure, they will set $\gamma = 0$ —as in our baseline model.

B.4 Ex-ante Heterogeneous Firms

The aim of this section is to show that our analysis and results carry through when firms are also heterogeneous *ex ante*, before they incur the idiosyncratic productivity shocks.

As an illustration, consider two observationally distinct sets of “high” (H) and “low” (L) quality firms of equal mass $1/2$, characterised by probabilities μ^H and μ^L of being unproductive

(*i.e.* of drawing $\omega_t(j) = 0$), with $\mu^H < \mu^L$. Households observe every firm's type H or L at the time they invest in equity and know μ^H and μ^L of productive firms. But they do not observe which firms are productive within each type. The rest of the model is unchanged.

In the presence of financial frictions, households may vary their equity investments across high and low quality firms. Let K_t^L and K_t^H denote low and high quality firms' respective initial capital stocks, with $K_t^L \neq K_t^H$.⁶⁵ The aggregate capital stock is $K_t = (K_t^H + K_t^L)/2$ and the share of K_t that is held by unproductive firms is

$$\mu_t \equiv \frac{\mu^H K_t^H + \mu^L K_t^L}{K_t^H + K_t^L} \quad (44)$$

The constant returns to scale imply that productive firms have the same realised return on capital r_t^k , irrespective of their type L or H and initial capital stock, K_t^L or K_t^H . Moreover, Proposition 1 shows that their initial capital stock does not affect firms' borrowing limit either: $\psi_t = (r_t^c + \delta)/(1 - \delta - \theta)$ and is the same across high and low quality firms. Put differently, once the $\omega_t(j)$ s are realised, what matters is whether a firm is productive, not its *ex ante* probability of being productive. It follows that the aggregate credit supply and demand schedules in normal times are given by

$$L^S(r_t^c) = \mu_t K_t$$

and

$$L^D(r_t^c) \in [-(1 - \mu_t)K_t, (1 - \mu_t)\psi_t K_t]$$

and normal times arise in equilibrium only if there exists a credit market rate r_t^c such that $r_t^c \leq r_t^k$ and

$$\mu_t K_t \in \left[-(1 - \mu_t)K_t, \frac{(1 - \mu_t)(r_t^c + \delta)}{1 - \delta - \theta} K_t \right]$$

which is the case if

$$\mu_t \leq \frac{(1 - \mu_t)(r_t^k + \delta)}{1 - \delta - \theta} \Leftrightarrow r_t^k \geq \frac{(1 - \theta)\mu_t - \delta}{1 - \mu_t} \quad (23c)$$

The above condition is similar to (23), meaning that the Y–M–K transmission channels of monetary policy are still present and operate the same way as in our baseline model. The only difference is that μ_t is now endogenously determined at end of period $t - 1$, *i.e.* that the share of capital invested in low *versus* high quality firms is yet another factor affecting financial

⁶⁵One can show that it is optimal for households to hold more equity from high quality firms than from low quality firms, so that $K_t^L < K_t^H$ and μ_t varies over time. So see why, first consider the case of a frictionless credit market. Absent financial frictions, firms perfectly hedge themselves against the idiosyncratic productivity shocks and all have the same return on equity: $r_t^q(j) = r_t^k$ for all j irrespective of the realization of the shock. As a consequence, firms' quality is irrelevant and the household does not discriminate across high and low quality firms, which thus all get the same equity funding: $K_t^H = K_t^L = K_t$. Hence, $\mu_t = (\mu_H + \mu_L)/2$ and is constant over time. In the presence of financial frictions, in contrast, the household understands that unproductive firms will distribute less dividends than productive firms if a crisis breaks out. It will invest in the equity of high and low quality firms until their marginal expected returns equate and no arbitrage is possible. Since low quality firms are less likely to be productive than high quality firms and the marginal return on equity decreases with the capital stock, it is optimal for the household to invest relatively more equity in high quality firms, especially so when the probability of a crisis goes up. It follows that $K_t^H > K_t^L$ and K_t^H/K_t^L increases with the crisis probability.

stability.⁶⁶ The upshot is that our results carry through to an economy with observationally *ex ante* heterogeneous firms, provided that there remains some residual *ex post* heterogeneity (here in the form of the idiosyncratic productivity shocks $\omega_t(j)$ s) and, therefore, a role for short term (intra-period) credit markets.

B.5 Non-Existence of a Rationing Equilibrium

The aim of this section is to show that there cannot be a credit market equilibrium with trade and rationing when the normal times equilibrium (equilibrium E in panel (i) of Figure 3) does not exist.⁶⁷

For this, we consider a separating equilibrium (where no borrower defaults) and a pooling equilibrium (where some borrowers may default) in turn. Throughout, we assume that the normal time equilibrium does not exist (*i.e.* that $r_t^k < \bar{r}^k$), look for an equilibrium with trade (*i.e.* with $r_t^c \leq r_t^k$), and argue by contradiction.

B.5.1 Separating Equilibrium

In a separating equilibrium, all unproductive firms want to sell their entire capital stock and lend the proceeds to productive firms. When $r_t^k < \bar{r}^k$, though, the supply of credit exceeds demand (as illustrated in panel (ii) of Figure 3) and not all unproductive firms can lend. Hence, a separating equilibrium entails the rationing of *lenders*.

Assume that every unproductive firm sells only a fraction λ_t of its capital stock —keeping the rest idle— and lends the proceeds of these sales to productive firms, where λ_t satisfies the equilibrium condition:

$$\lambda_t \mu K_t = (1 - \mu)(K_t^p - K_t) \quad (45)$$

In that case, an incentive compatibility constraint must ensure that no unproductive firm is better off if it borrows and defaults, *i.e.* that:

$$\begin{aligned} (1 - \delta)K_t^p - \theta(K_t^p - K_t) &\leq \lambda_t(1 + r_t^c)K_t + (1 - \lambda_t)(1 - \delta)K_t \\ \Leftrightarrow \frac{K_t^p - K_t}{K_t} &\leq \psi_t \equiv \max \left\{ \lambda_t \frac{r_t^c + \delta}{1 - \delta - \theta}, 0 \right\} \end{aligned}$$

which can be re-written using (45) as:

$$r_t^c \geq \bar{r}^k \equiv \frac{(1 - \theta)\mu - \delta}{1 - \mu}$$

with $r_t^c \leq r_t^k < \bar{r}^k$; hence a contradiction.

⁶⁶Since μ_t is predetermined, the effect of this additional channel can only be of second order compared to the Y–M–K channels.

⁶⁷Of course, the crisis equilibrium could be considered as a case of credit rationing, where unproductive firms do not lend *at all*. Here, the focus is on the existence of a rationing equilibrium *with trade*.

B.5.2 Pooling Equilibrium

Next, consider the case of a pooling equilibrium with trade (*i.e.* with $r_t^c \leq r_t^k$), *i.e.* a situation where both productive and unproductive firms prefer to borrow rather than lend and therefore where all firms apply for a loan. Since not all firms can borrow, such pooling necessarily entails the rationing of *borrowers*.

As lenders do not observe loan applicants' types, they accept/reject loan applications at random. Among the accepted applicants, the productive ones use the funds to purchase capital goods and increase the size of their project, produce, and repay their debt, while the unproductive ones divert the funds and default. Among the rejected applicants, the productive ones operate their initial project while the unproductive ones either sell their capital stock and lend the proceeds or keep their capital stock idle. Hence, in a pooling equilibrium, only the unproductive firms that did not obtain a loan may lend to other firms.

The question is whether such firms find it profitable to lend, even though some borrowers default. An unproductive firm will lend only if it is better off doing so than keeping its capital idle. Since unproductive and productive firms are equally likely to obtain a loan, the probability that a lender lends to a productive firm and is paid back is equal to $1 - \mu$, and the probability that it lends to an unproductive firm (that will default) is μ . Accordingly, an unproductive firm that has been denied a loan will lend its startup funds if and only if

$$(1 - \mu)(1 + r_t^c)K_t + \mu \times 0 \geq (1 - \delta)K_t \Leftrightarrow r_t^c \geq \frac{\mu - \delta}{1 - \mu} > \bar{r}^k \equiv \frac{(1 - \theta)\mu - \delta}{1 - \mu}$$

with $r_t^c \leq r_t^k < \bar{r}^k$; hence a contradiction.

B.6 Only One Financial Friction

Our baseline model features two standard financial frictions: moral hazard and asymmetric information between lenders and borrowers. This section shows that both frictions are needed for the aggregate equilibrium outcome to depart from the first best outcome.

B.6.1 Asymmetric Information as Only Friction

Assume first that firms cannot abscond with the proceeds of the sales of idle capital goods. Then unproductive firms always prefer to sell their capital stock and lend the proceeds, and have no incentive to borrow. As a result, productive firms face no borrowing limit: they borrow until the marginal return on capital equals the cost of credit and $r_t^\ell = r_t^k > -\delta$ in equilibrium. No capital is ever kept idle. The economy reaches the first best.

B.6.2 Moral Hazard as Only Friction

Assume next that firms' idiosyncratic productivities are perfectly observable at no cost. Then, only productive firms can borrow. But they must be dissuaded from borrowing $P_t(K_t^p - K_t)$ to purchase capital goods, keep them idle, and abscond. This will be the case if what they earn if

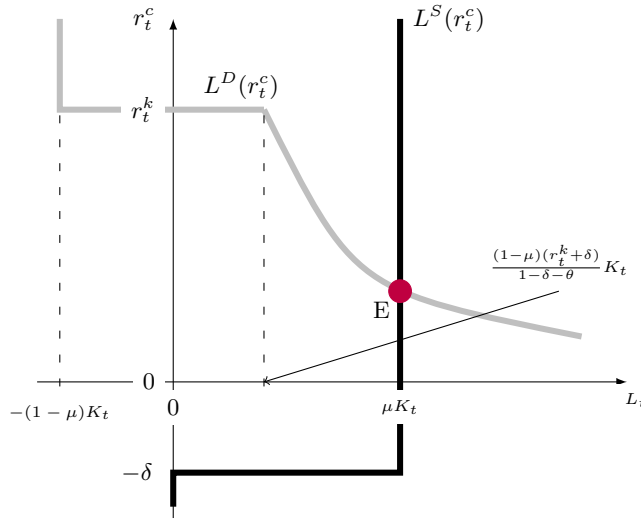
they abscond, $P_t(1 - \delta)K_t^p - P_t\theta(K_t^p - K_t)$ is less than what they earn if they use their capital stock in production, $P_t((1 + r_t^k)K_t^p - (1 + r_t^c)(K_t^p - K_t))$ (from (17)), which implies:

$$(1 - \delta)K_t^p - \theta(K_t^p - K_t) \leq (1 + r_t^k)K_t^p - (1 + r_t^c)(K_t^p - K_t)$$

$$\Leftrightarrow \frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \frac{r_t^k + \delta}{1 - \delta - \theta + r_t^c - r_t^k} \quad (46)$$

where the borrowing limit ψ_t now *decreases* with r_t^c : the higher the loan rate, the lower the productive firm's opportunity cost of borrowing and absconding, and hence the lower its incentive-compatible leverage.

Figure B.1: Credit Market Equilibrium Under Symmetric Information



Notes: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate credit demand (gray) curves, when credit contracts are not enforceable but information is symmetric.

The aggregate credit supply and demand schedules in Figure B.1 take the similar form as in (21) and (22), but with the borrowing limit ψ_t now given by (46) instead of Proposition 1. From Figure B.1 it is easy to see that there is only one equilibrium (E) and that the economy reaches the first best: no capital is ever kept idle. The only difference with the frictionless case is in terms of the distribution of equity returns across firms: in equilibrium E , productive firms' realised return on equity may be higher than that of unproductive firms.⁶⁸

⁶⁸To see this, notice that $K_t^u = 0$ in equilibrium E , implying (from (14)) that unproductive firms' return on equity is equal to r_t^c . Further notice that $r_t^k \geq r_t^c$ and $K_t^p > K_t$ in equilibrium E , which implies (from (17)) that productive firms' return on equity is equal to $r_t^c + (r_t^k - r_t^c)K_t^p/K_t \geq r_t^c$.

Previous volumes in this series

990 January 2022	Informational switching costs, bank competition, and the cost of finance	José Renato Haas Ornelas, Marcos Soares da Silva and Bernardus F Nazar Van Doornik
989 Januarz 2022	Central bank digital currencies (CBDCs) in Latin America and the Caribbean	Viviana Alfonso, Steven Kamin and Fabrizio Zampolli
988 January 2022	The premia on state-contingent sovereign debt instruments	Deniz Igan, Taehoon Kim and Antoine Levy
987 January 2022	Zombies on the brink: Evidence from Japan on the reversal of monetary policy effectiveness	Gee Hee Hong, Deniz Igan and Do Lee
986 January 2022	Platform-based business models and financial inclusion	Karen Croxson, Jon Frost, Leonardo Gambacorta and Tommaso Valletti
985 December 2021	Building benchmarks portfolios with decreasing carbon footprints	Eric Jondeau, Benoit Mojon and Luiz Awazu Pereira da Silva
984 December 2021	Credit constrained firms and government subsidies: evidence from a European Union program	Tirupam Goel, Péter Lang, Eszter Balogh, Ádám Banai, Martin Stancsics, Előd Takáts and Álmos Telegdy
983 November 2021	Losing traction? The real effects of monetary policy when interest rates are low	Rashad Ahmed, Claudio Borio, Piti Disyatat and Boris Hofmann
982 November 2021	Navigating by r^* : safe or hazardous?	Claudio Borio
981 November 2021	Back to the future: intellectual challenges for monetary policy	Claudio Borio
980 November 2021	What does machine learning say about the drivers of inflation?	Emanuel Kohlscheen
979 November 2021	Dampening the financial accelerator? Direct lenders and monetary policy	Ryan Banerjee and Jose-Maria Serena
978 November 2021	Financial crises and political radicalization: How failing banks paved Hitler's path to power	Sebastian Doerr, Stefan Gissler, José-Luis Peydró and Hans-Joachim Voth
977 November 2021	Does gender diversity in the workplace mitigate climate change?	Yener Altunbas, Leonardo Gambacorta, Alessio Reghezza and Giulio Velliscig

All volumes are available on our website www.bis.org.