Towards the Exploitation of Formal Methods for Information Fusion

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ABSTRACT

When an autonomous system has to act in or interact with an environment, a suitable representation of it is required. In the past decades, many different representation forms – especially spacial ones – have been proposed and even more information fusion techniques were developed in order to build these representations from multiple information sources. However, most of these algorithms do not exploit the full potential of the available information. This is caused by the fact that they are not able to handle the full complexity of all possible solutions compatible with the information and that they rely on restrictive assumptions (i.e. independencies) in order to make the computation feasible. In this work, a new methodology is envisioned that utilizes formal methods, in particular solvers for Pseudo-Boolean Optimization, to drop some of these assumptions. In order to illustrate the ideas, information fusion based on belief functions and occupancy grid maps are considered. It is shown that this approach allows for considering dependencies among multiple cells and thus significantly reduces the uncertainty in the resulting representation.

Keywords: information fusion, formal methods, belief function theory, occupancy grid maps, incomplete information, Boolean satisfiability

1. INTRODUCTION

Many autonomous systems rely on a representation of the environment in which they are applied. But particularly for in-field applications (e.g. robots which have to navigate through a particular terrain or floor plan), respective information on the "world" is often not or only partially available. Then, the system itself has to obtain a corresponding spatial representation during runtime, e.g. from data gathered by sensors, prior knowledge, etc. In order to derive the best possible understanding of the environment, the data from all available sources is eventually combined (*fused*) to an unified representation that can be used by the system to solve its tasks. This process is called *information fusion*.

For this purpose, *occupancy grid maps*[1] – one of the most popular spatial representations – can be utilized. They discretize the environment using a grid structure where each grid cell contains the state (e.g. *empty* or *occupied*) of the corresponding area. This representation is suitable for many tasks such as navigation and exploration. Grid maps have been applied to a wide field of applications, such as navigating mobile robots[2] and exploring extraterrestrial environments.[3] A significant amount of algorithms for the generation of those maps using many different sensors types (e.g. sonar[4] or laser range finders[5]) have been developed in the recent years.

However, since data is gathered from different sources the basis of the resulting representation is assumed to be very heterogeneous and inherently uncertain. While information fusion aims for reducing the uncertainty, the full potential of the methodologies mentioned above is usually not exploited. This is caused by the fact that several conclusions can only be drawn at high computational costs. Especially when online map building is needed, a system cannot perform these computations without violating its real-time requirements. Hence, information fusion is usually conducted by means of assumptions which hardly exploit the potential provided by the available data. Consequently less precise maps result (this is discussed and illustrated in more detail in Section 3).

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In this work, we envision the exploitation of formal methods to address this issue. We observe that the underlying problem (concluding as much as possible from the available information) eventually requires the traversal of huge search spaces to be considered. Formal methods such as *Binary Decision Diagrams* (BDDs[6]) or reasoning engines (e.g. Refs. 7–10) have proven to be an efficient technology for such tasks. As a representative, we propose the exploitation of solvers for *Pseudo-Boolean Optimization* (PBO[9,10]).

In the remainder of this paper, details on the envisioned methodology as well as its prospects are provided and discussed, respectively. For this purpose, Section 2 provides a brief review on occupancy grid maps, in particular based on belief functions,[11,12] and information fusion as it is conducted by today's state-of-the-art. Section 3 illustrates the considered problem in detail (complexity of concluding as much as possible from the available information), before the proposed methodology is described in Section 4. In order to ease the descriptions, the ideas and possible implementations are sketched by means of simple models and examples only – a thorough implementation and evaluation is left for future work. The required next steps for that are eventually outlined in Section 5 which also concludes the paper.

2. BACKGROUND

This section reviews the background on occupancy grid maps – the representative of spatial environment representations considered in this work. Besides that, the corresponding fusion process applied thus far in order to generate a more precise grid map from the combination of data is re-visited. In order to ease the description of the methodology envisioned in this work, we keep the considered models as simple as possible.

2.1 Occupancy Grid Maps

One particular and most frequently used form grid maps are *occupancy grid maps*,[13] which distinguishes between empty and occupied areas in the environment.

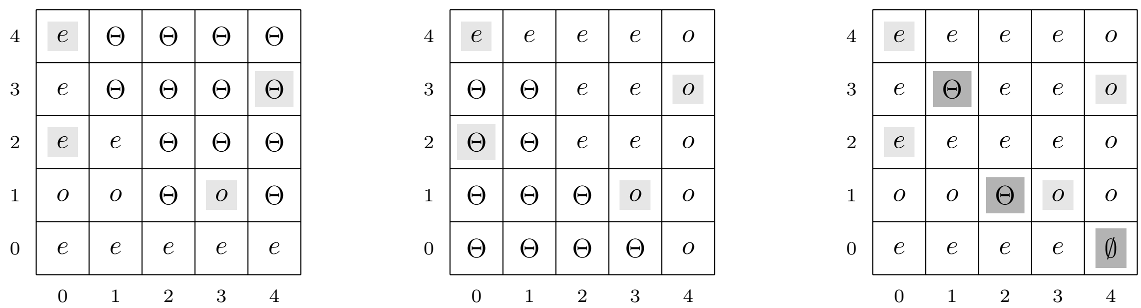
Definition 1. An *occupancy grid map* represents a spatial environment in terms of a discretized grid where each grid cell may either be empty (denoted by e) or occupied (denoted by o).

The information on whether a cell is empty or occupied is obtained from different sources, e.g. gathered by sensors, prior knowledge, etc. Because the sources of information may be afflicted with uncertainty (e.g. sensor noise, contradictory sensor measurements among different sensors or over time, vague expert knowledge, or the simple non-availability of information), a formalism to represent uncertain information in a given map is required. To this end, the state of a cell is usually modeled probabilistically with a single probability function $P(e)$.

Here, for example, $P(e) = 0.3$ represents that a cell in the grid may be empty with probability of 30%, what implies that it may be occupied with probability 70% ($P(o) = 0.7$). This approach, however, requires an explicit representation of different dimensions of uncertainty, since e.g. the complete lack of information cannot explicitly be expressed by probabilities. A uniform distribution ($P(e) = 0.5$ and $P(o) = 0.5$) could work, but bears the risk of being misinterpreted with the fact that the cell is empty/occupied with probability of 50%. Therefore, the belief function theory,[14,15] which is a generalization of the well known Bayesian probability theory, is used in this work. This theory allows to assign belief mass not only to the singletons of a hypotheses space (here e and o), but also to all subsets including $\{e, o\}$ and $\emptyset$. In the following, we only consider categorical mass functions which are defined as follows.

Definition 2. Let Θ be the frame of discernment, i.e. the hypotheses space, and $A \subseteq \Theta$ a hypothesis for a respectively given grid. Then, a *categorical mass function* m is a mapping $m : \mathcal{P}(\Theta) \to \{0, 1\}$ assigning a mass value to each hypothesis A of Θ such that

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$



(a) Initial map m_1 (b) Further map m_2 (c) Fused map $m_{1 \circledcirc 2}$

Figure 1: Obtained information in terms of occupancy grid maps

In terms of an occupancy grid map, the frame of discernment is defined as

$$\Theta = \{e, o\} \tag{2}$$

and a categorical mass function indicating the state of a grid cell as

$$m(A) = 1, \quad A \in \{\emptyset, e, o, \Theta\} \tag{3}$$

$$m(B) = 0, \quad \forall B \in \{\emptyset, e, o, \Theta\}, B \neq A. \tag{4}$$

The belief function theory allows for explicitly stating different dimensions of uncertainty.[16] More precisely,

- certain information can be represented by assigning mass to e or o,
- a lack of information can be expressed by assigning mass to the disjunction $\{e, o\}$ (i.e. Θ), and
- conflicting information can be expressed by assigning mass to the empty set (i.e. $\emptyset$).

In contrast, the probability theory can express uncertainty only by the ratio between $P(e)$ and $P(o)$.

Example 1. Fig. 1a shows an example for an occupancy grid map m_1 as it could have been derived from sensor data. The symbols in the cells indicate that, for this cell, the respective mass value is 1, what implies that it is 0 for all other $A \subseteq \Theta$ (see Eqs. 3 and 4). As shown, the bottom left part is well observed with mass assigned to empty (e) and occupied (o). However, a significant amount of uncertainty exist in the remainder of this map, where mass is assigned to Θ indicating a lack of information.

2.2 Information Fusion

In order to reduce the amount of uncertainty in the map and to build a complete representation of the environment, information is usually gathered from more than one source. The information sources can be from different types or from the same type and collected over time. This results in several maps which, eventually, have to be *fused* into a unified representation. This process is called *information fusion* or, in the context of multiple sensors, *multi-sensor fusion*. A consistent consideration of the uncertainty of the different maps is thereby required.

There are several works on information fusion to build occupancy grid maps with respect to the belief function framework.[17,18] But they focus on the mapping problem only and assume that the pose, i.e. the position and orientation of the robot gathering the data, is known. Accordingly, they do not consider the full joint estimation problem of building an environment map and simultaneously consider the respective *localization*. This is a problem since, e.g. in robotics, both issues frequently have to be considered at the same time. In other words, *Simultaneous Localization And Mapping* (SLAM[19]) is required.

In Ref. 11, an approach to model the SLAM problem in the belief function theory has been proposed as a generalization of the very successful FastSLAM algorithm.[20] Here, different fusion rules where compared in terms of the provided information. Among the considered rules, the *conjunctive rule of combination*[15] is the only one that yields mass on $\emptyset$ and, hence, allows for a representation of conflicting evidence. Because of that, this rule is applied in the following consideration as well. More precisely:

Definition 3. Let m_1 and m_2 be two mass functions defined over the same frame of discernment Θ and induced by two distinct pieces of evidence. The combination (*fusion*) of these two mass functions with the *conjunctive rule of combination* $\circledcap$ results in the mass function $m_{1\circledcap 2} = m_1 \circledcap m_2$ which is defined as

$$m_{1\circledcap 2}(A) = \sum_{B\cap C=A} m_1(B)m_2(C), \quad \forall A \subseteq \Theta. \tag{5}$$

For categorical mass functions in the context of occupancy grid maps (Eqs. 3 and 4) that are defined over $\Theta = \{e, o\}$ (Eq. 2), the conjunctive rule of combination reduces to

$$m_{1\circledcap 2}(A) = 1, \quad A = \begin{cases} o & \text{if } m_1(o) = m_2(o) = 1 \\ & \text{or } m_1(o) = m_2(\Theta) = 1 \\ & \text{or } m_1(\Theta) = m_2(o) = 1 \\ \\ e & \text{if } m_1(e) = m_2(e) = 1 \\ & \text{or } m_1(e) = m_2(\Theta) = 1 \\ & \text{or } m_1(\Theta) = m_2(e) = 1 \\ \\ \Theta & \text{if } m_1(\Theta) = m_2(\Theta) = 1 \\ \\ \emptyset & \text{else} \end{cases} \tag{6}$$

$$m_{1\circledcap 2}(B) = 0, \quad \forall B \subseteq \Theta, B \neq A. \tag{7}$$

Example 2. In order to reduce the uncertainty of the map m_1 from Fig. 1a, further data has been gathered – resulting in a second map m_2 as shown in Fig. 1b. Applying the conjunctive rule of combination (Eqs. 6 and 7), this eventually results in the fused representation $m_{1\circledcap 2}$ shown in Fig. 1c.

Here, all cells with mass on e or mass on o in *both* input maps obviously remain this mass in the fused map (see e.g. cell $(0, 4)$ or cell $(3, 1)$ in the grids from Fig. 1). But, beyond that, uncertainty is reduced in all cells with mass on Θ in only *one* of the input maps (see e.g. cell $(0, 2)$ or cell $(4, 3)$). In contrast, conflicts may arise when different evidence is represented in the input maps. This is the case e.g. for cell $(4, 0)$ where $m_1(o) = 1$ and $m_2(e) = 1$ is eventually fused to $m_{1\circledcap 2}(\emptyset) = 1$ – representing another dimension of uncertainty.

3. MOTIVATION

Information fusion as reviewed in the previous section is an effective method to combine different sources of information and to reduce the uncertainty of grid maps. However, the full potential of the gathered data is not exploited by simply fusing the gathered information to a combined representation. In fact, real world scenarios usually provide additional information in the form of background and expert knowledge which can be applied in order to further reduce the uncertainty of a map.

For example, in an office building, autonomous systems usually find a certain amount of empty space to work on (i.e. sub-grids of cells with mass on e) which, in turn, are divided by walls (i.e. sub-grids of cells with mass on o). This background and expert knowledge can be utilized to conclude that

- an uncertain cell which is surrounded by empty cells is empty as well and
- an uncertain cell within a "line" of occupied cells belongs to a wall and, hence, is occupied as well.

Typical structures like those can be utilized in terms of *further rules* which complement the fusion rule (i.e. the conjunctive rule of combination). By this, much more precise maps with less uncertainty may result.

Example 3. Consider again the representation of the fused information $m_{1 \circledcirc 2}$ shown in Fig. 1c. Applying the information provided by the rules sketched above allows to conclude that

- the unknown cell $(1, 3)$ with mass on Θ is empty, because most of the neighboring cells (in fact, all of them) are empty as well (i.e. have their mass on e),
- the unknown cell $(2, 1)$ with mass on Θ is occupied, because it is in a line with occupied cells (i.e. cells with mass on o), and
- the conflict in cell $(4, 0)$ with mass on $\emptyset$ may be resolved in favor of map m_2, since this cell is in a line with occupied cells (i.e. cells with mass on o) and, hence, very likely occupied as well.

Various of such rules can be derived. However, their application may lead to further conflicts, e.g. when a rule implies that a cell has to be empty while another rule implies that it has to be occupied. In fact, this might be the case for cell $(2, 1)$ in Fig. 1c. As discussed in Example 3, this cell is assumed to be occupied due to the "wall"-rule discussed above. But besides that, also the "empty subgrid"-rule (assuming this cell to be empty) might be applicable, since most of the neighboring cells are empty. Because of this, different priorities are assumed for each rule. In this work, we formalize this as follows:

Definition 4. Let R be a set of given rules which can be applied on a given grid map. For each rule $r \in R$, an additional *weight* w_r is provided which represents the priority of rule r against other rules $r' \in R \setminus \{r\}$.

Based on these priorities, a subset $\hat{R} \in \mathcal{P}(R)$ of rule combinations is desired which does not introduce new conflicts and maximizes the overall weight. More formally,

- for a given mass assignemnt m to a map, $\hat{R}$ shall not lead to new mass on $\emptyset$ and
- all remaining subsets $R' \in \mathcal{P}(R) \setminus \hat{R}$ which also do not introduce new conflicts have to have a smaller or equal overall weight, i.e. $w_{R'} \leq w_{\hat{R}}$ with $w_R = \sum_{r \in R} w_r$.

Following this optimization criteria, as much as possible further information is concluded while, at the same time, new conflicts are avoided.

However, determining $\hat{R}$ obviously is a computationally complex task: For each combination of rules, all possible mass assignments to all grid cells have to be checked. Since a total of $|\mathcal{P}(R)| = 2^{|R|}$ combinations are possible, this results in an exponential complexity. All this becomes even more crucial when all possible mass assignments to all cells in the grid map are considered. This results in the space of all possible maps with each cell defined over $\mathcal{P}(\Theta)$, i.e. a total of $|\mathcal{P}(\Theta)|^M = 2^{|\Theta|^M}$ combinations, where M is the number of cells in the map.

In order to handle this complexity, all solutions proposed thus far apply significant restrictions – including pure grid mappings with known pose and probabilistic grid maps,[13] the original FastSLAM algorithm,[20] and the evidential FastSLAM approach with belief function grid maps as considered here.[11,12] More precisely, all these approaches apply a so-called (conditional)[*] independence assumption which factorizes the joint distribution over all grid cells into marginal cell distributions. More precisely,

$$m(Y) = \prod_{i=1}^{M} m(Y_i) \tag{8}$$

is applied, where Y is the complete map, Y_i is a single cell, and M is the number of cells. This allows to update the cells independently of each other and reduces the complexity from the high-dimensional space of all maps $|\mathcal{P}(\Theta)|^M$ to single cells $M \cdot |\mathcal{P}(\Theta)|$. On the downside, however, it prevents to express any dependencies among cells. As a consequence, none of the rules sketched above can be applied under this assumption.

In this work, we envision a methodology which overcomes this limitation and, by this, leads to the best possible result based on the available information and rules.

[*]For simplification, the conditional part is omitted here. See Refs. 11 or 12 for the full equations.

4. PROPOSED METHODOLOGY

Performing information fusion as reviewed in Section 2.2 and, at the same time, considering the best possible utilization of the further rules as discussed in Section 3 is a computationally hard problem. For this purpose, all possible combinations of rules have to be considered. For a set of rules R, this multiplies together to a search space of $2^{|R|}$ different combinations to be explored – an exponential complexity.

To cope with this complexity, we propose to exploit the deductive power of formal methods – more precisely, of solvers for *Pseudo-Boolean Optimization* (PBO[9,10]). Their intelligent decision heuristics, powerful learning schemes, and fast implication methods enable to efficiently traverse large search spaces and have been proven to be very effective for many practically relevant problems such as equivalence checking,[21] property checking,[22] automatic test pattern generation,[23] or the design of biochips.[24] Our thesis is that this deductive power can also be utilized in order to determine the best solution from all the possible combinations discussed above.

In this section, we describe the envision methodology. For this purpose, we first briefly review the PBO problem. Afterwards, we sketch how the considered problem can be represented as a corresponding PBO instance.

4.1 Pseudo-Boolean Optimization

The *Pseudo-Boolean optimization* (PBO) problem is an extension of the *Boolean satisfiability* (SAT) problem. Both problems are defined as follows:

Definition 5. The *Boolean satisfiability problem* determines an assignment to the variables of a Boolean function $\Phi : \{0,1\}^n \to \{0,1\}$ such that Φ evaluates to 1 or proves that no such assignment exists. The function Φ is thereby given in *Conjunctive Normal Form* (CNF). Each CNF is a conjunction of clauses where each clause is a disjunction of literals and each literal is a propositional variable or its negation.

Definition 6. The *pseudo-Boolean optimization problem* determines a satisfying solution for a pseudo-Boolean function $\Psi : \{0,1\}^n \to \{0,1\}$ which – at the same time – minimizes an objective function $\mathcal{F}$. The *pseudo-Boolean function* Ψ is thereby a conjunction of constraints defined by $\sum_{i=1}^{n} c_i \dot{x}_i \geq c_n$, where $c_1 \ldots, c_n \in \mathbb{Z}$ and $\dot{x}_i$ either is a positive or a negative literal. The *objective function* $\mathcal{F}$ is defined by $\mathcal{F}(x_1, \ldots, x_n) = \sum_{i=1}^{n} m_i \dot{x}_i$ with $m_1, \ldots, m_n \in \mathbb{Z}$.

Example 4. Let $\Phi = (x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_3)(\overline{x}_2 + x_3)$. Then, $x_1 = 1, x_2 = 1$, and $x_3 = 1$ is a satisfying assignment solving the SAT problem.

Accordingly, let $\Psi = (2x_1 + 3x_2 + \overline{x}_3 \geq 3)(2x_1 + x_2 \geq 2)$ and $\mathcal{F} = x_1 + x_2 + x_3$. Then, $x_1 = 1, x_2 = 0$, and $x_3 = 0$ is a solution to the PBO problem, satisfying Ψ and, at the same time, minimizing $\mathcal{F}$.

Note that the correspondingly needed representation, i.e. CNF clauses as well as the PBO constraints, can easily be derived from any Boolean function in linear time (see e.g. Refs. 25 and 9, respectively). Hence, for sake of clarity we provide the following formulations in general pseudo-Boolean algebra.

4.2 Symbolic Formulation of the Fused Map

Instead of naively enumerating and checking all possible combinations, we are formulating the question what combination of rules leads to the best possible information as a PBO problem. For this purpose, we first symbolically represent all possible states of the fused map, i.e. all possible mass assignments to the respective cells, as follows:

Definition 7. Consider an $n \times n$-structure representing a fused grid map $m(Y)$ to be generated[†]. Then, all possible states of this map are symbolically represented by four-valued variables $c_{(x,y)}$ with $0 \leq x, y \leq n-1$ where

- $c_{(x,y)} = e$ represents that cell (x, y) is assumed to be empty (i.e. $m(Y_{(x,y)} = e) = 1$),
- $c_{(x,y)} = o$ represents that the cell (x, y) is assumed to be occupied (i.e. $m(Y_{(x,y)} = o) = 1$),

[†]For brevity, in the remainder of this paper, an $n \times n$-map is assumed. However, the proposed formulation can accordingly be extended to arbitrary $w \times h$-maps.

4	e	e	e	e	o
3	e	?	e	e	o
2	e	e	e	e	o
1	o	o	?	o	o
0	e	e	e	e	?
	0	1	2	3	4

(a) W/o further rules

4	e	e	e	e	o
3	e	e	e	e	o
2	e	e	e	e	o
1	o	o	o	o	o
0	e	e	e	e	o
	0	1	2	3	4

(b) W/ further rules

Figure 2: Solutions obtained by the PBO solver

- $c_{(x,y)} = \Theta$ represents that the state of the cell (x, y) is unknown (i.e. $m(Y_{(x,y)} = \Theta) = 1$), and
- $c_{(x,y)} = \emptyset$ represents that the state of the cell (x, y) is conflicting (i.e. $m(Y_{(x,y)} = \emptyset) = 1$).

Since this representation eventually is to be passed to a PBO solver – which usually accepts Boolean input variables only – we apply a four-valued formulation. That is, each variable $c_{(x,y)}$ is actually represented by two Boolean variables $c^1_{(x,y)}$ and $c^0_{(x,y)}$. All constraints proposed in the following are formulated accordingly.

The resulting set of $c_{(x,y)}$-variables constitutes an initial PBO instance which is entirely composed of unbounded variables and represents all possible states of the considered map. Next, this symbolic representation is restricted based on the available information. In fact, some assignments are already pre-defined by the fusion process. This is incorporated by employing constraints

$$c_{(x,y)} = e \tag{9}$$

or

$$c_{(x,y)} = o \tag{10}$$

for each cell (x, y) whose information is already certain (i.e. which has mass $m_{1 \circledcirc 2}(e) = 1$ or mass $m_{1 \circledcirc 2}(o) = 1$, respectively). All other variables remain unrestricted for now. This results in a PBO instance which symbolically represents all possible instances of the map left to be considered.

Example 5. Consider again the maps from Fig. 1. Applying the PBO formulation from above results in a symbolic representation as sketched in Fig. 2a. A "?" denotes cells whose information from the fusion process is considered uncertain[‡]. Certain information on these cells are now supposed to be determined by the PBO solver based on the further rules discussed in Section 3.

4.3 Incorporation of the Rules

Simply passing the PBO instance formulated above to a corresponding solving engine would lead to an arbitrary assignment to the remaining $c_{(x,y)}$-variables representing the state of the cells denoted by "?". Since we are not interested in arbitrary assignments, we employ a rule into the PBO formulation which, by default, sets all cells to their respective uncertainty value. More precisely, for each cell (x, y), either a constraint

$$r^{default}_{(x,y)} := c_{(x,y)} = \Theta \tag{11}$$

or a constraint

$$r^{default}_{(x,y)} := c_{(x,y)} = \emptyset \tag{12}$$

is added to the instance. Which constraint is chosen for a cell (x, y) depends on whether $m_{1 \circledcirc 2}$ has mass on Θ or $\emptyset$, respectively.

[‡]Due to the artificial nature of the considered example (aimed for illustrating all issues in a simple fashion), the amount of uncertainty is relatively small. Usually, maps include significantly larger uncertain information.

Note that $c_{(x,y)} = \Theta$ or $c_{(x,y)} = \emptyset$ are not explicitly enforced, but realized as a rule to be activated/deactivated during the search process of the PBO solver and associated with a very low weight.

Much more important are rules which are based on the background and expert knowledge, e.g. the "empty subgrid"-rule as well as the "wall"-rule as discussed in Section 3. They can be formulated as

$$r^{empty}_{(x,y)} := \left(\sum_{x' \in X'} \sum_{y' \in Y'} \left(c_{(x',y')} = e\right) = 6 \right) \Rightarrow \left(c_{(x,y)} = e\right), \tag{13}$$

where $X' = \{x-1, x+1\}$ and $Y' = \{y-1, y+1\}$, as well as

$$\begin{aligned} r^{wall}_{(x,y)} := & \left(\sum_{y'=0}^{n-1} \left(c_{(x,y')} = o\right) = n-1 \right) \\ & \vee \left(\sum_{x'=0}^{n-1} \left(c_{(x',y)} = o\right) = n-1 \right) \Rightarrow \left(c_{(x,y)} = o\right). \end{aligned} \tag{14}$$

In other words, Eq. 13 states that for cell (x, y) the mass is assigned e if at least 6 of its neighbors are assumed to be empty[§]. Similarly, Eq. 14 states that for cell (x, y) the mass is assigned o if all remaining cells in the same row or column are assumed to be occupied.

Eventually rules like this (as well as other ones derived from further background and expert knowledge) can be applied to each cell (x, y) which eventually defines the total set of rules R to be considered, i.e.

$$R = \bigcup_{x=0}^{n-1} \bigcup_{y=0}^{n-1} \{r^{default}_{(x,y)}, r^{empty}_{(x,y)}, r^{wall}_{(x,y)}\}. \tag{15}$$

But obviously not all of them can be enforced at the same time (as discussed several times above). Hence, additional constraints have to be added to the PBO formulation which enables the respective reasoning engine to activate or deactivate respective rules. For this purpose, further variables are introduced.

Definition 8. For each rule $r \in R$, a further Boolean variable a_r is added to the PBO instance. This variable is supposed to enforce rule r in case $a_r = 1$ (i.e. activate the rule) and, otherwise, to ignore rule r (i.e. deactivate the rule). This is ensured by adding the following constraint to the PBO formulation:

$$\bigwedge_{r \in R} a_r \Rightarrow r \tag{16}$$

Based on that, the PBO solver can determine various satisfying assignments to all variables. Since, the PBO solver could deactivate all the rules discussed above, this again would lead to an arbitrary assignment as discussed in Example 4.2. Hence, we finally employ an optimization function, namely

$$\mathcal{F} = \max : \sum_{r \in R} a_r \cdot w_r, \tag{17}$$

which guarantees the best possible assignment with respect to the given weights. In other words, the PBO solver does not only determine an arbitrary assignment to all variables, but an assignment which leads to the best combination of the rules according to their respective weights.

Example 6. Consider again the maps from Fig. 1 and the symbolic representation as sketched in Fig. 2a. Furthermore, it is assumed e.g. by an expert that the "wall"-rule has a higher priority than the "empty grid"-rule. The "default"-rule has the lowest priority. Employing the proposed PBO formulations together with the optimization function eventually leads to an optimal satisfying solution from which the map shown in Fig. 2b can be derived. Here, all the uncertainties are removed. More precisely:

[§]Note that, if cell (x, y) is in a corner or an edge of the grid, the value 6 is adjusted accordingly.

- Cell $(1,3)$ is assumed to be empty due to rule $r^{empty}_{(1,3)}$.
- For cell $(2,1)$, two rules can be applied, namely $r^{wall}_{(2,1)}$ and $r^{empty}_{(2,1)}$. However, since rule $r^{wall}_{(2,1)}$ is assumed to have a higher priority (and, hence, is associated with a higher weight), this cell is eventually assumed to be occupied.
- Finally, the conflict in cell $(4,0)$ is resolved since rule $r^{wall}_{(4,0)}$ has a higher priority than the rule $r^{default}_{(4,0)}$.

Results like this are expected to be obtained in a significantly more efficient fashion by PBO solvers than e.g. by simple enumeration. This provides the potential to overcome the drawbacks of existing solutions as discussed in Section 3.

5. CONCLUSION

In this work, we considered the process of information fusion in order to reduce the uncertainty in occupancy grid maps. It has been observed that the full potential of the available information is usually not exploited by existing approaches, since utilizing all background and expert knowledge eventually results in a computationally complex task. In order to overcome this drawback, we envisioned the exploitation of formal methods to address this issue. As a representative, solvers for PBO problems have been considered in detail and were applied to belief function based grid maps.

The resulting PBO formulation provides a promising direction how the considered problems can efficiently been solved in the future. However, investigations on the formalization of further rules (derived from background and expert knowledge) as well as an explicit evaluation on relevant problem instances are still missing. Those endeavors are left for future work. Nevertheless, preliminary feasibility studies already led to promising results, which motivate a deeper consideration of the application of formal methods for information fusion.

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