16-385 Computer Vision, Spring 2020

Take-home Quiz 1

Due Date: Monday February 3, 2020 23:59

1 Question 1

The continuous convolution of two functions $f(x)$ and $g(x)$ is given by

$$(f * g)(x) = \int_{-\infty}^{+\infty} f(y)\, g(x-y)\, \mathrm{d}y. \tag{1}$$

The Gaussian function at scale s is defined as

$$G_s(x) = \frac{1}{\sqrt{2\pi s}} \exp\left(-\frac{x^2}{2s}\right), \tag{2}$$

and has the property that

$$\int_{-\infty}^{+\infty} G_s(x)\, \mathrm{d}x = 1. \tag{3}$$

Prove that this class of functions satisfies the *semigroup property*: the convolution of one Gaussian with another produces a third Gaussian with scale equal to their sum, or

$$(G_{s_1} * G_{s_2})(x) = G_{s_1+s_2}(x). \tag{4}$$

2 Question 2

In class we derived a finite-difference approximation to the derivative of the univariate function $f(x)$ by considering the Taylor polynomial approximations of $f(x+h)$ and $f(x-h)$. We showed that

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2),$$

so that the derivative can be approximated by convolving a discrete version of $f(x)$—a vector of values $(\ldots, f(x_o - \Delta),\ f(x_o),\ f(x_o + \Delta), \ldots)$—with kernel $(1/2,\ 0,\ -1/2)$. This is termed a *central difference* because its interval is symmetric about a sample point.

1. Derive a higher order central-difference approximation to $f'(x)$ such that the truncation error tends to zero as h^4 instead of h^2. *Hint*: consider Taylor polynomial approximations of $f(x \pm 2h)$ in addition to $f(x \pm h)$.

2. What is the corresponding convolution (not correlation!) kernel?

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in this assignment. Please type your answers to theory questions.

4. **Submission:** Your submission for this assignment should be a zip file, `<andrew-id.zip>`, composed of your write-up.

   Your final upload should have the files arranged in this layout:

   `<AndrewID>.zip`

   - `<AndrewId>`
     - `<AndrewId>.pdf`