16-385 Computer Vision, Spring 2020

Take-home Quiz 10

Due Date: Sunday April 19, 2020 23:59

Question 1

Consider a simple convolutional neural network with one convolutional layer. Which of the following statements is true about this network?

1. It is scale invariant.
2. It is rotation invariant.
3. It is translation invariant.

Justify your answers.

Question 2

You are given a binary classification problem, for which you train a neural network whose final two layers are two non-linear activation functions: first a rectified linear unit (ReLU),

$$f(x) = \max(0, x), \tag{1}$$

and then a sigmoid function,

$$\tanh(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}}. \tag{2}$$

We assume that your test set consists of 50% samples belonging to the class with label +1, and another 50% belonging to the class with label −1. What will the classification accuracy of your network be?

Question 3

As we saw in class, the *brightness constancy* equation

$$I_x(x, y)u(x, y) + I_y(x, y)v(x, y) + I_t(x, y) = 0, \tag{3}$$

does not provide sufficient constraints for uniquely determining the optical flow $(u(x, y), v(x, y))$ at each pixel location (x, y) of an image. A standard way to deal with this ambiguity, first proposed by Lukas and Kanade in 1981, is to assume that, for all pixels $(x, y) \in W$ within a patch W of size $N \times N$, the optical flow is constant:

$$(u(x, y), v(x, y)) = (u_W, v_W) \equiv \mathbf{V}_W,\ \forall (x, y) \in W. \tag{4}$$

We will call this the *constant flow* assumption.

1. Show that the constant flow assumption can be equivalently expressed in the form of a heterogeneous linear system of the form $\mathbf{A}\mathbf{V}_W = \mathbf{b}$, and derive expressions for the matrix $\mathbf{A}$ and the vector $\mathbf{b}$.

2. Show that solving this heterogeneous linear system in the least-squares sense requires inverting the *covariance matrix*:

$$\mathcal{M}_W = \begin{bmatrix} \sum\limits_{(x,y)\in W} I_x(x, y) I_x(x, y) & \sum\limits_{(x,y)\in W} I_x(x, y) I_y(x, y) \\ \sum\limits_{(x,y)\in W} I_y(x, y) I_x(x, y) & \sum\limits_{(x,y)\in W} I_y(x, y) I_y(x, y) \end{bmatrix}. \tag{5}$$

3. What are the implications of the above about the patches W in an image for which we can reliably estimate the optical flow $\mathbf{V}_W$? *Hint:* Think about where else in class we have seen the covariance matrix $\mathcal{M}_W$, and about the properties this matrix needs to have in order for the system $\mathbf{A}\mathbf{V}_W = \mathbf{b}$ to have a stable solution.

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should be typese in LaTeXand should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in quizzes.

4. **Submission:** Your submission for this assignment should be a PDF file, `<andrew-id.pdf>`, composed of your write-up. **Please do not submit ZIP files.**