16-385 Computer Vision, Spring 2020

Take-home Quiz 2

Due Date: Monday February 10, 2020 23:59

1 Question 1

In class, we discussed how, given a windowing function $w(s,t)$, we can use the following *covariance* metric:

$$E_w(u,v;x,y) = \sum_{s,t} w(s,t)\left[I(x-s+u, y-t+v) - I(x-s, y-t)\right]^2, \tag{1}$$

in order to identify whether an image patch centered at (x,y) looks like a corner. In particular, large values of E_w for all possible displacements (u,v) of the window indicate that the patch is a corner.

1. Assuming that the displacements u and v are small, show that the metric of Equation (1) can be approximated as:

$$E_w(u,v;x,y) \approx [u,v] \cdot \mathcal{M}_w(x,y) \cdot [u,v]^T, \tag{2}$$

where $\mathcal{M}_w(x,y)$ is the *covariance matrix*:

$$\mathcal{M}_w(x,y) = \begin{bmatrix} \sum_{s,t} w(s,t) I_x(x-s,y-t) I_x(x-s,y-t) & \sum_{s,t} w(s,t) I_x(x-s,y-t) I_y(x-s,y-t) \\ \sum_{s,t} w(s,t) I_y(x-s,y-t) I_x(x-s,y-t) & \sum_{s,t} w(s,t) I_y(x-s,y-t) I_y(x-s,y-t) \end{bmatrix}. \tag{3}$$

2. Show that the covariance matrix can be writen equivalently as:

$$\mathcal{M}_w(x,y) = w(x,y) * \begin{bmatrix} I_x(x,y)I_x(x,y) & I_x(x,y)I_y(x,y) \\ I_y(x,y)I_x(x,y) & I_y(x,y)I_y(x,y) \end{bmatrix}, \tag{4}$$

where $*$ indicates convolution of the windowing functio $w(x,y)$ with each element of the matrix.

3. As we discussed a class, we can derive various "cornerness" metrics that take the form of functionals of *only* the product and sum of the eigenvalues of the covariance matrix. Pick your favorite one (or propose your own), and explain how you would compute this metric efficiently for the entire image, using only convolutions and element-wise operations between images. You can explain this either verbally, or using pseudocode.

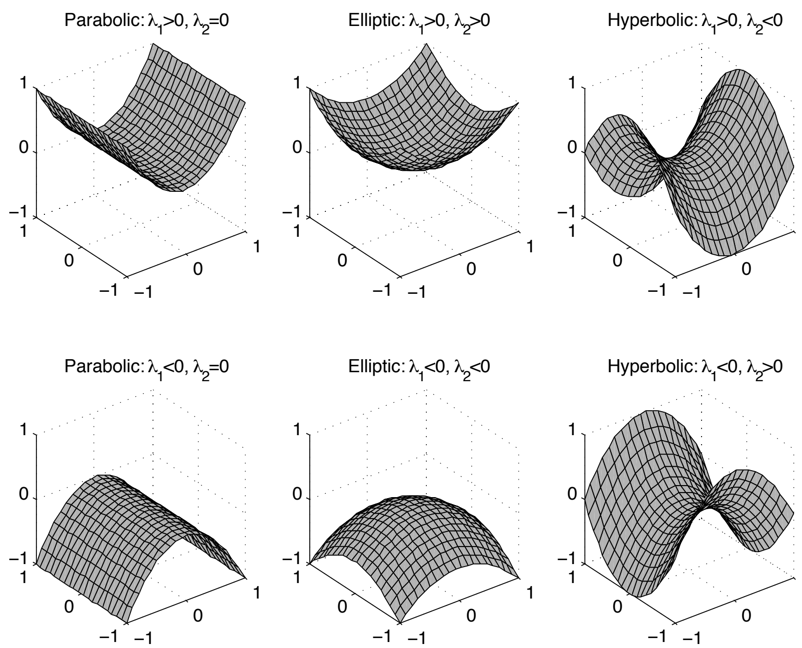

Figure 1: Eigenvalues (λ_1, λ_2) of the Hessian correspond to the principal curvatures of a surface and can be used to classify different surface types.

2 Question 2

In class we discussed a measure of "cornerness" based on the covariance matrix $\mathcal{M}_w$. An alternative approach to corner detection is based on another matrix, the *Hessian*:

$$\mathcal{H}(x,y) = \begin{bmatrix} I_{xx}(x,y) & I_{xy}(x,y) \\ I_{xy}(x,y) & I_{yy}(x,y) \end{bmatrix}. \tag{5}$$

Like the covariance matrix, the Hessian is real and symmetric and therefore has two real (*but not necessarily positive*) eigenvalues. The eigenvalues and eigenvectors of the Hessian have a different geometric interpretation, however, and this interpretation is depicted in Fig. 1. If we interpret the intensity function $I(x,y)$ as a surface, then the eigenvalues of $\mathcal{H}(x,y)$ correspond to the *principal curvatures* of the surface at the point (x,y), meaning the values of maximum and minimum curvature at that point. The eigenvectors are orthogonal and correspond to the directions of maximum and minimum curvature. Finally, the product of principal curvatures is the *Gaussian curvature*, and their average is the *mean curvature*. Figure 1 shows a visualization of these curvatures.

1. Explain what characteristics the eigenvalues of the Hessian matrix of a patch should have for the patch to be a corner. Draw a visualization similar to that of slide 72 of lecture 5, this time using the eigenvalues of the Hessian matrix instead of those of the covariance matrix.

2. Propose one-two simple metrics that use functionals of the Gaussian and mean curvature of the Hessian matrix to measure the “cornerness” of an image patch. How do these compare to the cornerness metrics we derived based on the covariance matrix?

3. Explain how you would compute these metrics efficiently for the entire image, using only convolutions and element-wise operations between images. You can explain this either verbally, or using pseudocode. How does this computation compare to the one you did for the cornerness metrics based on the covariance matrix in Question 1?

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in this assignment. Please type your answers to theory questions.

4. **Submission:** Your submission for this assignment should be a zip file, `<andrew-id.zip>`, composed of your write-up.

   Your final upload should have the files arranged in this layout:

   `<AndrewID>.zip`

   - `<AndrewId>`
     - `<AndrewId>.pdf`