16-385 Computer Vision, Spring 2020

Take-home Quiz 3

Due Date: Monday February 17, 2020 23:59

1 Question 1

Given a set of N points $\mathbf{p}_i = (x_i, y_i)$, $i = 1, \ldots, N$, in the image plane, we wish to find the best line passing through those points.

1. One way to solve this problem is to find (a, b) that most closely satisfy the equations $y_i = ax_i + b$, in a least-squares sense. Write these equations in the form of a *heterogeneous* least-squares problem $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{x} = (a,\ b)^\mathsf{T}$, and give an expression for the least-squares estimate of $\mathbf{x}$. Give a geometrical interpretation of the error being minimized, and use a simple graph to visualize the error. Does this make sense when fitting a line to points in an image?

2. Another way to solve the problem is to find $\boldsymbol{\ell} = (a,\ b,\ c)$ (defined up to scale) that most closely satisfies the equations $ax_i + by_i + c = 0$, in a least-squares sense. Write these equations in the form of a *homogeneous* least-squares problem $\mathbf{A}\boldsymbol{\ell} = \mathbf{0}$ where $\boldsymbol{\ell} = (a,\ b,\ c)^\mathsf{T}$ and $\boldsymbol{\ell} \neq \mathbf{0}$. This problem has a trivial solution (zero vector $\mathbf{0}$), which is not of much use. Describe some ways to avoid this trivial solution, and corresponding algorithms for solving the resulting optimization problem. Is this approach more or less useful than the previous approach in (1)? Why? (*Hint: Think about how we can express the distance of a point from a line.*)

3. We are now given N lines $\boldsymbol{\ell}_i = (a_i,\ b_i,\ c_i)$, $i = 1, \ldots, N$. These lines do not intersect at a point, but we can find a point that comes 'closest' to lying at their common intersection. Think of two ways to do this, one resulting in a heterogeneous and another in a homogeneous least-squares problem. Which of the two makes more sense geometrically (e.g., from the point of view of what error each approach is minimizing)?

2 Question 2

The equation for a conic in the plane using inhomogeneous coordinates (x, y) is

$$ax^2 + bxy + cy^2 + dx + ey + f = 0. \tag{1}$$

1. Suppose you are given a set of inhomogeneous points $\tilde{\mathbf{x}}_i = (x_i, y_i)$, $i = 1, \ldots, N$. Derive an expression for the least squares estimate of the conic $\mathbf{c} = (a, b, c, d, e, f)$ passing through those points. (Your expression may take the form of a null vector or eigenvector of a matrix; if so, you must provide expressions for the matrix elements.)

2. In general, what is the minimum value of N that allows a unique solution for $\mathbf{c}$?

3. "Homogenize" Eq. 1 by making the substitutions $x \to x_1/x_3$, $y \to x_2/x_3$, and show that in terms of homogeneous coordinates ($\mathbf{x} = (x_1, x_2, x_3)$) the conic can be expressed in matrix form,

$$\mathbf{x}^\top \mathbf{C} \mathbf{x} = 0,$$

with a symmetric matrix $\mathbf{C}$.

4. Suppose we apply a projective transformation to our points: $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$. The transformed points $\mathbf{x}'_i$ will lie on a transformed conic represented by a new symmetric matrix $\mathbf{C}'$. Write an equation that specifies the relationship between $\mathbf{C}'$ and $\mathbf{C}$, in terms of the homography $\mathbf{H}$.

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in this assignment. Please type your answers to theory questions.

4. **Submission:** Your submission for this assignment should be a zip file, `<andrew-id.zip>`, composed of your write-up.

   Your final upload should have the files arranged in this layout:

   `<AndrewID>.zip`

   - `<AndrewId>`
     - `<AndrewId>.pdf`