16-385 Computer Vision, Spring 2020

Take-home Quiz 4

Due Date: Sunday February 23, 2020 23:59

1 Question 1

As we discussed in class, homographies can be used, under certain conditions, to relate images of the same scene captured by different cameras.

1. Prove that there exists a homography $\mathbf{H}$ that satisfies

$$\mathbf{x}_1 \equiv \mathbf{H} \cdot \mathbf{x}_2, \tag{1}$$

   between the 2D points (in homogeneous coordinates) $\mathbf{x}_1$ and $\mathbf{x}_2$ in the images of a *plane* Π captured by two 3×4 camera projection matrices $\mathbf{P_1}$ and $\mathbf{P_2}$, respectively. The $\equiv$ symbol stands for equality *up to scale*. (*Note: A degenerate case happens when the plane* Π *contains both cameras' centers, in which case there are infinite choices of* $\mathbf{H}$ *satisfying Equation* (1). *You can ignore this special case in your answer.*)

2. Prove that there exists a homography $\mathbf{H}$ that satisfies Equation (1), given two cameras separated by a pure rotation. That is, for camera 1, $\mathbf{x_1} = \mathbf{K_1} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X}$, and for camera 2, $\mathbf{x_2} = \mathbf{K_2} \begin{bmatrix} \mathbf{R} & \mathbf{0} \end{bmatrix} \mathbf{X}$. Note that $\mathbf{K_1}$ and $\mathbf{K_2}$ are the 3×3 intrinsic matrices of the two cameras and are different. $\mathbf{I}$ is 3×3 identity matrix, $\mathbf{0}$ is a 3×1 zero vector and $\mathbf{X}$ is a point in 3D space. $\mathbf{R}$ is the 3×3 rotation matrix of the camera.

3. Suppose that a camera is rotating about its center $\mathbf{C}$, keeping the intrinsic parameters $\mathbf{K}$ constant. Let $\mathbf{H}$ be the homography that maps the view from one camera orientation to the view at a second orientation. Let θ be the angle of rotation between the two. Show that $\mathbf{H^2}$ is the homography corresponding to a rotation of 2θ.

4. Let I_0 be an image captured by a camera, and I_1 be an image of I_0 captured by another camera (an image of an image). Let the composite image be denoted I'. Show that the apparent camera center of I' is the same as that of I_0. Speculate on how this explains why a portrait's eyes "follow you around the room." (*Hint: The null space of an* $n \times n$ *invertible matrix* $\mathbf{A}$ *is empty, i.e.,* $\mathbf{Ax} = 0$ *if and only if* $\mathbf{x} = 0$.)

2 Question 2

In class we saw that a camera matrix satisfies the equation $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$, and that six 3D-2D matches $\mathbf{x} \leftrightarrow \mathbf{X}$ are sufficient to recover $\mathbf{P}$ using a linear (non-iterative) algorithm.

1. Find linear algorithms for computing the camera matrix **P** in the special cases when: i) the camera location (but not orientation) is known, and ii) the camera location and complete orientation are known.

2. Ignoring degenerate configurations, how many 2D-3D matches are required for there to be a unique solution in each case? Justify your answers.

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in this assignment. Please type your answers to theory questions.

4. **Submission:** Your submission for this assignment should be a zip file, `<andrew-id.zip>`, composed of your write-up.

   Your final upload should have the files arranged in this layout:

   `<AndrewID>.zip`

   - `<AndrewId>`
     - `<AndrewId>.pdf`