Take-home Quiz 5

Due Date: Sunday March 1, 2020 23:59

1 Question 1

As we discussed in class, two cameras are said to form a *rectified pair* if their camera coordinate systems differ only by a translation of their origins (the camera centers) along a direction that is parallel to either the x or y axis of the coordinate systems.

1. Prove that the epipolar lines of a rectified pair are parallel to the axis of translation.

2. Derive an expression for the essential matrix $\mathbf{E}$ of the rectified pair.

2 Question 2

Suppose two cameras fixate on a point P (see Figure 1) in space such that their optical axes intersect at that point. Show that if the image coordinates are normalized so that the coordinate system origin $(0,0)$ coincides with the principal point, the $\mathbf{F}_{33}$ element of the fundamental matrix $\mathbf{F}$ is zero.
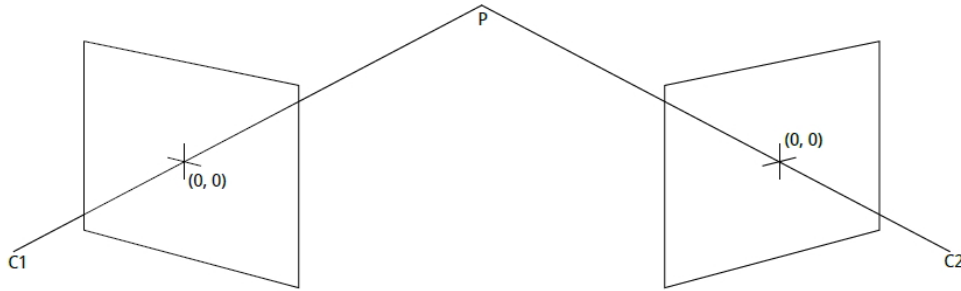

Figure 1: $C1$ and $C2$ are the optical centers. The principal axes intersect at point P.

3 Question 3

Consider three images I_1, I_2 and I_3 that have been captured by a system of three cameras, and suppose the fundamental matrices $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are known. (Notation: the matrix $\mathbf{F}_{ij}$ satisfies the equation $\mathbf{x}_j^\top \mathbf{F}_{ij} \mathbf{x}_i = 0$ for any correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}_j$ between images I_i and I_j.) In general, given a point $\mathbf{x}_1$ in I_1 and a corresponding point $\mathbf{x}_2$ in I_2, the corresponding point in $\mathbf{x}_3$ in I_3 is uniquely determined by the fundamental matrices $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$.

1. Write an expression for $\mathbf{x}_3$ in terms of $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$.

2. Describe a degenerate configuration of three cameras for which the point $\mathbf{x}_3$ cannot be uniquely determined by this expression.

Hint: Consider the epipolar geometry of the situation. Draw a picture!

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should be typese in LaTeXand should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in quizzes.

4. **Submission:** Your submission for this assignment should be a PDF file, `<andrew-id.pdf>`, composed of your write-up. **Please do not submit ZIP files.** (This is new in this quiz.)