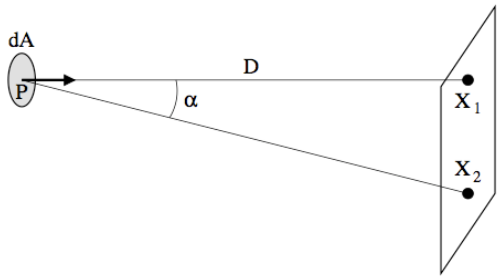
16-385 Computer Vision, Spring 2020

Take-home Quiz 6

Due Date: Sunday March 15, 2020 23:59

Question 1

A small Lambertian source dA is centered at P and emits radiance L. The orientation of this patch is the same as that of a plane containing two points, X_1 and X_2. The point X_1 is the point on this plane that is closest to P, and the distance from P to X_1 is D as shown.



1. Calculate the solid angle subtended by dA at points X_1 and X_2.
2. Calculate the irradiance E incident on the plane at points X_1 and X_2, and calculate the ratio $E(X_1)/E(X_2)$.

Question 2

As we discussed in class, the Lambertian and the specular BRDF are the two most commonly assumed reflectance models in physics-based vision.

1. For Lambertian surfaces, the BRDF is a constant function of the input and output directions. For such a material, we often describe the reflectance in terms of its *albedo*, which is given the symbol ρ. For a Lambertian surface, the BRDF and albedo are related by $f_r(\hat{\mathbf{v}}_i, \hat{\mathbf{v}}_o) = \rho/\pi$. Using conservation of energy, prove that $0 \leq \rho \leq 1$.
2. A specular surface perfectly reflects *radiance* in the *mirror direction*. Concretely, cosider a (non-absorbing) specular surface patch with normal $\hat{\mathbf{n}}$. For any incident direction $\hat{\mathbf{v}}_i$, the mirror direction equals $\hat{\mathbf{v}}_s = 2(\hat{\mathbf{n}}^\top \hat{\mathbf{v}}_i)\hat{\mathbf{n}} - \hat{\mathbf{v}}_i$, and $L(\hat{\mathbf{v}}_s) = L(\hat{\mathbf{v}}_i)$. Given this property, derive an expression for the specular BRDF.

Question 3

We observe a Lambertian cube with uniform albedo under unit-strength, directional illumination from an unknown direction $\hat{\mathbf{s}}$. The cube is viewed in such a way that three of its faces are visible, with known normal vectors $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$, and $\hat{\mathbf{n}}_3$. Show that the cube's albedo ρ and the illumination direction $\hat{\mathbf{s}}$ can be recovered from the observed radiance (I_1, I_2, and I_3) emitted from the three faces. Write expressions for ρ and $\hat{\mathbf{s}}$ in terms of $\{\hat{\mathbf{n}}_i\}$ and $\{I_i\}$.

Question 4

Consider a surface patch with BRDF

$$f_r(\hat{\mathbf{s}}, \hat{\mathbf{v}}, \hat{\mathbf{n}}) = \frac{1}{\sqrt{\hat{\mathbf{n}}^\top \hat{\mathbf{s}}} \sqrt{\hat{\mathbf{n}}^\top \hat{\mathbf{v}}}},$$

where $\hat{\mathbf{n}}$, $\hat{\mathbf{v}}$, and $\hat{\mathbf{s}}$ are the surface normal, view, and source directions, respectively. Suppose we view such a patch from a known direction $\hat{\mathbf{v}}$, and we capture two radiance measurements E_1 and E_2 under unit-strength distant lighting from known directions $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$.

1. Write expressions for the measurements E_1 and E_2 in terms of the normal, view, and source directions.

2. Show that we can recover the surface normal from these two measurements.

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should be typese in LaTeXand should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in quizzes.

4. **Submission:** Your submission for this assignment should be a PDF file, `<andrew-id.pdf>`, composed of your write-up. **Please do not submit ZIP files.** (This is new in this quiz.)