16-385 Computer Vision, Spring 2020

Take-home Quiz 7

Due Date: Sunday March 29, 2020 23:59

Question 1

A reasonable reflectance model for dielectric (non-conducting) surfaces is the so-called dichromatic model, according to which the spectral BRDF is written as a linear combination of a Lambertian diffuse component f_d and a wavelength-independent specular component f_s:

$$f(\lambda, \hat{\omega}_i, \hat{\omega}_o) = f_d(\lambda) + f_s(\hat{\omega}_i, \hat{\omega}_o).$$

When we image a scene consisting of such surfaces under a directional light source with spectral power distribution $I(\lambda)$ and direction $\hat{l} = (l_x, l_y, l_z)$, the RGB values $\vec{C}(\vec{u}) = (C_R(\vec{u}), C_G(\vec{u}), C_B(\vec{u}))$ recorded at pixel $\vec{u} = (u, v)$ can be expressed as

$$\vec{C}(\vec{u}) = \langle \hat{n}(\vec{u}), \hat{l} \rangle \vec{d}(\vec{u}) + g_s(\vec{u})\vec{s},$$

where $\langle \cdot, \cdot \rangle$ is the inner product operation, $\hat{n}(\vec{u})$ is the surface normal at the scene point imaged by pixel $\vec{u}$, and $g_s(\vec{u})$ is a function that depends non-linearly on $\hat{n}(\vec{u})$ (as well as view and lighting directions) through the specular component of the BRDF.

1. Assuming that the spectral sensitivities of a camera's three filters are $(c_R(\lambda), c_G(\lambda), c_B(\lambda))$ and that the BRDF at the surface point imaged at pixel $\vec{u}$ is $f(\lambda, \hat{\omega}_i(\vec{u}), \hat{\omega}_o(\vec{u})) = f_d(\lambda, \vec{u}) + f_s(\hat{\omega}_i(\vec{u}), \hat{\omega}_o(\vec{u}))$, write expressions for the elements of the *diffuse color vectors* $\vec{d}(\vec{u})$ and the *source color vector* $\vec{s}$.

2. Suppose you are given two unit-length three-vectors $\hat{r}_1$ and $\hat{r}_2$ that are orthogonal to $\vec{s}$. Show that the two-channel image given by the per-pixel inner products $\vec{J}(\vec{u}) = (\langle \hat{r}_1, \vec{C}(\vec{u}) \rangle, \langle \hat{r}_2, \vec{C}(\vec{u}) \rangle)$:

   (a) does not depend on the specular components of the BRDFs, $f_s(\hat{\omega}_i(\vec{u}), \hat{\omega}_o(\vec{u}))$.

   (b) depends linearly on the surface normals, $\hat{n}(\vec{u})$.

3. Show that the two properties from part (b) are also satisfied by the single-channel (grayscale) image $J(\vec{u}) = ||\vec{J}(\vec{u})||$.

4. Figure 1 shows the results of computing the grayscale image $J(\vec{u})$ from some original RGB image. Explain why $J(\vec{u})$ might be more useful than the original RGB image to a computer vision system.

Figure 1: Left: Original RGB image. Right: Grayscale image produced with the procedure of Question 1.

Question 2

In 1941, Marcel Minnaert introduced one of the first analytic expressions for a non-Lambertian BRDF. His motivation was to allow a better interpretation of observations of the brightness of the moon. Minnaert's BRDF model has two parameters, ρ and k, with the latter (k) taking a value between 0 and 2. Using $\boldsymbol{n}$ to represent a unit-length surface normal vector, the Minnaert BRDF model can be written:

$$f(\boldsymbol{\omega_i}, \boldsymbol{\omega_o}) = \rho \left((\boldsymbol{n}^\top \boldsymbol{\omega_i})(\boldsymbol{n}^\top \boldsymbol{\omega_o}) \right)^{k-1}.$$

1. Prove that the Lambertian BRDF model is a special case of the Minnaert BRDF model.

2. As shown on the left in the figure below, a full moon that is high in the sky is perceived as being a flat disk instead of a round ball, because its edges do not darken like those of a Lambertian sphere that is rendered with frontal view and light directions (shown right). What value(s) of $k \in [0, 2]$ would predict this moon-like behavior? Explain your answer in words, supported by any necessary equations or diagrams.

Instructions

1. **Integrity and collaboration:** Students are encouraged to work in groups but each student must submit their own work. If you work as a group, include the names of your collaborators in your write up. Plagiarism is strongly prohibited and may lead to failure of this course.

2. **Questions:** If you have any questions, please look at Piazza first. Other students may have encountered the same problem, and it may be solved already. If not, post your question on the discussion board. Teaching staff will respond as soon as possible.

3. **Write-up:** Your write-up should be typese in LaTeXand should consist of your answers to the theory questions. Please note that we **DO NOT** accept handwritten scans for your write-up in quizzes.

4. **Submission:** Your submission for this assignment should be a PDF file, `<andrew-id.pdf>`, composed of your write-up. **Please do not submit ZIP files.** (This is new in this quiz.)