Deterministic Abortable Mutual Exclusion with Sublogarithmic Adaptive RMR Complexity

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ABSTRACT

We present a deterministic abortable mutual exclusion algorithm for a cache-coherent (CC) model with read, write, Fetch-And-Add (F&A), and CAS primitives, whose RMR complexity is $O(\log_W N)$, where W is the size of the F&A registers. Under the standard assumption of $W = \Theta(\log N)$, our algorithm's RMR complexity is $O(\frac{\log N}{\log\log N})$; if $W = \Theta(N^\epsilon)$, for $0 < \epsilon < 1$ (as is the case in real multiprocessor machines), the RMR complexity is $O(1)$. Our algorithm is adaptive to the number of processes that abort. In particular, if no process aborts during a passage, its RMR cost is $O(1)$.

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1 INTRODUCTION

Mutual exclusion [9] is a fundamental problem in distributed computing. A mutual exclusion object (henceforth, *lock*) prevents simultaneous entry to a *critical section* of code, in which some shared resource is accessed. To execute the critical section, a process must first *acquire* the lock by executing an *entry section*. The lock algorithm guarantees that at any time, at most one process holds the lock. After acquiring the lock and executing the critical section, a process *releases* the lock by executing an *exit section*.

Classic locks do not allow a process waiting in the entry section to abort its lock acquisition attempt and quit the lock protocol. Several use cases, however, require this feature: (1) a process blocked on a lock may wish to abandon its work chunk and switch to working on a different work chunk not subjected to serialization [8]; (2) database systems use aborts to recover from deadlocks and to deal with preemption of a lock holding process [25]; and (3) low-priority processes can abort to expedite lock handoff to a high-priority process [8, 24]. To meet these demands, an *abortable lock* [24, 25] additionally allows a process to abort its lock acquisition attempt in a finite number of its own steps.

We investigate the *remote memory references* (RMR) complexity of abortable locks. The RMR complexity measure captures the fact that the cost of a memory reference on shared-memory multiprocessor machines is not uniform. Some references can be satisfied quickly from memory local to the processor, whereas the rest must be satisfied from remote memory. For example, in a *cache-coherent* (CC) system, each processor stores local copies of the shared variables it accesses in its cache; a cache coherence protocol maintains the consistency of the copies in the different caches. A memory access to a cached variable is local; otherwise, it is remote. In a *distributed shared-memory* (DSM) system, each shared variable is permanently locally accessible to a single processor and remote to all other processors. References to remote memory are orders of magnitude slower than local memory accesses, as they must traverse the system's interconnect, and so the performance of lock algorithms critically depends on the number of remote memory references they generate [5, 21]. The RMR complexity measure (or *RMR cost*) therefore charges an algorithm only for RMRs; local memory accesses are considered free. RMR cost has been used almost exclusively as the complexity measure in shared-memory mutual exclusion research over the last 20 years [4, 6, 7, 10–12, 14, 15, 17, 23].

There is a gap between the known RMR cost of locks and of abortable locks. For an N-process system with read, write, comparison primitives [4] such as Compare-And-Swap (CAS), or LL/SC, Yang and Anderson's lock [26] and Jayanti's abortable lock [17] both have $O(\log N)$ RMR cost, which is optimal [6]. For mutual exclusion, the $\Omega(\log N)$ lower bound can be defeated by leveraging additional synchronization primitives. The MCS lock [21], for example, uses Fetch-And-Store (SWAP) in addition to the aforementioned primitives, and has RMR cost of $O(1)$. For abortable mutual exclusion, however, no algorithm with worst-case sublogarithmic RMR cost is known. Lee's abortable lock [19] leverages SWAP and Fetch-And-Add (F&A) primitives to obtain an RMR cost of $O(A^2)$, where A is the number of processes that abort. Lee's algorithm therefore incurs $O(1)$ RMRs if no process aborts, but its worst-case RMR cost is $O(N^2)$. Sublogarithmic RMR cost can also be achieved using randomization, both for locks [7, 11, 14, 15] and for abortable locks [12, 23], but this paper considers deterministic algorithms.

Our Contribution. We show that, as with mutual exclusion, abortable locks can leverage additional primitives to obtain sublogarithmic worst-case RMR cost. We present an abortable lock algorithm for a CC model with F&A in addition to read, write, and CAS primitives, whose worst-case RMR cost is $O(\log_W N)$, where W is the size of the F&A registers. Under the standard assumption of $W = \Theta(\log N)$, this time/space tradeoff implies that our algorithm's RMR cost is $O(\frac{\log N}{\log\log N})$. The RMR cost becomes $O(1)$ if $W = \Theta(N^\epsilon)$ for some $0 < \epsilon < 1$, as is the case in realistic multiprocessor systems.[1] Our algorithm is *adaptive to the number of processes that abort.* The RMR cost of a complete *passage* (entry and corresponding exit of the critical section) is $O(\log_W A_i)$, where A_i is the number of processes that abort during the passage. If no

[1] E.g., for a system with a billion processes and 64-bit memory words, $W \approx N^{1/5}$.

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Algorithm		Model	Primitives	Space (#words)	Fairness Guarantee	RMR cost of a passage through CS		
						Worst-case	No aborts	Adaptive bound
Scott [24]		CC/DSM	SWAP, CAS	unbounded	FCFS	unbounded	$O(1)$	$O(\#A)$, where $\#A$ is the number of aborts during the execution
Jayanti [17]		CC/DSM	LL/SC or CAS	$O(N)$	FCFS	$O(\log N)$	$O(\min(k, \log N))$	$O(\min(k, \log N))$, where k is the maximum number of processes active concurrently during the passage (i.e., *point contention*)
Lee [19]		CC	F&A, SWAP	$O(N^2)$	FCFS	$O(N^2)$	$O(1)$	$O(A_i \cdot A_t)$
This Work	**One-Shot**	**CC/DSM**	**F&A**	$\boldsymbol{O(N)}$	**FCFS**	$\boldsymbol{O(\log_W N)}$	$\boldsymbol{O(1)}$	$\boldsymbol{O(\log_W A_i)}$ **for complete passage** $\boldsymbol{O(\log_W A_t)}$ **for aborted passage**
	Long-Lived	**CC**	**F&A, CAS**	$\boldsymbol{O(N^2)}$	**Starvation Freedom**			

Table 1: ***Comparison of abortable locks, showing their system model, required primitives, complexity, and fairness.***

process aborts during a passage, its RMR cost is $O(1)$. The RMR cost of an aborted passage is $O(\log_W A_t)$, where A_t is the number of processes that abort during the entire execution. (Note that our notion of adaptivity differs from that of Jayanti's algorithm [17], in which the RMR cost of a passage depends on the maximum number of processes that are concurrently active during the passage, not only on those that abort.)

We obtain our algorithm by composing two constructions. We first design a *one-shot* abortable lock, which each process can attempt to acquire at most once. This lock has the aforementioned RMR cost and linear space complexity. We next present a generic transformation that converts a one-shot abortable lock algorithm with space complexity $s(N)$, where $s(N)/2^W = O(1)$, into a long-lived algorithm with the same asymptotic RMR cost and space complexity $O(N \cdot s(N) + N^2)$. Our final algorithm thus uses $O(N^2)$ memory words. The transformation does not maintain fairness guarantees. While our one-shot algorithm is first-come-first-served (FCFS) [18], the final algorithm is starvation-free. Table 1 compares the final algorithm to prior work.

2 MODEL AND PROBLEM STATEMENT

Model. We consider an asynchronous shared-memory model, in which a set of N deterministic processes communicate by executing atomic operations on shared W-bit words that support read, write, CAS, and F&A operations. $CAS(w, o, n)$ atomically changes w's value to n if w contains o, and returns *true*; otherwise, it returns *false* without modifying w. $F\&A(w, x)$ atomically updates the value stored in w from v to $v + x$, and returns v. A *configuration* consists of the state of all processes and memory words. An *execution* is a (possibly infinite) sequence of *steps*. Each *step* consists of a process invoking an operation on a shared variable and receiving its return value, thereby moving the system to a new configuration.

RMR complexity. In a cache-coherent (CC) model, each processor maintains local copies of shared variables it accesses in its cache, and a coherence protocol ensures the consistency of cached variables by invalidating cached copies when a variable is written. A memory access to an uncached variable is called a *remote memory reference* (RMR): Each write, CAS, or F&A incurs an RMR. A read by process p of a shared variable w incurs an RMR if (1) it is p's first read of w, or (2) after p's last read of w, another process performed a write, CAS, or F&A to w. In a distributed shared-memory (DSM) model, each register is (forever) local to some processor and remote to all others. An access to a remote register is an RMR.

Abortable mutual exclusion. An *abortable lock* supports the methods $Enter$ and $Exit$. A process *attempts* to acquire the lock by executing $Enter$. If $Enter$ returns *true*, the process *acquires* the lock and *enters* the critical section (CS). While inside $Enter$, a process can receive an external signal to *abort* its attempt, in which case $Enter$ may return *false*.[2] Once a process completes the CS, it *exits* the CS by invoking $Exit$ and thereby *releases* the lock. A *passage* is the sequence of steps in which a process executes $Enter$, the CS, and $Exit$. A process not executing $Enter$, $Exit$, or the CS is said to be in the *remainder section.*

Problem statement. The goal is to design an abortable lock algorithm satisfying the following requirements: (1) *mutual exclusion:* at any time, at most one process is in the CS; (2) *starvation-freedom:* if no process crashes outside the remainder section and every process that enters the CS eventually leaves it, then if a process p invokes $Enter$ and does not abort its attempt, p eventually enters the CS in that attempt; (3) *bounded exit:* a process completes an $Exit$ call in a finite number of its own steps; and (4) *bounded abort:* if a process p busy waiting in $Enter$ receives a signal to abort its attempt, then p's execution of $Enter$ returns (either *true* or *false*) within a finite number of p's steps.

3 ONE-SHOT ALGORITHM

Here, we describe our one-shot abortable lock, which is the main building block of our algorithm. Unless noted otherwise, we assume the CC complexity model.

The one-shot lock implements an array-based queue lock [5, 13], augmented with a data structure that tracks which processes have aborted and thus given up their place in the queue. Our high-level design is similar to Jayanti's algorithm [17], with the main difference being in the augmenting data structure. Whereas Jayanti uses a linearizable priority queue [16], we use a tree-based ordered set that is *not* linearizable. The semantics of our $Tree$ data structure cannot be cleanly captured by a sequential specification, as they depend on the concurrency between operations.

Figure 1 shows the pseudo-code of the lock algorithm. Its underlying data structures are an array-based queue, *go*, and a $Tree$ whose implementation we describe in Section 4. We assume that each process attempts to perform at most one pass through the critical section. To acquire the lock, a process first increments $Tail$ using F&A to obtain an index to a slot in the *go* array. We will identify each process with its index $0 \le i \le N-1$. Process i spins

[2] Returning *false* is not mandatory because a process can receive the signal after being handed the lock, but before noticing the handoff. Formulations in which an abort signal moves the process to some $Abort$ method [17] similarly require this method to detect and handle such a scenario.

Shared Variables:
$Head = 0$; $Tail = 0$; $LastExited = -1$; $go = [1, 0, \ldots, 0]$; $Tree$ (see Section 4)

Algorithm 3.1 $Enter()$

```
i ← F&A(Tail, 1)
while ¬go[i] do
    if AbortSignal then
        Abort(i)
        return false
Head ← i
return true
```

Algorithm 3.2 $Exit()$

```
head ← Head
LastExited ← head
SignalNext(head)
```

Algorithm 3.3 $Abort(i)$

```
Tree.Remove(i)
head ← Head
if head ≠ LastExited then
    return
SignalNext(head)
```

Algorithm 3.4 $SignalNext(head)$

```
j ← Tree.FindNext(head)
if j ∈ {⊤, ⊥} then
    return
go[j] ← true
```

Figure 1: ***One-shot abortable lock algorithm***

on its slot until being signaled that it owns the lock, and then sets the queue's $Head$ to i and enters the CS. (Initially, $go[0]$ is set, so process 0 immediately enter the CS.)

When process i exits the CS, it hands the lock off to the *next slot* in the queue, which is the minimal $j > i$ such that slot j has not been abandoned by its process due to an abort. The $Tree$ data structure facilitates this handoff. It maintains the (ordered) set of queue slots that have not been abandoned by aborting processes. (Initially, $Tree = \{0, \ldots, N-1\}$.) The handoff of process i's lock ownership to the next slot is performed by $SignalNext(i)$. This procedure calls $Tree.FindNext(i)$ to find the next slot, j, in order to set $go[j]$ to *true*. $FindNext(i)$ might return $\bot$, indicating that all possible successor slots have been abandoned and thus the lock is now unusable. Finally, $FindNext(i)$ may also return $\top$, indicating that its successor search "crossed paths" with an aborting process k removing itself from $Tree$. In such a case, as described next, some aborting process will *assume responsibility* for performing the handoff on behalf of process i.

Aborts are performed by the $Abort()$ procedure, which a process busy waiting in *Enter* calls when it detects the external *AbortSignal*. An aborting process i abandons its queue slot by removing itself from $Tree$. It then reads $Head$ to obtain the id of the process in the CS, h, and compares h to the *LastExited* variable, which contains the id of the last process to release the lock. If $h = LastExited$, then process h may be in the middle of exiting the CS, and its $FindNext()$ might have crossed paths with process i's *Remove* and thus returned $\top$. Therefore, process i *assumes responsibility* for h's lock handoff and executes $SignalNext(h)$. Of course, it can then cross paths with some aborting process j and so fail to complete the handoff, but then process j would assume responsibility for the handoff. The crux of our correctness proofs is to show that eventually, some aborting process that assumes responsibility for the handoff manages to complete it.

DSM variant. In the DSM model, we cannot guarantee that the *go* slot of a process is local, since the slot is determined at run time. As a result, a process might incur an unbounded number of RMRs while busy waiting. We use indirection to address this problem, by having the process spin on a local spin bit that it publishes in an *announce* array. To synchronize with a concurrent lock handoff, process i publishes its spin bit s by writing $announce[i] = s$, and then, if $go[i] \neq 1$, spinning locally on s . A $SignalNext()$ hands the lock to process i by writing $go[i] = 1$, reading $s = announce[i]$, and if $s \neq \bot$, writing $s = 1$.

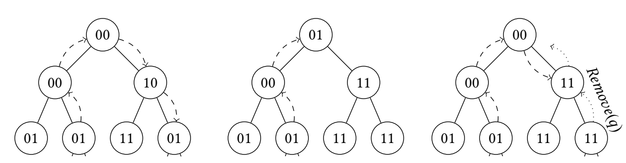
4 TREE DATA STRUCTURE

The $Tree$ data structure maintains a W-ary tree with N leafs, whose height is $H = \lceil \log_W N \rceil$. A tree node u contains a W-bit word, initially 0, in which the j-th most significant bit is associated with u's j-th child, counting from the left (so the leftmost bit in u is associated with u's leftmost child). We number the leafs from left to right starting with 0, and identify leaf p with process p (equivalently, queue slot p). $Tree$ has the property that if some queue slot in the subtree rooted at u has not been abandoned by an aborting process, then the bit associated with u in u's parent is clear. To maintain this property, an aborting process p ascends the tree starting from its leaf, performing a F&A to set the bit associated with its subtree in each visited node u. If all bits in u are then set, the process keeps ascending to u's parent; otherwise, it stops.

$FindNext(p)$ needs to find the first leaf to the right of p that has not been abandoned. To find this leaf, it simply walks up the tree until finding a clear bit to the right of p, and then walks down towards the relevant leaf. If $FindNext(p)$ does not find a zero bit during its ascent, it returns $\bot$. If it encounters a node in which all bits are set after starting to descend, then it has "crossed paths" with a $Remove()$ ascending the subtree and so returns $\top$. Figure 2 depicts these scenarios.

Figure 3 presents the pseudo-code of the algorithm. Because the tree structure is static, we do not need pointers in the nodes; parent or child nodes are computed by the processes. Leaf nodes act as static sentinels. Only the values stored in non-leaf nodes need to be stored in shared memory. The size of the tree is thus $O(\frac{N}{W})$ words. For $0 \leq j \leq W-1$, we denote the j-th child (from left to right) of node u by $Child(u, j)$; if u is a leaf, $Child(u, j) = \bot$. We denote the parent of node u by $Parent(u)$; if u is the root, $Parent(u) = \bot$. The i-th leaf is denoted $Leaf(i)$. The W bits maintained in a node are stored in a field named *value*. For leafs, *value* contains the id of the associated process (i.e., $Leaf(p).value = p$ for $0 \leq p \leq N-1$).

We use the following notation to associate bits with processes (equivalently, queue slots). The *level* of node u, denoted $Lvl(u)$, is 0 if u is a leaf; otherwise, $Lvl(u) = Lvl(Child(u, 0)) + 1$. Let $1 \leq l \leq H$. The *node of process p in level l*, denoted $Node(p, l)$, is $Leaf(p)$ if $l = 0$; otherwise, $Node(p, l) = Parent(Node(p, l-1))$. The *offset of process p in level l*, denoted $Offset(p, l)$ is the number o such that $Child(Node(p, l), o) = Node(p, l-1)$. The *bit of process p in level l*, denoted $Bit(p, l)$, is the o-th MSB of $Node(p, lvl).value$, where $o = Offset(p, l)$.



(a) Found successor (return q) **(b) No successor (return $\bot$)** **(c) Cross paths with ascending *Remove* (return $\top$)**

Figure 2: ***Possible*** $FindNext(p)$ ***scenarios***

Algorithm 4.1 *Tree.FindNext(p)*

```
20: for lvl ← 1 to H do
21:     node ← Node(p, lvl)
22:     offset ← Offset(p, lvl)
23:     snap ← node.value
24:     if HasZeroToTheRight(snap, offset) then
25:         break
26: if HasZeroToTheRight(snap, offset) = false then
27:     return ⊥        // reached root and found no candidate
28: index ← GetFirstZeroToTheRight(snap, offset)
29: node ← Child(node, index)
30: for dummy ← lvl − 1 to 1 do
31:     snap ← node.value
32:     if snap = EMPTY then
33:         return ⊤
34:     index ← GetFirstZero(snap)
35:     node ← Child(node, index)
36: return node.value
```

Algorithm 4.2 *Tree.Remove(p)*

```
37: for lvl ← 1 to H do
38:     j ← word with only Offset(p, lvl)-th MSB set
39:     snap ← F&A(Node(p, lvl).value, j)
40:     if snap + j ≠ EMPTY then
41:         break
```

Algorithm 4.3 *Tree.AdaptiveFindNext(p)*

```
42: node ← Node(p, 1)
43: offset ← Offset(p, 1)
44: for lvl ← 1 to H do
45:     if offset = W − 1 then
46:         node ← RightCousin(node)
47:         offset ← −1
48:     snap ← node.value
49:     if HasZeroToTheRight(snap, offset) then
50:         break
51:     if offset = −1 then
52:         offset ← offsetAtParent(node) − 1
53:     else
54:         offset ← offsetAtParent(node)
55:     node ← Parent(node)
56: Continue as in FindNext() (from Line 26 of Algorithm 4.1)
```

- *HasZeroToTheRight(snap, offset)* returns **true** if and only if there is a zero bit in *snap* to the right of *offset*.
- *GetFirstZeroToTheRight(snap, offset)* returns the offset of the first zero bit in *snap* to the right of *offset*.
- *GetFirstZero(snap)* returns the offset of the first zero bit in *snap*.
- *EMPTY* is the all-ones word, $2^W - 1$.
- *RightCousin(node)* is the node to the right of *node* at the same level.
- *offsetAtParent(node)* is the offset of the bit associated with *node* at its parent, i.e., o such that $Child(Parent(node), o) = node$.

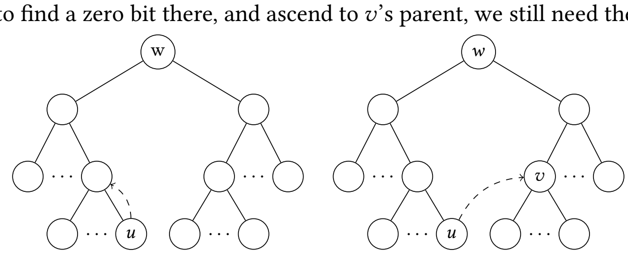
Figure 3: ***Tree data structure***

4.1 Adaptive *FindNext*()

It is easy to see that the RMR cost of *Remove*() is $O(\log_W A_t)$, where A_t is the number of processes that abort during the execution. However, *FindNext*() is not adaptive to the number of aborts. When invoked on a leaf v that is the rightmost node in its subtree, *FindNext*() ascends to the root of the subtree—which can be of height $\log_W N$—even if the leaf of the next non-aborted process is immediately to the right of v (just in another subtree).

We introduce a novel (though small) change to the way *FindNext*(p) walks up the tree that improves its RMR cost to $O(\log_W A_i(p))$, where $A_i(p)$ is the number of processes that abort during process p's passage. Algorithm 4.3 presents the pseudo-code of the adaptive algorithm. Instead of ascending along a path from the leaf to the root, whenever we reach a node u that is the rightmost child of its parent, we "sidestep" to v, the node right to u's parent at the same level, as depicted in Figure 4. The idea is that because we cannot hope to find a zero bit to the right of u's bit in *Parent*(u), we instead optimistically check whether v has a zero bit. If this is the case, then *FindNext*(p)'s ascent would have stopped at w, the lowest common ancestor of u and v, and would have then started descending until eventually reaching v. If, however, v does not contain a zero bit, then all leafs between p and the rightmost leaf in v's subtree have been abandoned. Therefore, it is safe to resume the ascent from v.

A subtle point in Algorithm 4.3 is that if we "sidestep" to v, fail to find a zero bit there, and ascend to v's parent, we still need the search for a zero bit in v's parent to include v's subtree. (Hence the use of $offsetAtParent(v) - 1$ in line 52, after "sidestepping.") The reason is that the *Remove*() which has set the last bit in v might not have set v's bit in the parent yet. In such a case, *FindNext*() would have returned ⊤, as it would descend towards v (which it would find *EMPTY*) after going through *Parent*(v) (where v's bit is zero). Therefore, *AdaptiveFindNext*() mimics this property.



(a) *FindNext* **ascent** **(b)** *AdaptiveFindNext* **ascent**

Figure 4: ***Comparison of*** *FindNext*() ***ascent algorithms***

We can thus show that *AdaptiveFindNext*() is equivalent to *FindNext*() in the following sense: (The proof appears in the full version [3].)

Lemma 1. *Let E be an execution of the one-shot algorithm using AdaptiveFindNext(). Then there exists an execution E' of the one-shot algorithm using FindNext(), such that (1) process p invokes FindNext(q) in E' if and only if it invokes AdaptiveFindNext(q) in E; (2) process p's FindNext(q) in E' returns $x \in [N] \cup \{\bot, \top\}$ if and only if its AdaptiveFindNext(q) in E returns x; and (3) the order of FindNext() invocation/responses in E' is the same as the order of AdaptiveFindNext() invocation/responses in E.*

5 CORRECTNESS AND COMPLEXITY OF THE ONE-SHOT ALGORITHM

Here, we prove that our one-shot algorithm satisfies mutual exclusion, starvation freedom, and the first-come-first-served (FCFS) fairness condition (formally defined later). Then we show that the algorithm has sublogarithmic adaptive RMR cost. We assume that $W \geq \lceil \log N \rceil$. The following theorem summarizes these properties.

Theorem 2. *The one-shot lock of Figure 1 and Figure 3 satisfies mutual exclusion, starvation freedom, bounded exit, bounded abort, and FCFS. Each passage of the algorithm incurs $O(\log_W A_i)$ RMRs, where A_i is the number of processes that abort during the passage. An aborted attempt incurs $O(\log_W A_t)$ RMRs, where A_t is the number of processes that abort during the entire execution.*

Lemma 1 implies that any correctness result obtained for the one-shot algorithm using *FindNext*() (Algorithm 4.1) also holds for $AdaptiveFindNext()$ (Algorithm 4.3). Therefore, for simplicity, the following proofs consider the one-shot lock with $FindNext()$.

Notation. Let E be an execution. We denote by E_t the prefix of E consisting of the first t steps in E. The *value of variable v at time t* is the value of v in the configuration following E_t. Step s *occurs at time t* if it is the t-th step in E. We identify a process with the index it obtains from the F&A on $Head$ (Algorithm 3.1, line 1).

5.1 Tree Properties

We begin the proof of Theorem 2 by proving some properties of the $Tree$ data structure. We assume a *well-formed* execution, in which (1) each process invokes $FindNext()$ at most once; (2) $Remove(p)$ is invoked only by process p, and at most once; and (3) if process p invokes both $Remove(p)$ and $FindNext(u)$, then it invokes them in this order. An execution of the one-shot algorithm is well-formed.

5.1.1 Preliminaries. Abusing notation, we say that $p \in u$ if $Node(p, Lvl(u)) = u$, i.e., if $Leaf(p)$ is in the subtree rooted at u.

Definition 1. *The* lowest common level *of leafs p and q, denoted $LCL(p,q)$, is the lowest level in which p and q have a common ancestor, i.e., $LCL(p,q) = \min\{lvl \| Node(p,lvl) = Node(q,lvl)\}$. The* lowest common level *of leaf p and internal node u is $LCL(p,u) = LCL(p,q)$ for some leaf $q \in u$.*

The structure of the tree immediately implies that for any leafs $p < q$ with $L = LCL(p,q)$, the following hold: (1) $Offset(p,L) < Offset(q,L)$; (2) $\forall lvl > L. Offset(p,lvl) = Offset(q,lvl)$; (3) $\forall lvl \geq L. Node(p,lvl) = Node(q,lvl)$; and (4) $\forall p < r < q. LCL(p,r) \leq L, LCL(r,q) \leq L$.

Claim 3. *In any execution E, $Bit(p,lvl) = 1$ if and only if some process performs F&A$(Node(p,lvl).value, j)$ in $Remove()$ (Algorithm 4.2, line 39), where j is a word whose only set bit is the $Offset(p,lvl)$-th MSB.*

Proof. Appears in the full version of the paper [3]. □

Lemma 4. *For every execution E, process p, and $1 \leq lvl \leq H$, if $Bit(p,lvl) = 1$ at time t, then for all $l < lvl$ and $m \in Node(p, lvl-1)$, $Bit(m,l) = 1$ at time t.*

Proof. We prove by induction over lvl. The base case of $lvl = 1$ is vacuously true. For the induction step, assume the claim holds for levels $1, \dots, lvl-1$. Suppose that $Bit(p,lvl) = 1$ at time t. Claim 3 implies that $Bit(p,lvl) = 1$ because of a F&A performed on $Node(p,lvl).value$ by some process q. Then $Node(q,lvl) = Node(p,lvl)$ and $Offset(q,lvl) = Offset(p,lvl)$. Therefore, $q \in Node(p, lvl-1)$. For process q to have accessed $Node(p,lvl)$ to set $Bit(p,lvl)$, it must have reached iteration lvl in $Remove(q)$. Consider iteration $lvl-1$ of $Remove(q)$. Process q wrote $Node(q, lvl-1).value = EMPTY$ at time $t_1 < t$. Because $Node(q,lvl-1) = Node(p,lvl-1)$, this means that for every $m \in Node(p,lvl-1)$, $Bit(m,lvl-1) = 1$ at time $t_1 < t$. Let $m \in Node(p,lvl-1)$. We know $Bit(m,lvl-1) = 1$ at time t_1. Since $m \in Node(m,lvl-2)$, by the induction assumption, for every $l < lvl-1$, $Bit(m,l) = 1$ at time t_1. The claim follows, as $t_1 < t$. □

Corollary 5 (Remove Invariant). *For every execution E, process p, and $1 \leq lvl \leq H$, if $Bit(p,lvl) = 1$ at time t, then (1) for every $q \in Node(p,lvl-1)$, process q invokes $Remove(q)$ at some time $t' \leq t$; and (2) if $Remove(p)$ is not invoked in E_t, then for all $1 \leq lvl \leq H$, $Bit(p,lvl) = 0$ in E_t.*

5.1.2 FindNext() Properties. Let E be an execution of the algorithm. We now prove several properties provided by $Tree$ during E. We identify the *invocation* (respectively, *completion*) of a $Tree$ method with the first (respectively, last) memory operation performed by the method's code. If process p invokes method M, we denote by M_p the subsequence of E that starts at M's invocation and ends at its completion (or, if M does not complete, at the end of E). We say that M_p *happens before* M'_q, denoted $M_p \to M'_q$, if the completion of M_p occurs before the invocation of M'_q (i.e., p performs the last memory operation in M_p's code before q performs the first memory operation of M'_q). We say that a method M_p *starts before method M'_q completes*, denoted $M_p \rightsquigarrow M'_q$, if the invocation of M_p occurs before the completion of M'_q in E. Note that $M_p \rightsquigarrow M'_q \iff \neg(M'_q \to M_p)$. We use the fact implied by this equivalence, that if $A \rightsquigarrow B$ and $B \to C$, then $A \rightsquigarrow C$.

The first three properties say that $FindNext(p)_a$ returns the first process $q > p$ that did not yet invoke $Remove(v)$.

Property 6. *If $FindNext(p)_a$ returns $q \notin \{\bot, \top\}$, then $p < q$.*

Property 7. *If $FindNext(p)_a$ returns $q \notin \{\bot, \top\}$ and $Remove(q)$ is invoked in E, then $FindNext(p)_a \to Remove(q)$.*

Proof. Let t_1 be the time a executes the last step of $FindNext(p)$. Code inspection shows that this step reads $Bit(q,1) = 0$. The first step of $Remove(q)$ is a F&A that sets $Bit(q,1)$ to 1. It follows that $Remove(q)$ cannot perform its first step before t_1, i.e., $FindNext(p)_a \to Remove(q)$. □

Corollary 8. *If $Remove(q) \rightsquigarrow FindNext(p)$ then $FindNext(p)$ does not return q.*

Property 9. *If $FindNext(p)_a$ returns $q \notin \{\bot, \top\}$, then for all $p < w < q$, $Remove(w) \rightsquigarrow FindNext(p)_a$.*

Proof. Let t be the time a executes the last step of $FindNext(p)$. Assume towards a contradiction, that for some $p < w < q$, $Remove(w)$ does not perform its first memory operation in E_t. It follows from the remove invariant that for all $1 \leq lvl \leq H$, $Bit(w,lvl) = 0$ at time t. Let $l_{p,q} = LCL(p,q)$, $l_{p,w} = LCL(p,w)$, and $l_{w,q} = LCL(w,q)$. Code inspection shows that a ascends until level $l_{p,q}$, observing only 1 bits to the right of $Offset(p,l)$ at all levels $l < l_{p,q}$. At level $l_{p,q}$, a observes 0 at bit $Offset(q, l_{p,q})$ and 1 at every bit between $Offset(p,l_{p,q})$ and $Offset(q,l_{p,q})$. Then a starts descending. In each level l, a observes 0 at bit $Offset(q,l)$ and 1 at every bit to the left of that offset. Finally, a reaches $Leaf(q)$ and returns q.

If $l_{p,w} < l_{p,q}$, then $Offset(w,l_{p,w}) > Offset(p,l_{p,w})$. Yet a ascends through level $l_{p,w}$, implying that it observes $Bit(w,l_{p,w}) = 1$ during E_t, a contradiction. If $l_{p,w} = l_{p,q} = l_{w,q}$, then $Offset(p,l_{p,w}) < Offset(w,l_{p,q}) < Offset(q,l_{p,w})$. Thus, a reads $Bit(w,l) = 1$ during E_t, a contradiction. Otherwise, $l_{p,q} > l_{w,q} > 0$, and therefore $Offset(w,l_{w,q}) < Offset(q,l_{w,q})$. Yet

a descends level $l_{w,q}$, implying that it observes $Bit(w, l_{w,q}) = 1$ during E_t, a contradiction. ◻

The next property says that if $FindNext(p)$ returns $\bot$, then every process $q > p$ has invoked $Remove(q)$ (and thus no next process should be signaled).

Property 10. *If $FindNext(p)_a$ returns $\bot$, then for all $p < q < N$, $Remove(q) \rightsquigarrow FindNext(p)_a$.*

Proof. Assume towards a contradiction, that for some $p < w < n$, $Remove(w)$ does not perform its first step in E_t. It follows from the remove invariant that for all $1 \leq lvl \leq H$, $Bit(w, lvl) = 0$ at time t. Let $l_{p,w} = LCL(p, w) \leq H$. At level $l_{p,w}$, a observes 1 at all offsets greater than $Offset(p, l_{p,w})$ at some time $t_1 \leq t$. We know $Offset(p, l_{p,w}) < Offset(w, l_{p,w})$, implying a observed $Bit(w, l_{p,w}) = 1$ at time $t_1 \leq t$, which is a contradiction. ◻

The next property says that the process ids returned by non-overlapping $FindNext(p)$ executions are monotonically increasing.

Property 11. *If $FindNext(p)_a \rightarrow FindNext(p)_b$ and $FindNext(p)_a, FindNext(p)_b$ respectively return $q_a, q_b \notin \{\top, \bot\}$, then $q_a \leq q_b$.*

Proof. Assume towards a contradiction that $q_a > q_b$. From code inspection, a ascends until level $l_{p,q_a} = LCL(p, q_a)$ and then descends until level 0, and b ascends until level $l_{p,q_b} = LCL(p, q_b)$ and then descends until level 0. From Property 6, we have $p < q_b < q_a$, and therefore $l_{p,q_b} \leq l_{p,q_a}$. If $l_{p,q_b} < l_{p,q_a}$, then (1) a reads $Bit(q_b, l_{p,q_b}) = 1$, as a ascends past level l_{p,q_b}; and (2) b reads $Bit(q_b, l_{p,q_b}) = 0$, as it stops its ascent at level l_{p,q_b}. However, $FindNext(p)_a \rightarrow FindNext(p)_b$, and therefore any bit observed by a as 1 cannot be observed by b as 0, so this is a contradiction.

If $l_{p,q_b} = l_{p,q_a}$, then both a and b stop their ascent at this level, but descend into different subtrees. Let $l = LCL(q_a, q_b)$. If $l = l_{p,q_b} = l_{p,q_a}$, then clearly $Offset(p, l) < Offset(q_b, l) < Offset(q_a, l)$, and therefore a reads $Bit(q_b, l) = 1$ but b reads $Bit(q_b, l) = 0$, which is a contradiction, since $FindNext(p)_a \rightarrow FindNext(p)_b$. Therefore, $l < l_{p,q_b} = l_{p,q_a}$ and $Offset(q_b, l) < Offset(q_a, l)$. Thus, a reads $Bit(q_b, l) = 1$, as it descends towards a, but b reads $Bit(q_b, l) = 0$. Since $FindNext(p)_a \rightarrow FindNext(p)_b$, this is a contradiction. ◻

The last property refers to scenarios in which process a's $FindNext(p)$ is about to return some value $q > p$, but a $Remove(q)$ crosses paths with a's $FindNext(p)$ execution and causes it to return $\top$. The property says that in such a case, there exists a process b that assumes responsibility for p's lock handoff. We reason about the responsibility for the handoff with the following *responsibility relation.*

Definition 2 (Responsibility relation). *Given processes a, b, we say that process a* has less p-responsibility than b, *denoted $a <_p^R b$, if the following 4 conditions hold in E:*

1. *$Remove(b)$ is invoked by b in E.*
2. *$FindNext(p)_a$ is invoked by a in E.*
3. *$FindNext(p)_a \rightsquigarrow Remove(b)$.*
4. *For every $p < d < max\{a, b\}$, $Remove(d) \rightsquigarrow Remove(b)$.*

Lemma 12. *The relation $<_p^R$ is a strict partial order.*

Proof. A strict partial order is irreflexive and transitive.

Irreflexive: Assume $Remove(a)$ and $FindNext(p)_a$ are both invoked in E by some process a. The execution is well-formed, and so we have $Remove(a) \rightarrow FindNext(p)_a$. Therefore, $\neg(FindNext(p)_a \rightsquigarrow Remove(a))$. Therefore, Conditions 1–3 in Definition 2 cannot all hold together for a, and thus $a <_p^R a$ cannot hold.

Transitive: Assume $a <_p^R b$ and $b <_p^R c$ for some processes a, b, and c. From Conditions 1 and 2 in Definition 2, $Remove(c)$ and $FindNext(p)_a$ are invoked in E. Therefore, Conditions 1 and 2 also hold for a and c. From Condition 1, $Remove(b)$ is invoked by b in E. From Condition 2, $FindNext(p)_b$ is invoked by b in E. The execution is well-formed, so we have $Remove(b) \rightarrow FindNext(p)_b$. From Condition 3, $FindNext(p)_a \rightsquigarrow Remove(b)$. From Condition 3, $FindNext(p)_b \rightsquigarrow Remove(c)$. Therefore, we conclude that $FindNext(p)_a \rightsquigarrow Remove(b) \rightarrow FindNext(p)_b \rightsquigarrow Remove(c)$, and thus $FindNext(p)_a \rightsquigarrow Remove(c)$ holds. Therefore Condition 3 holds for a, c. Let d such that $p < d < max\{a, c\}$. If $p < d < max\{b, c\}$, then from Condition 4 we get $Remove(d) \rightsquigarrow Remove(c)$ as needed. Otherwise, $p < d < max\{a, b\}$, and from Condition 4, we know $Remove(d) \rightsquigarrow Remove(b)$. We can thus conclude that $Remove(d) \rightsquigarrow Remove(b) \rightarrow FindNext(p)_b \rightsquigarrow Remove(c)$, which implies $Remove(d) \rightsquigarrow Remove(c)$ as needed. Therefore, Condition 4 holds for a, c, implying $a <_p^R c$.

◻

Corollary 13. *If $a <_p^R b$, then there exists a maximal u for a, i.e., u such that $a <_p^R u$ and for all c, $\neg(u <_p^R c)$.*

Property 14. *Fix processes p and $a \geq p$. If for every $p < d < a$, $Remove(d) \rightsquigarrow Remove(a)$, and $FindNext(p)_a$ returns $\top$, then for some process $b \neq a$, $a <_p^R b$*

Proof. Since $FindNext(p)_a$ returns $\top$, a reads $EMPTY$ from $node.value$ for some node $node$ at time t (Algorithm 4.1, line 31). Let b be the process whose F&A in $Remove(b)$ writes $EMPTY$ to $node.value$. Clearly, $b \in node$ and $b \neq a$. We claim that $a <_p^R b$.
Condition 1: Let l^- be the level of $node$, i.e., the level in which p stops its descent. Then at time t_1, a reads $Bit(l^- + 1, b) = 0$, and at time $t_2 > t_1$, a reads $Bit(b, l^-) = 1$. The latter fact implies, from the remove invariant, that $Remove(b)$ is invoked by b in E.
Condition 2: Immediate from the premise.
Condition 3: Consider $Remove(b)$'s execution. Since b sets $node.value$ to $EMPTY$, it ascends to level $l^- + 1$, with the intent to set $Bit(l^- + 1, b)$ to 1. However, at time t_1, a reads $Bit(l^- + 1, b) = 0$, implying that $Remove(b)$ did not perform a F&A at level $l^- + 1$ by then. Therefore, $Remove(b)$ does not complete before $FindNext(p)_a$ is invoked, i.e., $FindNext(p)_a \rightsquigarrow Remove(b)$.
Condition 4: Let $p < d < max\{a, b\}$. Suppose $p < d < a$. We have $Remove(d) \rightsquigarrow Remove(a)$, $Remove(a) \rightarrow FindNext(p)_a$ and $FindNext(p)_a \rightsquigarrow Remove(b)$. Therefore, $Remove(d) \rightsquigarrow Remove(b)$, as needed. Otherwise, $p \leq a \leq d < b$. Let l^+ be the level at which a stops its ascent and starts descending towards b. Let $l_{d,b} = LCL(d, b)$, and $l_{p,d} = LCL(p, d)$.

Suppose that $l_{d,b} > l^-$. If $l_{d,b} < l^+$, a descends through $Node(b, l_{d,b}) = Node(d, l_{d,b})$ and observes $Bit(d, l_{d,b}) = 1$ since it descends towards b. If $l_{d,b} = l^+$ and $l_{p,d} < l^+$, a ascends through

$Node(p, l_{p,d}) = Node(d, l_{p,d})$ and observes $Bit(d, l_{p,d}) = 1$, as it keep ascending. If $l_{d,b} = l^+$ and $l_{p,d} = l^+$, a stops ascending at $Node(b, l_{d,b}) = Node(d, l_{p,d})$ and observes $Bit(d, l_{p,d}) = 1$, as a then descends towards b. In any case, we have that at some time $t_3 \leq t_1$, a reads $Bit(d, l_{d,b}) = 1$ or $Bit(d, l_{p,d}) = 1$. The remove invariant implies that d performs the first step of $Remove(d)$ at some time $t_4 \leq t_3$. Above we have established that $Remove(b)$ does not complete before $t_1 \geq t_3 \geq t_4$, implying that $Remove(d) \rightsquigarrow Remove(b)$.

Suppose, then, that $l_{d,b} \leq l^-$. Then $Remove(b)$ ascends through level $l_{d,b}$ before setting $node.value$ to $EMPTY$ at time $t_5 < t_1$. Thus, at time t_5, $Bit(d, l^-) = 1$, and so the remove invariant implies that d performs the first step of $Remove(d)$ at some time $t_6 < t_5$. Above we have established that $Remove(b)$ does not complete before $t_1 > t_5 > t_6$, and so $Remove(d) \rightsquigarrow Remove(b)$.

Thus all four conditions of Definition 2 hold, and indeed $a <^R_p b$. □

5.2 Mutual Exclusion

Lemma 15. *For every execution E and process i, if $go[i] =$ **true** at time t_0 then for all $j < i$, j performs the first step of either Abort() or Exit() at time $t < t_0$.*

Proof. We prove by contradiction. Let E be an execution in which the claim is false, and let t_0 be the first time in which the above condition is violated. Then at time t_0, some process k writes $go[i] =$ **true**, but there exists some process $j < i$ that has not yet performed the first step of either $Abort()$ or $Exit()$.

We claim that $LastExited < j$ at all times $t < t_0$. Otherwise, at some time $t < t_0$, some process $k' \geq j$ writes $LastExited = k'$, implying that k' exits the CS. By definition of j, $k' \neq j$. Process k' reads $go[k'] =$ **true** at time $t' < t < t_0$, even though $j < k'$ and j has not performed the first step of either $Abort()$ or $Exit()$ at t'. This contradicts t_0 being the first time in which such a violation occurs.

Now, since k writes $go[i] =$ **true** at time t_0, it completes a $FindNext(h)$ invocation, for some h, at time $t_1 < t_0$ and receives i as the response. Clearly, $k \neq j$, because by time t_0, process k has invoked $FindNext()$ but j has not. Process k invokes $SignalNext(h)$ either from $Abort()$ or from $Exit()$. In either case, for $SignalNext(h)$ to be invoked, it must hold that at some time $t_2 < t_1$, process k reads $LastExited$ and observes its value to be h (either because of line 13 in Algorithm 3.3 or line 9 of Algorithm 3.2). Thus, $h < j$. Now, $FindNext(h)_k$ returns i at time $t_1 < t_0$. Since $h < j < i$, it follows from Property 9 that $Remove(j) \rightsquigarrow FindNext(h)_k$, i.e., that process j performs the first step of $Remove(j)$ at some time $t' < t_1 < t_0$, which is a contradiction. □

Corollary 16. *The one-shot algorithm satisfies mutual exclusion.*

5.3 FCFS and Starvation Freedom

For a one-shot abortable lock, the *first-come-first served* (FCFS) fairness condition [17] requires that (1) *Enter* starts with a *doorway*, which is a wait-free section of code (i.e., that can be completed in a finite number of the executing process' steps); and (2) if process p completes the doorway before q starts executing the doorway, and if p does not abort, then p enters the CS before q does.

Lemma 17. *The one-shot algorithm satisfies FCFS.*

Proof. We define the doorway to be the F&A operation (Algorithm 3.1, line 1). Process j receives index j from this F&A and proceeds to the "waiting" section, in which it waits for $go[j]$ to become **true**. Let i be a process that executes the doorway after process j, receiving index $i > j$. Suppose that at time t, process i observes $go[i] =$ **true** and enters the CS. Lemma 15 implies that by time t, process j has invoked either $Exit()$ or $Abort()$, which establishes FCFS. □

Lemma 18. *The one-shot algorithm satisfies starvation freedom.*

Proof. By contradiction. Let E be an execution in which no process crashes outside the remainder section and every process that enters the CS eventually leaves it, but there exists a minimal i such that process i invokes but does not complete the execution of $Enter()$. This means that i does not invoke $Abort()$, $Exit()$, or $SignalNext()$. It is easy to verify that $LastExited$ and $Head$ are both strictly increasing. Let m be the largest value of $Head$ in E. Then $m < i$. Otherwise, some process $m > i$ writes to $Head$, and the assumption on E implies that m eventually enters the CS, which violates FCFS. It is easy to verify that at any time $LastExited \leq Head$ and so $LastExited \leq m$ in E. The assumption on E implies that process m eventually, at some time t_0, reaches line 9 and sets $LastExited = m$. Therefore, from time t_0 onwards, $LastExited = Head = m$. Let k be the maximal value such that $go[k] =$ **true** in E. Then $m \leq k < i$.

Claim 19. *There exists q such that $k <^R_m q$.*

Proof. Suppose that $k = m$. Then process k executes $Exit()$, invoking $FindNext(k)_k$, which returns b. If $b \notin \{\top, \bot\}$, then from Property 6, $b > k$ and k eventually writes $go[b] =$ **true**, contradicting k's maximality. If $b = \bot$, then Property 10 implies that i invokes $Remove(i)$, which can only be invoked from $Abort(i)$, which is a contradiction. Therefore, $b = \top$. The preconditions for Property 14 hold vacuously, as there is no d such that $m = k < d < k$. It follows that for some process q, $k <^R_m q$.

Alternatively, suppose that $k > m$. We will show that the preconditions of Property 14 hold, and thus for some process q, $k <^R_m q$. Process k does not enter the CS, as that would contradict m's maximality. Therefore, process k invokes $Abort(k)$ and, in turn, $Remove(k)$. Consider the process p that wrote $go[k] =$ **true**. p executes $FindNext(v)_p$ and receives k. From the definition of m, $v \leq m$. If $v < m < k$, then Property 9 implies that m invoked $Abort$, which is a contradiction. Therefore, $v = m$ and p executes $FindNext(m)_p$, receiving k in response. From Property 7, $FindNext(m)_p \rightarrow Remove(k)$. Process k executes $Remove(k)$ and then $FindNext(m)_k$, receiving b. Therefore $FindNext(m)_p \rightarrow FindNext(m)_k$. If $b \notin \{\top, \bot\}$, then from Property 11, $k \leq b$. From Corollary 8, $b \neq k$. Thus, $b > k$ and k writes $go[b] =$ **true**, contradicting k's maximality. If $b = \bot$, Property 10 implies that i invokes $Remove(i)$, which is a contradiction. Therefore $b = \top$. Consider any d such that $m < d < k$. By Property 9, $Remove(d) \rightsquigarrow FindNext(m)_p$. We also have $FindNext(m)_p \rightarrow Remove(k)$. Therefore, $Remove(d) \rightsquigarrow Remove(k)$. The preconditions of Property 14 thus hold. □

Claim 19 shows that $k <_m^R q$ for some q. By Corollary 13, there exists a maximal r such that $k <_m^R r$. The definition of $<_m^R$ implies that $Remove(r)$ is invoked in E and that $FindNext(m)_k \rightsquigarrow Remove(r)$. Therefore, at time t_0 (when m sets $LastExited = m$), $Remove(r)$ has not yet completed. Once $Remove(r)$ returns, r thus reads $Head = LastExited = m$ at line 13 of Algorithm 3.3 and invokes $FindNext(m)_r$, receiving some value c.

If $c = \bot$, this again implies i invokes $Remove(i)$, which is a contradiction. If $c = \top$, let d such that $m < d < r$. We have $u < d < max\{k, r\}$ and $k <_m^R r$, and so by the definition of $<_m^R$, $Remove(d) \rightsquigarrow Remove(r)$. The preconditions for Property 14 thus hold for m and $r > m$. This implies that for some process $\hat{r} \neq r$, $r <_m^R \hat{r}$, contradicting r's maximality.

Finally, suppose that $c \notin \{\top, \bot\}$. If $c > k$ then process r writes $go[c]$ = **true**, contradicting k's maximality. Therefore $c \leq k$. From condition 3 in the definition of $<_m^R$, we have $FindNext(m)_k \rightsquigarrow Remove(r)$. So $Remove(k) \rightarrow FindNext(m)_k \rightsquigarrow Remove(r) \rightarrow FindNext(m)_r$, and therefore $Remove(k) \rightsquigarrow FindNext(m)_r$. From Corollary 8, we have that $FindNext(m)_r$ does not return k, and therefore $c < k$. From Property 6, we have $c > m$. So $m < c < max\{k, r\}$. Condition 4 in the definition of $<_m^R$ implies that $Remove(c) \rightsquigarrow Remove(r)$. Therefore, $Remove(c) \rightsquigarrow FindNext(m)_r$. From Corollary 8, $FindNext(m)_r$ does not return c, which is a contradiction. □

5.4 Complexity Analysis

The one-shot algorithm performs $O(1)$ RMRs in addition to the RMRs performed by $Tree$ operations. Each $Tree$ operation is wait-free and takes $O(\log_W N)$ steps, and thus at most $O(\log_W N)$ RMRs. Given an execution E, let R denote the number of processes that invoke $Remove()$ in E. In the following, some proofs are relegated to the full version [3] due to space limitations.

Claim 20. *The RMR cost of Remove() is $O(\log_W R)$.*

Denote by $R_p(t)$ the number of processes $r \geq p$ that invoke $Remove(r)$ in E_t.

Claim 21. *The RMR cost of $AdaptiveFindNext(p)_q$ is $O(\log_W R_p(t))$, where t is the time in which $AdaptiveFindNext(p)_q$ completes.*

Proof. Consider an execution of $AdaptiveFindNext(p)$ by process q that performs $l > 2$ iterations of the loop at line 44 of Algorithm 4.3. Consider the execution of $HasZeroToTheRight$ at line 49 in iteration $l-1$, and let $node$ be the node q accesses at this iteration. The check at line 45 guarantees that the value of the $offset$ argument is less than $W-1$, and that $p \notin Child(node, W-1)$ (i.e., p is not the rightmost child). Since process q does not break at this iteration, we have that it receives **false** from this $HasZeroToTheRight$. This implies that the least significant (rightmost) bit of $node.value$ is 1. It follows from the remove invariant that for all $r \in Child(node, W-1)$, process r invokes $Remove(r)$ before $AdaptiveFindNext(p)_q$ completes at time t, i.e., in E_t. Because $p \notin Child(node, W-1)$, for every $r \in Child(node, W-1)$, $r > p$. There are W^{l-2} leafs in the subtree rooted at $Child(node, W-1)$. The number of processes $r \geq p$ that invoke $Remove(r)$ in E_t is $R_p(t)$. Therefore, $W^{l-2} \leq R_p(t)$, and so $l \leq 2 + \log_W R_p(t)$. Each iteration of the loop performs a constant number of RMRs, and the claim follows. □

Corollary 22. *Each successful passage of the one-shot algorithm incurs $O(\log_W A_i)$ RMRs, where A_i is the number of processes that abort during the passage. An aborted attempt incurs $O(\log_W A_t)$ RMRs, where A_t is the number of processes that abort during the entire execution.*

6 FROM ONE-SHOT TO LONG-LIVED LOCK

We present a generic transformation that converts a one-shot abortable lock algorithm L, with space complexity $s(N)$ (where $s(N)/2^W = O(1)$) into a long-lived algorithm with the same asymptotic RMR cost as L, and space complexity $O(N \cdot s(N) + N^2)$. Our transformation preserves starvation-freedom, but not FCFS.

Figure 5 presents the pseudo-code of the transformation. For simplicity, we assume a system with unbounded word and memory size, in which allocating a new (and initialized) instance of the one-shot lock L is free of charge. Section 6.2 removes these assumptions.

The long-lived lock uses an instance of L to solve mutual exclusion, and dynamically switches to new instances to keep processes from accessing the same one-shot lock instance twice. We represent the long-lived lock as a tuple ($Lock, Spn, Refcnt$), where $Lock$ is a pointer to the instance of L; Spn is a pointer to a *spin node* associated with this instance, which contains a single boolean field *go*; and $Refcnt$ is a $\lceil \log N \rceil$-bit *reference count*, indicating the number of processes currently accessing the one-shot lock instance. The tuple is stored in a single memory word $LockDesc$, which enables its fields to be manipulated atomically, as follows.

We store $Refcnt$ in the lower bits of $LockDesc$, allowing processes to use F&A to increment/decrement it while simultaneously obtaining a snapshot of the tuple. To acquire the lock, a process p uses F&A on $LockDesc$ to increment $Refcnt$, obtaining the instance l pointed to by $Lock$ in response (line 62), which p then attempts to acquire. Once p finishes its attempt (either due to aborting or after releasing l), it uses a F&A on $LockDesc$ to decrement $Refcnt$ (line 70). If p decrements $Refcnt$ to 0, it uses a CAS to atomically switch $Lock$ and Spn to point to new one-shot lock and spin node instances, conditioned on $Refcnt = 0$ (line 76).

However, p will fail to switch the lock from l if another process increments $Refcnt$ between lines 70 and 76. This scenario can lead to p's next acquisition attempt occurring while $Lock$ still points to l, but p cannot access the one-shot lock l again. We use the spin node spn associated with l to efficiently prevent processes from accessing l again: p saves spn in a local variable when its attempt to acquire l finishes (line 70), and in the next acquisition attempt, p busy waits on $spn.go$ if $LockDesc.Spn = spn$ (lines 57–59). Once the lock is switched from (l, spn) to new instances, $spn.go$ is set (line 77), signalling the processes busy waiting on $spn.go$ that $LockDesc.Lock \neq l$, and so they may attempt to acquire the one-shot lock. Using the spin nodes thus establishes that $LockDesc.Lock$ changed in $O(1)$ RMRs. Without them, it would require accessing $LockDesc$, which could incur $N-1$ RMRs, as $Refcnt$ can change N times before $LockDesc.Lock$ changes.

6.1 Correctness of the Transformation

Here, we prove:

Theorem 23. *Let L be a one-shot starvation-free abortable lock. Our transformation yields a long-lived starvation-free abortable lock.*

Shared variables:
$LockDesc$: ($Lock$: ptr to L instance, Spn : ptr to $SpinNode$, $Refcnt$: int)
initially, $LockDesc$ = (fresh L instance, fresh $SpinNode$, 0)
Local variables: $oldSpn$: ptr to $SpinNode$, initially $oldSpn = \bot$

Algorithm 6.1 $Enter()$

```
57: (l, spn, v) ← LockDesc
58: if spn = oldSpn then
59:     while ¬spn.go do
60:         if AbortSignal then
61:             return false
62: (l, spn, refcnt) ← F&A(LockDesc, 1)        // read Lock, Spn & inc Refcnt
63: completed ← l.Enter()
64: if completed = false then
65:     Cleanup()
66: return completed
```

Algorithm 6.2 $Exit()$

```
67: (l, spn, r) ← LockDesc
68: l.Exit()
69: Cleanup()
```

Algorithm 6.3 $Cleanup()$

```
70: (oldLock, oldSpn, refcnt) ← F&A(LockDesc, −1)
71: if refcnt = 1 then
72:     newLock ← AllocateOneShotLockInstance()
73:     newSpn ← AllocateSpinNode()
74:     old ← (oldLock, oldSpn, 0)
75:     new ← (newLock, newSpn, 0)
76:     if CAS(LockDesc, old, new) then
77:         oldSpn.go ← true
```

Figure 5: Transformation of a one-shot abortable lock algorithm L into a long-lived abortable lock

Claim 24. *Suppose process p invokes $l.Enter()$ (line 63). Let (l, s, r) be the LockDesc value p obtains at line 62 prior to this $l.Enter()$ call. Then LockDesc remains $(l, s, \_)$ until p executes the F&A at line 70.*

Proof. Every process increments and decrements $LockDesc.Refcnt$ once per acquisition attempt (lines 62 and 70, respectively). Therefore, $LockDesc.Refcnt \geq 1$ from p's F&A execution at line 62 prior to invoking $l.Enter()$ and until it executes the F&A at line 70 afterwards (after either aborting or releasing l). Now, $LockDesc.Spn$ and $LockDesc.Lock$ change only if a CAS at line 76 succeeds. However, a *CAS* at line 76 can succeed only if $LockDesc.Refcnt = 0$. The claim follows. ◻

Claim 25. *Let E be an execution of the long-lived algorithm. Then no process executes $Enter()$ twice on the same one-shot lock object.*

Lemma 26. *The long-lived algorithm satisfies mutual exclusion.*

Proof. Assume towards a contradiction that processes p, q are both in the CS at time t. Prior to entering the CS of the one-shot lock, both processes execute the *F&A* at line 62 and obtain *LockDesc* values $(l_p, s_p, \_)$ and $(l_q, s_q, \_)$. WLOG, assume that p performs its *F&A* first, at time t_0. Since p is in the CS at time t, Claim 24 implies that $LockDesc.Lock = l_p$ at time t. Therefore, $l_q = l_p$, as q performs its *F&A* at time t_1, $t_0 < t_1 < t$. Both processes are thus in l's CS at time t, contradicting mutual exclusion of the one-shot lock l. ◻

Lemma 27. *The long-lived algorithm satisfies starvation freedom.*

Proof. Let E be an execution in which no process crashes outside the remainder section and every process that enters the CS eventually leaves it. Suppose that some process p is spinning at line 59 at time t. Let l be the one-shot lock instance pointed to by p's *oldSpn* variable at time t. Then at some time $t' < t$, p executed a F&A at line 70 that returned $(l, \_, \_)$. Claim 24 implies that if a process executes a F&A at line 70 that returns $(l, \_, \_)$, then the previous execution of line 63 by the process invoked $l.Enter()$. Therefore, Claim 25 implies that each process executes a F&A at line 70 that returns $(l, \_, \_)$ at most once. Let q be the last process to execute such a F&A. Since q is last, this F&A returns $(l, \_, 1)$, and so eventually q executes the *CAS* at line 76. This CAS cannot fail because some process increments $LockDesc.Refcnt$ (while $LockDesc.L = l$) as such a process will later call $Cleanup()$ and execute line 70, in contradiction. Therefore, either q's CAS succeeds or it fails because another process executes a CAS at line 76 that succeeds. In either case, eventually some process sets $oldSpn.go$ to **true**, and so eventually p breaks out of its spin loop and invokes $Enter()$ on some instance l' of the one-shot lock. Starvation-freedom of l' implies that if p does not abort, it will enter the CS. ◻

6.2 Bounding Space and RMR Complexity

We augment the long-lived lock with memory management schemes that safely recycle one-shot lock and spin node instances, so that the overall number of objects used by the algorithm is $O(N)$ one-shot locks and $O(N^2)$ spin nodes. These bounds enable maintaining *LockDesc* in a W-bit memory word, assuming that $W = \Omega(\log N)$. We only provide an overview of the schemes, due to space constraints. Details appear in the full version [3].

Recycling one-shot locks. A process that successfully installs a new one-shot lock instance holds on to the instance l that it replaced, and uses l to satisfy its next allocation. (This is safe because other processes attempt to acquire l only if $LockDesc.Lock$ points to it, which is not the case after switching to a new one-shot lock instance.) The main problem is how to reset the *variables* of l so that when l gets reused they contain their initial values, but without having a single reset operation that incurs $s(N)$ RMRs. To this end, we use a *lazy* reset scheme, in which the processes reset the variables of l as they are accessed inside the one-shot algorithm.

Our scheme borrows ideas from the scheme of Aghazadeh et al. [1, § 4], but does not "steal" bits from the words being reset. The idea of their scheme is to add a version number for l, which is incremented each time l is reused, and to encode a version number in each of l's memory words, enabling processes to detect "stale" values. A process reading value x from a word uses x only if the version encoded in it equals l's current version; otherwise, the process proceeds as if it read the word's initial value. A process that writes to a word augments the write with l's current version. In our scheme, for each word w of l we maintain a word V_w that contains a pair (v_w, b_w), where v_w is w's version and b_w is an *incarnation* bit. We also maintain two words, w_0 and w_1, and guarantee that w's *next incarnation*, w_{1-b_w}, always contains w's initial value. When a process p first needs to access w, it reads (v_w, b) from V_w. If $v_w = v$, where v is l's current version, p will access w_b whenever it needs to access w from then on. Otherwise, if $v_w \neq v$, p updates V_w to $(v, 1-b)$ and resets w_b to w's initial value. It will subsequently access w_{1-b} whenever it needs to access w.

To prevent wraparound of l's version from making a stale word appear valid, we (like Aghazadeh et al. [1, § 4]) additionally reset $s(N)/2^W$ of l's words each time l gets reused, which guarantees that after l's version wraps around, each word has been reset. In summary, our scheme (1) increases the one-shot lock's space complexity from $s(N)$ to $O(s(N))$; (2) adds $O(1)$ RMRs to the first access a process makes in the one-shot algorithm; and (3) adds $O(s(N)/2^W)$ RMRs to the cost of allocating a one-shot lock instance.

Recycling spin nodes. Since a spin node might be accessed by a busy waiting process even after *LockDesc* no longer points to it, recycling spin nodes requires a *safe memory reclamation* scheme that recycles a spin node only if no process busy waits on it. We use the memory reclamation scheme of Aghazadeh et al. [2], which is similar to hazard pointers [22] but has constant worst-case RMR cost. Each process allocates spin nodes from a local pool. The reclamation scheme replenishes the pool as follows: A process p that switches *LockDesc*.*Spn* to point from spin node *spn* to another node *retires* the node *spn*. Once the scheme ascertains that no process is busy waiting on *spn*, it places *spn* into p's pool. The reclamation scheme guarantees that a process can have at most N spin nodes that it retired but have not yet been placed into its pool. We therefore guarantee that pools never become empty by using pools of size $N + 1$. Overall, we manage $O(N^2)$ spin nodes, and the reclamation scheme requires $O(N)$ words for its shared variables.

The following result follows from the above discussion.

Claim 28. *If L has RMR cost $t(A_i, A_t)$ and space complexity $s(N)$, and if $s(N)/2^W = O(1)$, then the long-lived lock has RMR cost $O(t(A_i, A_t))$ and space complexity $O(N \cdot s(N) + N^2)$.*

7 RELATED WORK

Randomized abortable locks can obtain $O(\frac{\log N}{\log \log N})$ expected RMR cost [23] or $O(1)$ expected amortized RMR cost [12]. Both algorithms are for the CC model; use reads, writes, and CAS; and work against an adversary that is slightly weaker than the *strong adaptive adversary*, which can make scheduling decisions based on all past events, including the latest coin-flips.

Lee [20] obtains an $O(\log N)$-RMR (non-adaptive) abortable lock by modifying Yang and Anderson's lock [26], making each two-process lock in their binary tree abortable. It is thus not clear that his construction can be generalized to make the tree W-ary. Jayanti's adaptive $O(\log N)$-RMR abortable lock [17] hinges on a binary tree-based f-array [16]. While the f-array can be made W-ary, the RMR complexity of such a W-ary tree of height $H = \log_W N$ is $O(W \cdot H)$, which is not sublogarithmic. In contrast, with F&A we can aggregate certain information about a node's children with one RMR, while the LL/SC-based f-array requires $O(\#\ \text{children})$ RMRs. Further, unlike prior work, we simplify the problem to a one-shot variant, which we then (generically) transform to a long-lived lock.

8 CONCLUSION

Applying the transformation of Section 6 to the one-shot abortable lock of Section 3 yields a starvation-free (but not FCFS) abortable lock with space complexity $O(N^2)$, in which the RMR cost of a successful passage is $O(\log_W A_i)$ and the RMR cost of an aborted attempt is $O(\log_W A_t)$. Assuming $W = \Theta(\log N)$, the worst-case RMR cost of the composed lock is $O(\frac{\log N}{\log \log N})$. This paper thus shows that, as with mutual exclusion, abortable locks can leverage additional primitives (beyond read, write and CAS) to obtain sublogarithmic worst-case RMR cost.

Several interesting questions remain. Our algorithm is for the CC model; the problem in the DSM model remains open. Our algorithm achieves $O(1)$ RMR cost only if $W = \omega(\log N)$, whereas lock algorithms such as the MCS lock [21] obtain $O(1)$ RMR cost with $W = \Theta(\log N)$. Is this an inherent difference between abortable locks and regular locks? Finally, Jayanti's $O(\log N)$-RMR abortable lock [17] satisfies FCFS and is adaptive to point contention; our algorithm does not have these properties. Can this gap be bridged?

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