Model Counting meets Distinct Elements

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Model Counting

- Given
  - Boolean variables $X_1, X_2, \cdots X_n$
  - Formula φ over $X_1, X_2, \cdots X_n$
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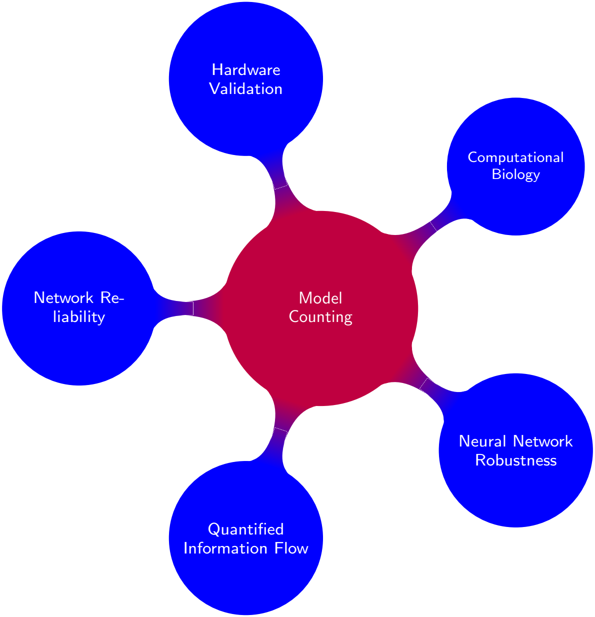
Problem Compute (ε, δ) approximation of $|\mathsf{Sol}(\varphi)|$
Concern Number of NP Queries

Applications across Computer Science



Distinct Elements

- Given a stream $\mathbf{a} = a_1, a_2, \ldots a_m$ where $a_i \in \{0,1\}^n$
- $\text{DE}(\mathbf{a}) = |\cup_i a_i|$
  - Also known as F_0 estimation

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- Example $\mathbf{a} = 1, 2, 1, 1, 2, 1, 3, 5, 1, 2, 1, 3$
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  Problem Compute (ε, δ) approximation of F_0

  Concern Space Complexity

Hashing-Based Techniques

Model Counting (S83,GSS06,GHSS07,CMV13b,EGSS13b,CMV14,CDR15,CMV16,ZCSE16,AD16 KM18,ATD18,SM19,ABM20,SGM20)

Distinct Elements (FM85,AMS99,GT01,BKS02,BJKST02, CM03,CLKB04,PT07, TW12,SP09)

2-wise independent Hashing

- Let H be family of 2-wise independent hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$

$$\forall y_1, y_2 \in \{0,1\}^n, \alpha_1, \alpha_2 \in \{0,1\}^m, h \xleftarrow{R} H$$
$$\Pr[h(y_1) = \alpha_1] = \Pr[h(y_2) = \alpha_2] = \left(\frac{1}{2^m}\right)$$

$$\Pr[h(y_1) = \alpha_1 \wedge h(y_2) = \alpha_2] = \left(\frac{1}{2^m}\right)^2$$

2-wise independent Hash Functions

- Variables: $X_1, X_2, \cdots X_n$
- To construct $h : \{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
  - $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$
  - Expected size of each XOR: $\frac{n}{2}$

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- To choose $\alpha \in \{0,1\}^m$, set every XOR equation to 0 or 1 randomly

$$X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2} = 0 \qquad (Q_1)$$

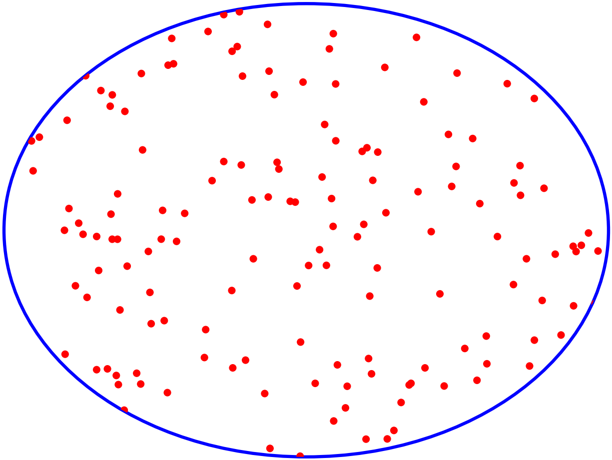
$$X_2 \oplus X_5 \oplus X_6 \cdots \oplus X_{n-1} = 1 \qquad (Q_2)$$

$$\cdots \qquad (\cdots)$$

$$X_1 \oplus X_2 \oplus X_5 \cdots \oplus X_{n-2} = 1 \qquad (Q_m)$$

- Therefore, $h(X) = \alpha$ can be represented as $AX = b$

As Simple as Counting Dots



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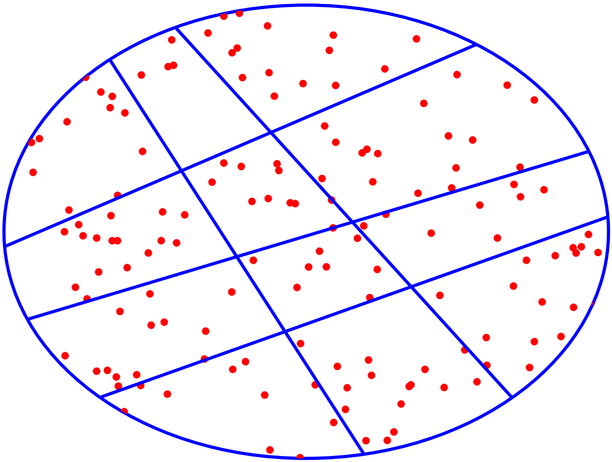
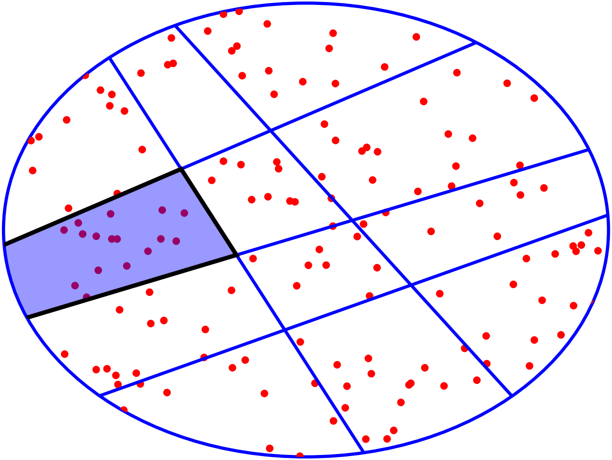


As Simple as Counting Dots

Pick a random cell



Estimate = Number of models in a cell × Number of cells

Challenges

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Challenge 2 How many cells?

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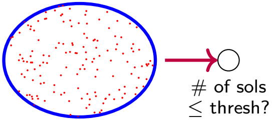
2-wise independent hash functions

ApproxMC



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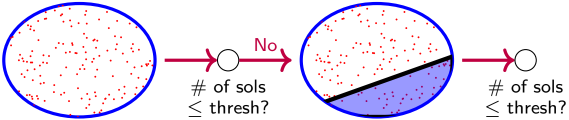

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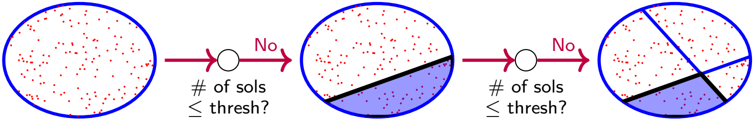

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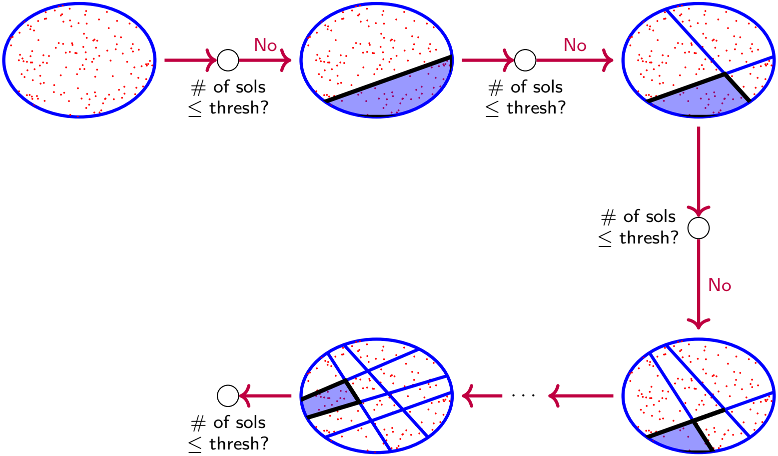

No
# of sols
≤ thresh?
No
# of sols
≤ thresh?
# of sols
≤ thresh?
No
...
# of sols
≤ thresh?

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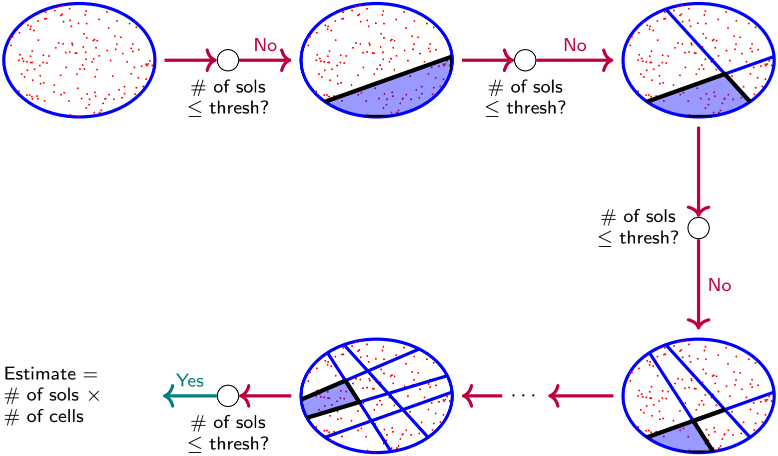
No
# of sols
≤ thresh?
No
# of sols
≤ thresh?
# of sols
≤ thresh?
No
Estimate =
# of sols ×
# of cells
Yes
# of sols
≤ thresh?
· · ·

Distinct Elements

Choose $h : \{0,1\}^n \mapsto \{0,1\}^n$
minhash $\leftarrow 2^n$;
for $a_i \in$ **a do**
    if $h(a_i) <$ minhash **then**
        minhash $= h(a_i)$
    end if
end for
return $\frac{2^n}{\text{minhash}}$

Is there more than meets the eyes?

- From Distinct Elements to Counting
- From Counting to Distinct Elements

Hashing-based Distinct Elements

$h \leftarrow$ ChooseHashFunctions
$\mathcal{S} \leftarrow \{\}$
for $a_i \in$ **a do**
    ProcessUpdate(S, h, a_i)
end for
Est $\leftarrow$ ComputeEst($\mathcal{S}$)
Return Est

Hashing-based Distinct Elements

```
h ← ChooseHashFunctions
S ← {}
for a_i ∈ a do
    ProcessUpdate(S, h, a_i)
end for
Est ← ComputeEst(S)
Return Est
```

Different Algorithms based on ProcessUpdate

- Minimum: Keep track of minimum $h(a_i)$
- Bucketing

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Different Algorithms based on ProcessUpdate

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From Distinct Elements to Counting: A Two Step Recipe

$\mathbf{a}_u$: set of all distinct elements of the stream $\mathbf{a}$.

Key Idea The formula φ can viewed as symbolic representation of some set $\mathbf{a}_u$ such that $\mathsf{Sol}(\varphi) = \mathbf{a}_u$.

Step 1 Capture the relationship $\mathcal{P}(\mathcal{S}, h, \mathbf{a}_u)$ between the sketch $\mathcal{S}$, h, and the set $\mathbf{a}_u$ at the end of stream.

Step 2 Given a formula φ and hash function h, design an algorithm to construct sketch $\mathcal{S}$ such that $\mathcal{P}(\mathcal{S}, h, \mathsf{Sol}(\varphi))$ holds. And now, we can estimate $|\mathsf{Sol}(\varphi)|$ from $\mathcal{S}$.

Min-based Estimation

Choose $h : \{0,1\}^n \mapsto \{0,1\}^n$
minhash $\leftarrow 2^n$;
for $a_i \in$ **a do**
    if minhash $< h(a_i)$ **then**
        minhash $= h(a_i)$
    end if
end for
return $\frac{2^n}{\text{minhash}}$

Application I: Min-based Counting Algorithm

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Bucketing-based Streaming Algorithm

```
Choose $h : \{0,1\}^n \mapsto \{0,1\}^n$
$\ell \leftarrow 0; \mathcal{B} \leftarrow \emptyset$
for $a_i \in$ a do
    if $h(a_i) \mod 2^\ell = 0^\ell$ then
        $\mathcal{B}$.Append($a_i$)
        if $|\mathcal{B}| \geq$ thresh then
            $\ell$++
            Filter($\mathcal{B}, h, \ell$)
        end if
    end if
end for
return $|\mathcal{B}| \times 2^\ell$
```

Bucketing-based Streaming Algorithm

Choose $h : \{0,1\}^n \mapsto \{0,1\}^n$
$\ell \leftarrow 0; \mathcal{B} \leftarrow \emptyset$
for $a_i \in \mathbf{a}$ **do**
    if $h(a_i) \mod 2^\ell = 0^\ell$ **then**
        $\mathcal{B}$.Append(a_i)   Elements that satisfy XOR
        if $|\mathcal{B}| \geq$ thresh **then**
            ℓ++
            Filter($\mathcal{B}, h, \ell$)   Add another XOR
        end if
    end if
end for
return $|\mathcal{B}| \times 2^\ell$

Application II: Bucketing-based Counting Algorithm

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This is ApproxMC!

From Distinct Elements to Counting: Implications

Given a formula φ and hash function h, design an algorithm to construct sketch $\mathcal{S}$ such that $\mathcal{P}(\mathcal{S}, h, \mathsf{Sol}(\varphi))$ holds.

Theorem (FPRAS)

If construction of sketch $\mathcal{S}$ is in PTIME for a class of formulas, then there is FPRAS for the corresponding class. E.g.: DNF, Union of XORs

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Theorem (Lower Bounds)

Lower bounds for Distributed Streaming translate to lower bounds for Distributed DNF counting

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- A general scheme for structured sets
- Encompasses models such as ranges, affine spaces
- Application: Distinct Elements over Range
  - Every item $[a_i, b_i]$ can be represented using a DNF formula.
  - So just apply FPRAS for DNF

Conclusion

Summary

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Future Directions

- Practical scalability of newly devised counting techniques
- Lifting Sparse Hashing techniques to streaming
- What is the analogue for higher moments (F_k)