Sampling Techniques for Constraint Satisfaction and Beyond

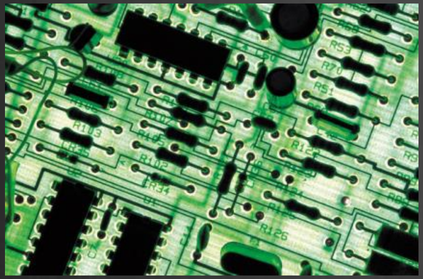
Kuldeep S. Meel

Rice University

Joint work with Supratik Chakraborty (IITB), Daniel J. Fremont(UCB), Dror Fried (Rice), Sanjit A. Seshia (UCB), Moshe Y. Vardi (Rice)

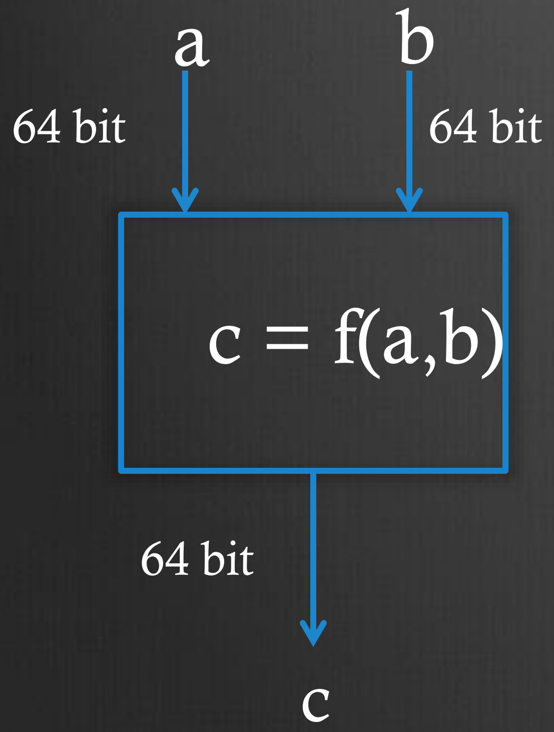
@Princeton June-23-2014

Life in The 21st Century!



How do we guarantee that the systems work *correctly* ?

Motivating Example



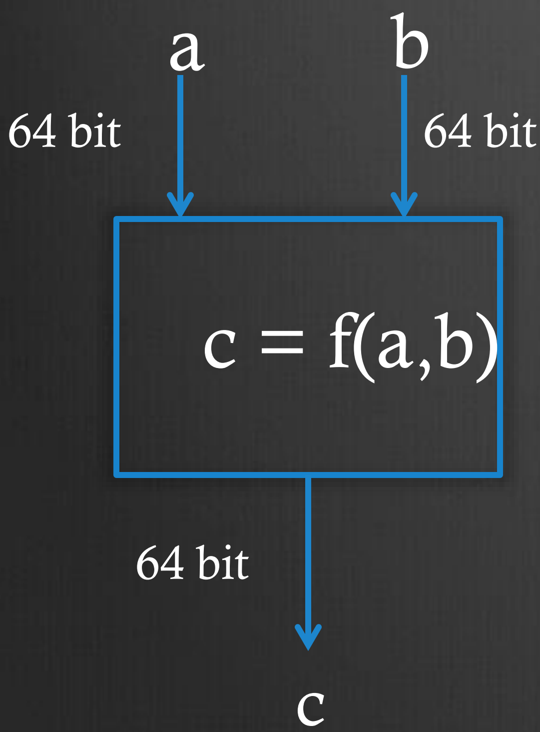
How do we verify that this circuit works ?

- Try for all values of a and b
  - 2^{128} possibilities (10^{22} years)
  - Not scalable

Simulation-Based Verification

- Dominant paradigm in recent years
- Hardware design is simulated with test vectors
- Test vectors represent different verification scenarios

Constrained-Random Simulation

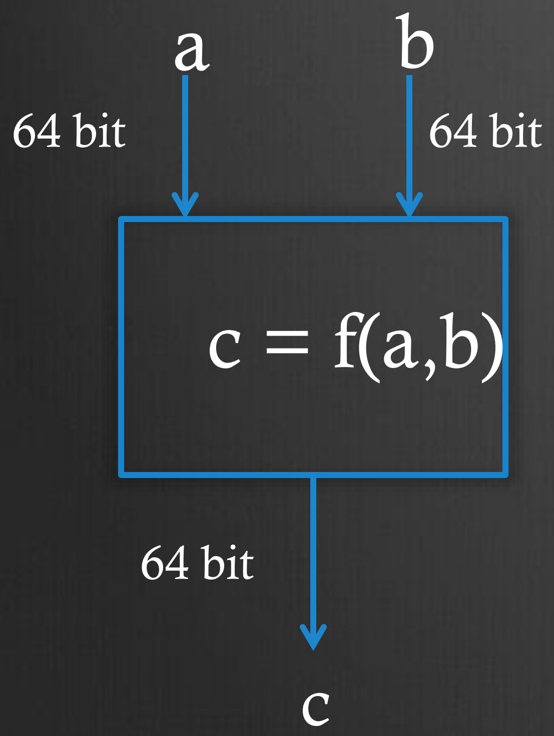


Sources for Constraints

- Designers:
  1. $100 < b < 200$
  2. $300 < a < 451$
  3. $40 < a < 50$ and $30 < b < 40$
- Past Experience:
  1. $400 < a < 2000$
  2. $120 < b < 230$
- Users:
  1. $1000 < a < 1100$
  2. $20000 < b < a < 22000$

Problem: How can we uniformly sample the values of a and b satisfying the above constraints?

Problem Formulation



Set of Constraints

↓

SAT Formula

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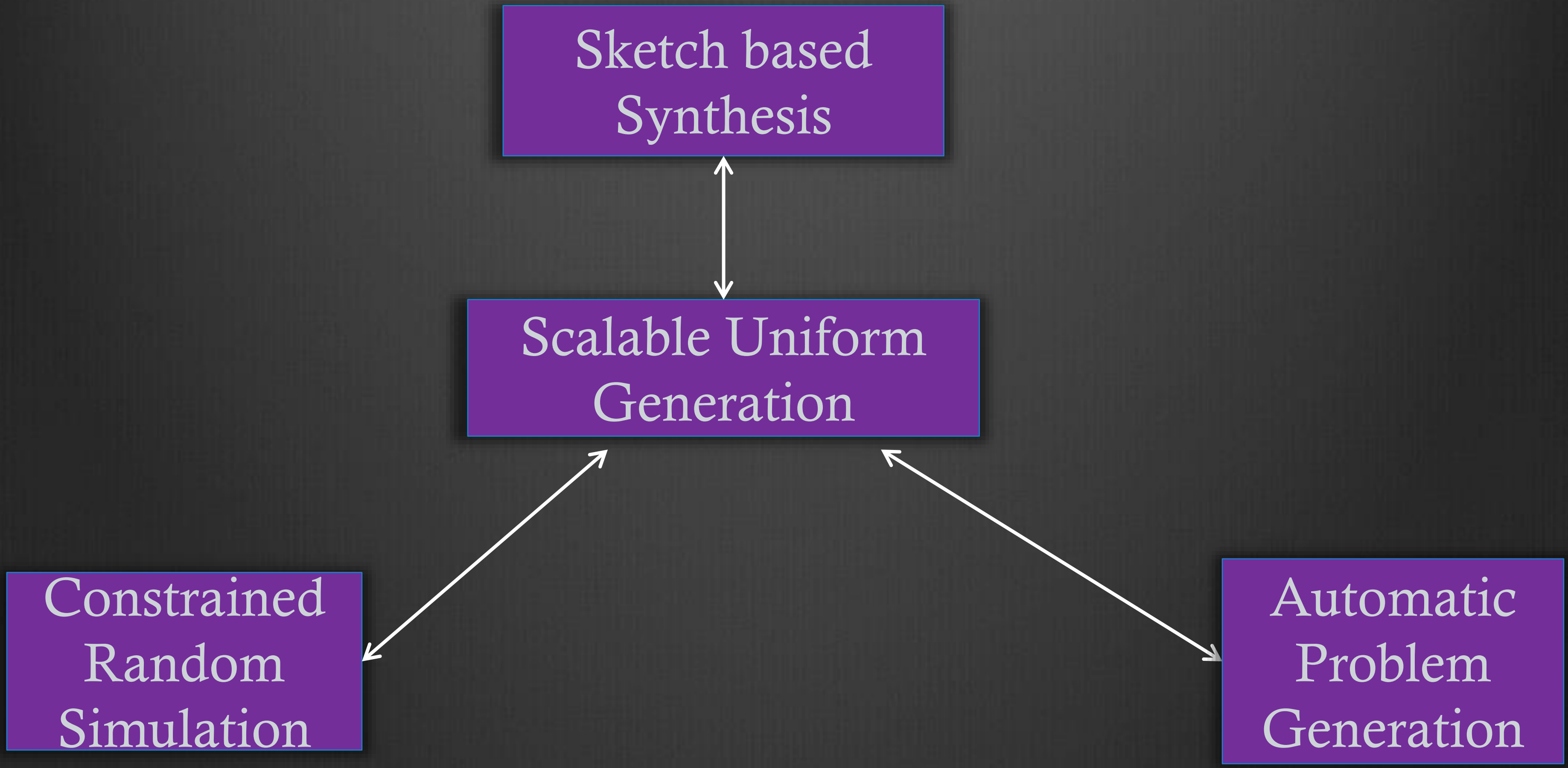
Given a SAT formula, can one uniformly sample solutions without enumerating all solutions while scaling to real world problems?

Scalable Uniform Generation of SAT-Witnesses

Uniform Generation of SAT-Witnesses



Sketch-Based Synthesis

- Given: Sketch and correctness condition
- Large space of programs that satisfy the correctness conditions
- Goal: Get the optimal program (running time, memory)
- Uniformly sample from the space of programs

Outline

- Sampling Techniques via Uniform Generation
- Extension to model counting and biased sampling
- Discussion on hashing
- Future Directions

Uniform Generation

Ref: “A Scalable Near-Uniform Generator” (CAV 2013)
“Balancing Scalability and Uniformity in SAT-Witness Generator” (DAC 2014)

Prior Work

BDD-based **Guarantees: strong** Performance: weak	SAT-based heuristics Guarantees: weak **Performance: strong**	INDUSTRY
Theoretical Work **Guarantees: strong** Performance: weak	Heuristic Work Guarantees: weak **Performance: strong**	ACADEMIA
BGP Algorithm	**XORSample'**	

Our Contribution

BDD-based
Guarantees: strong
Performance: weak

SAT-based heuristics
Guarantees: weak
Performance: strong

INDUSTRY

UniGen
Guarantees : strong
Performance: strong

Theoretical Work
Guarantees: strong
Performance: weak

Heuristic Work
Guarantees: weak
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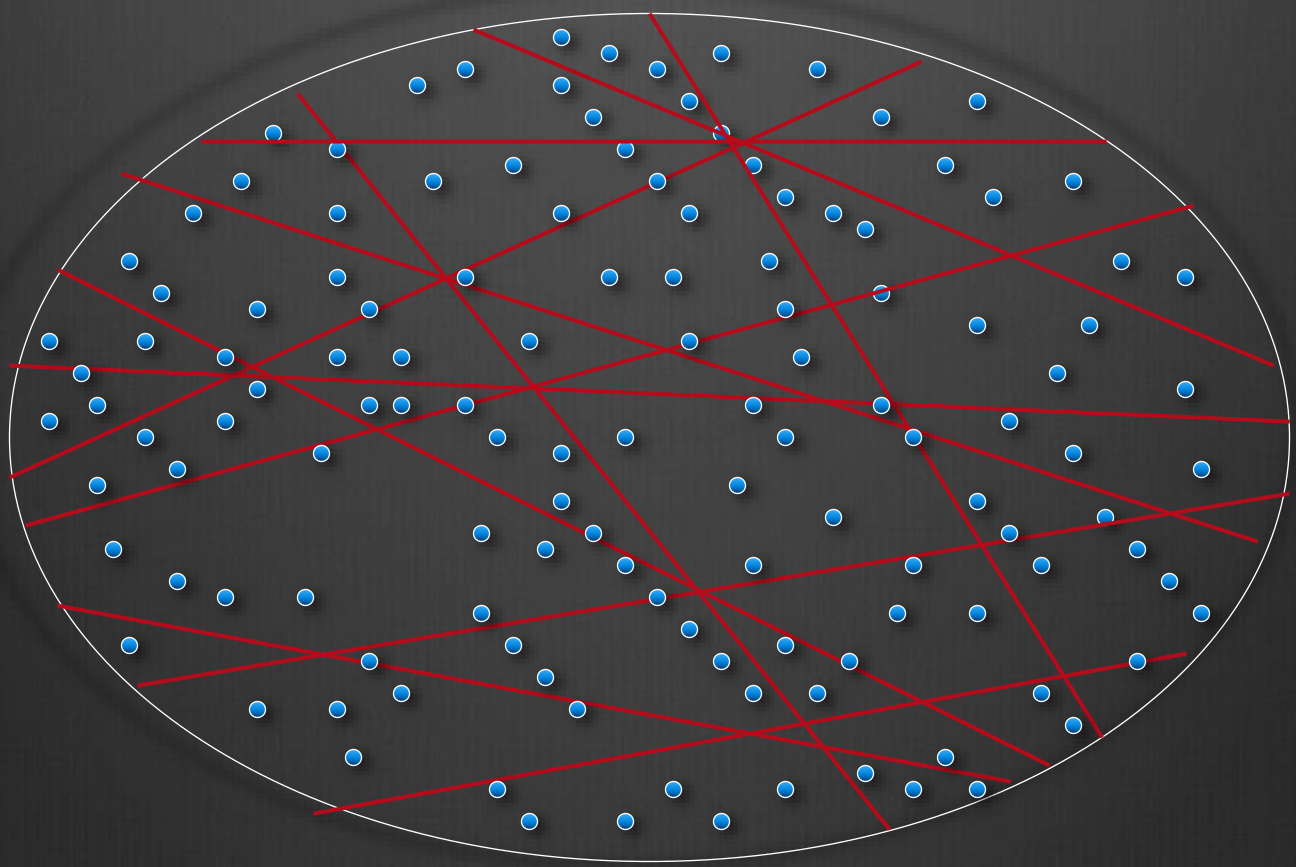
ACADEMIA

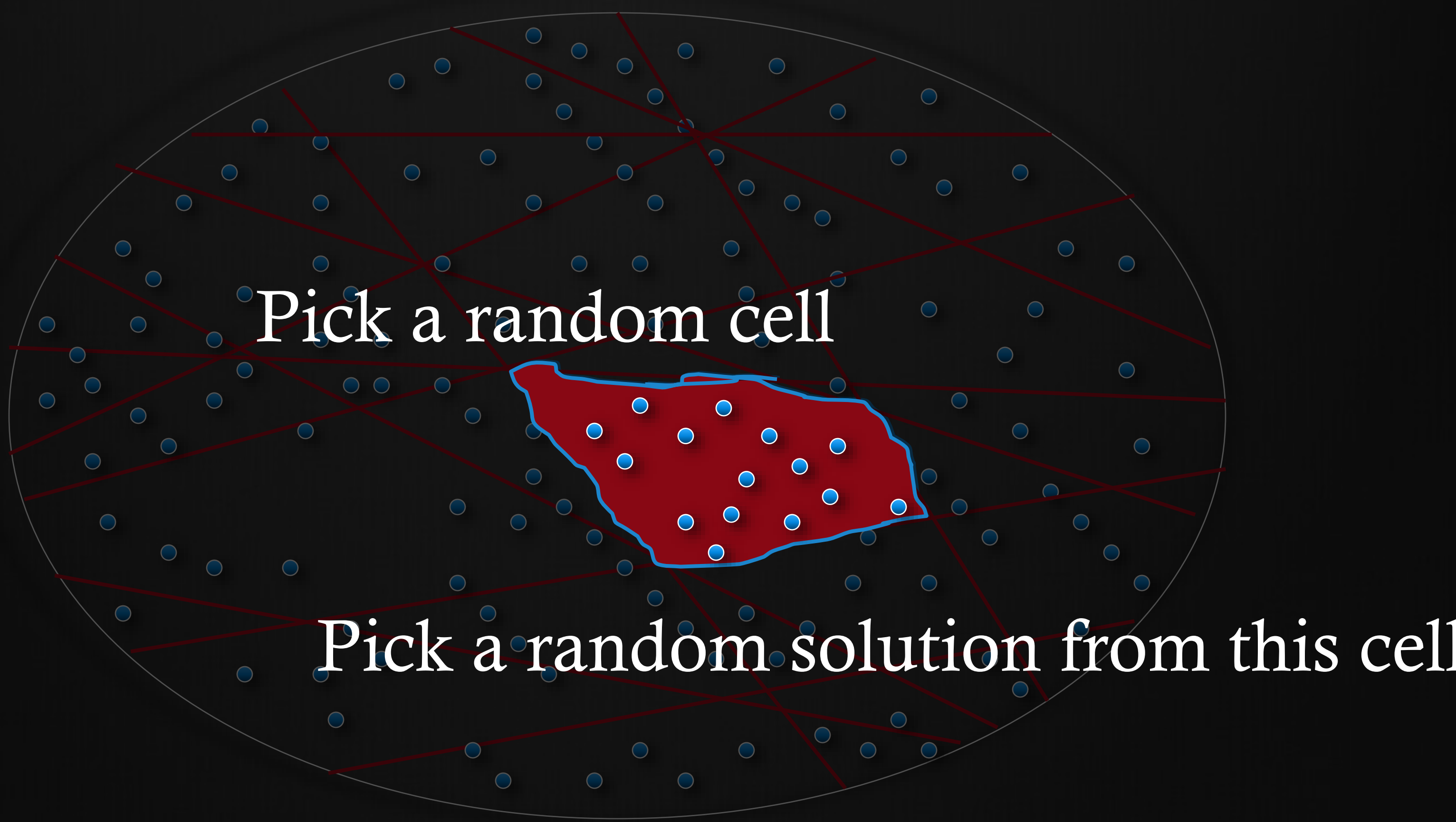
BGP Algorithm

XORSample'

Partitioning into equal "small" cells



Partitioning into equal "small" cells



How to Partition?

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

Universal Hashing [Carter-Wegman 1979, Sipser 1983]

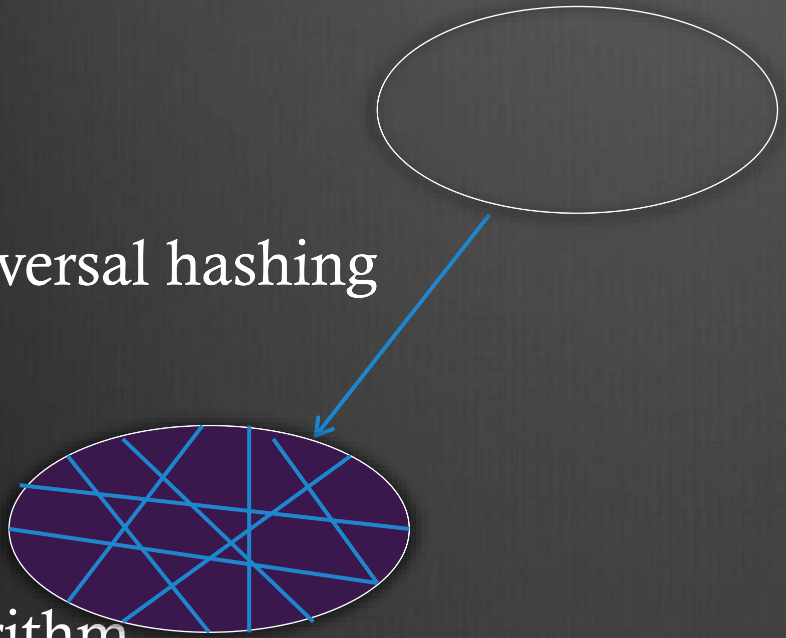
Universal Hashing

- Hash functions from mapping $\{0,1\}^n$ to $\{0,1\}^m$
  - (2^n elements to 2^m cells)
- Random inputs => All cells are *roughly* equal
- Universal hash functions:
  - Adversarial (any distribution) inputs => All cells are *roughly* equal
- Need stronger bounds on range of the size of cells

Lower Universality ➡ Lower Complexity

- H(n,m,r): Family of r-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)
- Higher the r => Stronger guarantees on range of size of cells
- r-wise universality => Polynomials of degree r-1
- Lower universality => lower complexity

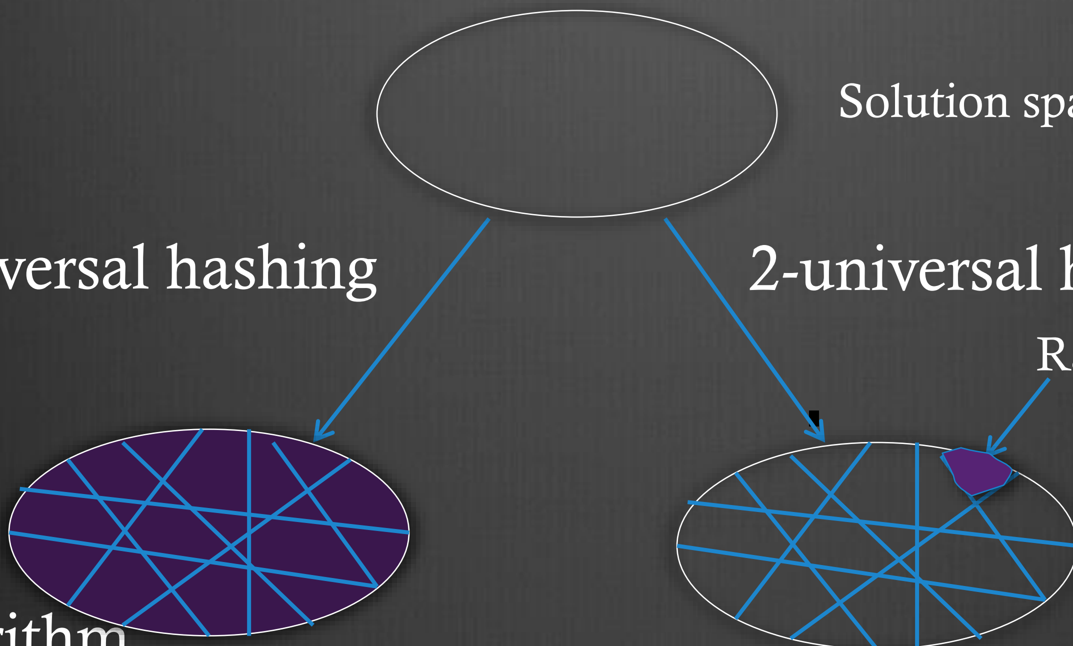
Hashing-based Approaches



All cells should be small

Uniform Generation

Scaling to Thousands of Variables



BGP Algorithm

UniGen

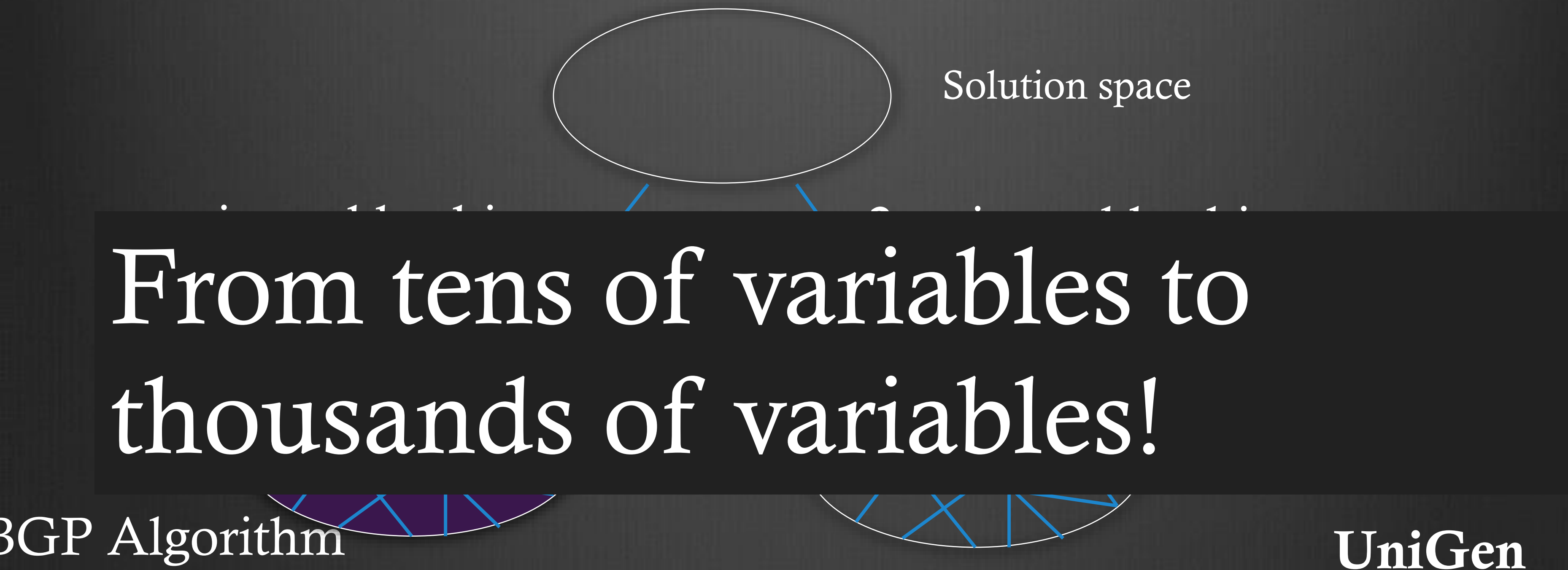
All cells should be small

Only a randomly chosen cells needs to be "small"

Uniform Generation

Near-Uniform Generation

Scaling to Thousands of Variables



From tens of variables to thousands of variables!

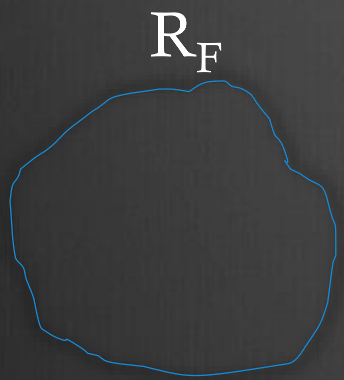
All cells should be small

Uniform Generation

Only a randomly chosen cells needs to be "small"

Near-Uniform Generation

UniGen



UniGen

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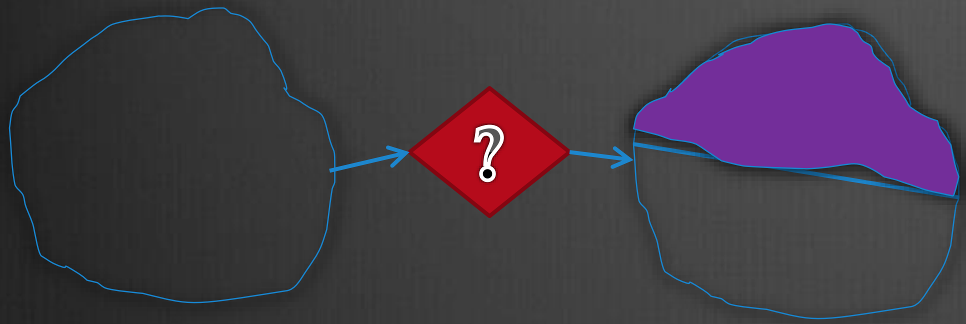

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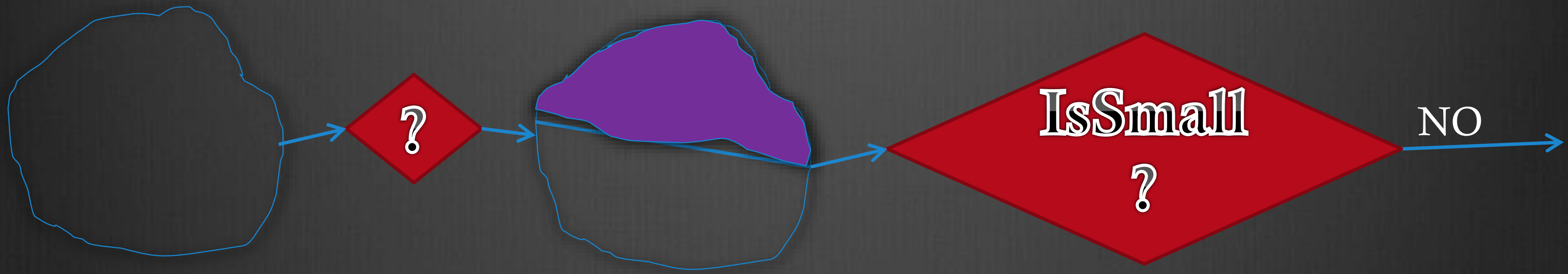

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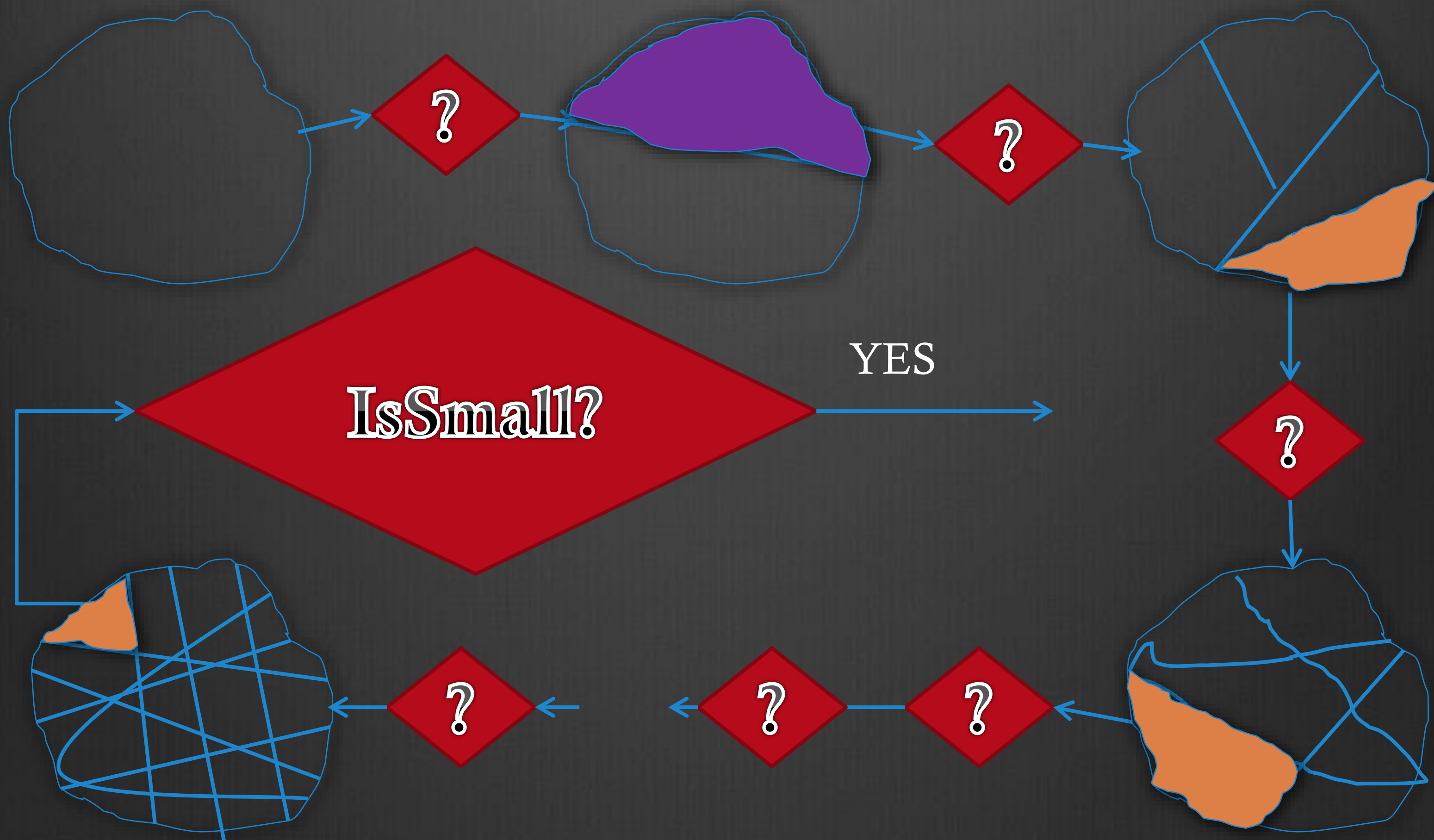

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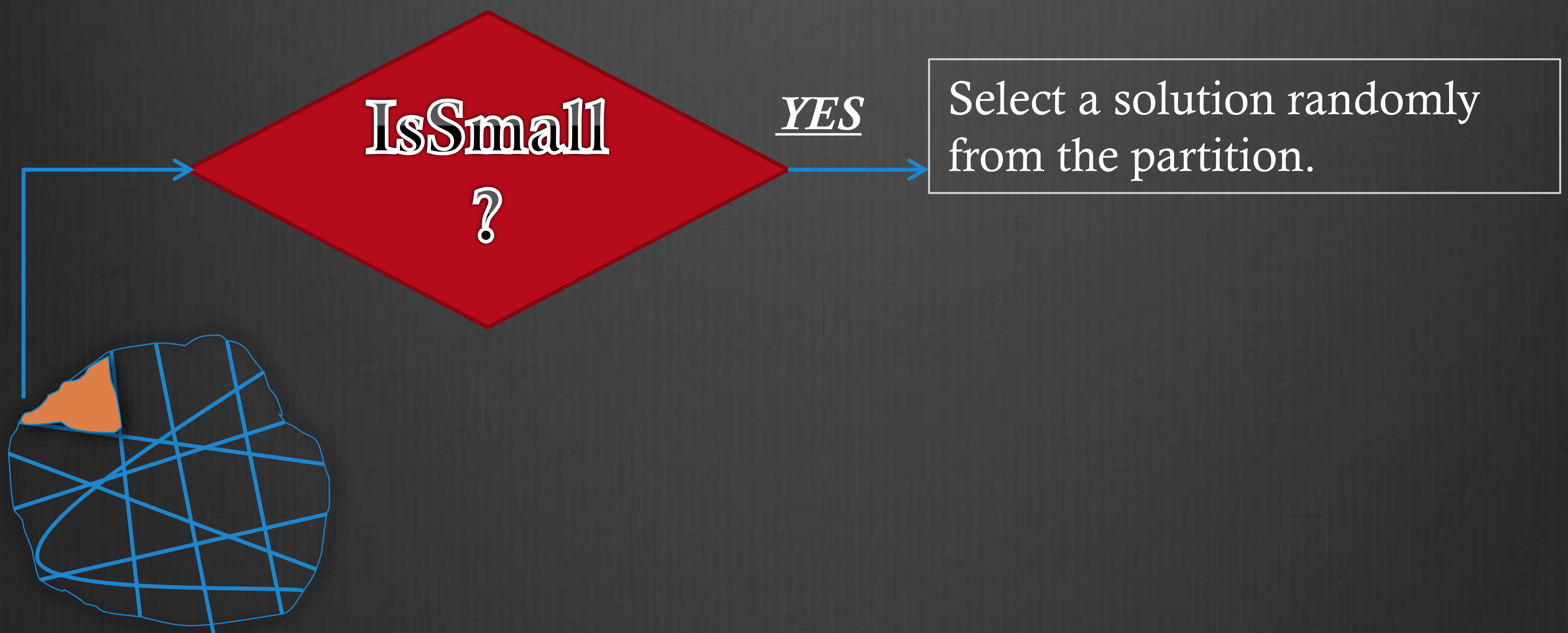


Strong Theoretical Guarantees

- Near-Uniformity

For every solution y of R_F

$1/(6.84+\varepsilon) \times 1/|R_F| \le \Pr[\text{y is output}] \le (6.84+\varepsilon)/|R_F|$

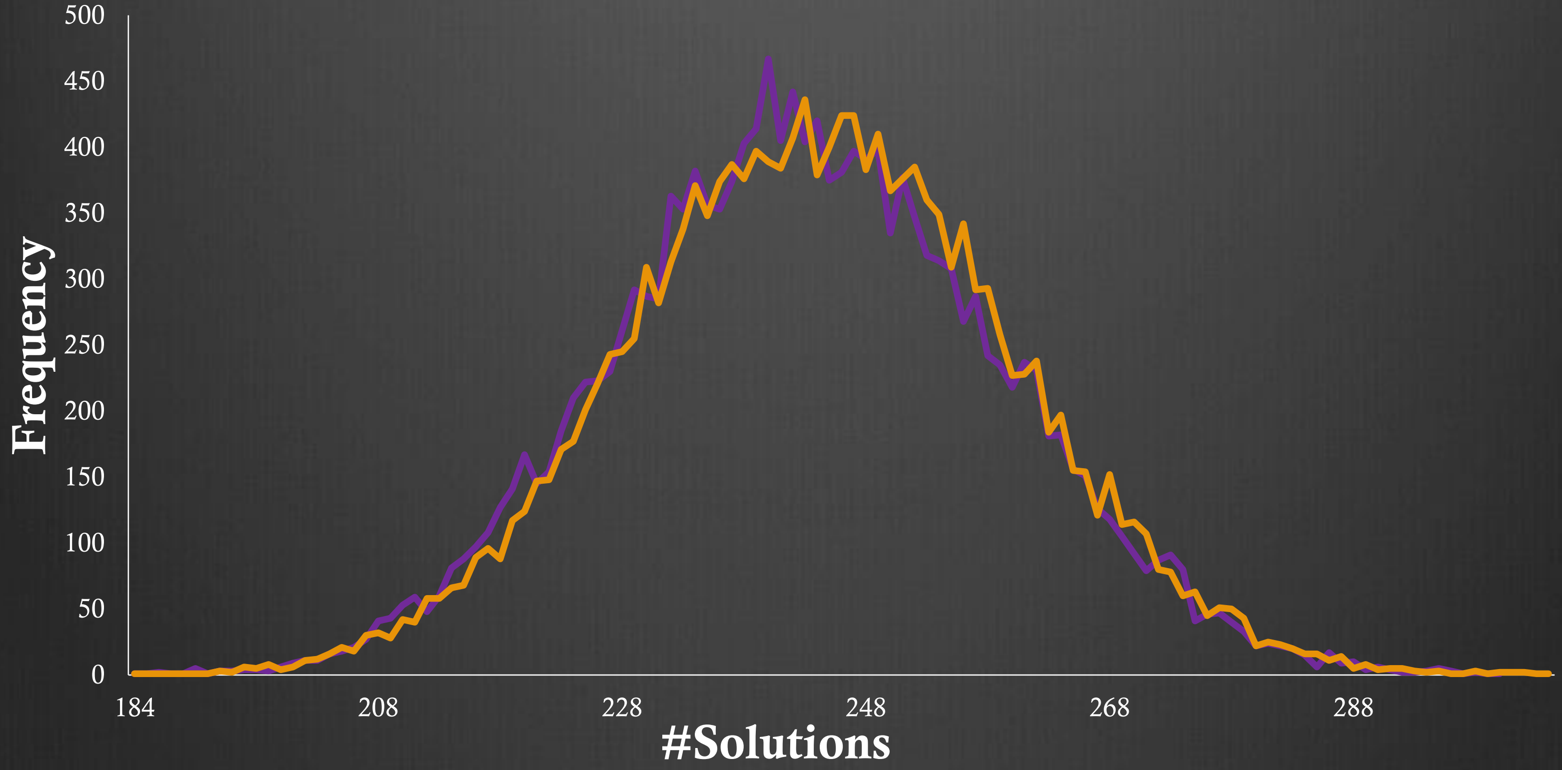
- Success Probability

UniGen succeeds with probability at least 0.52

  - In practice, succ. probability > 0.9

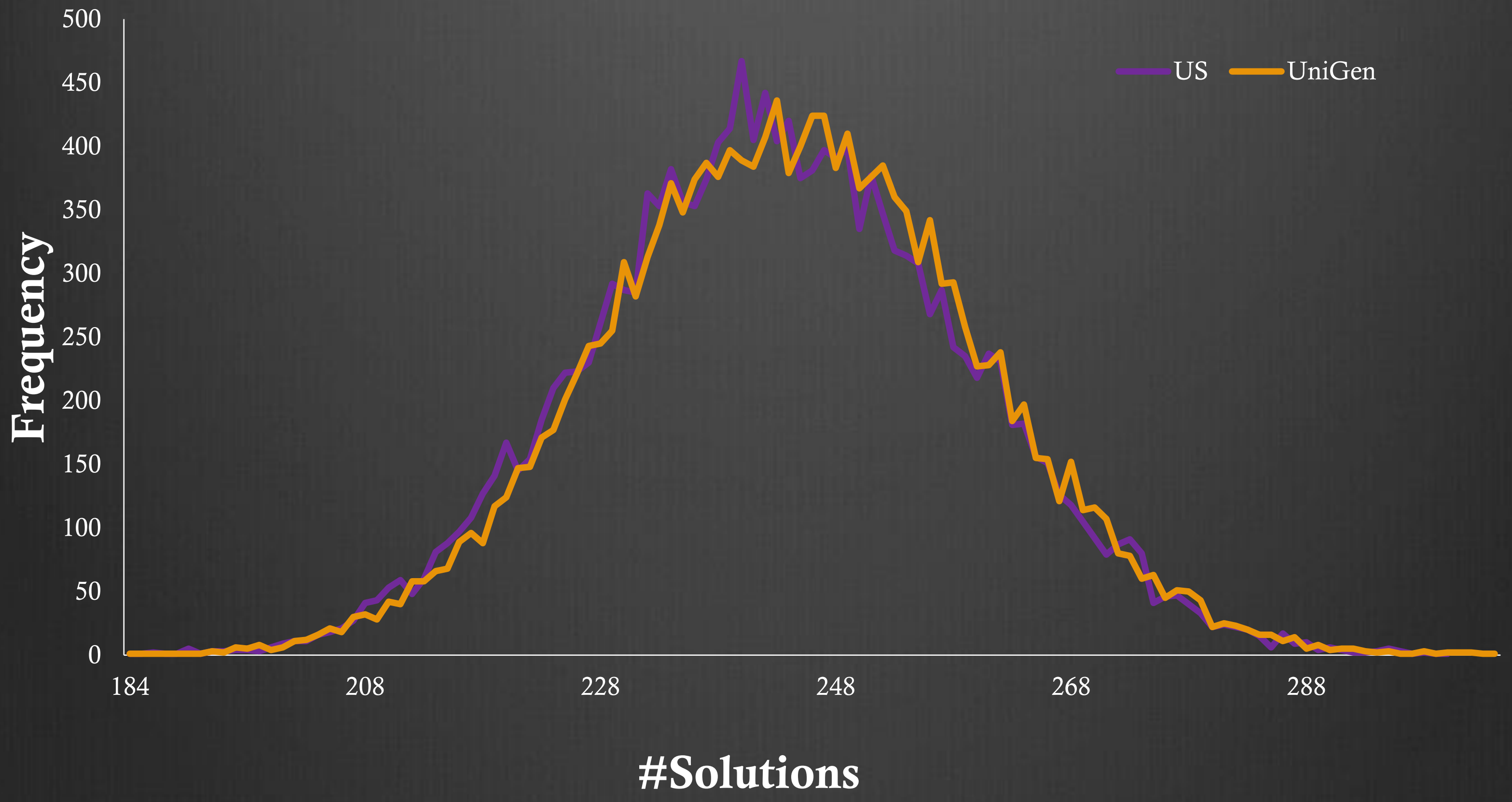
- Polynomial calls to SAT Solver

Results: Uniformity



- Benchmark: case110.cnf;   #var: 287; #clauses: 1263
- Total Runs**: $4x10^6$;** Total Solutions **: 16384**

Results: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
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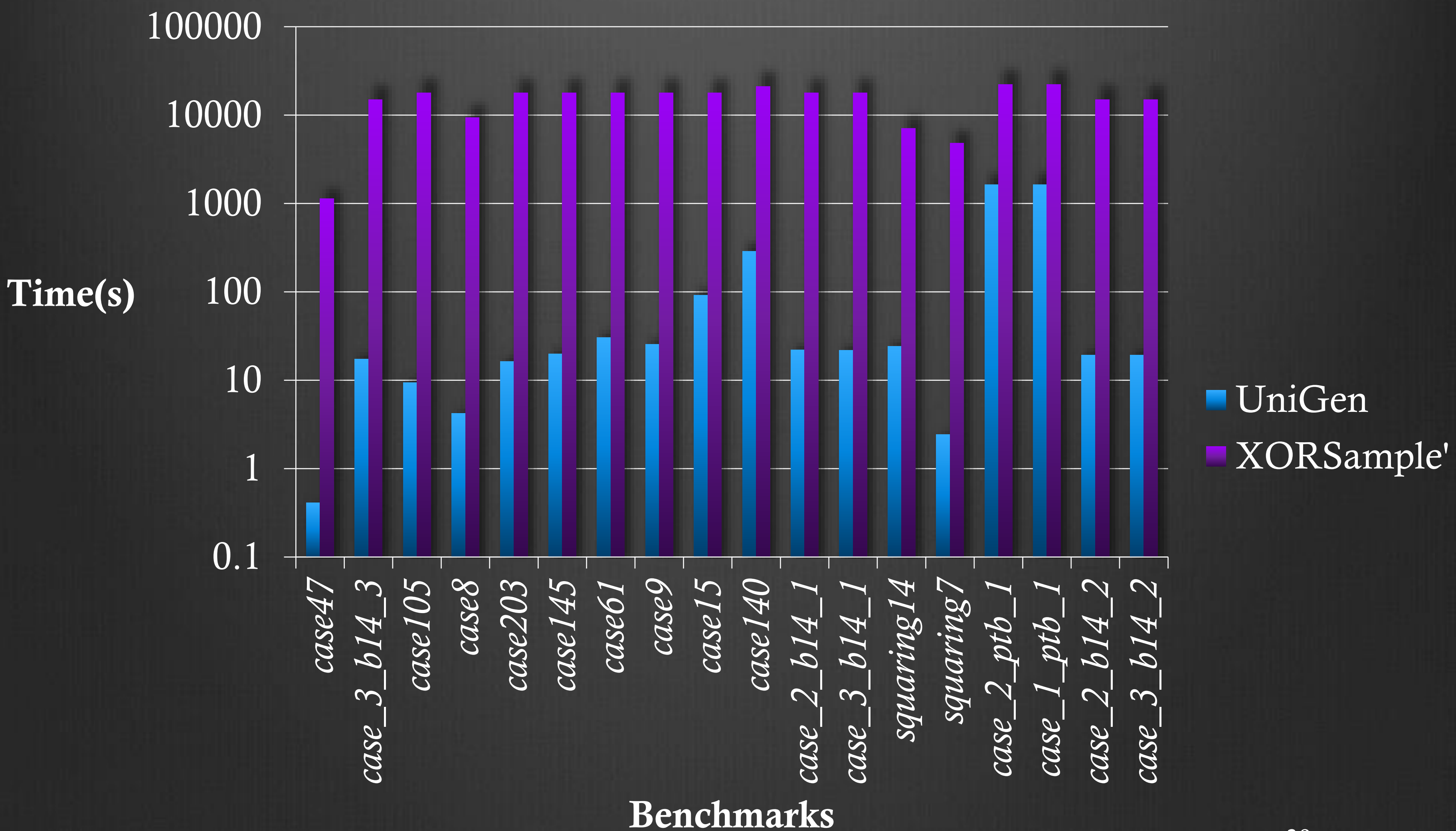
2-3 Orders of Magnitude Faster



Outline

- Sampling Techniques via Uniform Generation
- ***<u>Extension to model counting and biased sampling</u>***
- Discussion on hashing
- Future Directions

Approximate Model Counting

Ref: “A Scalable Approximate Model Counter” (CP 2013)

What is Model Counting?

- Given a SAT formula F
- R_F: Set of all solutions of F
- Problem (#SAT): Estimate the number of solutions of F (#F) i.e., what is the cardinality of R_F?
- E.g., F = (a v b)
- $R_F = \{(0,1), (1,0), (1,1)\}$
- The number of solutions (#F) = 3

#P: The class of counting problems for decision problems in NP!

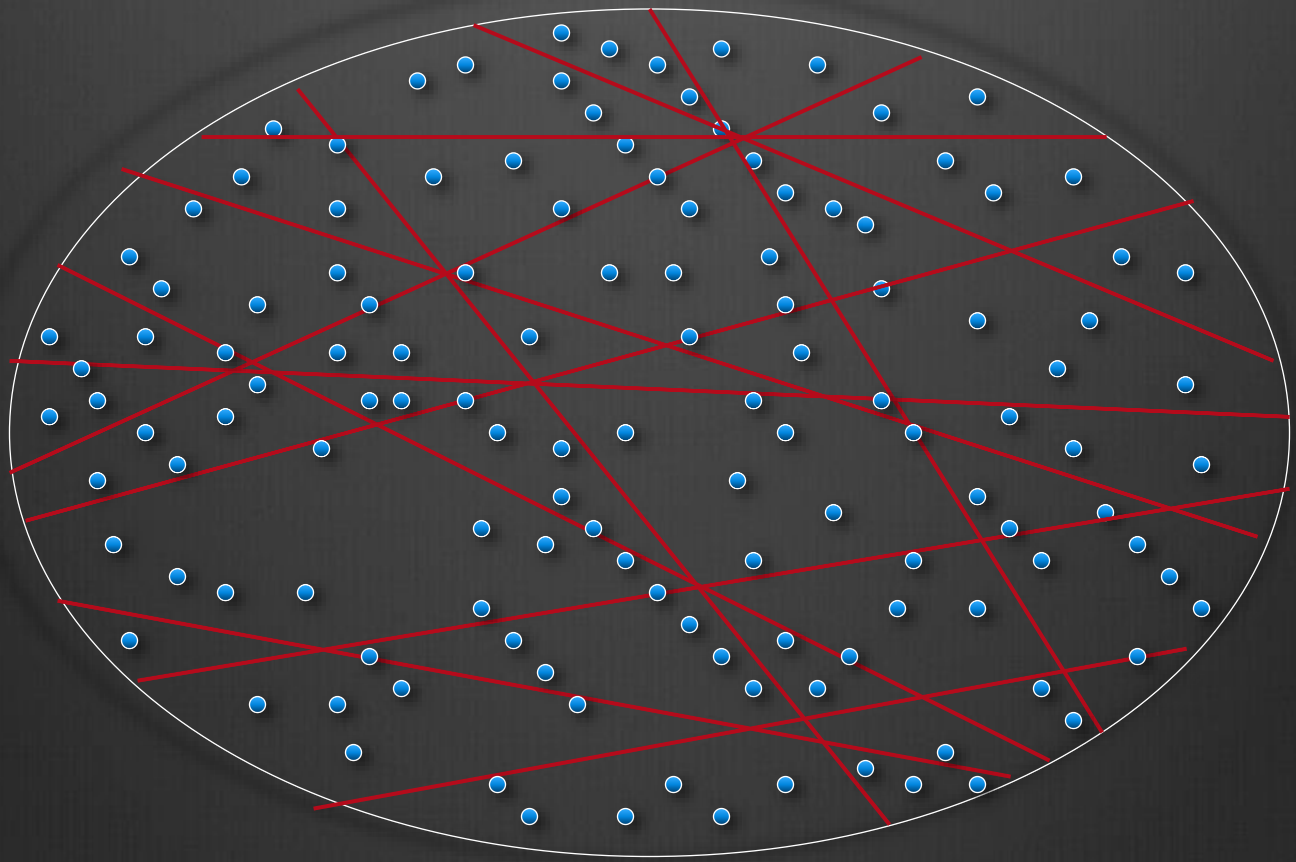
Practical Applications

Exciting range of applications!

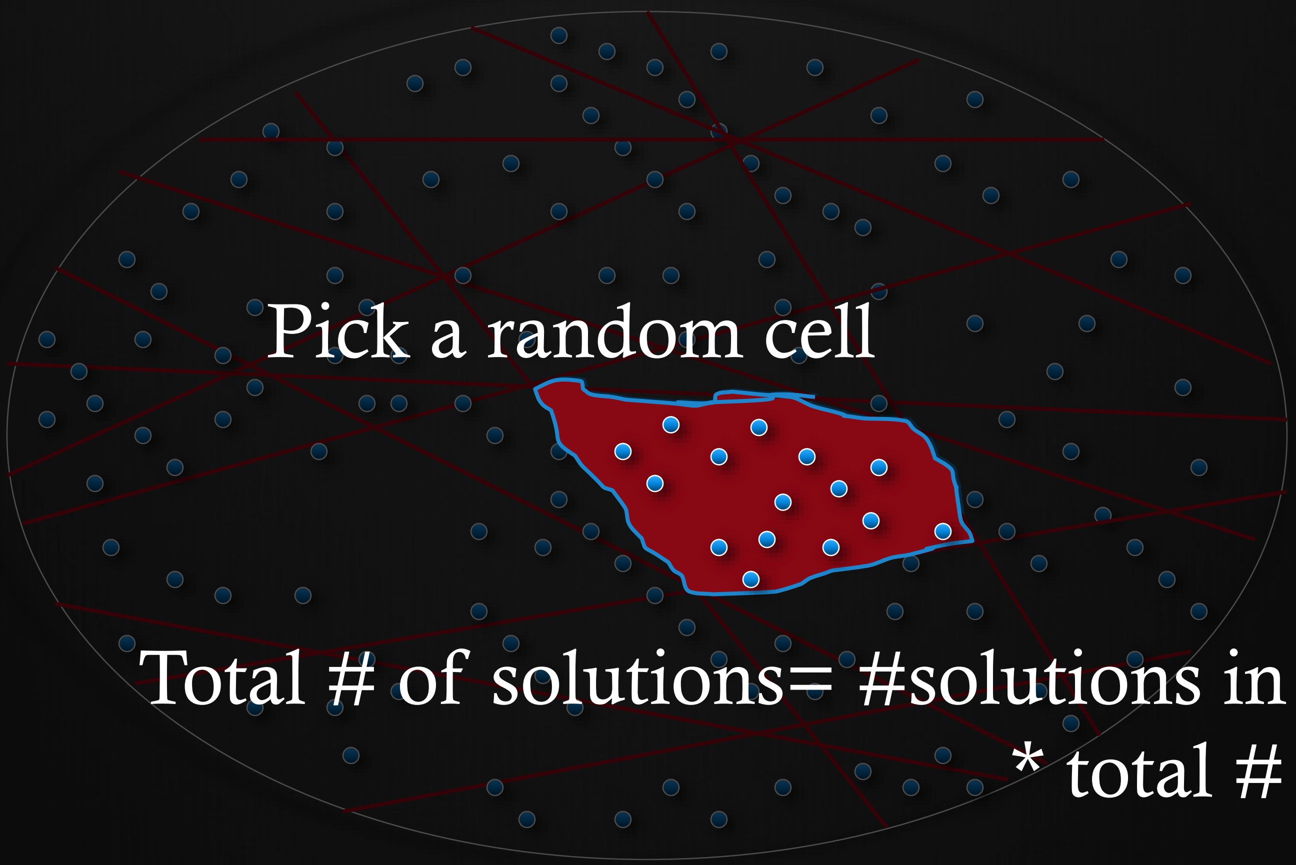
- Probabilistic reasoning/Bayesian inference
- Planning with uncertainty
- Multi-agent/ adversarial reasoning

[Roth 96, Sang 04, Bacchus 04, Domshlak 07]

Counting through Partitioning



Counting through Partitioning

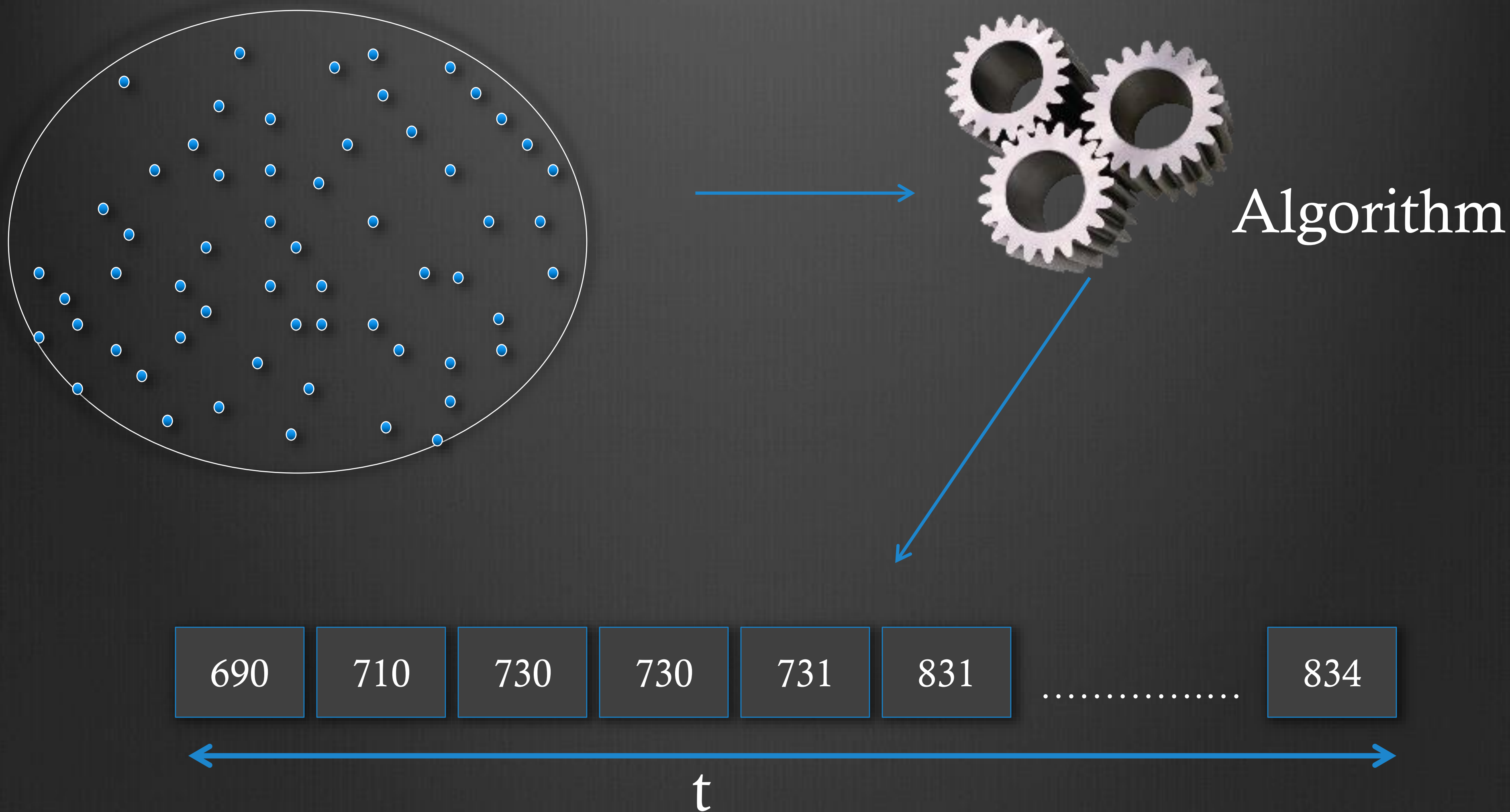


Pick a random cell

Total # of solutions= #solutions in the cell

* total # of cells

ApproxMC in Action



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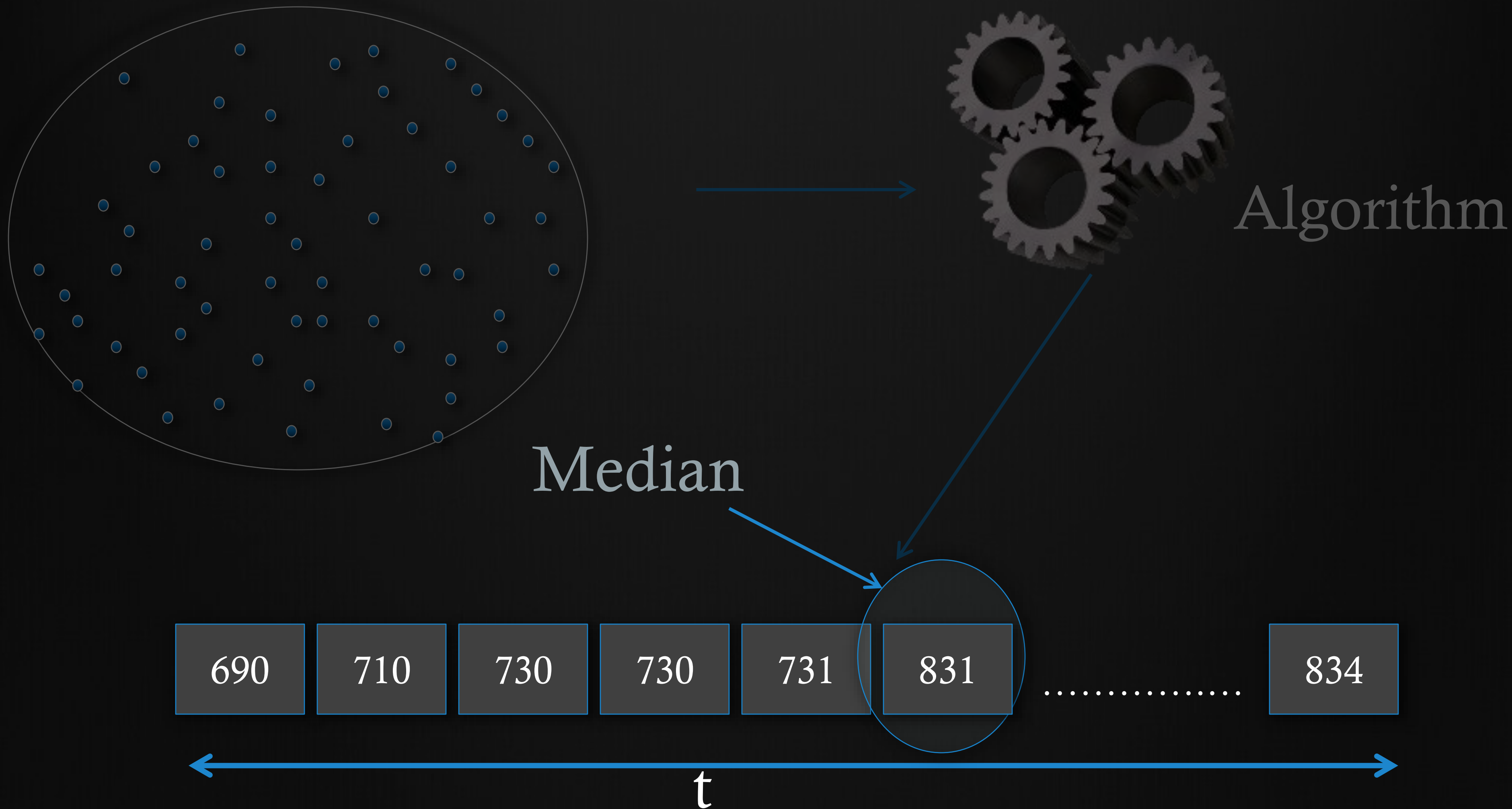


Strong Theoretical Results

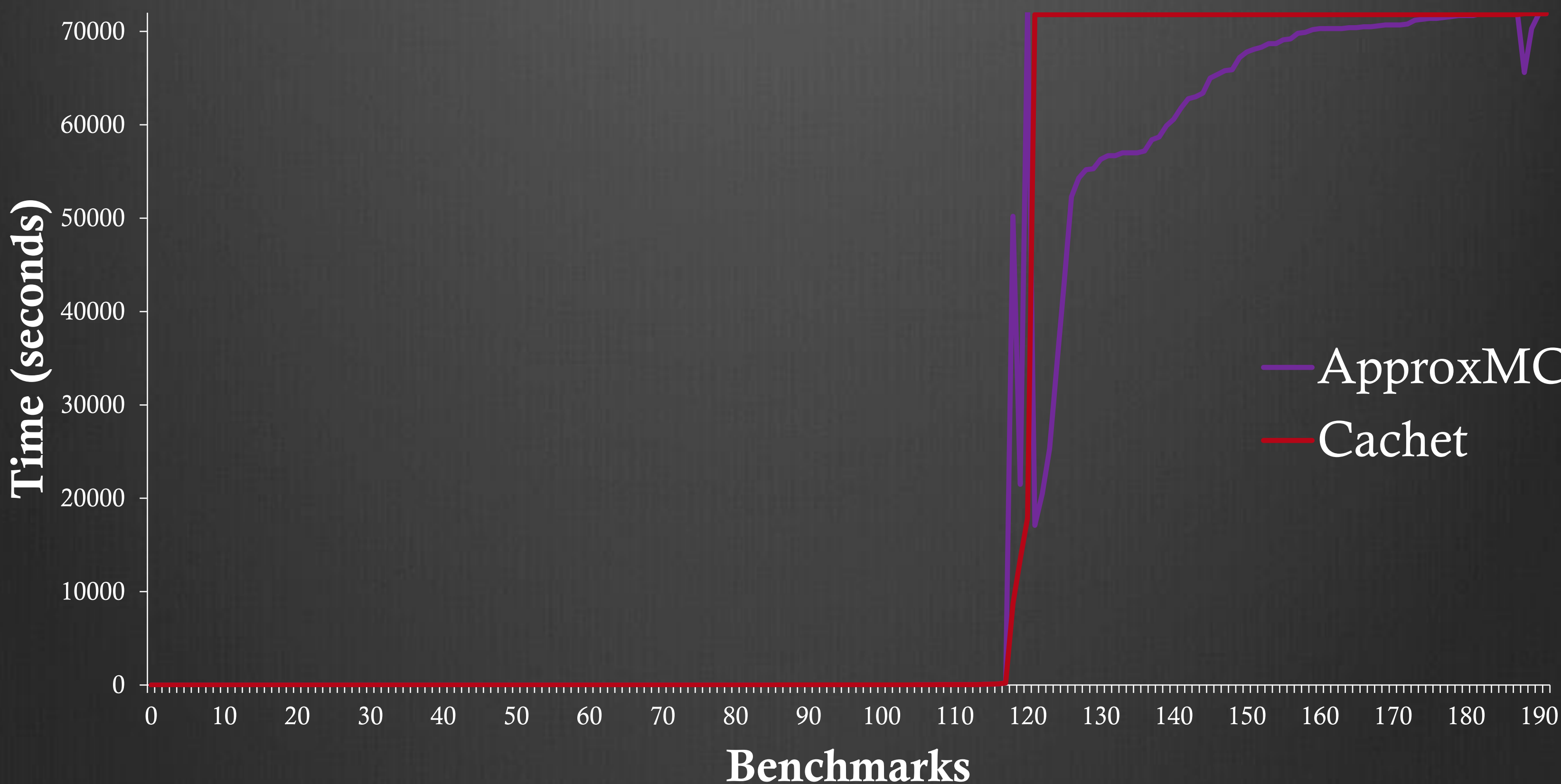
ApproxMC (CNF: F, tolerance: ε, confidence:δ)

Suppose ApproxMC(F,ε,δ) returns C. Then,

$$\mathbf{Pr}\ [\ \#F/(1+\varepsilon) <= C\ \ <= (1+\varepsilon)\ \#F\ ] \geq \delta$$

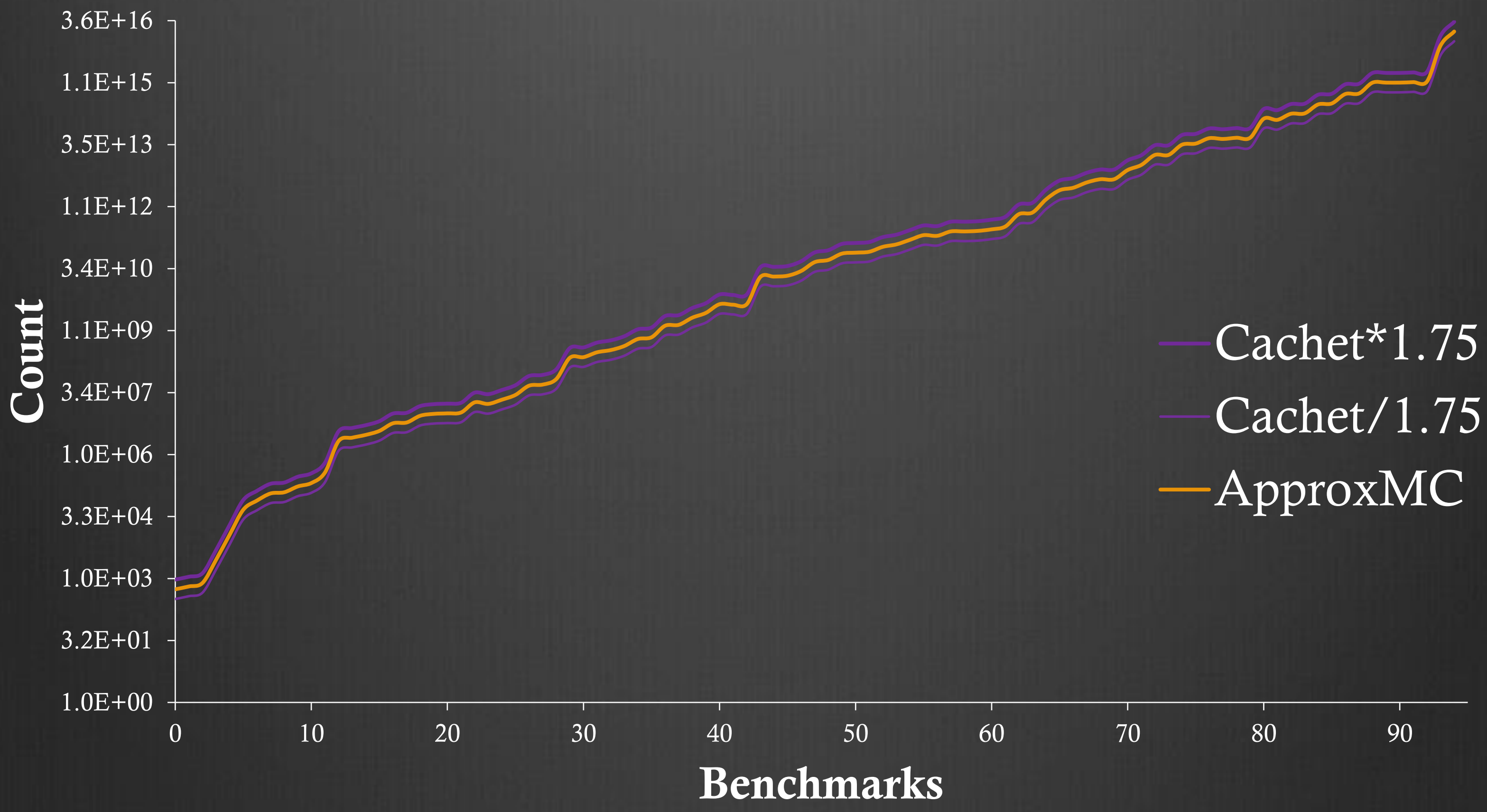
ApproxMC runs in time polynomial in $\log (1-\delta)^{-1}$, $|F|$, ε^{-1} relative to SAT oracle

Can Solve a Large Class of Problems



Large class of problems that lie beyond the exact counters but can be computed by ApproxMC

Mean Error: Only 4% (allowed: 75%)

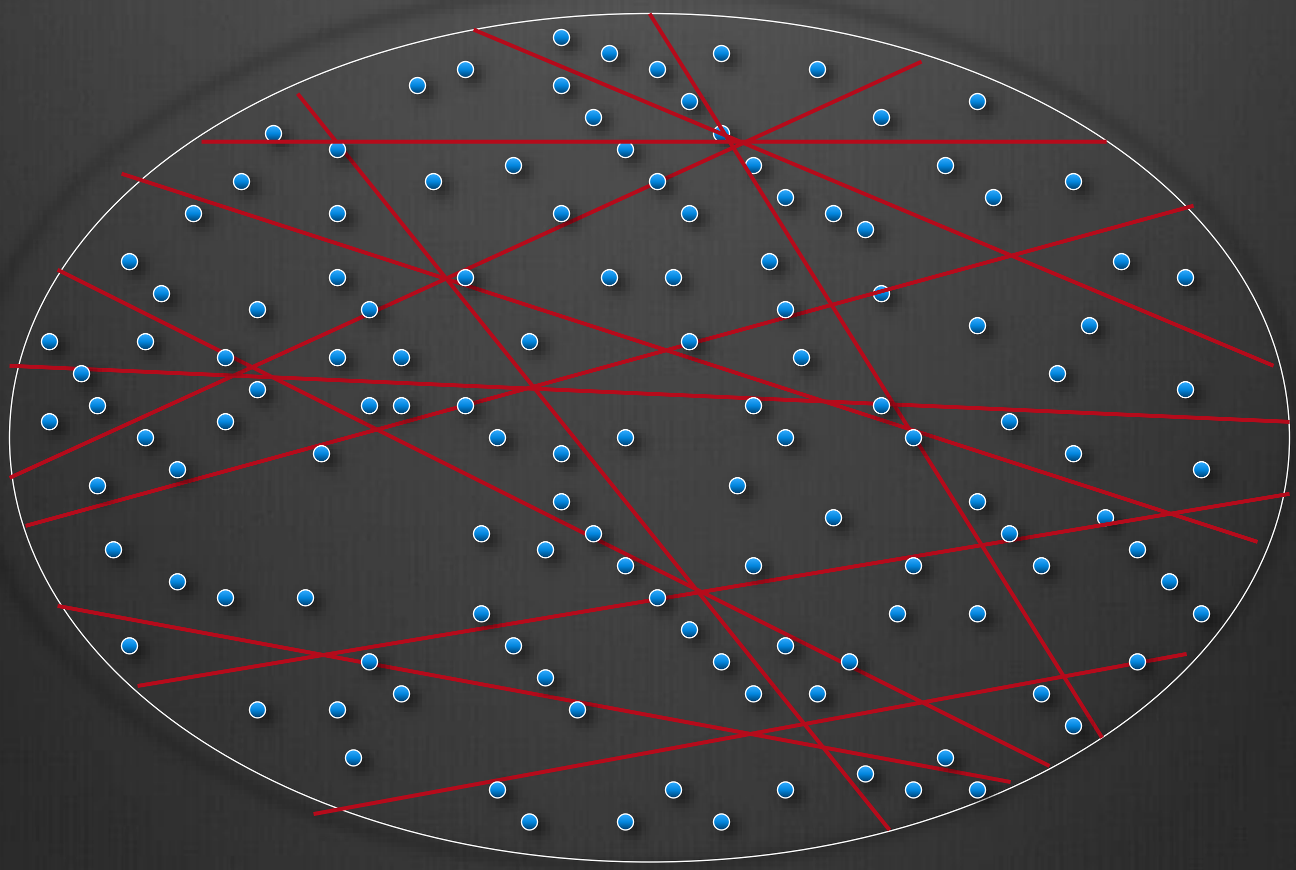


Mean error: 4% – much smaller than the theoretical guarantee of 75%

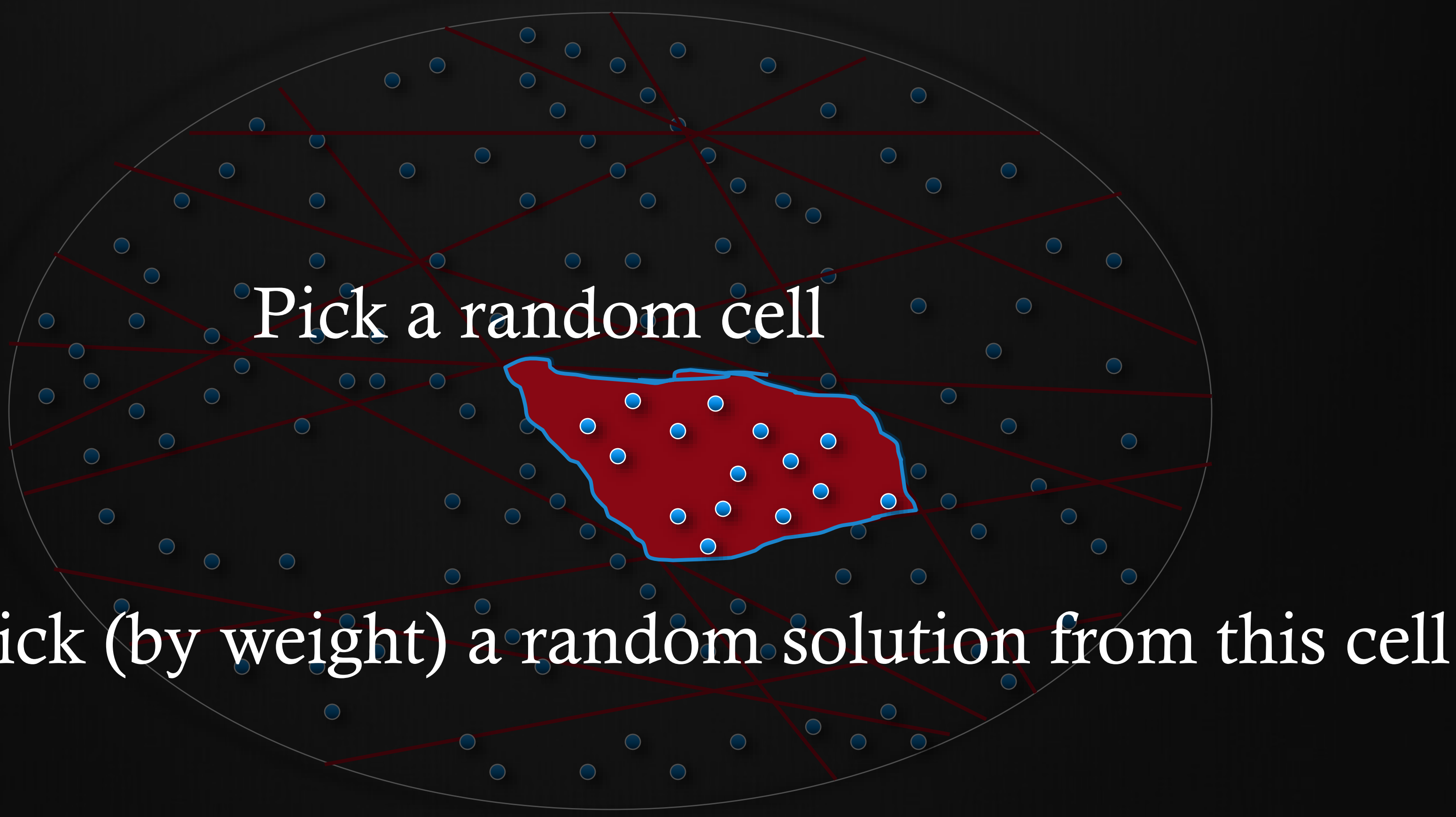
Weighted/Biased Sampling

Ref: "Distribution-Aware Sampling and Weighted Model Counting for SAT" (To Appear in AAAI 204)

Partition into (weighted) equal "small" cells



Partition into (weighted) equal "small" cells



Projection Counting/Sampling

- What if I care about only few variables?
- (a=0, b = 0, c = 1), (a = 0, b = 0, c=0), (a = 0, b = 1, c=0)
- Partition only on the projected subspace

Outline

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- Extension to model counting and biased sampling
- <u>*Discussion on hashing*</u>
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XOR-Based Hashing

- 3-universal hashing
- Partition 2^n space into 2^m cells
- Variables: $X_1, X_2, X_3, \ldots.., X_n$
- Pick every variable with prob. ½ ,XOR them and equate to 0/1 with prob. ½
- $X_1+X_3+X_6+\ldots.\ X_{n-1} = 0$ (Cell ID: 0/1)
- m XOR equations -> 2^m cells
- The cell: F && XOR (CNF+XOR)

XOR-Based Hashing

- CryptoMiniSAT: Efficient for CNF+XOR
- Avg Length : n/2
- Smaller the XORs, better the performance

How to shorten XOR clauses?

Independent Variables

- Set of variables such that assignments to these uniquely determine assignments to rest of variables for formula to be true
- (a V b = c) ➔ Independent Support: {a, b}
- # of auxiliary variables introduced: 2-3 orders of magnitude
- Hash only on the independent variables (huge speedup)

Future Directions

Extension to More Expressive Domains (SMT, CSP)

- Efficient 3-independent hashing schemes
  - Extending bit-wise XOR to SMT provides guarantees but no advantage of SMT progress
- Solvers to handle F + Hash efficiently
  - CryptoMiniSAT has fueled progress for SAT domain
  - Similar solvers for other domains?

Exploring CNF+XOR

- Very little understanding as of now
- Can we observe phase transition?
- Eager/Lazy approach for XORs?
- How to reduce size of XORs further?

Potentially New Connections

- Near-Uniformity

For every solution y of R_F

$1/(6.84+\varepsilon) \times 1/|R_F| <= \Pr$ [y is output] $<= (6.84+\varepsilon) / |R_F|$

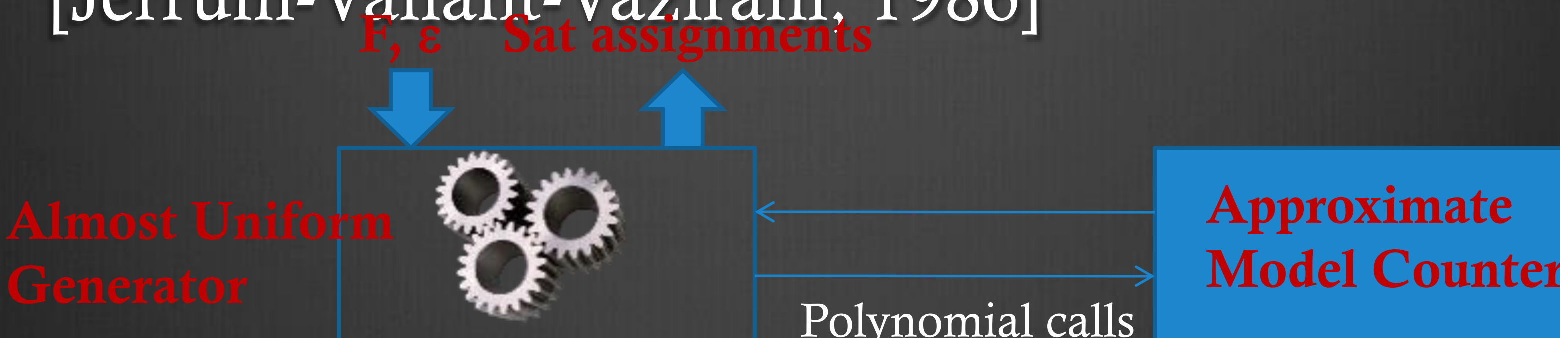
- Almost-Uniformity

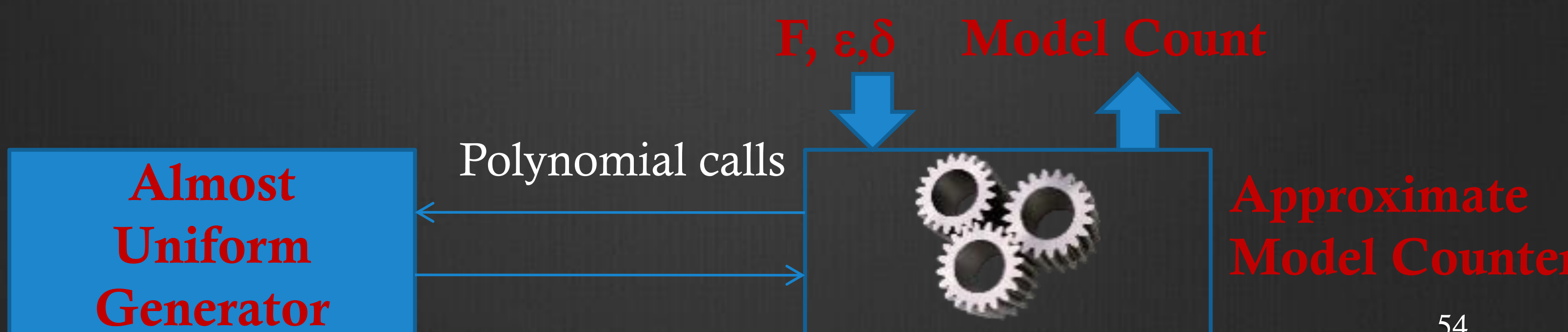
For every solution y of R_F

$1/(1+\varepsilon) \times 1/|R_F| <= \Pr$ [y is output] $<= (1+\varepsilon) / |R_F|$

Potentially New Connections

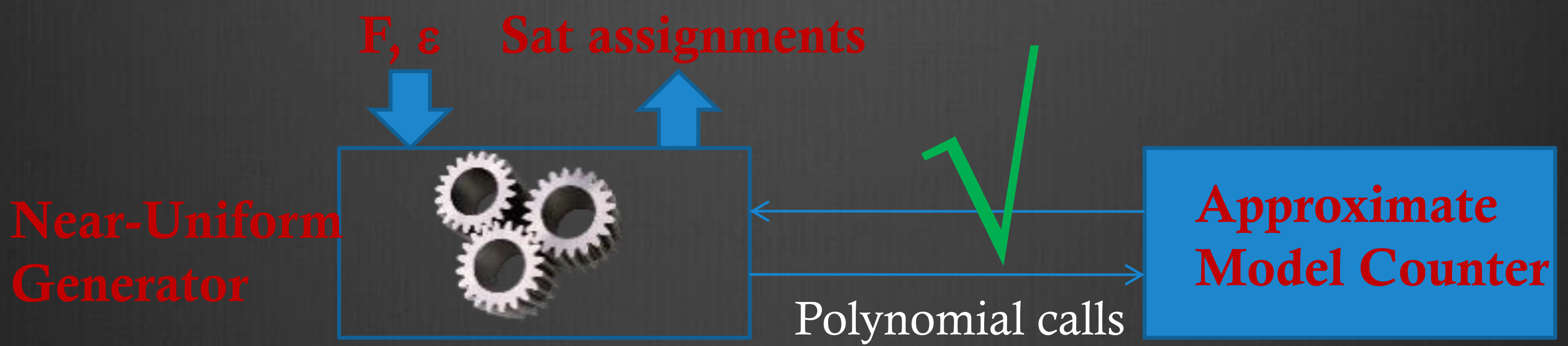
- Polynomial inter-reducibility of near-uniform generation and approximate model counting [Jerrum-Valiant-Vazirani, 1986]

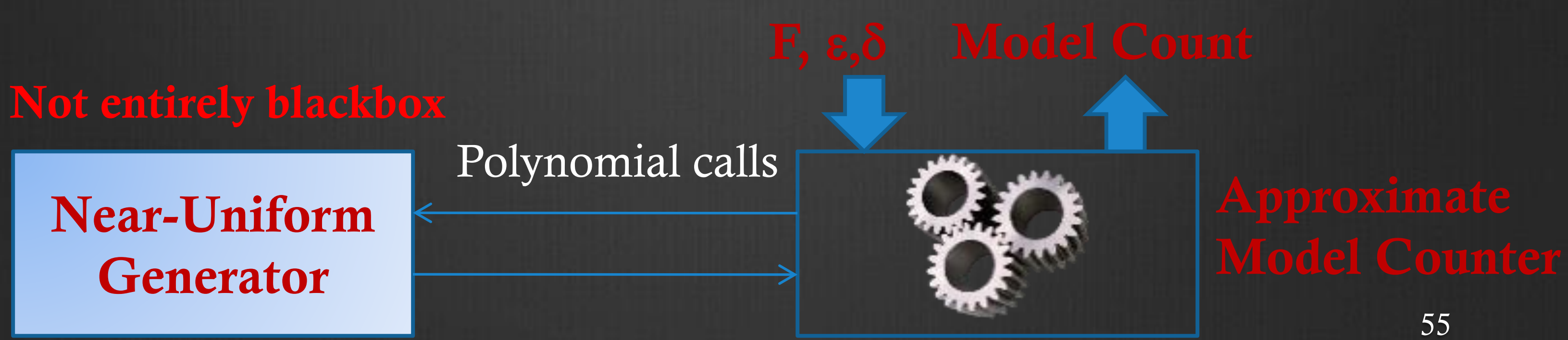




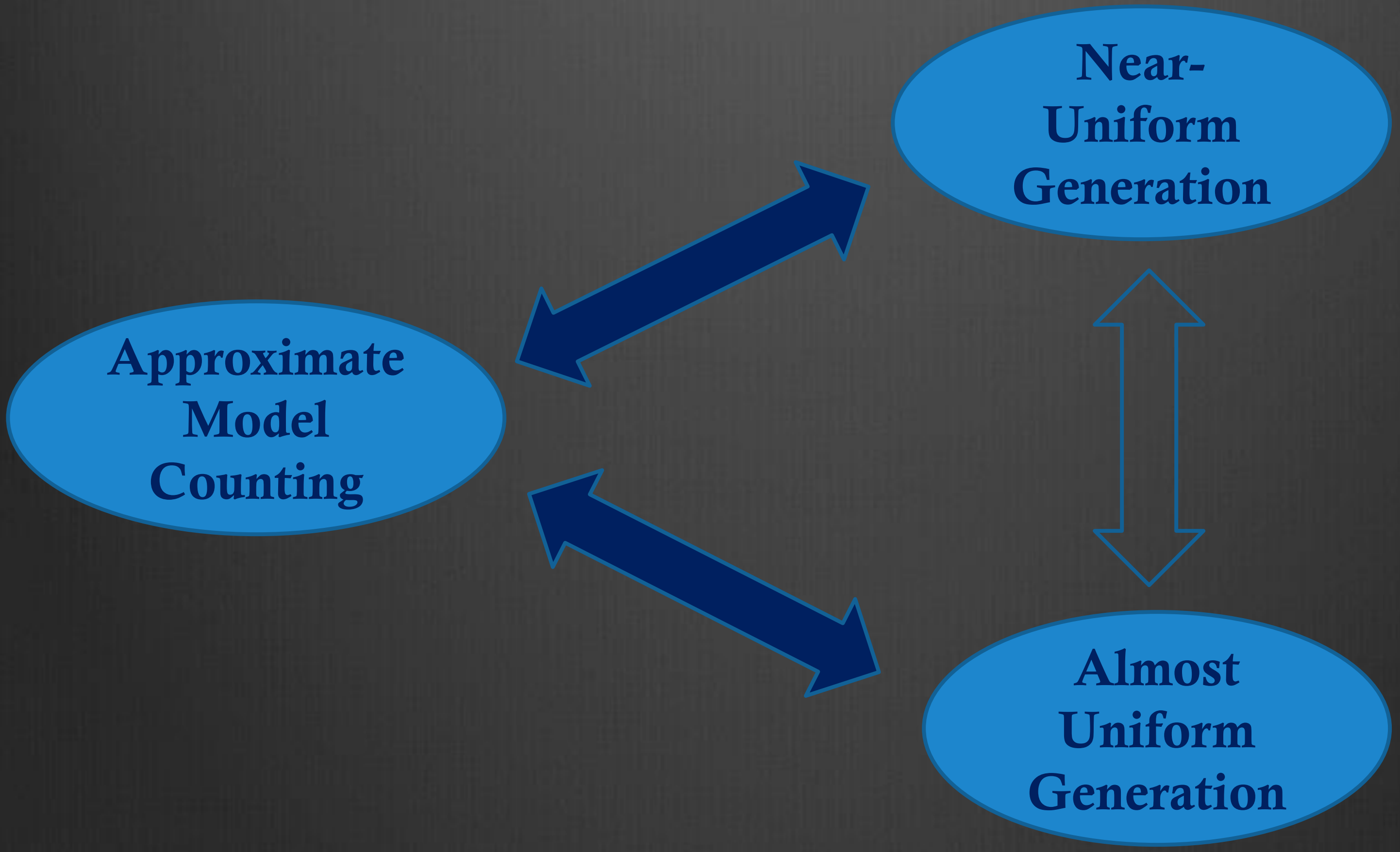
Potentially New Connections

- Is there a similar relation between near-uniform generation (much weaker than almost uniform generation) and approximate model counting?





Some Questions?



Publications

- S. Chakraborty, D. J. Fremont, K.S. Meel, S.A. Seshia, M.Y. Vardi "Distribution-Aware Sampling and Weighted Model Counting for SAT " In Proc. of AAAI 2014

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- S. Chakraborty, K.S. Meel, M.Y. Vardi "A Scalable Approximate Model Counter" In Proc. of CP 2013

Collaborators

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Impact of Independent Variables