A Round-Efficient Distributed Betweenness Centrality Algorithm

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Betweenness Centrality

- Betweenness Centrality (BC) used to determine relative importance of node in graph
- Applications
  - Key actor detection in terrorist nets
  - Disease studies
  - Power grid analysis
  - River flow confluence
- Distributed implementations necessary
  - Large graphs with billions of nodes/edges
  - BC takes hours to complete even if approximating

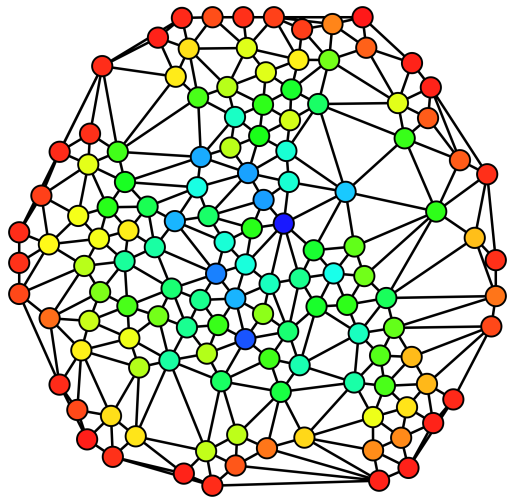
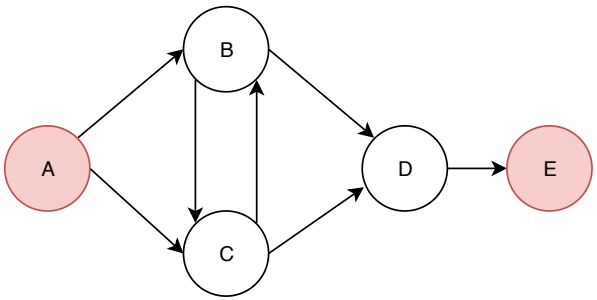


Figure Credit: Claudio Rocchini, Creative Commons Attribution 2.5 Generic

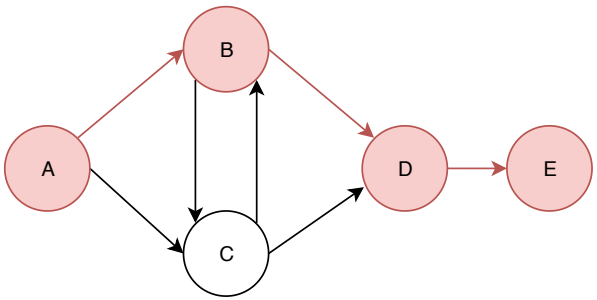
Betweenness Centrality Definition



- BC: fraction of shortest paths in which node appears

- Example: consider the 2 shortest paths from A to E:
  - B appears in 1: $\frac{1}{2}$; C appears in 1: $\frac{1}{2}$; D appears in 2: $\frac{2}{2} = 1$

Betweenness Centrality Definition



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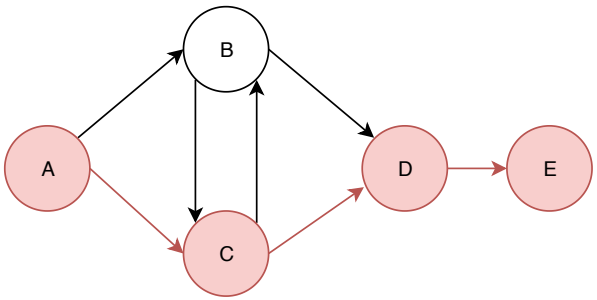

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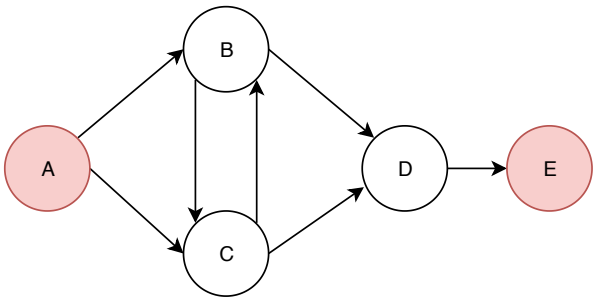

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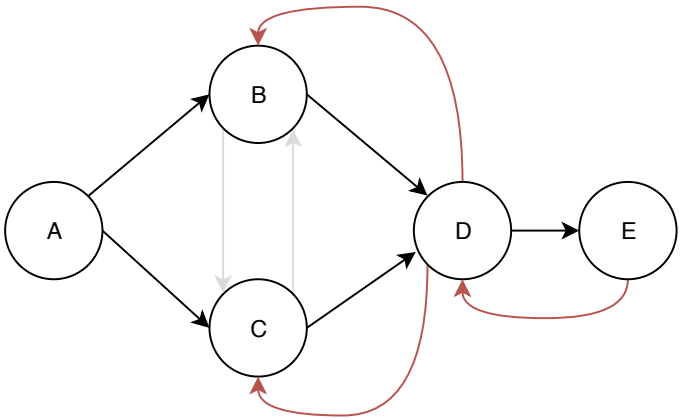
σ_{st}, number of shortest paths from s to t; $\sigma_{st}(v)$, number of shortest paths from s to t passing through v, $v \neq s \neq t$.

Betweenness Centrality (BC)

$$BC(v) = \sum_{s \neq t \neq v} \frac{\sigma_{st}(v)}{\sigma st}$$

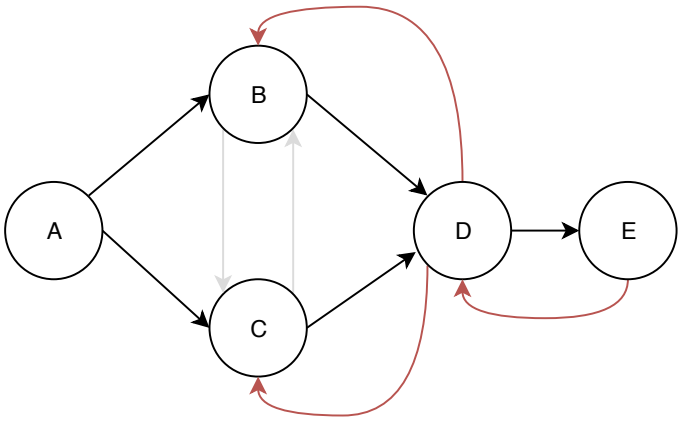
From definition: about n^3 operations (n is number of vertices)

Brandes Betweenness Centrality



- Shortest-path DAG with shortest path counts rooted at node s: propagate dependencies ($\delta_{s\bullet}$) along DAG predecessors

Brandes Betweenness Centrality



- Shortest-path DAG with shortest path counts rooted at node s: propagate dependencies ($\delta_{s\bullet}$) along DAG predecessors

BC from Dependencies of a Node

$$BC(v) = \sum_{s \neq v} \delta_{s\bullet}(v)$$

$$\text{where} \quad \delta_{s\bullet}(v) = \sum_{w:v \in P_s(w)} \frac{\sigma_{sv}}{\sigma_{sw}} \cdot (1 + \delta_{s\bullet}(w))$$

$P_s(w)$ are predecessors of w in DAG

- Brandes BC [1]: sum dependencies from all DAGs: $O(nm)$ operations (m is number of edges)
- All-pairs shortest paths (APSP) or k-source shortest paths (k-SSP, shortest paths for subset of k nodes) to find DAGs

[1] U. Brandes. A Faster Algorithm for Betweenness Centrality. Journal of Mathematical Sociology 2001.

Related APSP and BC Work

- APSP
  - $O(n)$ round undirected, unweighted APSP algorithms [2,3,4]
    - Lenzen-Peleg: prior best unweighted APSP
- BC
  - Asynchronous Brandes BC (ABBC): asynchronous, shared-memory [5]
  - Maximal Frontier BC (MFBC): distributed, sparse-matrix Brandes BC [6]
  - Hua et al.: distributed BC for undirected, unweighted graphs [7]

[2] S. Holzer and R. Wattenhofer. Optimal Distributed All Pairs Shortest Paths and Applications. PODC 2012.
[3] D. Peleg, L. Roditty, and E. Tal. Distributed Algorithms for Network Diameter and Girth. ICALP 2012.
[4] C. Lenzen and D. Peleg. Efficient Distributed Source Detection with Limited Bandiwidth. PODC 2013
[5] D. Prountzos and K. Pingali. Betweenness centrality: algorithms and implementations. PPoPP'13.
[6] E. Solomonik, M. Besta, F. Vella, and T. Hoefler. Scaling Betweenness Centrality Using Communication-efficient Sparse Matrix Multiplication.
[7] Q. S. Hua, H. Fan, M. Ai, L. Qian, Y. Li, X. Shi, and X. Jin. Nearly Optimal Distributed Algorithm for Computing Betweenness Centrality. ICDCS 2016.

Motivation for Our Work

- Practical implementations of theoretical, distributed O(n)-round APSP/BC algorithms do not exist
- Existing distributed BC mainly use SSSP/k-SSP with Brandes BC
  - High amount of bulk-synchronous parallel (BSP) rounds with expensive communication barriers

Tradeoff exploration:
decreasing number of rounds at cost of increasing computation per round

Our Contributions: Theory

Min-Rounds APSP and Min-Rounds Betweenness Centrality (MRBC) for directed and undirected unweighted graphs

- CONGEST: (known) n nodes, m edges, diameter D: APSP in $min(n + O(D), 2n)$ rounds and $mn + O(m)$ messages

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- In systems that detect termination: k-SSP in at most $k + H$ rounds and $m \cdot k$ messages, H is largest finite shortest path distance for the k sources

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- In systems that detect termination: k-SSP in at most $k + H$ rounds and $m \cdot k$ messages, H is largest finite shortest path distance for the k sources
- BC: at most twice the rounds/messages as APSP/k-SSP

Our Contributions: Practice

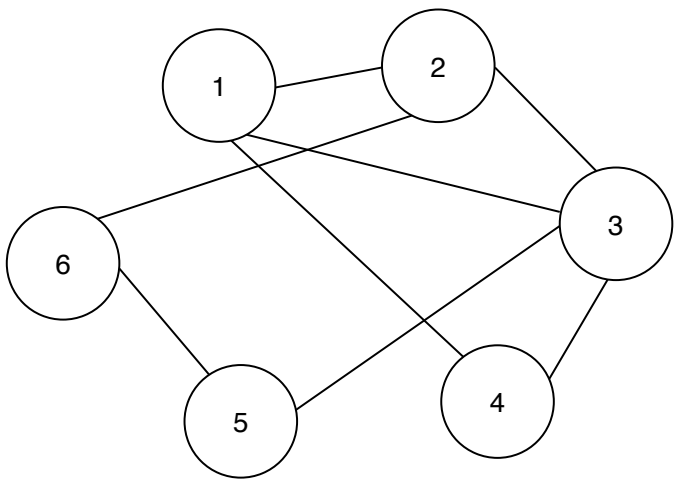
- MRBC implementation in D-Galois[8] with communication optimization exploiting MRBC properties
- MRBC evaluation
  - 3× faster than prior state-of-the-art MFBC
  - 2.8× speedup over Brandes BC on high diameter graphs

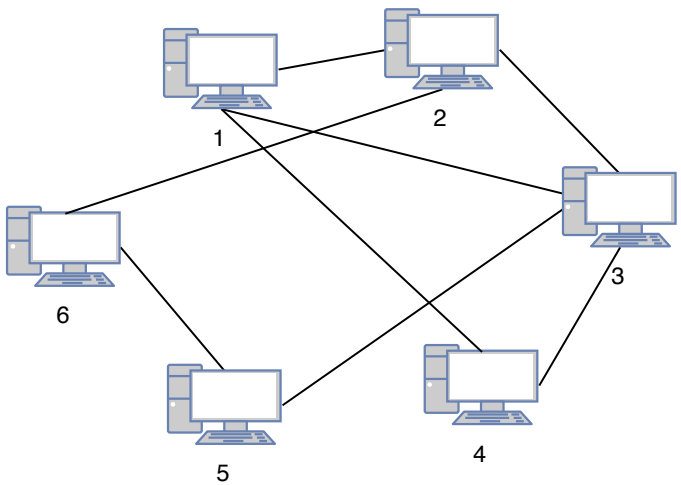
[8] R. Dathathri, G. Gill, L. Hoang, H.V. Dang, A. Brooks, N. Dryden, M. Snir, K. Pingali. Gluon: A Communication-Optimizing Substrate for Distributed Heterogeneous Graph Analytics. PLDI 2018.

Outline

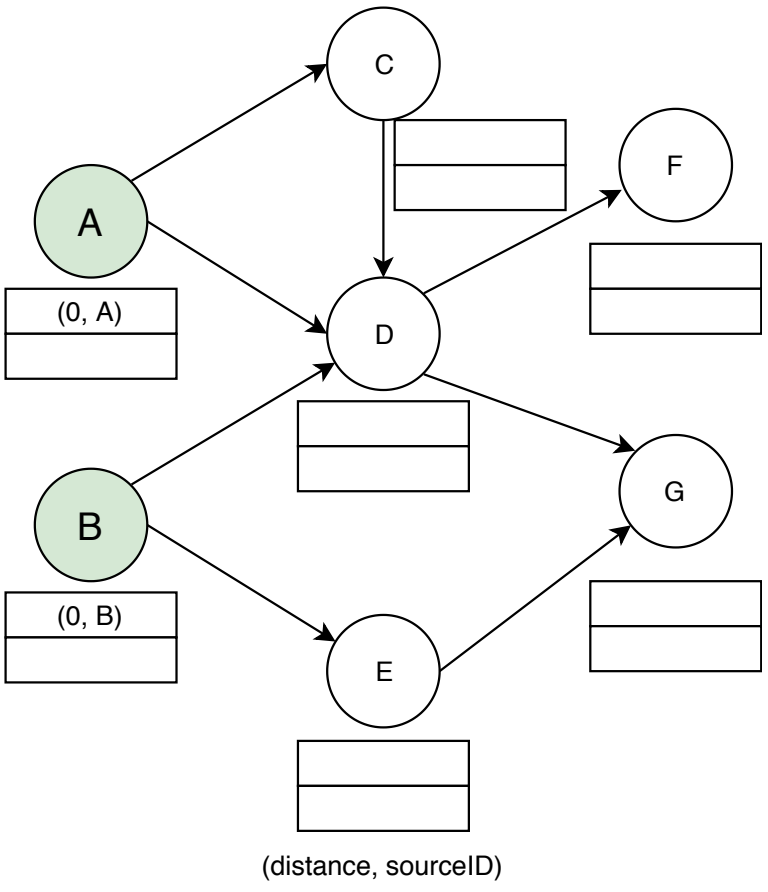
CONGEST Model for Distributed Algorithms

- Machines are nodes, edges are communication channels
- Send message (constant number of words) per round to do updates





k-SSP Example: Initial State



- Left: Initial State of k-SSP where $k = 2$ sources A and B
- Vertices store current distance from a source to self in lexicographically sorted vector
- Every round, vertex chooses 1 (distance, source) pair to send along outgoing edges

APSP: When To Send A Pair?

- Problem: sent distance may not be final distance associated with source

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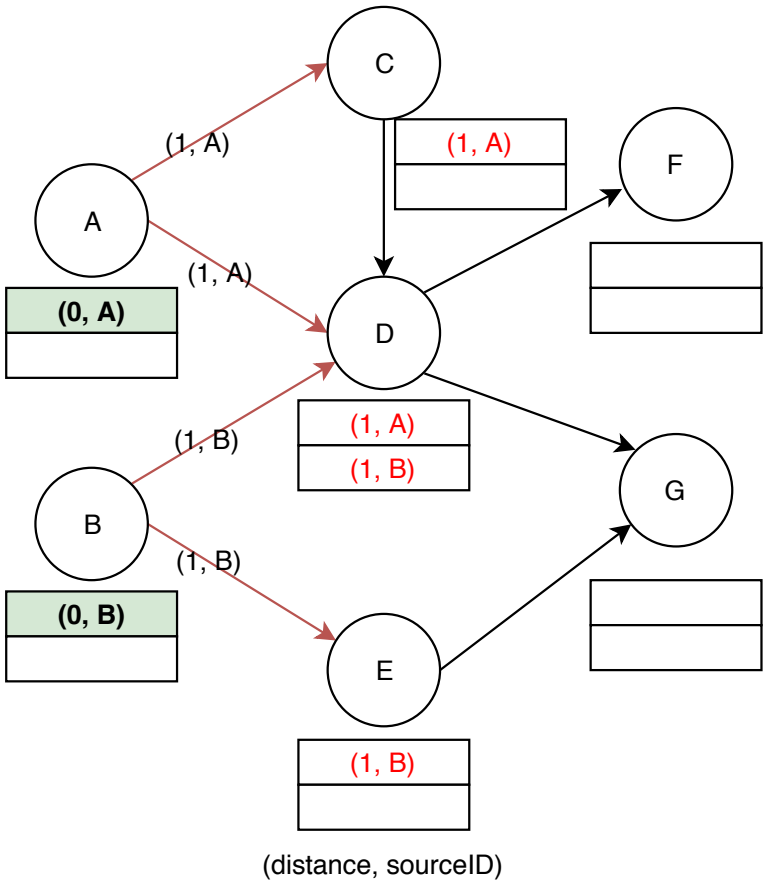
Min-Rounds APSP New Insight: Message Send Rule

Send unsent distance d with position p on sorted vector with corresponding source in round r if $p + d = r$

- Like Dijkstra: sends only final distance
- Resulting algorithm pipelines messages: orchestrates updates across edges and reduces amount of messages sent

k-SSP Example: Round 1

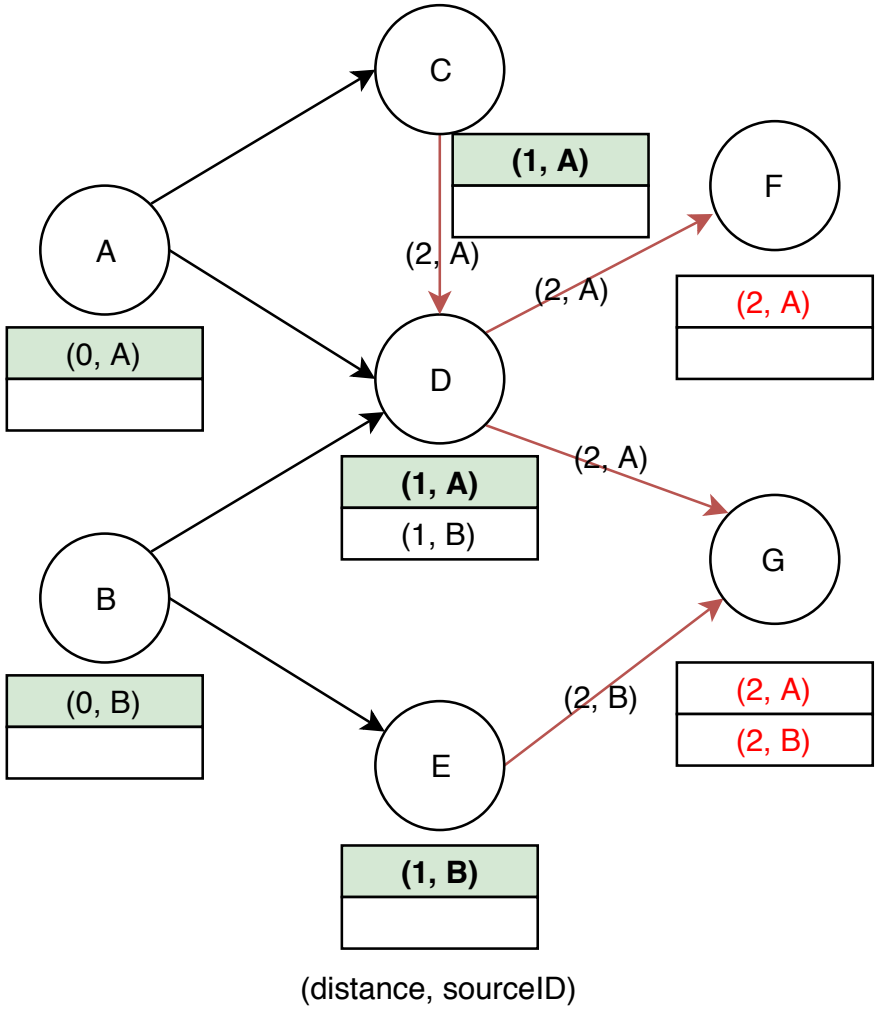


Message Send Rule

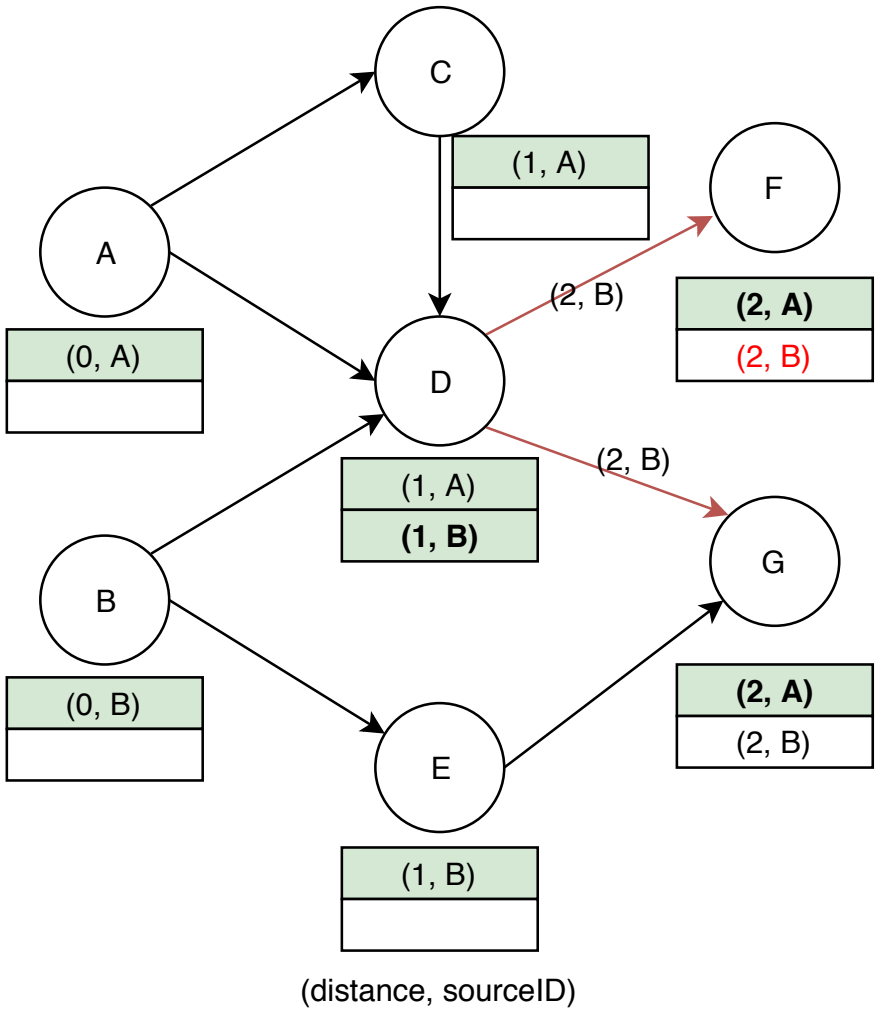
Send unsent distance d with position p on sorted vector with corresponding source in round r if $p + d = r$

- Example: $(0, A)$ chosen because $0 + 1$ (1 is position on vector) equals round 1

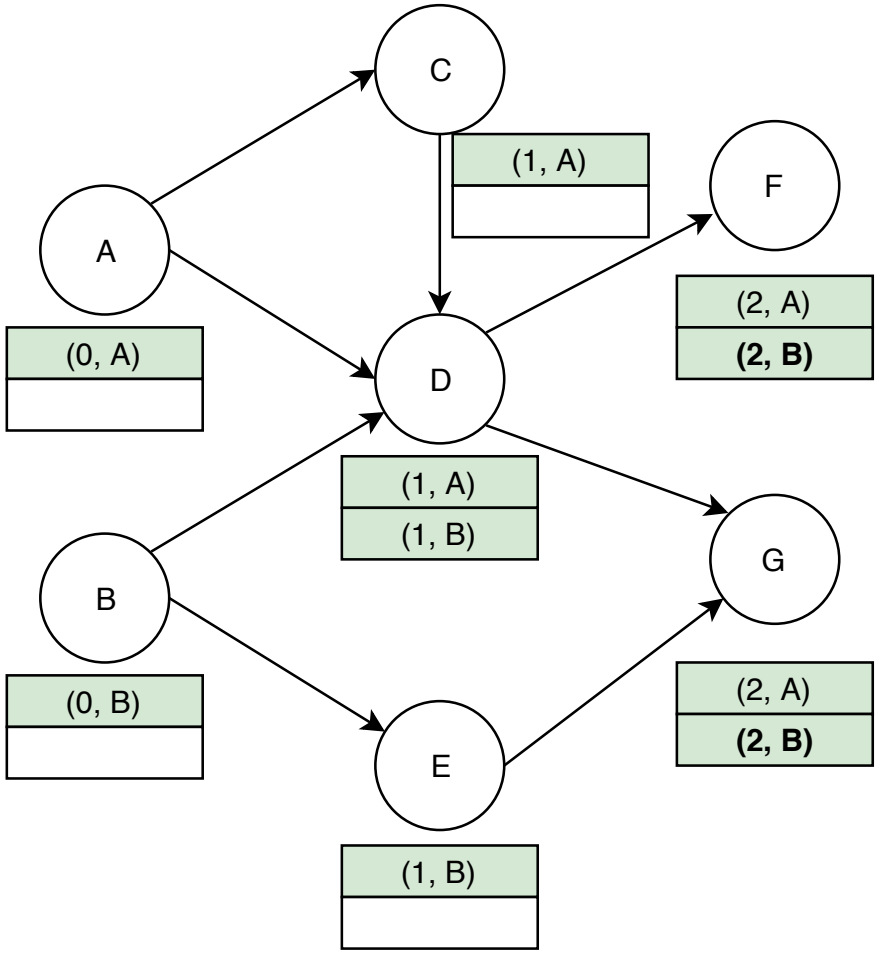
k-SSP Example: Round 2



k-SSP Example: Round 3



k-SSP Example: Round 4 (Final)



APSP for Brandes BC

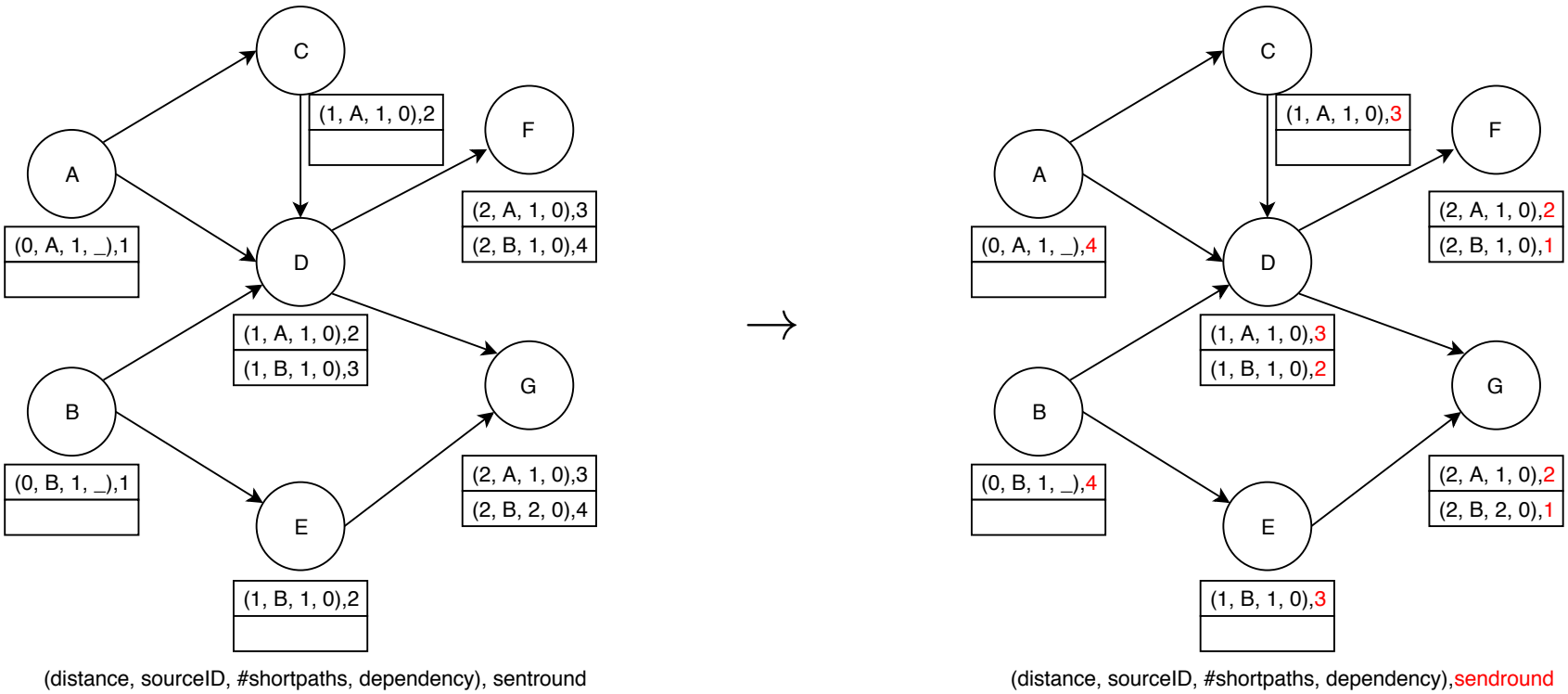
Min-Rounds APSP as subroutine for Brandes BC backward accumulation

Three Additions to APSP

- Send shortest path count with distance/source ID in APSP
- Timestamp round number in which message is sent
- Track predecessors of shortest path DAG for each source

Min-Rounds BC: Reversing Global Delays

Insight: leverage saved timestamps, send final values

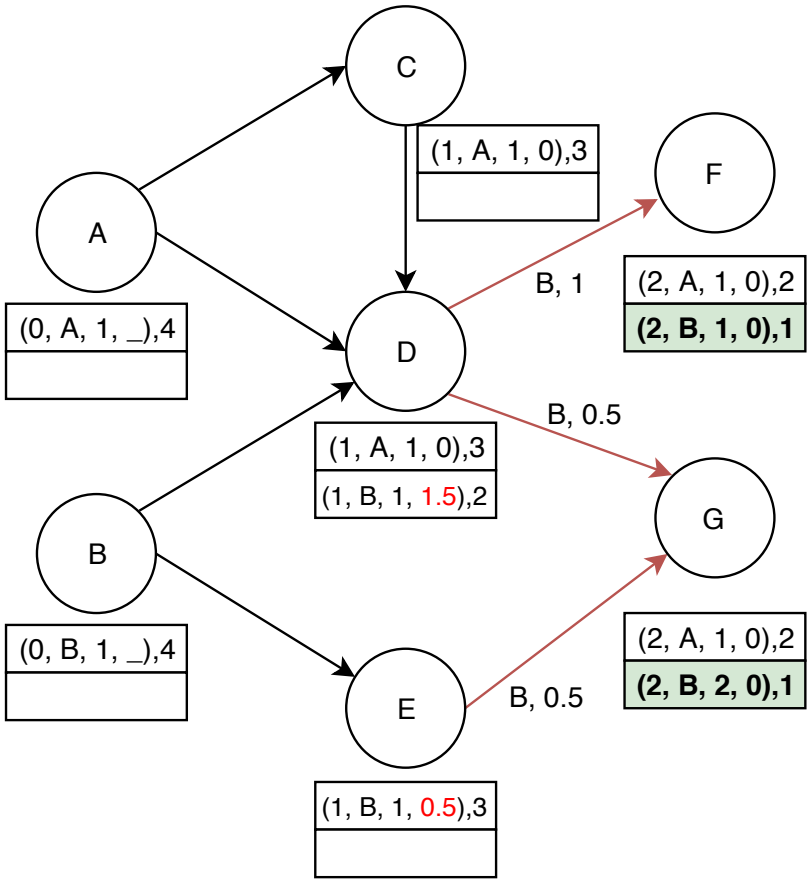


Timestamp Pipelining By Reversing Global Delay

Send source's dependency value to predecessors in source's DAG in *reverse round order*: total rounds + 1 - timestamp

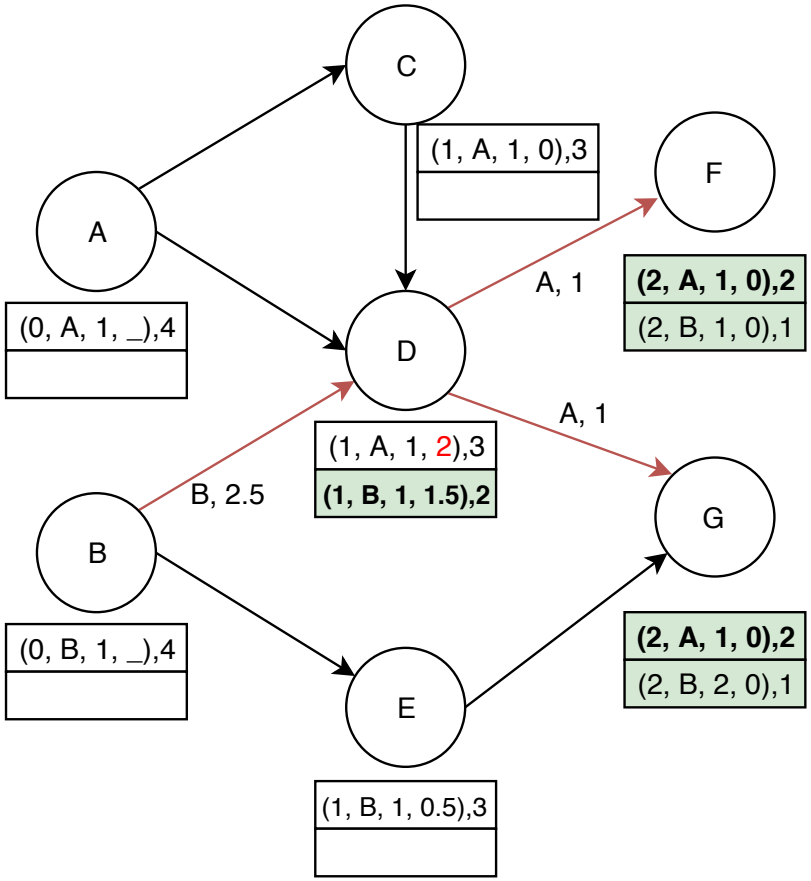
Backward Accumulation: Round 1

Brandes formulation to propagate finalized dependencies



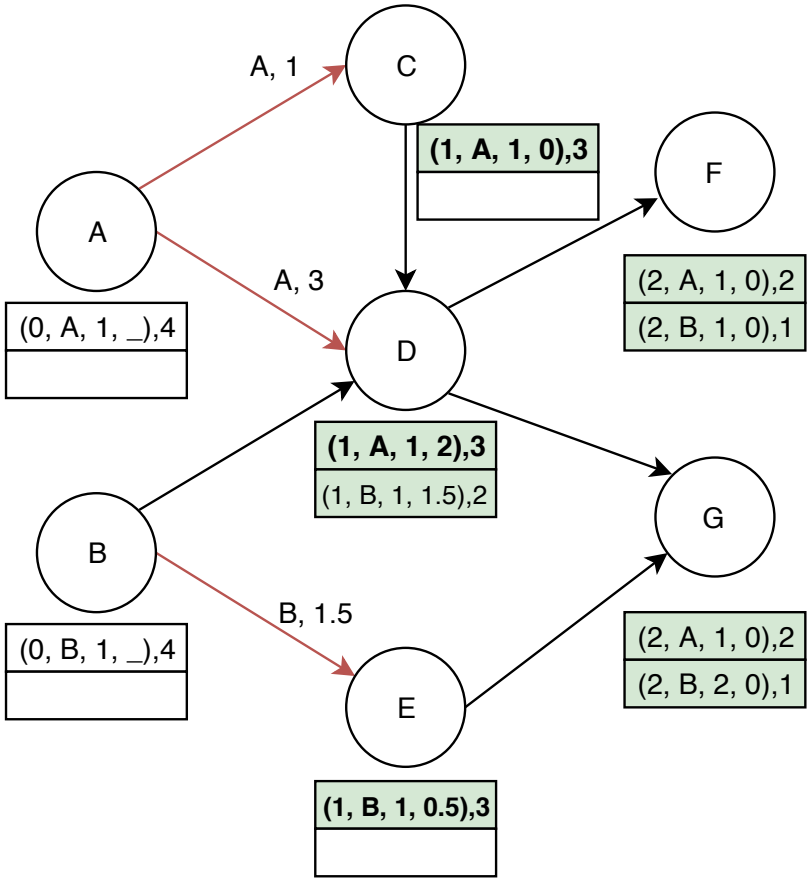
(distance, sourceID, #shortpaths, dependency),sendround

Backward Accumulation: Round 2



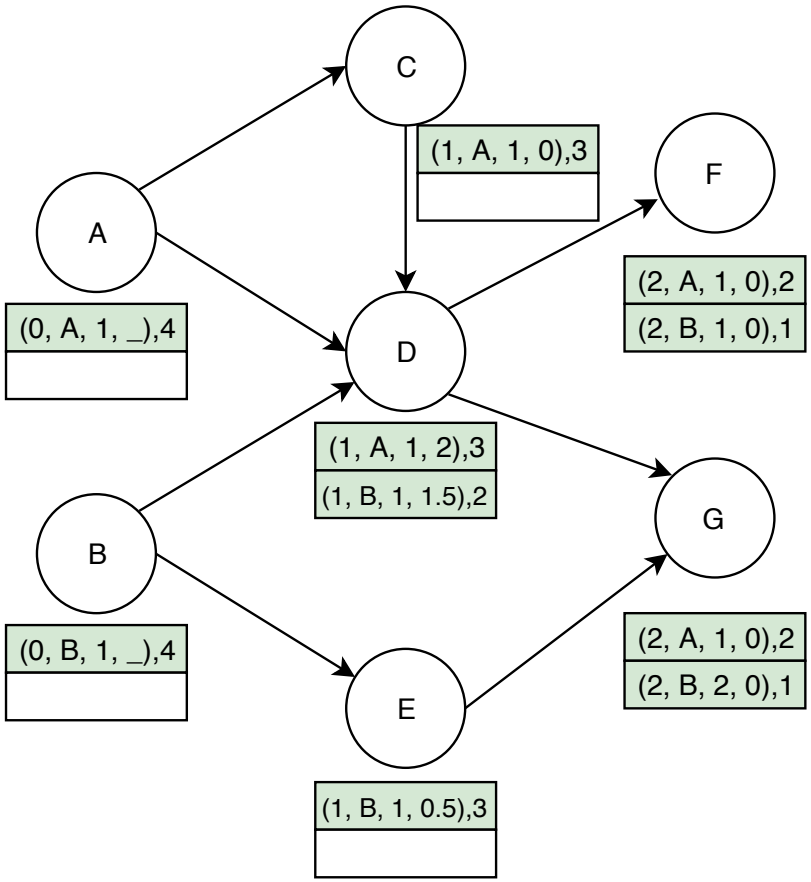
(distance, sourceID, #shortpaths, dependency),sendround

Backward Accumulation: Round 3



(distance, sourceID, #shortpaths, dependency),sendround

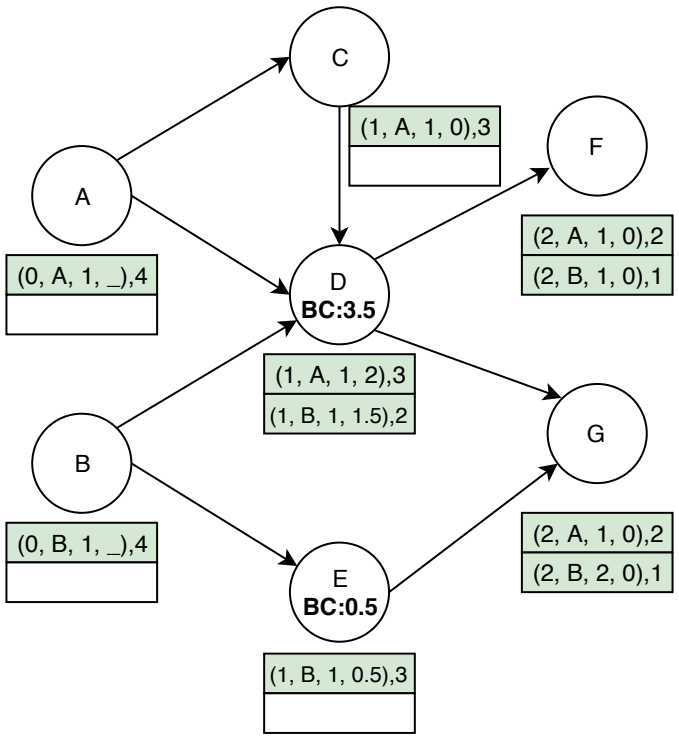
Backward Accumulation: Round 4



(distance, sourceID, #shortpaths, dependency),sendround

Final Result

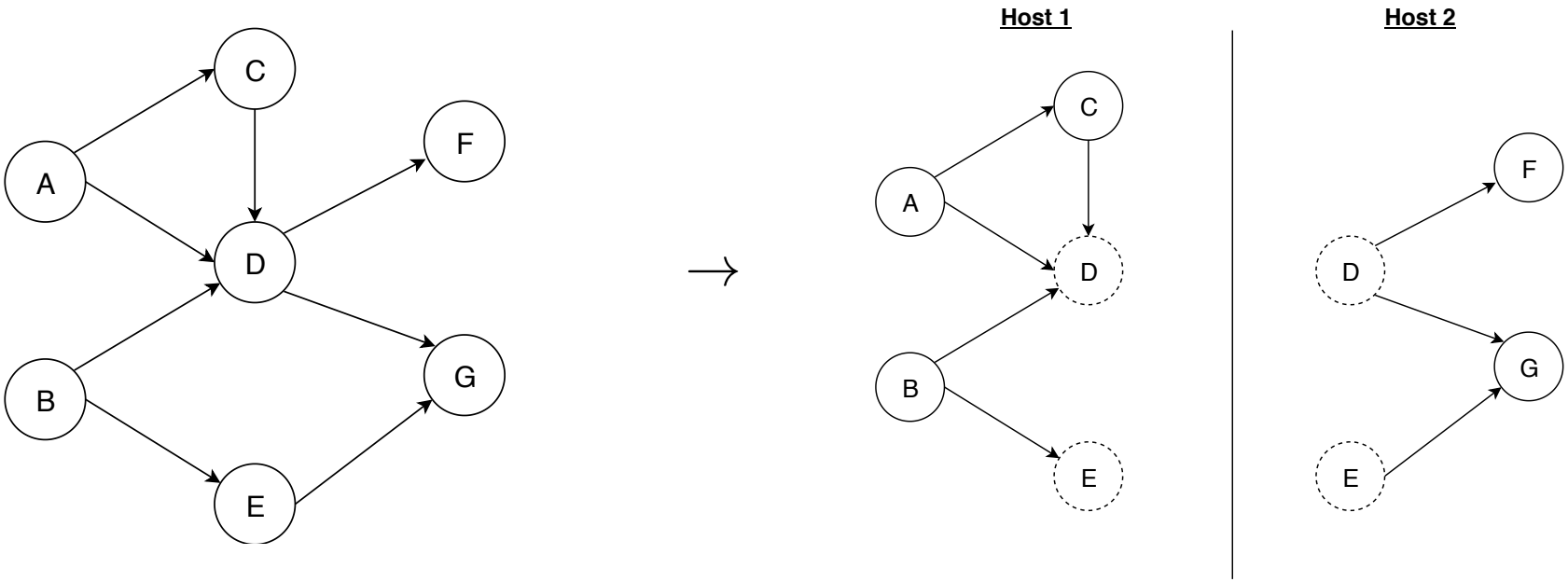
Add source dependencies to get BC contribution



To get BC, use APSP rather than k-SSP

D-Galois and the Execution Model

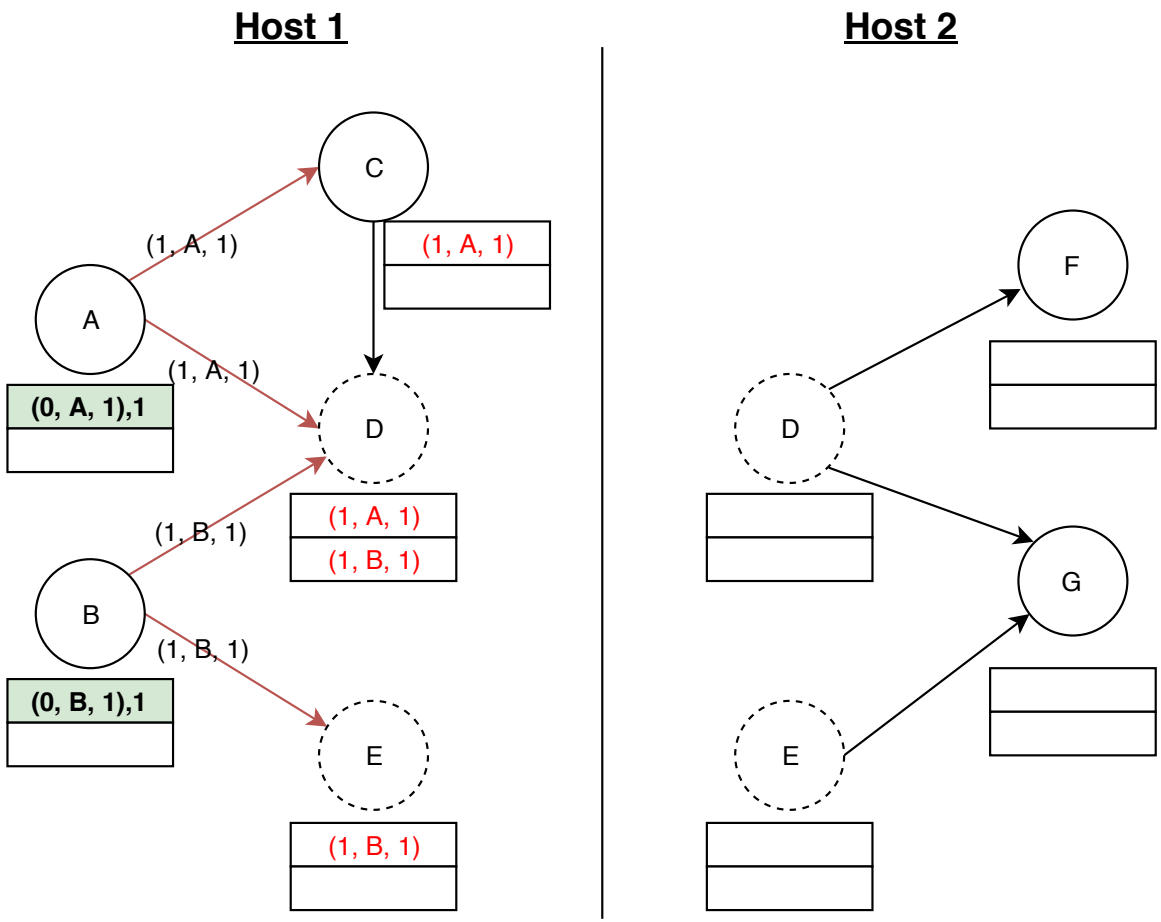
- D-Galois: distributed graph analytics system using shared-memory Galois and Gluon communication substrate



- Distribute edges from graph; cached-copies (proxies) of endpoints created
- Execution in bulk-synchronous parallel (BSP) rounds: computation then communication to sync proxies

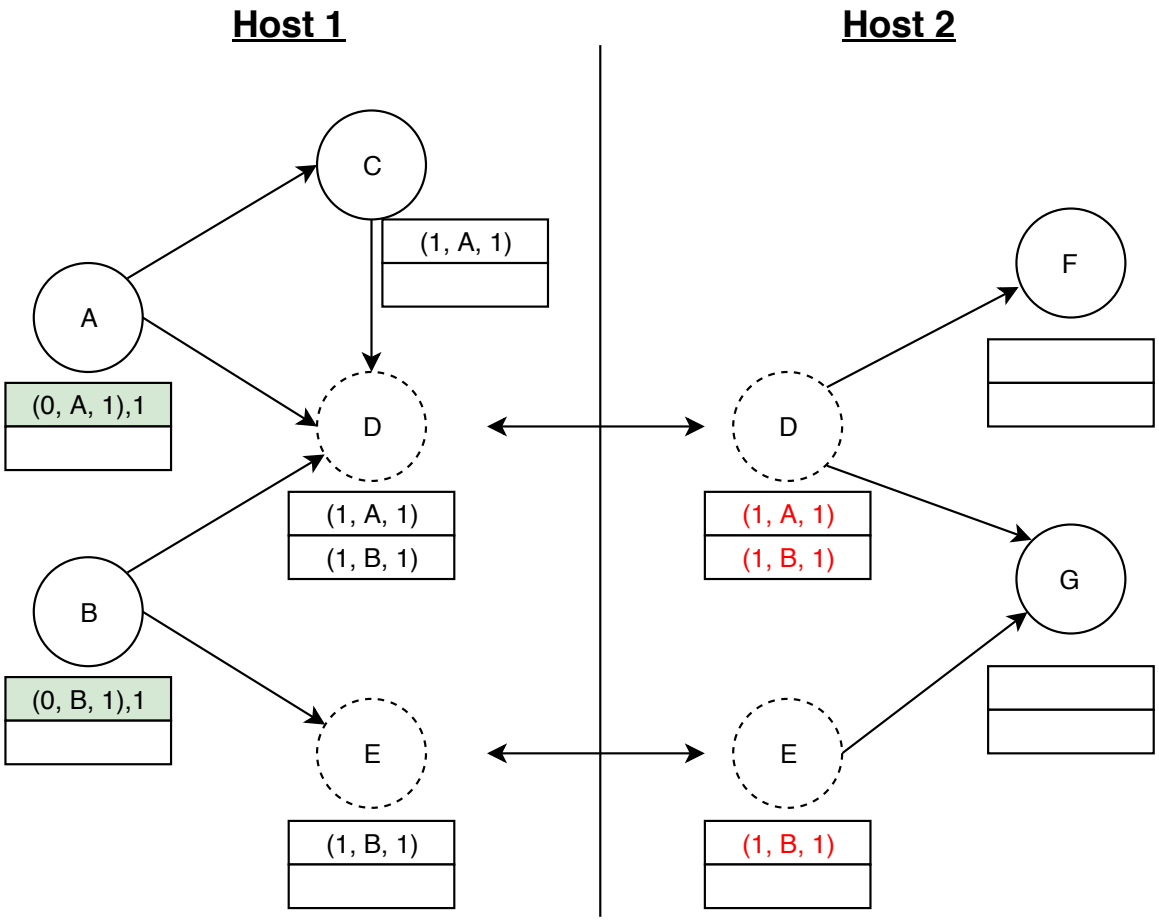
Example Execution in D-Galois: Round 1 Compute

Computation: CONGEST "message sends" along edges



Example Execution in D-Galois: Round 2 Sync

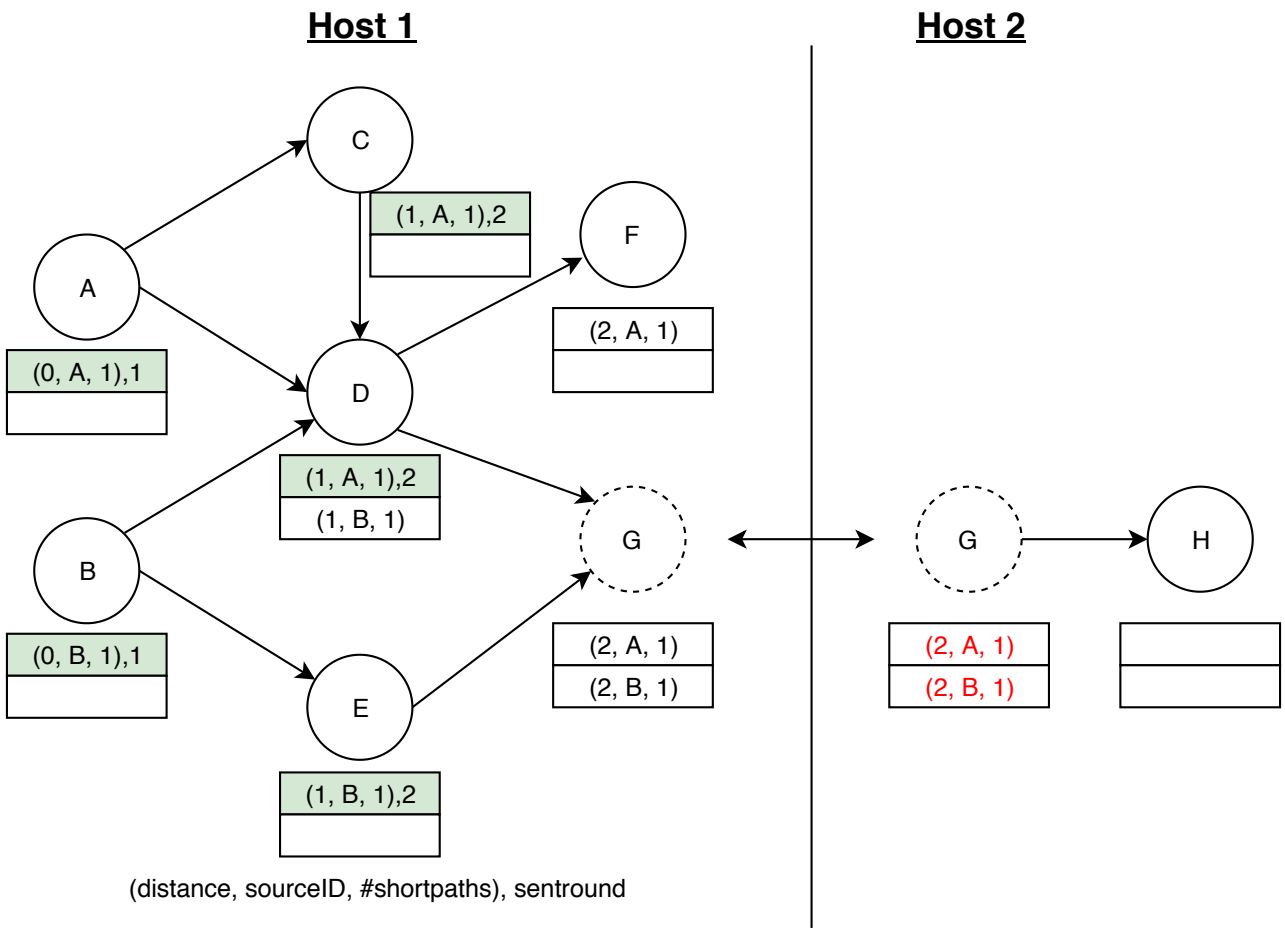
Synchronization of proxies D and E after computation



(distance, sourceID, #shortpaths), sentround

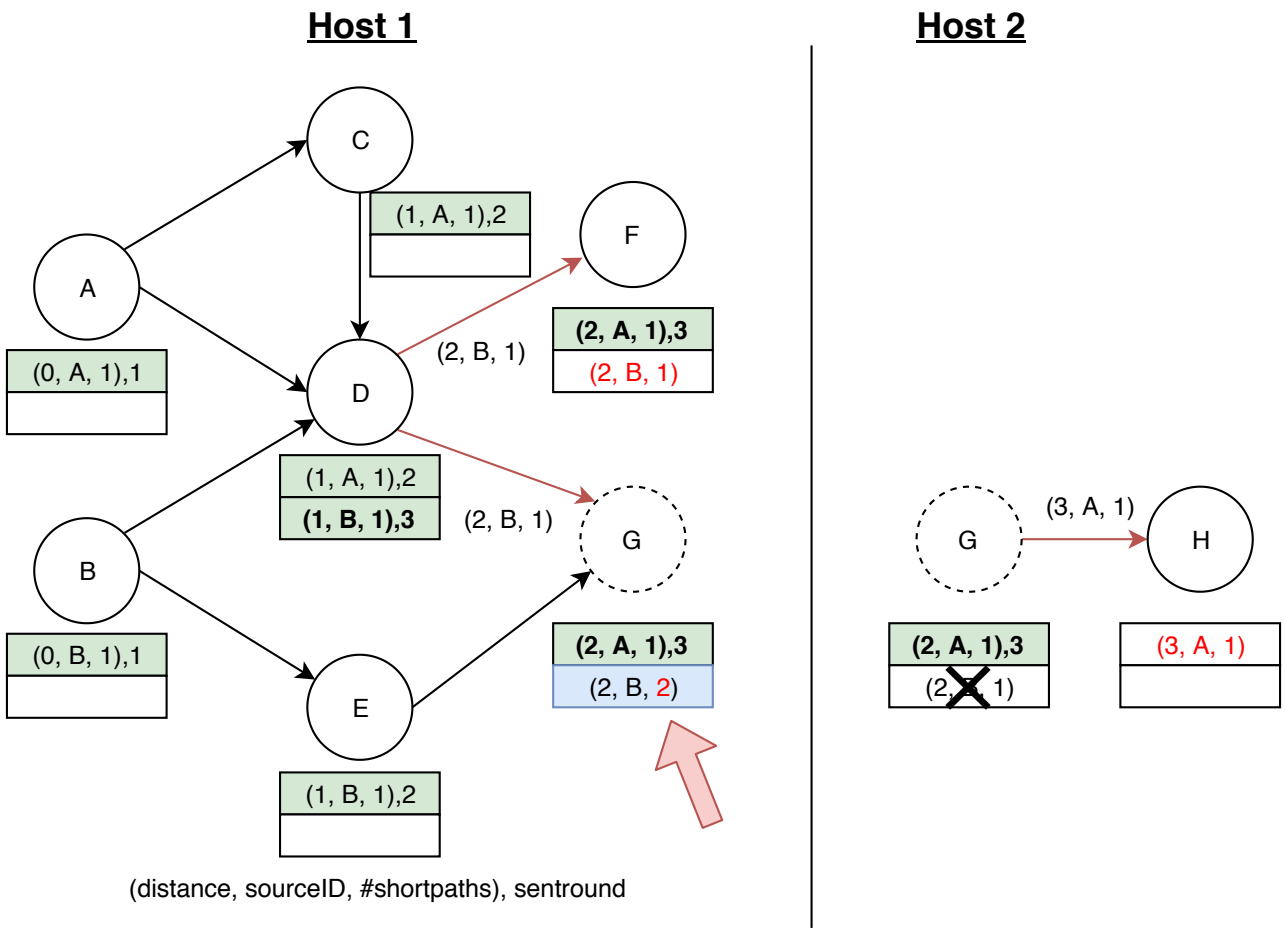
Redundant Synchronization (I)

Beginning of Round 3: Synchronize all data on proxy *G*



Redundant Synchronization (II)

After compute, stale value on host 2: needs synchronization again!

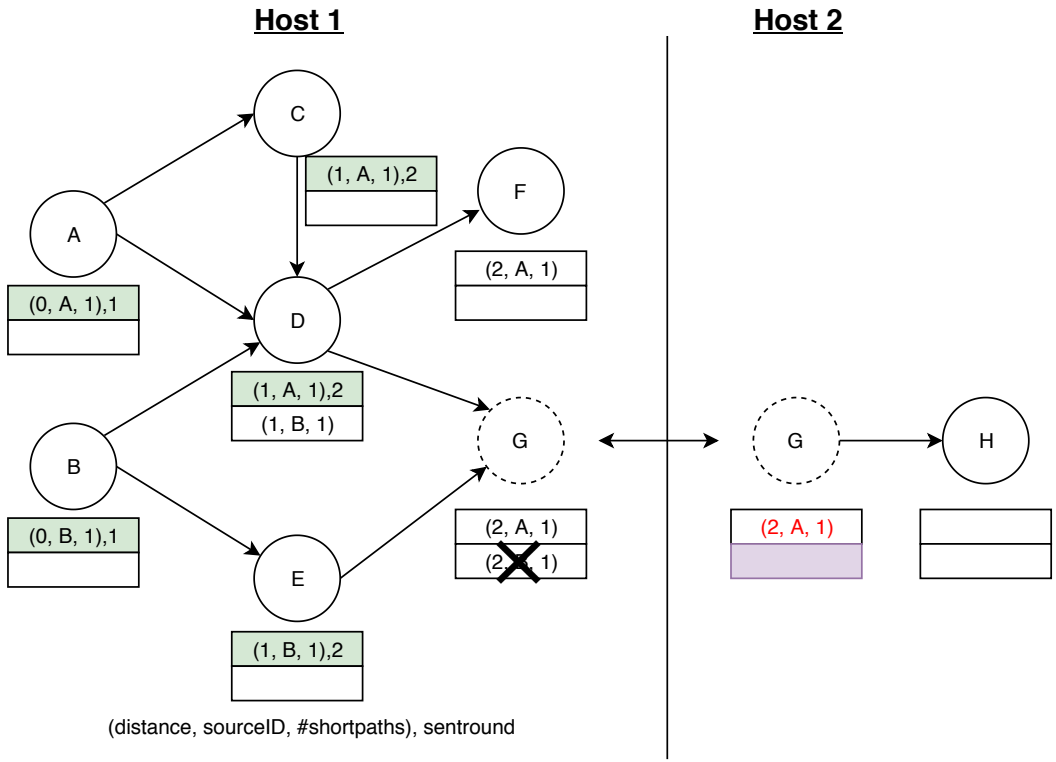


Optimization: Delayed Synchronization in D-Galois

Delayed Synchronization

Synchronize updated data associated with a source on a proxy only if that data meets the message send rule's conditions

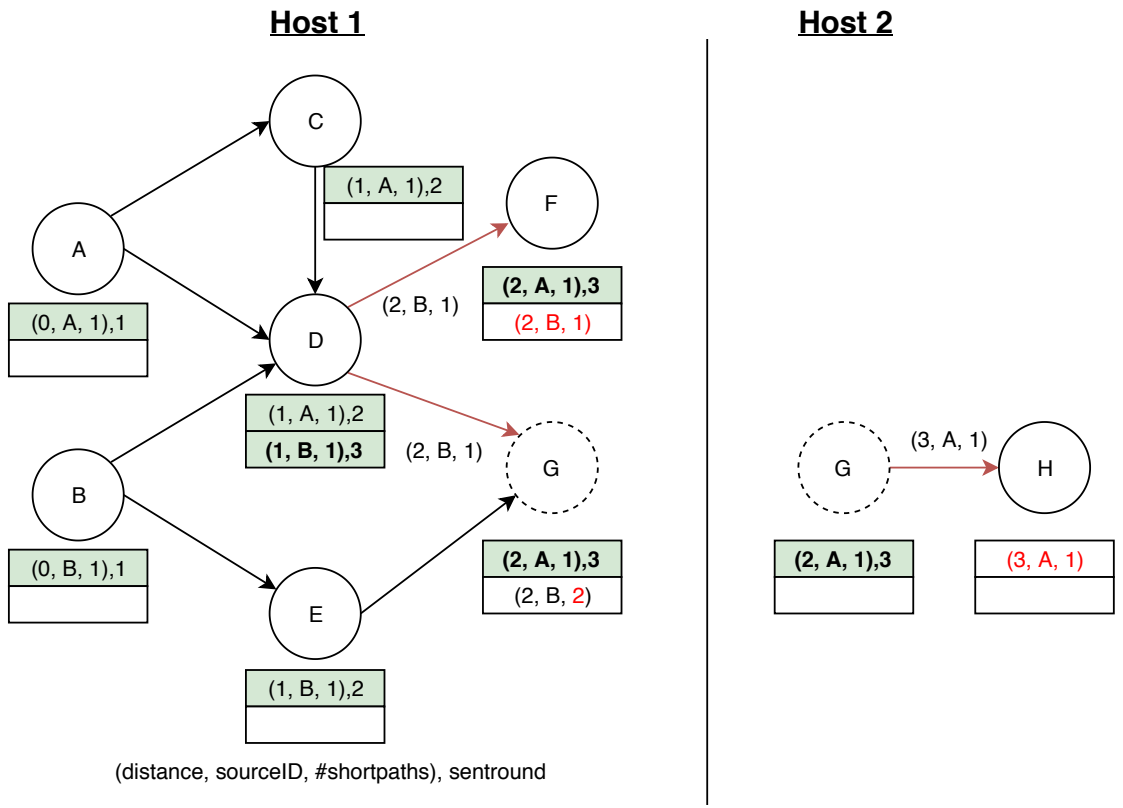
Beginning of Round 3, sync only source A data on G
(distance 2 + position 1 = round 3)



- Intuition: data not read until round it is sent
- Availability of proxies allows delaying synchronization

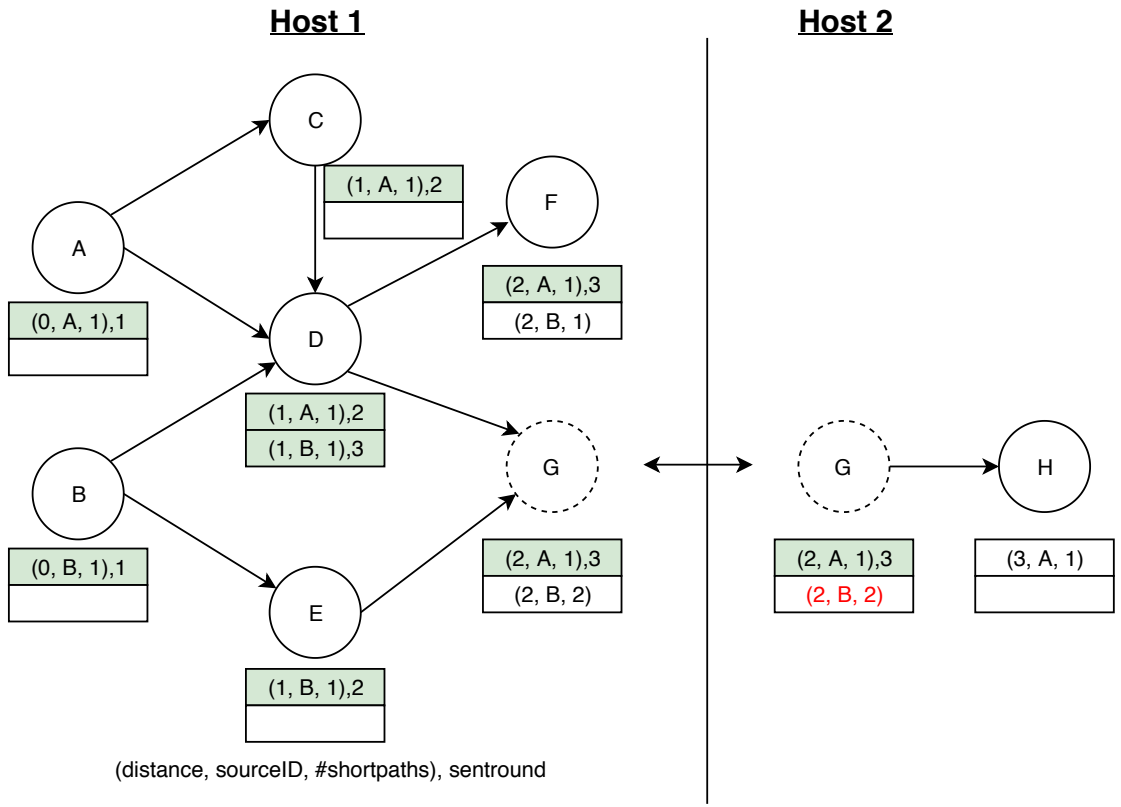
Delayed Synchronization Example Continued (I)

Round 3 compute



Delayed Synchronization Example Continued (II)

Beginning of Round 4: synchronize source B data on proxy G (distance 2 + position 2 = round 4)



Delayed sync reduces network congestion and communication volume

Outline

Experimental Setup: Evaluated Algorithms

1. Asynchronous Brandes BC (ABBC)
2. Maximal Frontier BC (MFBC), sparse-matrix-based
3. Synchronous Brandes BC (SBBC), Brandes in D-Galois
4. Min-Rounds BC (MRBC)

	System	(A)synchronous?	Distributed?	Batching?
ABBC	Galois	Async	N	N
MFBC	CTF	Sync	Y	Y
SBBC	D-Galois	Sync	Y	N
MRBC	D-Galois	Sync	Y	Y

We focus on SBBC and MRBC

- ABBC excellent for high diameter graphs if fits in memory
- MFBC performs moderately well, slows as graphs grow

Experimental Setup: Platform

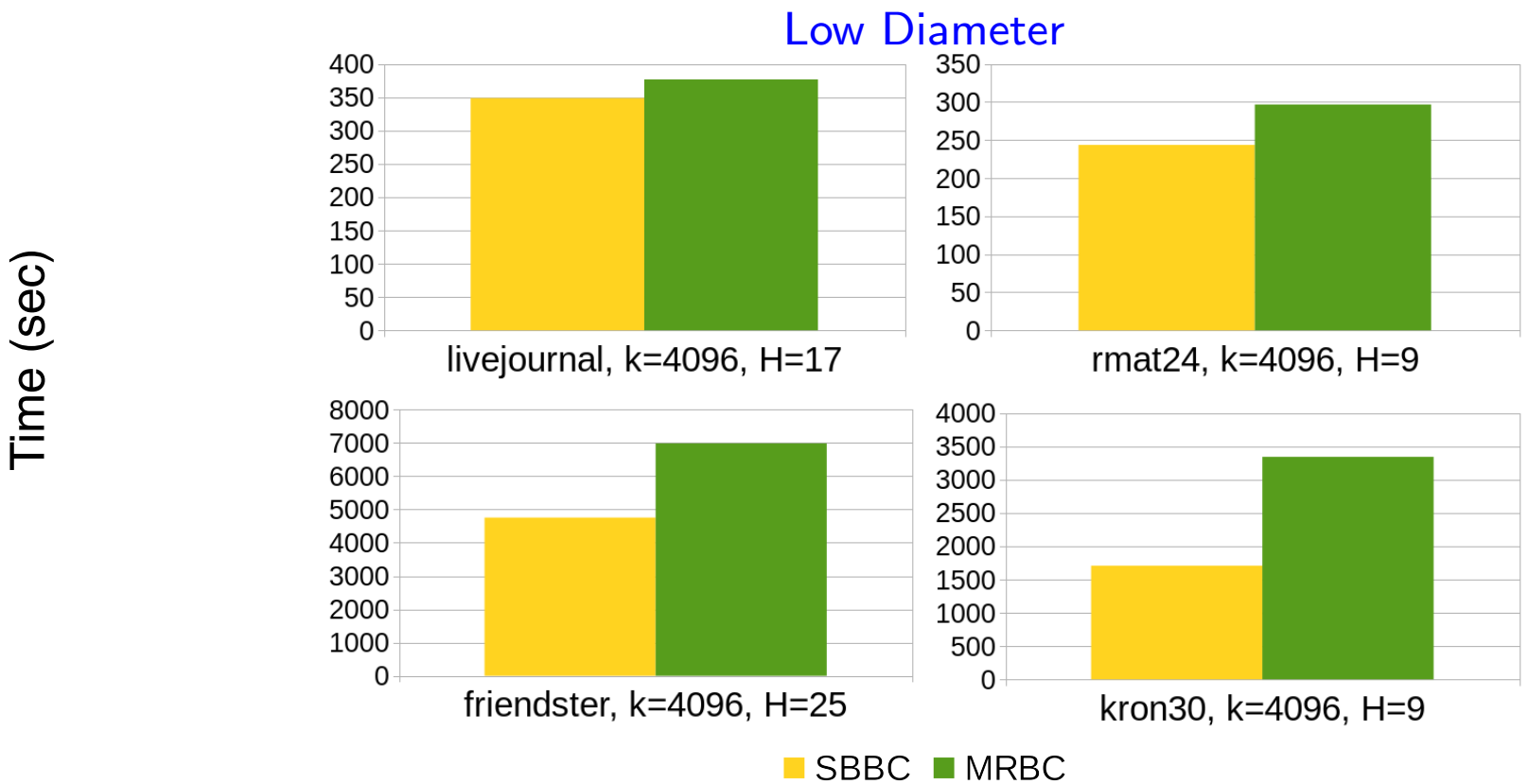
	Low Diameter				High Diameter			
	livejournal	rmat24	friendster	kron30	indochina04	road-europe	gsh15	clueweb12
$\|V\|$	4.8M	17M	66M	1,073M	7.4M	174M	988M	978M
$\|E\|$	69M	268M	3,612M	17,091M	194M	348M	33,877M	42,574M
H (Estimated Diameter)	17	9	25	9	45	22541	103	501

- Platform: Stampede2's Skylake cluster
  - Intel Xeon Platinum 8160, 48 cores on 2 sockets per machine
  - 2.1GHz clock rate, 192GB DDR4 RAM
- Graphs run on up to 256 machines
- Low diameter graphs ≤ 25, high diameter greater than 25
  - Web crawls (such as clueweb12) also high-diameter

Execution Times, Low Diameter Graphs

SBBC: $O(k \cdot H)$ BSP rounds **MRBC**: $O(k + H)$ BSP rounds

H = largest shortest path distance for k sources



- MRBC round reduction (which leads to communication improvements) does not outweigh compute overhead

Execution Times, High Diameter Graphs

SBBC: $O(k \cdot H)$ BSP rounds **MRBC**: $O(k + H)$ BSP rounds

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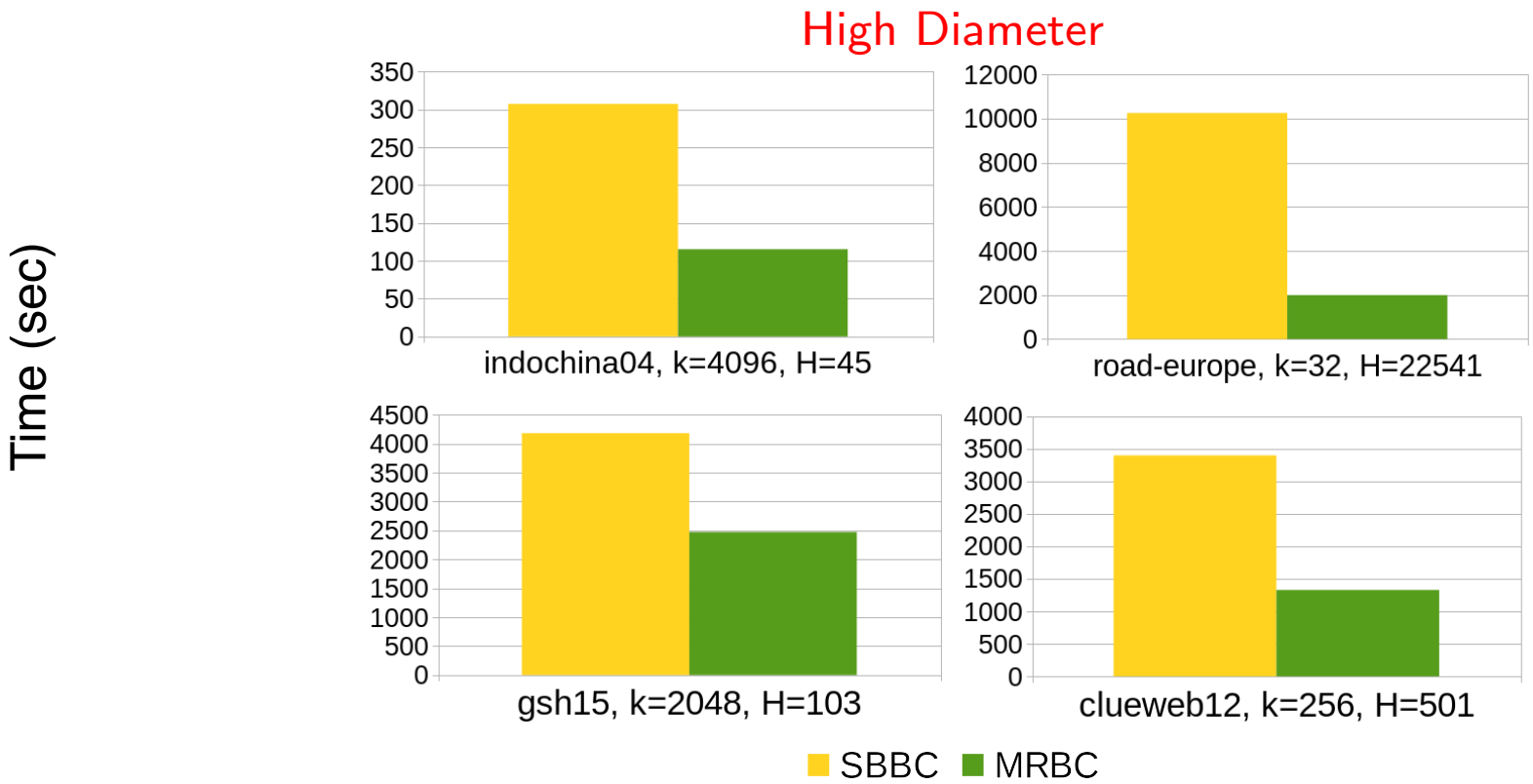
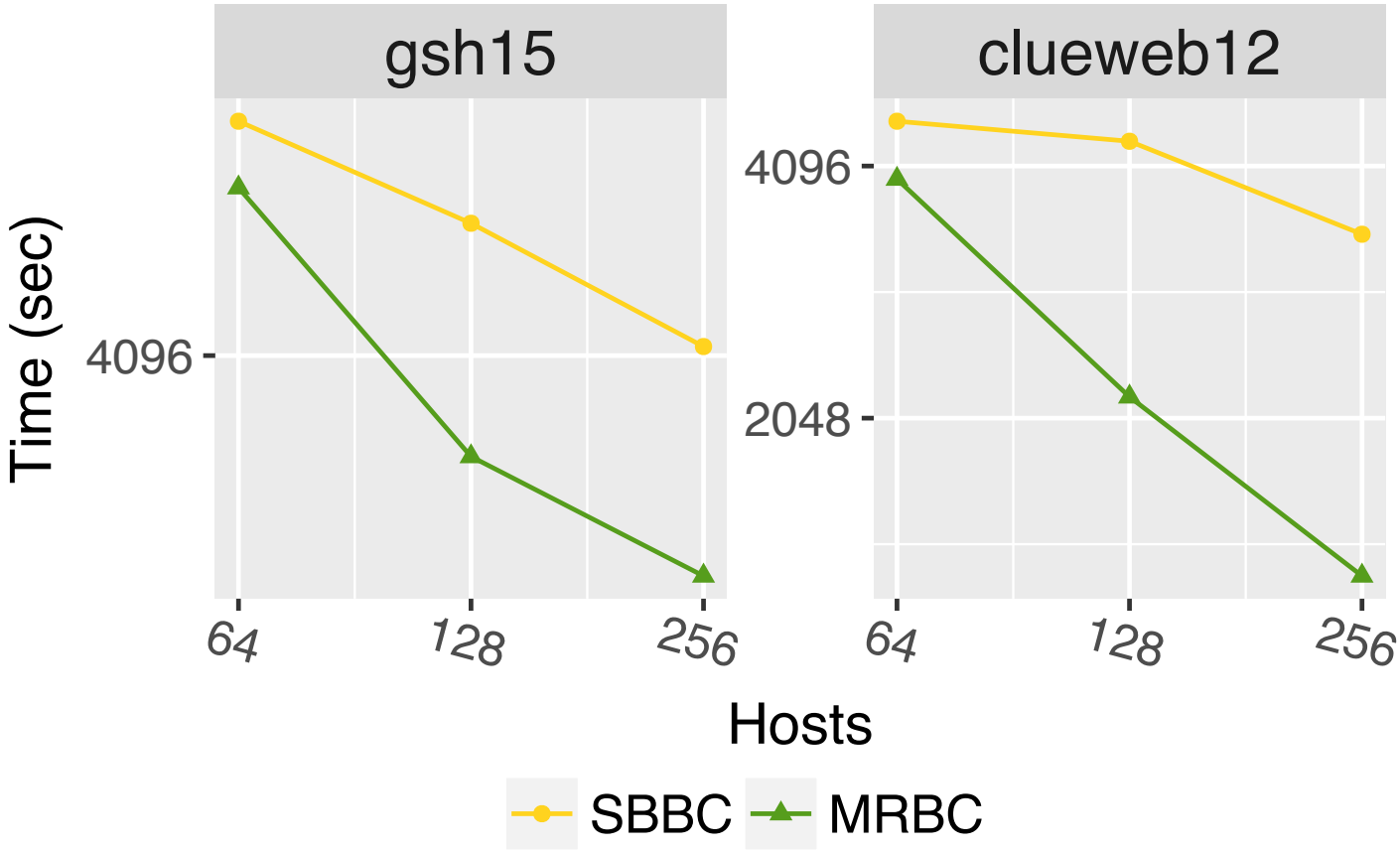


- MRBC outperforms SBBC (2.8× faster): round reduction and communication improvement more significant

Execution Time of SBBC/MRBC from 64 to 256 Hosts



- Both SBBC and MRBC scale as number of hosts increase
- Communication time of SBBC does not scale as well compared to MRBC

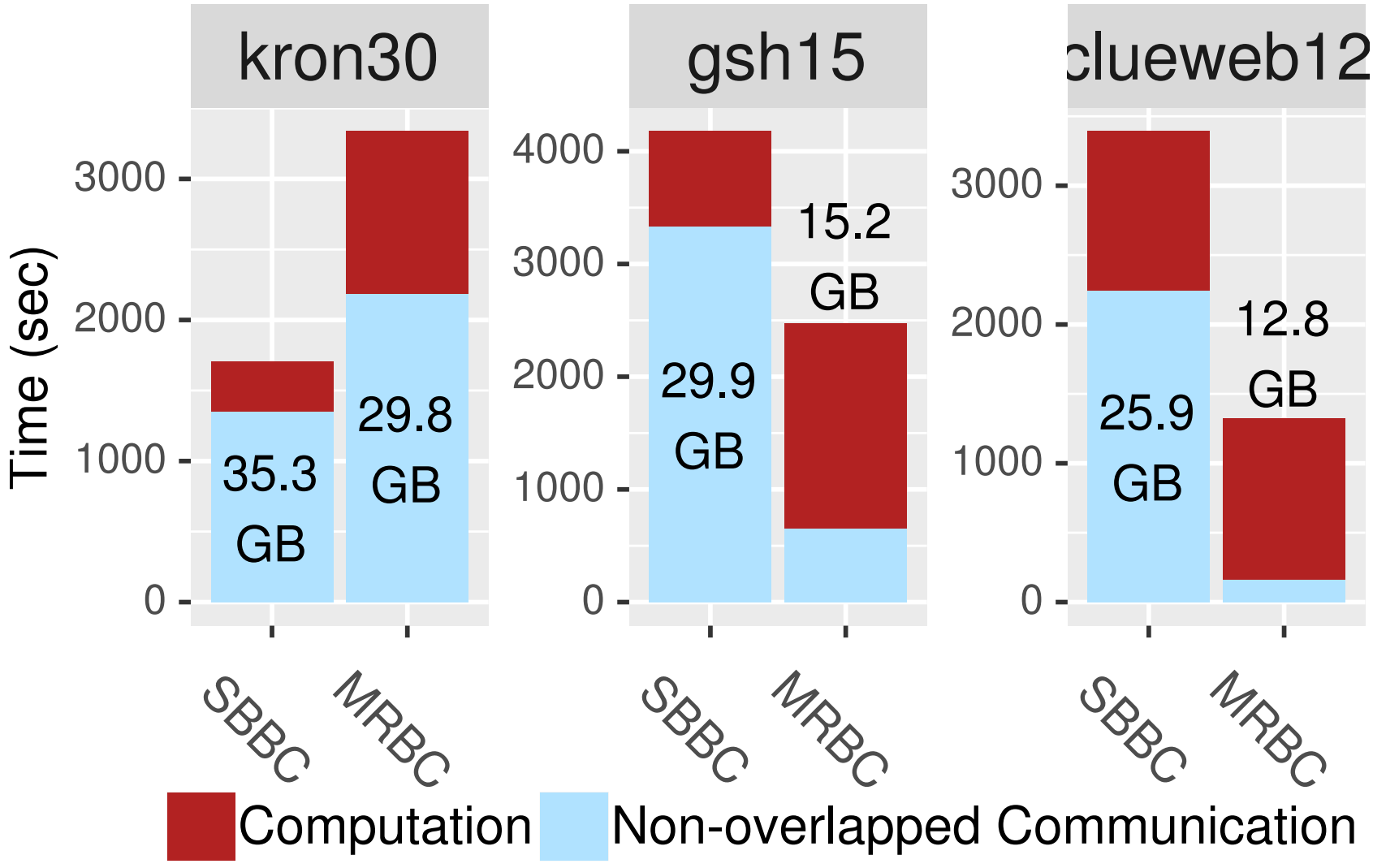
Conclusion

- Presented round-efficient distributed APSP and BC algorithm (MRBC) that improves communication by pipelining message sends
- MRBC in D-Galois over Brandes BC: 14× reduction in rounds, 2.8× speedup for high-diameter graphs

Source Code:
`https://github.com/IntelligentSoftwareSystems/Galois/`
Artifact:
`https://zenodo.org/record/2399798`

Backup Slides

Execution Time of SBBC/MRBC at 256 Hosts, Breakdown



More Topics Covered in the Paper

- Termination detection routine for Min-Rounds APSP which reduces the round complexity and termination detection in D-Galois
- Proofs of correctness for the algorithm and its optimizations
- More detailed analysis of experiments

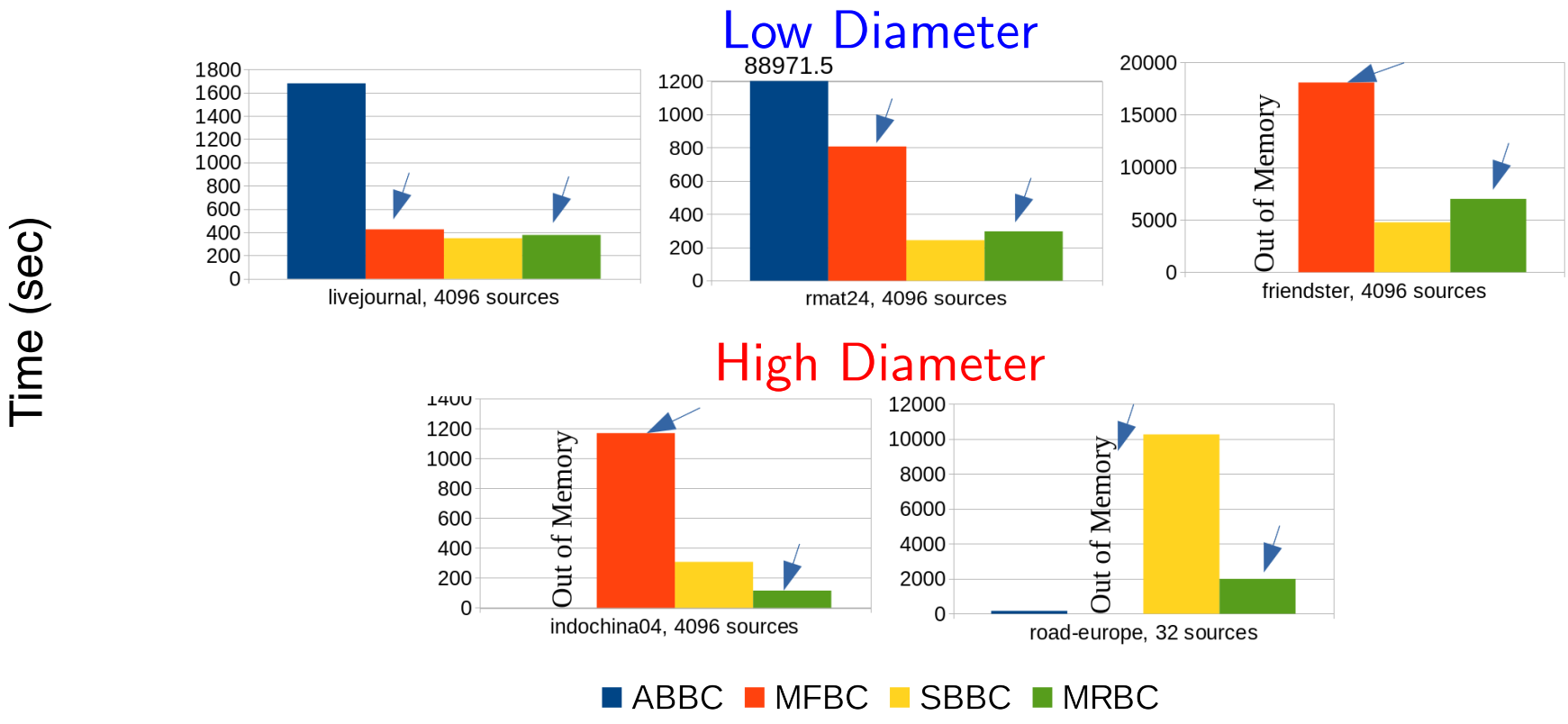
Effect of D-Galois Optimization

Note difference in CONGEST model and D-Galois model

- Number of Rounds
  - MRBC reduces rounds over SBBC in both models (same bounds apply)
- Messages Sent
  - CONGEST: messages sent along edges in SBBC/MRBC are same (only final value is sent)
  - D-Galois
    - SBBC: proxy distance from source updated/sent only once (updated value is final value)
    - MRBC: proxy distance updated multiple times before finalization, i.e. communicate every update, not just when value is finalized

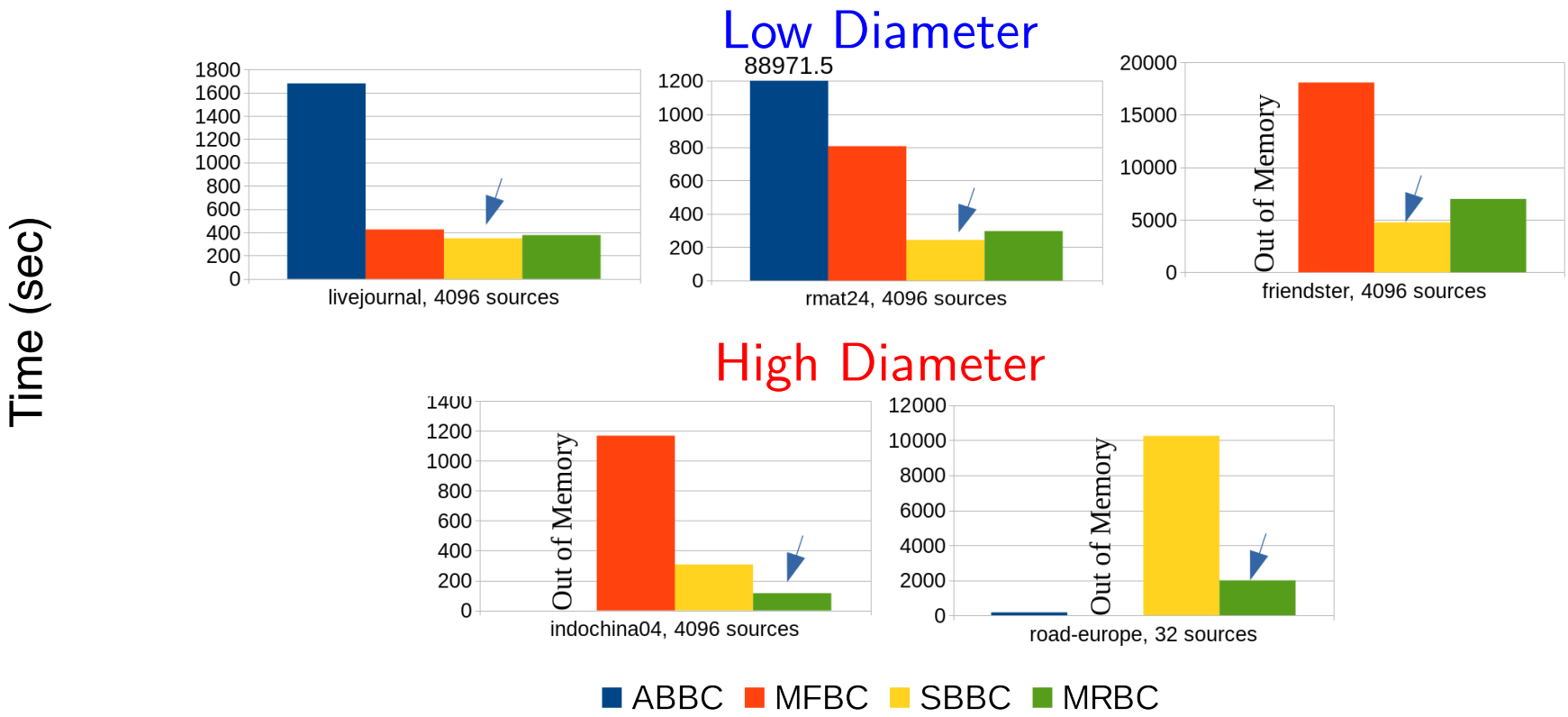
Without optimization, MRBC may send more messages; expected to perform worse

Best Execution Times (1, 32 Hosts) on Small Graphs (I)



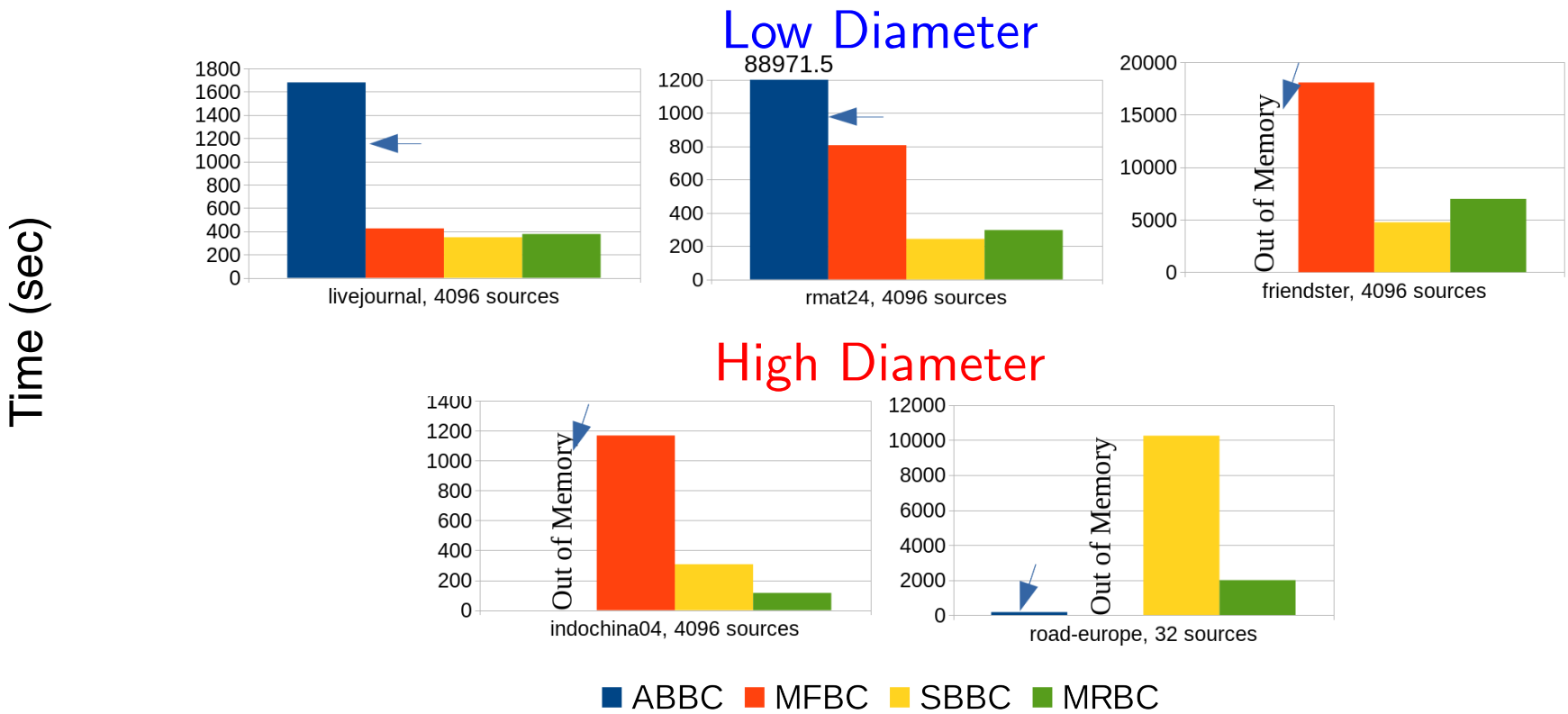
- MRBC is 3× faster than MFBC on average
- SBBC also outperforms MFBC

Best Execution Times (1, 32 Hosts) on Small Graphs (II)



- SBBC best for graphs with low-diameter
- MRBC better for high-diameter

Best Execution Times (1, 32 Hosts) on Small Graphs (III)



- ABBC fast on high diameter graphs
- ABBC extremely slow otherwise