# Differentiable ocean models from large to small scales

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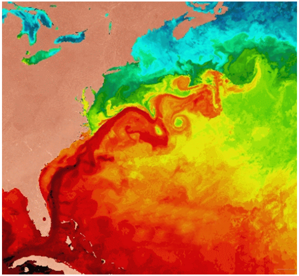
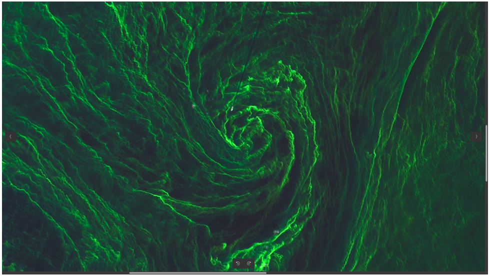
# Ocean dynamics: a large range of scales





$10^6$ m to $10^2$ m

# Hierarchy of dynamical equations

| **Quasi-geostr.** | **Shallow-water** | **Primitive Eq.** | **Boussinesq NH** |
|---|---|---|---|
| $\mathbf{q}$ | $\mathbf{u}, \mathbf{v}, \boldsymbol{\eta}$ | $\mathbf{u}, \mathbf{v}, \boldsymbol{\eta}, \mathbf{T}, \mathbf{s}$ | $\mathbf{u}, \mathbf{v}, \mathbf{w}, \boldsymbol{\eta}, \mathbf{T}, \mathbf{s}$ |
| slow dynamic | constant density horizontally | hydrostatic equilibrium | $\rho(T, s) \sim \rho_0$ |

Physical phenomena variety →

← Simplifications

**I. Dynamical core**

→ *high-order non-linear advection schemes*
→ *prognostic variables continuity*
→ *automatic differentiation*

**II. Data assimilation**

→ *4D Var*
→ *deep denoiser priors*

**III. Machine learning**

→ *emulators*
→ *physical params*

# Starting from simple models...

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# Quasi-geostrophic equations

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Physical phenomena variety →

← simplifications

**Quasi-geostrophic scaling of Shallow-water eqs**

- $\mathbf{q}$: potential vorticity
- $\mathbf{u}, \mathbf{v}$: horizontal velocity
- $\mathbf{u}, \mathbf{v} = \nabla^{\perp}(\Delta - \lambda Id)^{-1}\mathbf{q}$ (elliptic eq.)

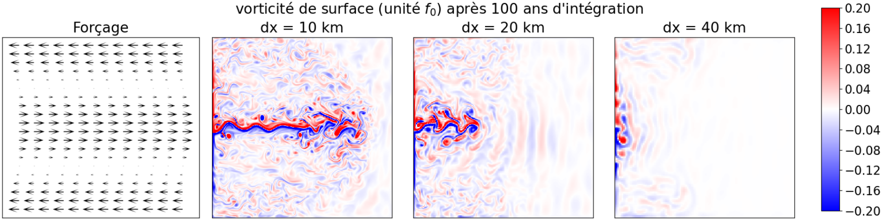
$$\partial_t \mathbf{q} + \nabla \cdot \left( \mathbf{q} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} \right) = \underset{\text{forcing}}{Q}$$

# Quasi-geostrophic equations

Usual discretization (Uchida et al. 2022)

- Advection: second order linear Arakawa (1981)
- Additional bilaplacian dissipation (hand-tuned coefficient)
- **github.com/louity/qgm_pytorch** (400 lignes)

Idealized Gulf-stream configuration, $R_d = 40$ km.

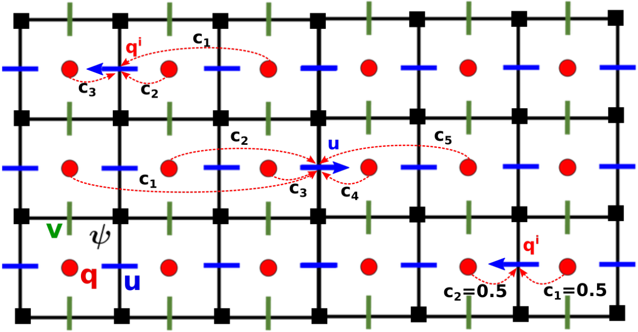



dx $\longrightarrow R_d$: Gulf-stream severely impacted

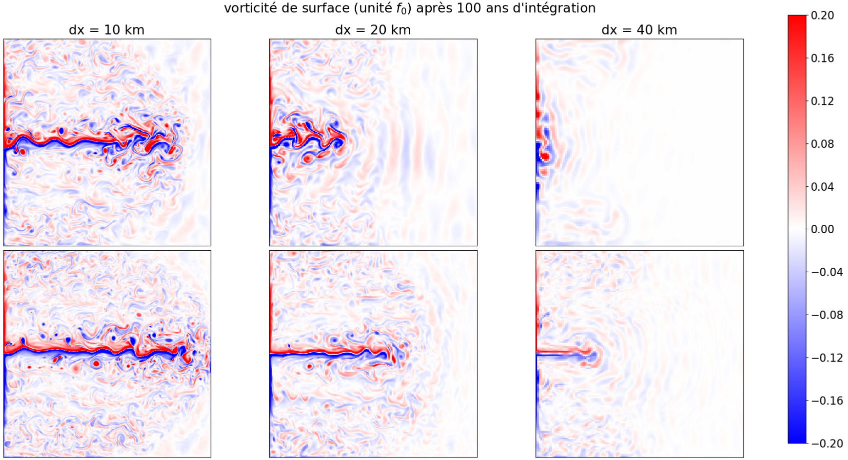
# Quasi-geostrophic equations

Working on the advection scheme (Thiry et al., 2023)

- Finite-volume, staggered grid
- Second-order discretization for $\nabla\cdot$, $\nabla\wedge$, $\nabla^{\perp}$
- Flux: non-linear order 3/5 WENO (Borges et al., 2008)
- Implicit dissipation $\implies$ **no explicit dissipation, no tuning**



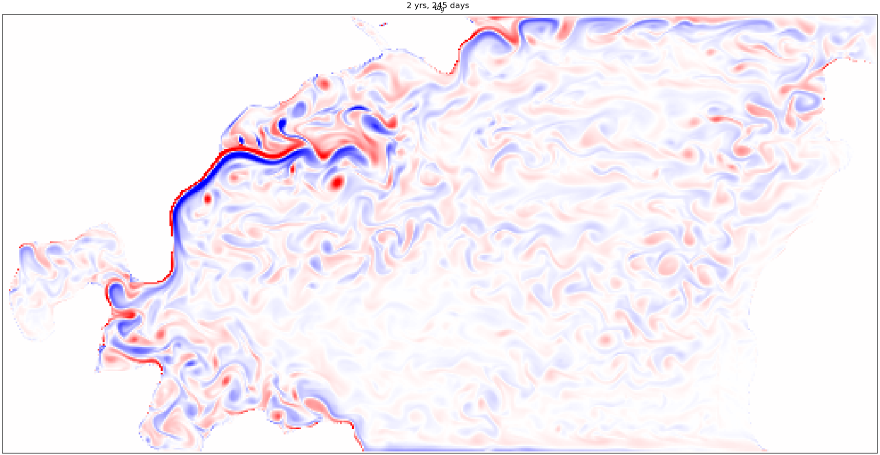
# Quasi-geostrophic equations




Usual (top) vs. our (bottom) QG discretization on idealized Gulf-stream config (ours costs $\sim 2\times$ usual)

# Quasi-geostrophic equations

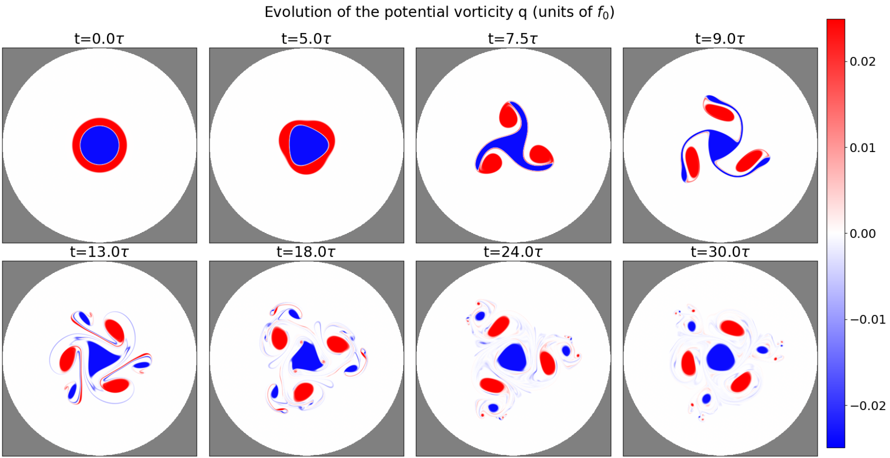
## Non-rectangular domain with capacitance matrix method




Realistic Gulf-stream simulation (20min runtime)

# Quasi-geostrophic equations

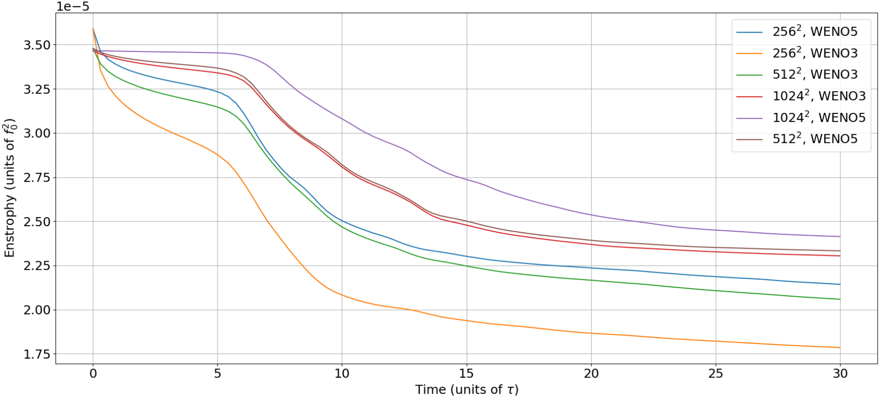
## Non-rectangular domain with capacitance matrix method




Rankine vortex shear instability resolved in tripole

# Quasi-geostrophic equations

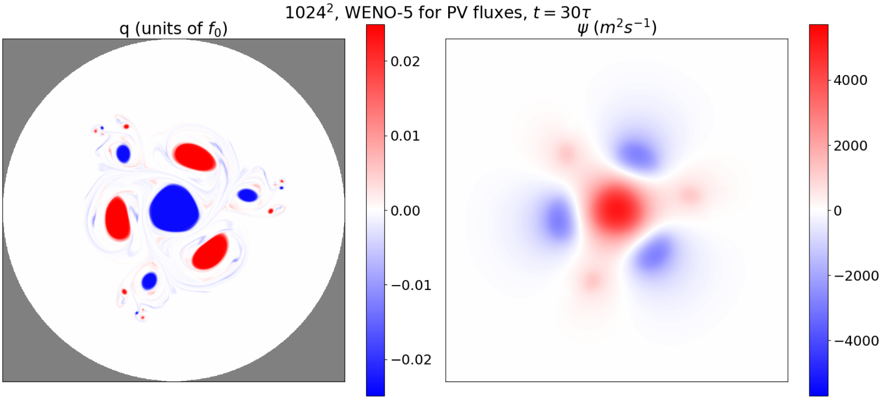
What is the point of WENO-5 vs WENO-3 ?




Evolution of $\|\mathbf{q}\|_2$ at different resolution using WENO-3/5

# Quasi-geostrophic equations

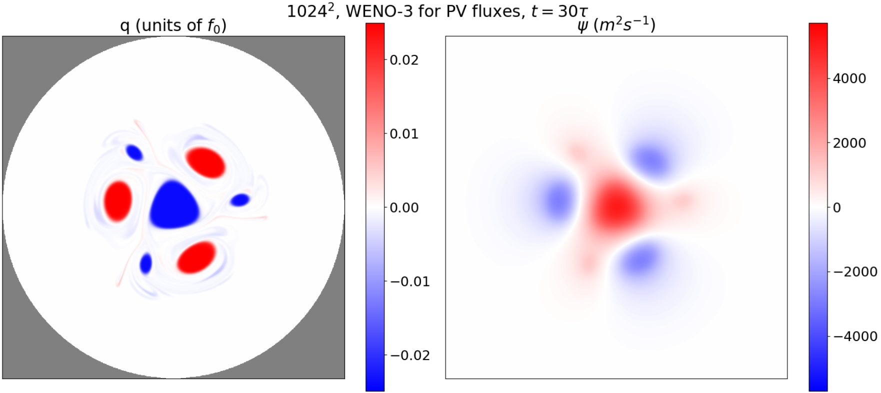
WENO-5 vs WENO-3




Final state, $1024^2$ WENO-5, 7min30 runtime

# Quasi-geostrophic equations

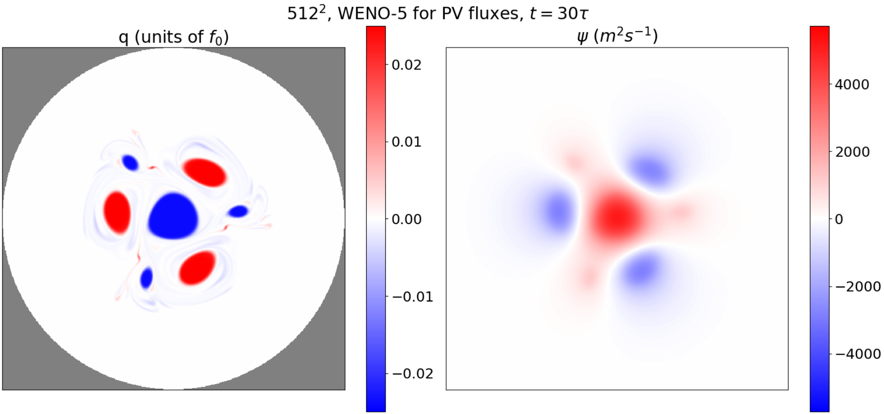
WENO-5 vs WENO-3




Final state, $1024^2$ WENO-3, 6min runtime

# Quasi-geostrophic equations

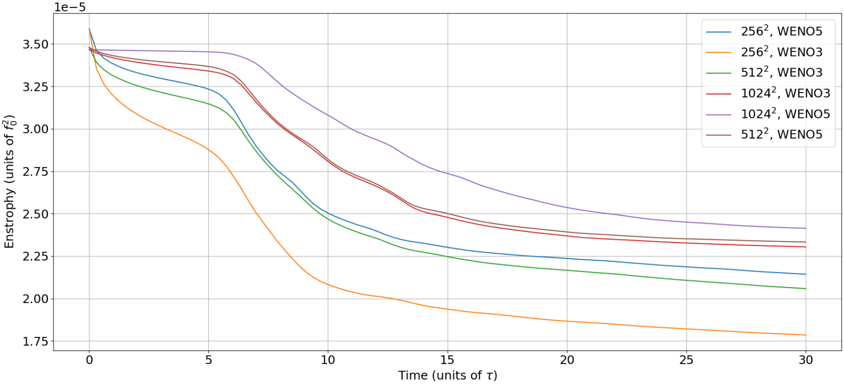
WENO-5 vs WENO-3




Final state, $512^2$ WENO-5, 1min runtime

# Quasi-geostrophic equations

$$\partial_t \mathbf{q} + \nabla \cdot \left( \mathbf{q} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} \right) = 0$$




Why does the enstrophy decrease ? $\rightarrow$ **numerical dissipation**

# Lessons from our QG new implementation

| Quasi-geostr. | Shallow-water | Primitive Eq. | Boussinesq NH |
|---|---|---|---|
| $\mathbf{q}$ | $\mathbf{u}, \mathbf{v}, \eta$ | $\mathbf{u}, \mathbf{v}, \eta, \mathbf{T}, \mathbf{s}$ | $\mathbf{u}, \mathbf{v}, \mathbf{w}, \eta, \mathbf{T}, \mathbf{s}$ |
| slow dynamic | constant density horizontally | hydrostatic equilibrium | $\rho(T, s) \sim \rho_0$ |

Physical phenomena variety →

← simplifications

- Benefits of high-order non-linear advection scheme
  - Large-scale structures (Gulf-stream)
  - Small-scale structures (Eddies and filaments)
  - No parameter tuning (Bilaplacian eddy viscosity)
- Extension to non-rectangular geometries → realistic Gulf-stream
- Implicit dissipation diagnostic (G. Roullet)

# Moving to the right...

QG variable: $\mathbf{q} \neq$ Shallow-water variables: $\mathbf{u}, \mathbf{v}, \boldsymbol{\eta}$ (free-surface)

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**A new formulation of Shallow-water equations**

$$\partial_t^{\mathrm{qg}}(\mathbf{u}, \mathbf{v}, \boldsymbol{\eta}) = P(\partial_t^{\mathrm{sw}}(\mathbf{u}, \mathbf{v}, \boldsymbol{\eta}))$$

$$P = G \circ (Q \circ G)^{-1} \circ Q \qquad \text{(projecteur QG)}$$

Continuous version in Frederic Charve Thesis.

# Benefits for implementation

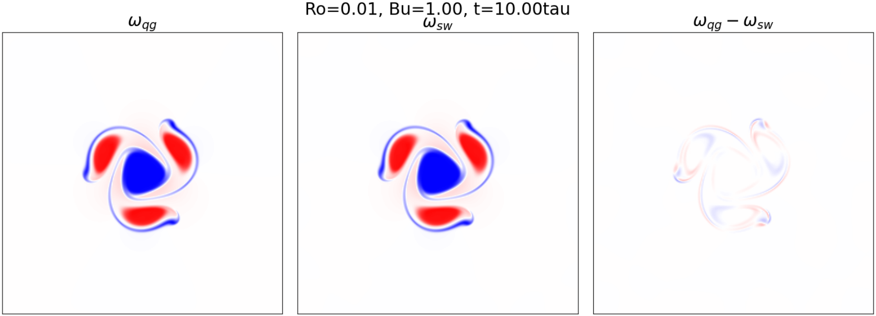
Quasi-Geostrophic scaling $\iff$ class inheritance

```
class SW:
    """Concise implementation of multilayer shallow-water model."""
    def __init__(self, param):
        ...
    def compute_time_derivatives(self):
        self.compute_diagnostic_variables()
        dt_h = self.advection_h()
        dt_u, dt_v = self.advection_momentum()
        return dt_u, dt_v, dt_h

class QG(SW):
    """QG as projected SW."""
    def __init__(self, param):
        super().__init__(param)
        ...
    def compute_time_derivatives(self):
        dt_u_sw, dt_v_sw, dt_h_sw = super().compute_time_derivatives()
        self.dt_u_sw, self.dt_v_sw = dt_u_sw, dt_v_sw
        dt_u_qg, dt_v_qg, dt_h_qg = self.compute_qg_projection(dt_u_sw, dt_v_sw, dt_h_sw)
        return dt_u_qg, dt_v_qg, dt_h_qg
```

# Comparing QG and SW on a small system

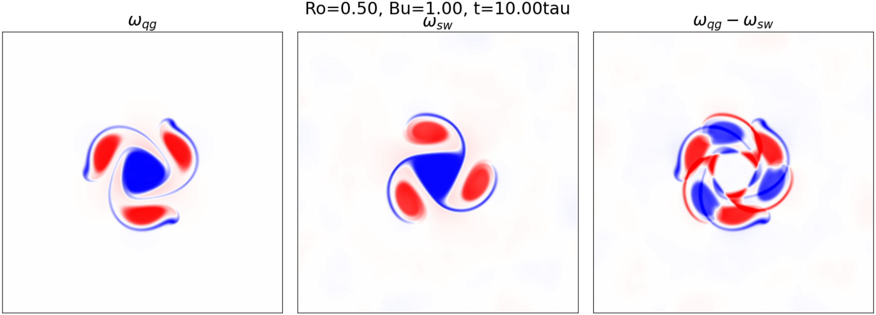
QG equations valid for $Bu \leq 1$ and $Ro \ll 1$.




Vortex shear instability $Ro = 0.01$ and $Bu = 1$

# Vortex shear instability $Ro = 0.5$ and $Bu = 1$

QG equations valid for $Bu \leq 1$ and $Ro \ll 1$.




Vortex shear instability $Ro = 0.5$ and $Bu = 1$

# Lessons from our SW new implementation

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Physical phenomena variety →

← simplifications

- Restoring the continuity in variables
- QG implementation via class inheritance, simply with a projection
- Compare QG and SW physics with same numerics

# Data assimilation

| Quasi-geostr. | Shallow-water | Primitive Eq. | Boussinesq NH |
|---|---|---|---|
| $\mathbf{u}, \mathbf{v}, \eta$ | $\mathbf{u}, \mathbf{v}, \eta$ | $\mathbf{u}, \mathbf{v}, \eta, \mathbf{T}, \mathbf{s}$ | $\mathbf{u}, \mathbf{v}, \mathbf{w}, \eta, \mathbf{T}, \mathbf{s}$ |
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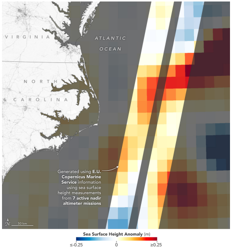
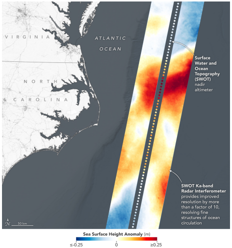
# Data assimilation

Ocean observations

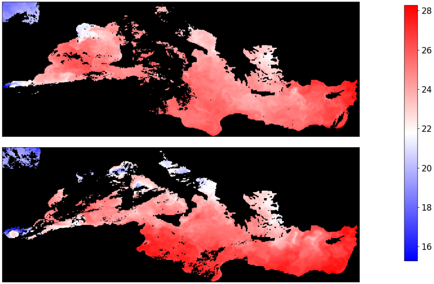
- Sea surface height (SSH)
- Sea surface temperature (SST)




(a) SSH NADIR




(b) SSH SWOT




(c) SST les 1 et 30 sept. 22

Source: NASA earth observatory, Copernicus Marine.

# Data assimilation with 4D Var

| Quasi-geostr. | Shallow-water | Primitive Eq. | Boussinesq NH |
|---|---|---|---|
| $\mathbf{u}, \mathbf{v}, \eta \quad (\mathbf{q})$ | $\mathbf{u}, \mathbf{v}, \eta$ | $\mathbf{u}, \mathbf{v}, \eta, \mathbf{T}, \mathbf{s}$ | $\mathbf{u}, \mathbf{v}, \mathbf{w}, \eta, \mathbf{T}, \mathbf{s}$ |

← **lower dimension**

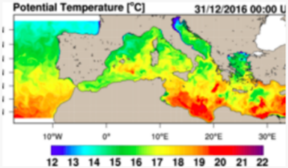
$$\mathbf{X}^* = \underset{\mathbf{X_{t_0}}}{\text{argmin}} \ \|\mathcal{H}(\mathbf{X}) - \mathbf{y}\|^2 + \mathcal{L}_{\text{prior}}(\mathbf{X})$$

- $\mathbf{X_{t_0}} \to \mathbf{X}$ with the model
- argmin ? automatic differentiation (PyTorch)

**Simple models (QG, SW) $\implies$ low-dimension , strong regularization**

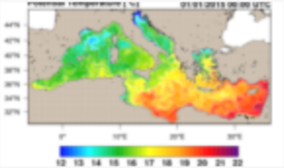
# Current products on copernicus marine

**Products** 3



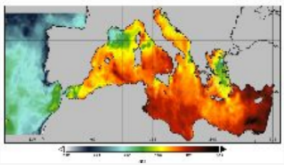

**Mediterranean Sea Physics Analysis and Forecast**

MEDSEA_ANALYSISFORECAST_PHY_006_013
Models
Med Sea, 0.042° × 0.042° × 141 levels
Since 29 Nov 2020, sub-hourly, hourly, daily,...
Mixed layer thickness, salinity, sea surface height, temperature, velocity



**Mediterranean Sea Physics Reanalysis**

MEDSEA_MULTIYEAR_PHY_006_004
Models
Med Sea, 0.042° × 0.042° × 141 levels
Since 1 Jan 1987, hourly, daily, monthly, yearly...
Mixed layer thickness, salinity, sea surface height, temperature, velocity



**Mediterranean Sea - High Resolution Diurnal Subskin Sea...**

SST_MED_PHY_SUBSKIN_L4_NRT_010_036
Satellite (L4)
Med Sea, 0.0625° × 0.0625°
Since 1 Jan 2019, hourly
Temperature

- Observation: 6km, surface only
- Reanalysis: primitive equations, 4km $\times$ 141 vertical levels (!!!)

# Machine learning

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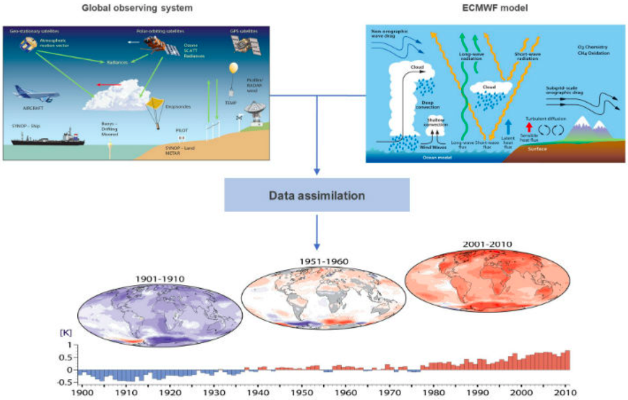
# Available Data for Machine Learning

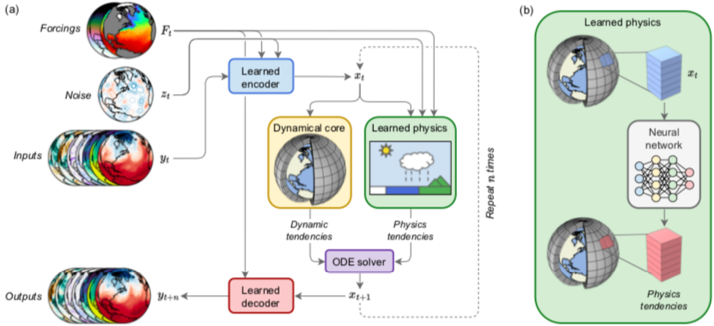
**Ocean**: only surface observations, Copernicus poor quality reanalysis
**Atmosphere**: lots of observations, ECMWF ERA5 high-quality reanalysis




- january 1940 to present
- whole atmosphere, res. 25km, 137 vertical levels
- time resolution: 1h

# NeuralGCM: hybrid physics + ML model

Differentiable physical dynamical core + machine learned physics on the vertical




https://github.com/google-research/dinosaur , 8609 of Python-JAX code.

# Questions ?

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# References I


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