Mining and Forecasting of Big Time-series Data

Yasushi Sakurai (Kumamoto University)
Yasuko Matsubara (Kumamoto University)
Christos Faloutsos (Carnegie Mellon University)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 1

Kumamoto U — CMU CS

Roadmap

- Motivation
- **Similarity search, pattern discovery and summarization** — **Part 1**
- Non-linear modeling and forecasting — **Part 2**
- Extension of time-series data: tensor analysis — **Part 3**

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 2

Part 1

Similarity search, pattern discovery and summarization

Yasushi Sakurai (Kumamoto University)
Yasuko Matsubara (Kumamoto University)
Christos Faloutsos (Carnegie Mellon University)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 3

Kumamoto U — CMU CS

Part 1 - Roadmap

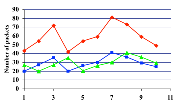
- Motivation
- Similarity Search and Indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 4

Kumamoto U — CMU CS

Problem definition

- Given: one or more sequences
  $x_1, x_2, \ldots, x_t, \ldots$
  $(y_1, y_2, \ldots, y_t, \ldots)$
- Find
  - similar sequences; forecasts
  - patterns; clusters; outliers

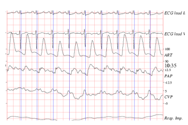


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 5

Kumamoto U — CMU CS

Motivation - Applications

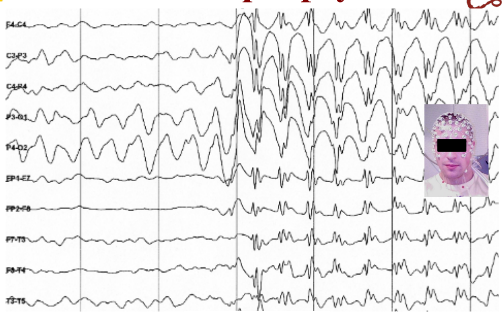
- Financial, sales, economic series
- Medical
  - reactions to new drugs
  - elderly care
  - ECG ('physionet.org')



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 6

Kumamoto U CMU CS

EEG - epilepsy



from wikipedia

Kumamoto U CMU CS

Motivation - Applications (cont' d)

- 'Smart house'
  - sensors monitor temperature, humidity, air quality
- Video surveillance

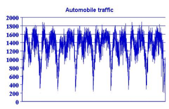
Kumamoto U CMU CS

Motivation - Applications (cont' d)

- Civil/automobile infrastructure
  - bridge vibrations [Oppenheim+02]

Tokyo Gate Bridge

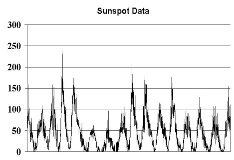
  - road conditions / traffic monitoring



Kumamoto U CMU CS

Motivation - Applications (cont' d)

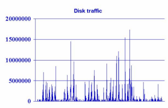
- Weather, environment/anti-pollution
  - volcano monitoring
  - air/water pollutant monitoring



Kumamoto U CMU CS

Motivation - Applications (cont' d)

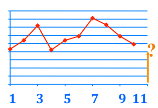
- Computer systems
  - 'Active Disks' (buffering, prefetching)
  - web servers (ditto)
  - network traffic monitoring
  - ...

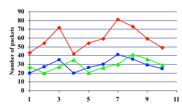


Kumamoto U CMU CS

Wish list

- Problem 1: find patterns/**rules**
- Problem 2: **forecast**
  - Problem 2': **similarity** search
- Problem 3: find patterns/rules/forecast, with **many** time sequences



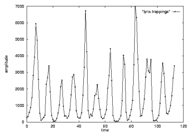


Kumamoto U CMU CS

Problem #1:

Goal: given a signal (eg., #packets over time)
Find: patterns, periodicities, and/or compress

count



lynx caught per year
(packets per day;
temperature per day)

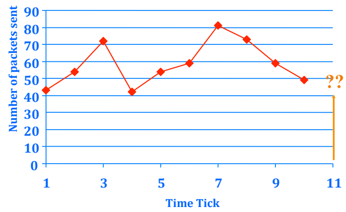
year

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 13

Kumamoto U CMU CS

Problem#2: Forecast

Given x_t, x_{t-1}, …, forecast x_{t+1}

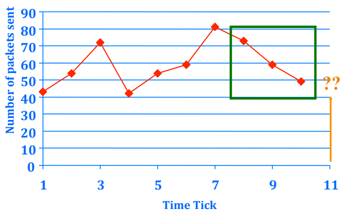


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 14

Kumamoto U CMU CS

Problem#2': Similarity search

Eg., Find a 3-tick pattern, similar to the last one

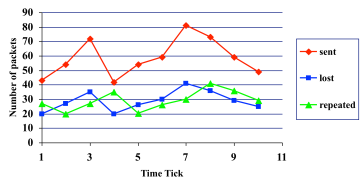


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 15

Kumamoto U CMU CS

Problem #3:

- Given: A set of **correlated** time sequences
- Forecast 'Sent(t)'

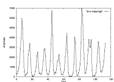


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 16

Kumamoto U CMU CS

Important observations

Patterns, outliers, forecasting and similarity indexing are closely related:



- For forecasting, we need
  – patterns/rules
  – similar past settings
- For outliers, we need to have forecasts
  – (outlier = too far away from our forecast)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 17

Kumamoto U CMU CS

Important topics NOT in this tutorial:

- Continuous queries
  – [Babu+Widom] [Gehrke+] [Madden+]
- Categorical data streams
  – [Hatonen+96]
- Outlier detection (discontinuities)
  – [Breunig+00]

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 18

Kumamoto U CMU CS

Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

Kumamoto U CMU CS

Roadmap

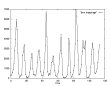
- Motivation
- Similarity Search and Indexing
  - distance functions: Euclidean;Time-warping
  - indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

Kumamoto U CMU CS

Importance of distance functions

Subtle, but **absolutely necessary**:

- A 'must' for similarity indexing
  - (-> forecasting)
- A 'must' for clustering

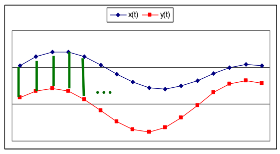


Two major families

- Euclidean and Lp norms
- time warping and variations

Kumamoto U CMU CS

Euclidean and Lp



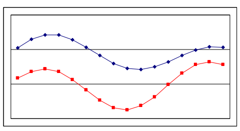
$$D(\vec{x},\vec{y}) = \sum_{i=1}^{n} (x_i - y_i)^2$$

$$L_p(\vec{x},\vec{y}) = \sum_{i=1}^{n} |x_i - y_i|^p$$

- L_1: city-block = Manhattan
- L_2 = Euclidean
- L_∞

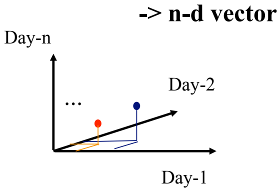
Kumamoto U CMU CS

Observation #1



- **Time sequence**

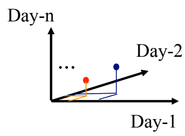
  -> n-d vector

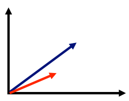


Kumamoto U CMU CS

Observation #2

- Euclidean distance is closely related to
  - cosine similarity
  - dot product
  - 'cross-correlation' function





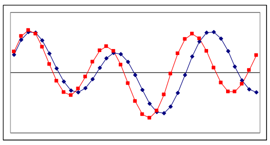
Kumamoto U CMU CS

Time Warping

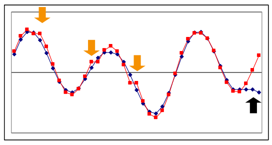
- allow accelerations - decelerations
  - (with or w/o penalty)
- THEN compute the (Euclidean) distance (+ penalty)
- related to the string-editing distance
- fast search methods [Yi+98] [Keogh+02] [Sakurai +05]

Kumamoto U CMU CS

Time Warping



'stutters' :



Kumamoto U

Time Warping DETAILS

Q: how to compute it?

A: dynamic programming

$D(i, j)$ = cost to match

prefix of length i of first sequence x with prefix of length j of second sequence y

Kumamoto U

Time Warping DETAILS

Thus, with no penalty for stutter, for sequences

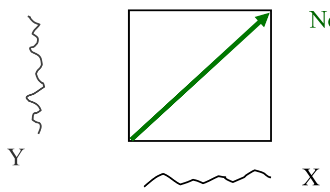
$x_1, x_2, \ldots, x_{i,:} \qquad y_1, y_2, \ldots, y_j$

$$D(i,j) = \|x[i] - y[j]\| + \min \begin{cases} D(i-1, j-1) & \text{no stutter} \\ D(i, j-1) & \text{x-stutter} \\ D(i-1, j) & \text{y-stutter} \end{cases}$$

Kumamoto U CMU CS

Time Warping

- Time warping matrix & optimal path:

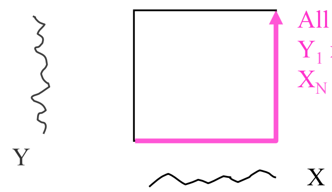


No stutters

Kumamoto U CMU CS

Time Warping

- Time warping matrix & optimal path:



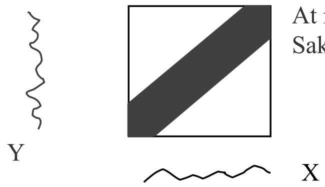
All stutters

Y_1 x N times;

X_N x M times

Time Warping - variations

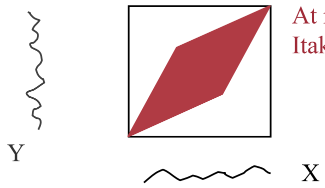
- Time warping matrix & optimal path:



At most k stutters:
Sakoe-Chiba band

Time Warping - variations

- Time warping matrix & optimal path:

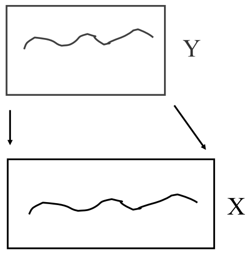


At most x% stutters:
Itakura parallelogram

Time warping

- Complexity: O(M*N) - quadratic on the length of the strings
- **Many** variations (penalty for stutters; limit on the number/percentage of stutters; …)
- popular in voice processing [Rabiner+Juang]

A variation: Uniform axis scaling



- Stretch / shrink time axis of Y, up to p%, for free
- THEN compute Euclidean distance
- [Keogh+, VLDB04]

Other Distance functions

- piece-wise linear/flat approx.; compare pieces [Keogh+01] [Faloutsos+97]
- 'cepstrum' (for voice [Rabiner+Juang])
  - do DFT; take log of amplitude; do DFT again!
- Allow for small gaps [Agrawal+95]

More distance functions.

- Chen + Ng [vldb' 04]: ERP 'Edit distance with Real Penalty' : give a penalty to stutters
- Keogh+ [kdd' 04]: VERY NICE, based on information theory: compress each sequence (quantize + Lempel-Ziv), using the **other** sequences' LZ tables

On The Marriage of Lp-norms and Edit Distance, Lei Chen, Raymond T. Ng:, VLDB' 04

Towards Parameter-Free Data Mining, E. Keogh, S. Lonardi, C.A. Ratanamahatana, KDD' 04

Conclusions

- Prevailing distances:
  - Euclidean and
  - time-warping

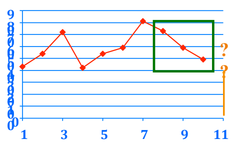
Roadmap

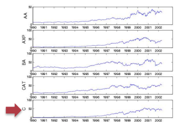
- Motivation
- Similarity Search and Indexing
  - distance functions: Euclidean;Time-warping
  - indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

Indexing

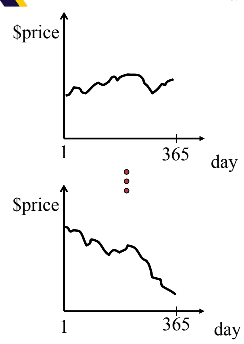
Problem 2':

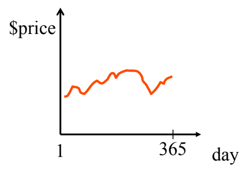
- given a set of time sequences,
- find the ones similar to a desirable query sequence





Indexing





distance function: by expert

(Euclidean; DTW; …)

Idea: 'GEMINI'

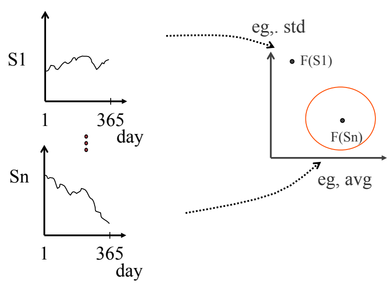
Eg., *'find stocks similar to MSFT'*

Seq. scanning: too slow

How to accelerate the search?

[Faloutsos96]

'GEMINI' - Pictorially



Kumamoto U　CMU CS

GEMINI

Solution: Quick-and-dirty' filter:

- extract *n* features (numbers, eg., avg., etc.)
- map into a point in *n*-d feature space
- organize points with off-the-shelf spatial access method ('SAM' – R-tree, etc)
- discard false alarms

Kumamoto U　CMU CS

Examples of GEMINI

- Time sequences: DFT (up to 100 times faster) [SIGMOD94];
- [Kanellakis+], [Mendelzon+]

Kumamoto U　CMU CS

Indexing - SAMs

Q: How do Spatial Access Methods (SAMs) work?

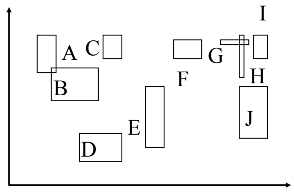
A: they group nearby points (or regions) together, on nearby disk pages, and answer spatial queries quickly ('range queries', 'nearest neighbor' queries etc)

For example:

Kumamoto U　CMU CS

R-trees

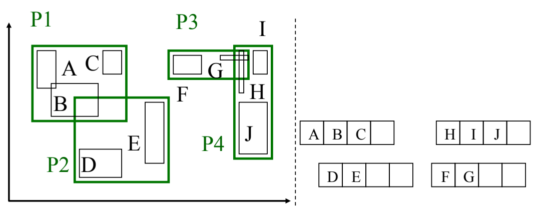
- [Guttman84] eg., w/ fanout 4: group nearby rectangles to parent MBRs; each group -> disk page



Kumamoto U　CMU CS

R-trees

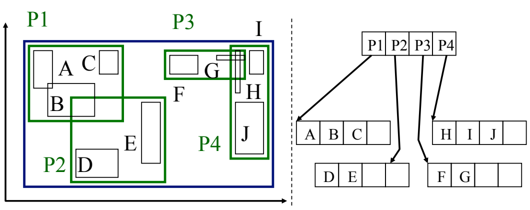
- eg., w/ fanout 4:



Kumamoto U　CMU CS

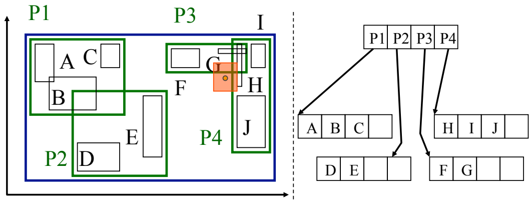
R-trees

- eg., w/ fanout 4:



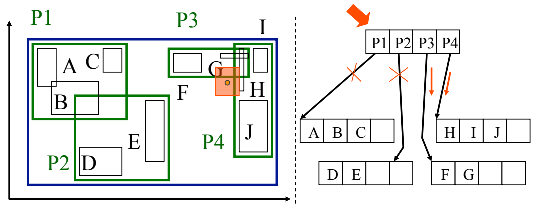
Kumamoto U　　CMU CS

R-trees - range search?



Kumamoto U　　CMU CS

R-trees - range search?



Kumamoto U　　CMU CS

Recent work

- Rakthanmanon+ [kdd' 12]: EXCELLENT Software, the UCR Suite for ultrafast subsequence search
- Zoumpatianos+ [sigmod'14]: ADS+, exploratory analysis
- Camerra+ [KAIS' 14]: iSAX2+, indexing for bulk loading

Eamonn Keogh (UCR)

Searching and Mining Trillions of Time Series Subsequences under Dynamic Time Warping, T. Rakthanmanon, B. Campana, A. Mueen, G. Batista, B. Westover, Q. Zhu, J. Zakaria, E. Keogh:,KDD' 12

Indexing for Interactive Exploration of Big Data Series, K. Zoumpatianos, S. Idreos, T. Palpanas:, SIGMOD' 14

Beyond One Billion Time Series: Indexing and Mining Very Large Time Series Collections with iSAX2+, A. Camerra, J. Shieh, T. Palpanas, T. Rakthanmanon, E. Keogh:, KAIS 2014

Kumamoto U　　CMU CS

Conclusions

- Fast indexing: through GEMINI
  - feature extraction and
  - (off the shelf) Spatial Access Methods [Gaede +98]

Kumamoto U　　CMU CS

Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

Kumamoto U　　CMU CS

Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
  - DFT, DWT, DCT (data independent)
  - SVD, ICA (data independent)
  - MDS, FastMap
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

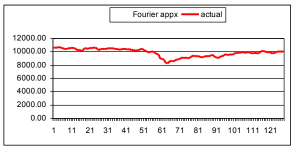
Kumamoto U | CMU CS

DFT and cousins

- very good for compressing real signals
- more details on DFT/DCT/DWT: later

Kumamoto U | CMU CS

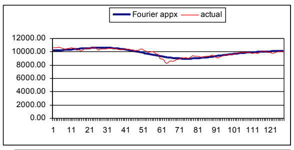
DFT and stocks



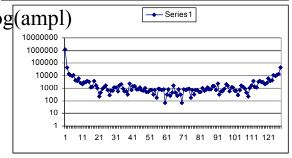
- Dow Jones Industrial index, 6/18/2001-12/21/2001

Kumamoto U | CMU CS

DFT and stocks



Log(ampl)



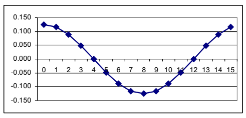
freq

- Dow Jones Industrial index, 6/18/2001-12/21/2001
- just 3 DFT coefficients give very good approximation

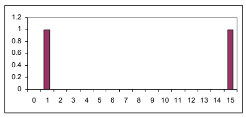
Kumamoto U | CMU CS

DFT (and DWT)

- Many more details, soon



time



freq

Kumamoto U | CMU CS

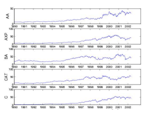
Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
  - DFT, DWT, DCT (data independent)
  - SVD, ICA (data independent)
  - MDS, FastMap
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

Kumamoto U | CMU CS

SVD

- THE optimal method for dimensionality reduction
  - (under the Euclidean metric)
- Given: many time sequences
- Find: the latent ('hidden') variables



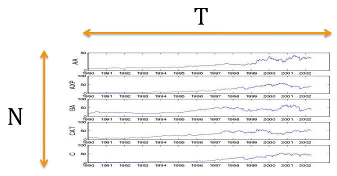
Kumamoto U | CMU CS

SVD

Two (equivalent) interpretations:

– Geometric (each sequence -> point in T-d space)

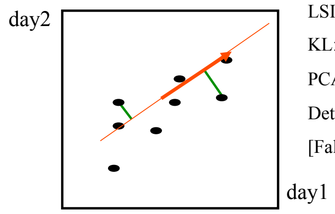
– Matrix algebra (N x T matrix)



Kumamoto U | CMU CS

Singular Value Decomposition (SVD)

- SVD (~LSI ~ KL ~ PCA ~ spectral analysis...) – Geometric interpretation



LSI: S. Dumais; M. Berry

KL: eg, Duda+Hart

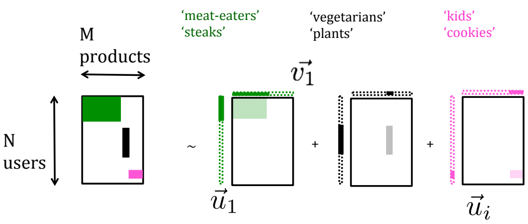
PCA: eg., Jolliffe

Details: [Press+], [Faloutsos96]

Kumamoto U | CMU CS

Reminder:

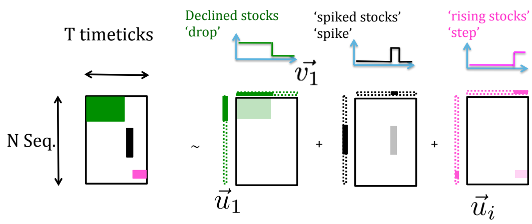
- SVD -> matrix factorization: finds blocks



Kumamoto U | CMU CS

SVD – matrix interpretation

- SVD -> matrix factorization: finds blocks



Kumamoto U | CMU CS

SVD

- **Extremely** useful tool
  - (also behind PageRank/google and Kleinberg's algorithm for hubs and authorities)

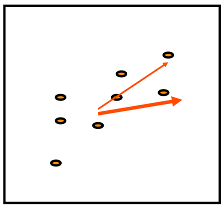
Kumamoto U | CMU CS

SVD

- **Extremely** useful tool
  - (also behind PageRank/google and Kleinberg's algorithm for hubs and authorities)
- But may be slow: O($N * M * M$) if $N>M$
- any approximate, faster method?

SVD shorcuts

- random projections (Johnson-Lindenstrauss thm [Papadimitriou+ pods98])

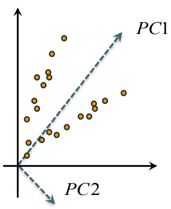


Random projections

- pick 'enough' random directions (will be ~orthogonal, in high-d!!)
- distances are preserved probabilistically, within epsilon
- (also, use as a pre-processing step for SVD [Papadimitriou+ PODS98])

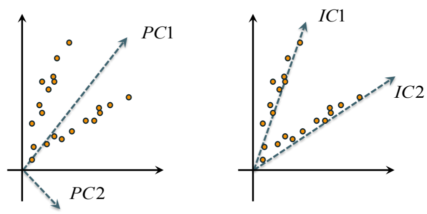
SVD & improvement

- Q: Can we do even better?
- A: sometimes, yes – by shooting for sparsity



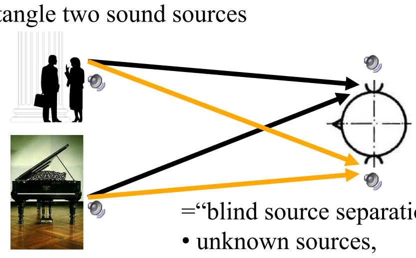
Independent Component Analysis (ICA)

- PCA sometimes misses essential features
  - PCA vs. ICA



A.k.a.: BSS = coctail party problem Find hidden variables

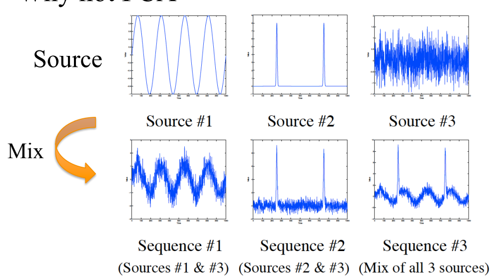
- Untangle two sound sources

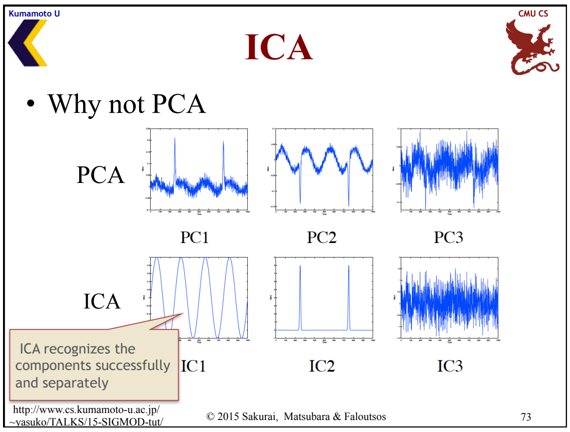


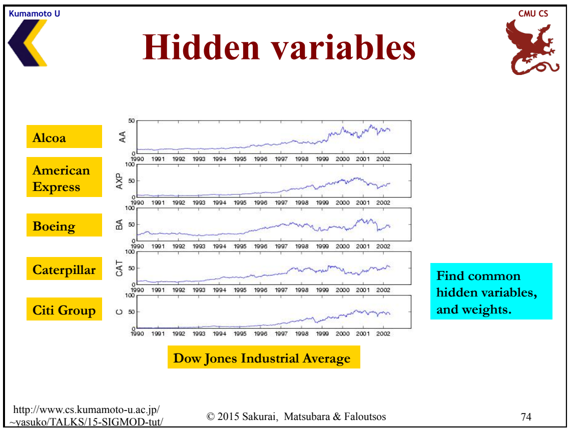
="blind source separation"
- unknown sources,
- unknown mixing

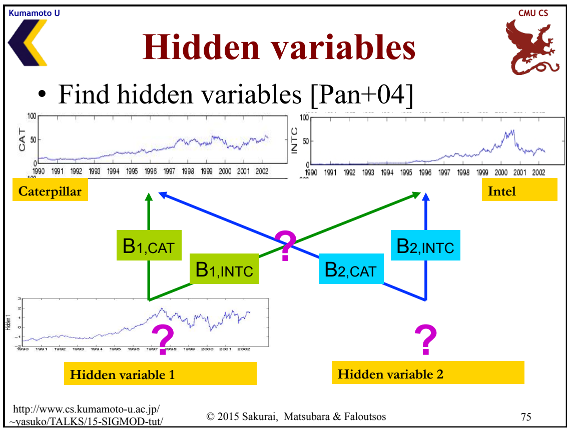
ICA

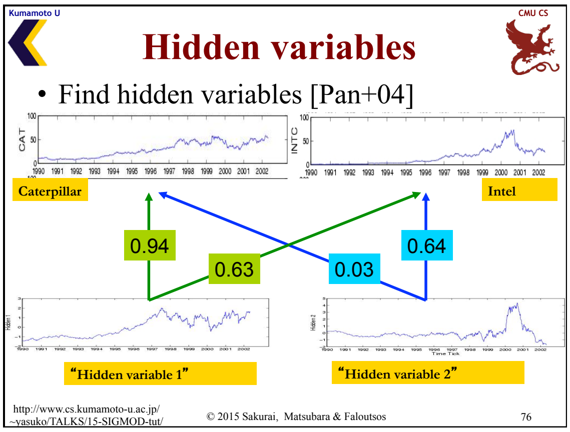
- Why not PCA

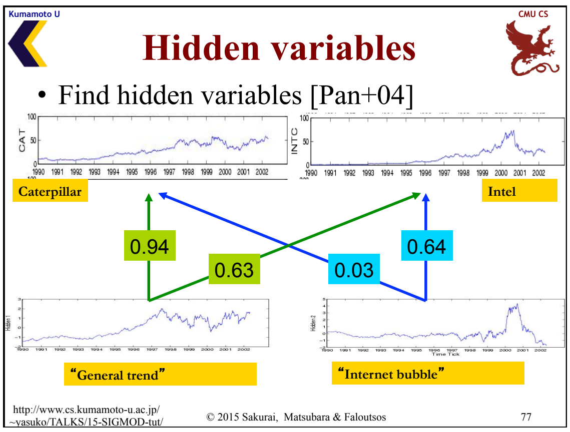


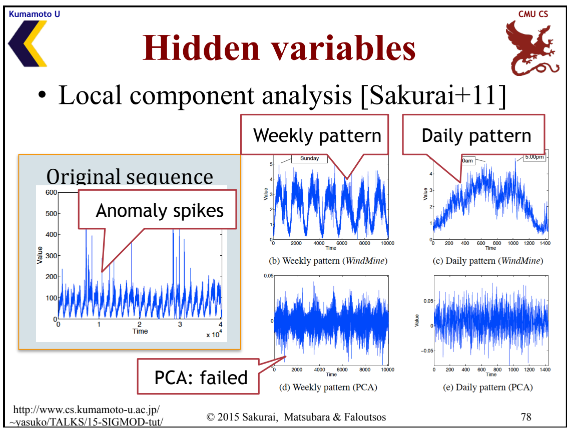

Kumamoto U
CMU CS
ICA
• Why not PCA
PCA
PC1
PC2
PC3
ICA
IC1
IC2
IC3
ICA recognizes the components successfully and separately
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/
© 2015 Sakurai, Matsubara & Faloutsos
73


Kumamoto U
CMU CS
Hidden variables
Alcoa
American Express
Boeing
Caterpillar
Citi Group
Find common hidden variables, and weights.
Dow Jones Industrial Average
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/
© 2015 Sakurai, Matsubara & Faloutsos
74


Kumamoto U
CMU CS
Hidden variables
• Find hidden variables [Pan+04]
Caterpillar
Intel
B1,CAT
B1,INTC
B2,CAT
B2,INTC
?
Hidden variable 1
Hidden variable 2
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/
© 2015 Sakurai, Matsubara & Faloutsos
75


Kumamoto U
CMU CS
Hidden variables
• Find hidden variables [Pan+04]
Caterpillar
Intel
0.94
0.63
0.03
0.64
"Hidden variable 1"
"Hidden variable 2"
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/
© 2015 Sakurai, Matsubara & Faloutsos
76


Kumamoto U
CMU CS
Hidden variables
• Find hidden variables [Pan+04]
Caterpillar
Intel
0.94
0.63
0.03
0.64
"General trend"
"Internet bubble"
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/
© 2015 Sakurai, Matsubara & Faloutsos
77

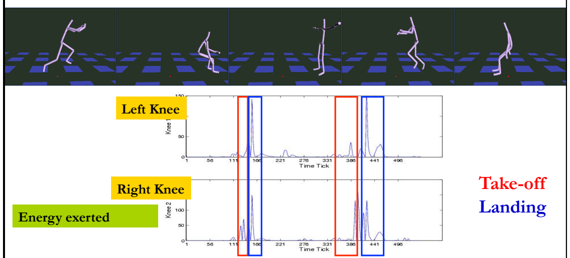

Kumamoto U
CMU CS
Hidden variables
• Local component analysis [Sakurai+11]
Weekly pattern
Daily pattern
Original sequence
Anomaly spikes
(b) Weekly pattern (WindMine)
(c) Daily pattern (WindMine)
PCA: failed
(d) Weekly pattern (PCA)
(e) Daily pattern (PCA)
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/
© 2015 Sakurai, Matsubara & Faloutsos
78

Motivation: Find hidden variables

- ICA: also known as 'Blind Source Separation'
- 'cocktail party problem'
  - in a party, we can hear two concurrent conversations,
  - but separate them (and tune-in on one of them only)
- http://www.cnl.salk.edu/~tewon/Blind/blind_audio.html
- (in stocks: one 'discussion' is the general economy trend; the other 'discussion' is the tech-stock boom

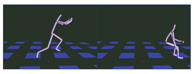
Motivation: Find patterns in data

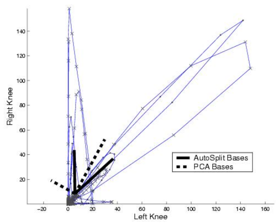
- Motion capture data (broad jumps)



Motivation: Find patterns in data

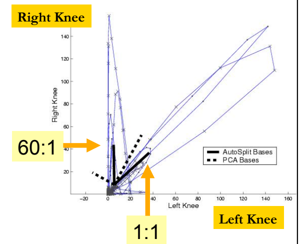
- Best SVD axis: not always meaningful!





Motivation: Find patterns in data

- Human would say
  - Pattern 1: along diagonal
  - Pattern 2: along vertical axis
- How to find these automatically?



Problem formulation

- Given n data items, each has m attributes
- Find the m hidden variables and the m bases

$$\begin{bmatrix} X_{11}, X_{12}, \dots, X_{1m} \\ \dots \\ X_{n1}, X_{n2}, \dots, X_{nm} \end{bmatrix} = \begin{bmatrix} H_{11}, H_{12}, \dots, H_{1m} \\ \dots \quad ? \\ H_{n1}, H_{n2}, \dots, H_{nm} \end{bmatrix} \begin{bmatrix} B_{11}, B_{12}, \dots, B_{1m} \\ \dots \quad ? \\ B_{m1}, B_{m2}, \dots, B_{mm} \end{bmatrix}$$

Samples of the m-th hidden variable

X=HB

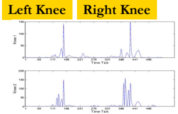
Formulation: (Q1) Find patterns in data

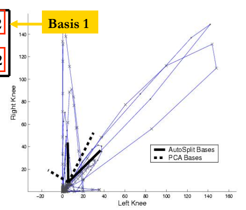
$$\begin{bmatrix} X_{11}, X_{12} \\ \dots \\ X_{n1}, X_{n2} \end{bmatrix} = \begin{bmatrix} H_{11}, H_{12} \\ \dots ? \\ H_{n1}, H_{n2} \end{bmatrix} \begin{bmatrix} B_{11}, B_{12} \\ ? \\ B_{21}, B_{22} \end{bmatrix}$$

Basis 1

Left Knee

Right Knee

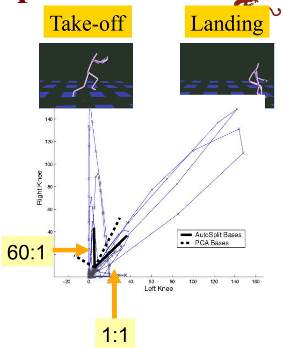




Kumamoto U CMU CS

Q1: Find patterns

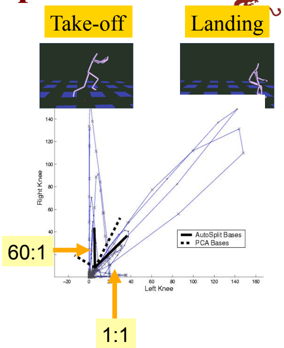
- Patterns found



Kumamoto U CMU CS

Q1: Find patterns

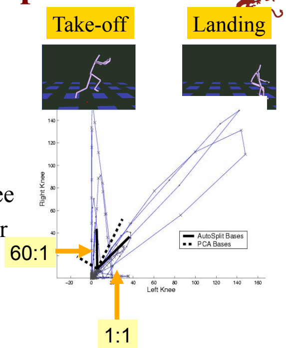
- Patterns found
  - Landing: both knees



Kumamoto U CMU CS

Q1: Find patterns

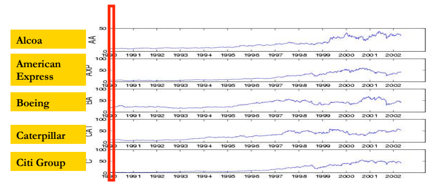
- Patterns found
  - Landing: both knees
  - Take-off: right knee
  - Right-handed actor



Kumamoto U CMU CS

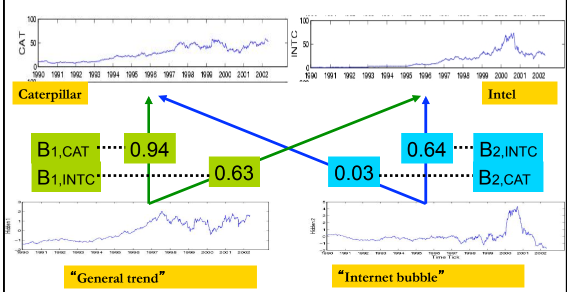
Q2: Find hidden variables (DJIA stocks)

- Weekly DJIA closing prices
  - 01/02/1990-08/05/2002, n=660 data points
  - A data point: prices of 29 companies at the time



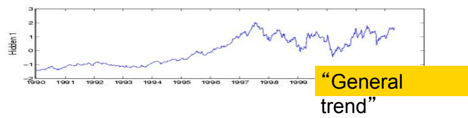
Kumamoto U CMU CS

(Q2) Characterize hidden variable by the companies it influences



Companies related to hidden variable 1

$B_{1,j}$			
Highest		Lowest	
Caterpillar	0.938512	AT&T	0.021885
Boeing	0.911120	WalMart	0.624570
MMM	0.906542	Intel	0.638010
Coca Cola	0.903858	Home Depot	0.647774
Du Pont	0.900317	Hewlett-Packard	0.658768



Citation

- *AutoSplit: Fast and Scalable Discovery of Hidden Variables in Stream and Multimedia Databases,* **Jia-Yu Pan**, Hiroyuki Kitagawa, Christos Faloutsos and Masafumi Hamamoto, PAKDD 2004, Sydney, Australia.
- *WindMine: Fast and Effective Mining of Web-click Sequences*, Yasushi Sakurai, Lei Li, Yasuko Matsubara, Christos Faloutsos, SDM 2011, Mesa, Arizona.

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos #91

Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
  - DFT, DWT, DCT (data independent)
  - SVD, ICA (data independent)
  - MDS, FastMap
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 92

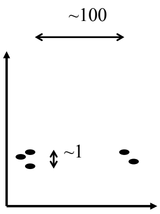
MDS / FastMap

- but, what if we have NO points to start with? (eg. Time-warping distance)
- A: Multi-dimensional Scaling (MDS) ; FastMap

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 93

MDS/FastMap

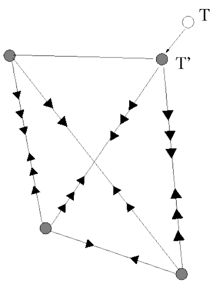
	01	02	03	04	05
01	0	1	1	100	100
02	1	0	1	100	100
03	1	1	0	100	100
04	100	100	100	0	1
05	100	100	100	1	0



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 94

MDS

Multi Dimensional Scaling



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 95

FastMap

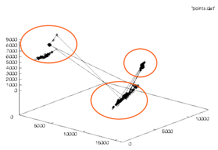
- Multi-dimensional scaling (MDS) can do that, but in O(N**2) time
- FastMap [Faloutsos+95] takes O(N) time

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 96

Kumamoto U　CMU CS

FastMap: Application

VideoTrails [Kobla+97]

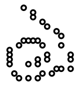



scene-cut detection (about 10% errors)

Kumamoto U　CMU CS

Variations

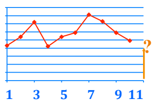
- Isomap [Tenenbaum, de Silva, Langford, 2000]
- LLE (Local Linear Embedding) [Roweis, Saul, 2000]
- MVE (Minimum Volume Embedding) [Shaw & Jebara, 2007]

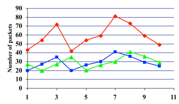


Kumamoto U　CMU CS

Wish list

- Problem 1: find patterns/**rules**
- Problem 2: **forecast**
  - ✔ Problem 2': **similarity** search
- Problem 3: find patterns/rules/forecast, with **many** time sequences





Kumamoto U　CMU CS

Conclusions - Practitioner's guide

Similarity search in time sequences

1) establish/choose distance (Euclidean, time-warping,…)

2) extract features (SVD, ICA, DWT), and use an SAM (R-tree/variant, or a Metric Tree M-tree)

2') for high intrinsic dimensionalities, consider sequential scan (it might win…)

Kumamoto U　CMU CS

Books

- William H. Press, Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery: *Numerical Recipes in C*, Cambridge University Press, 1992, 2nd Edition. (Great description, intuition and code for SVD)
- C. Faloutsos: *Searching Multimedia Databases by Content*, Kluwer Academic Press, 1996 (introduction to SVD, and GEMINI)

Kumamoto U　CMU CS

References

- Agrawal, R., K.-I. Lin, et al. (Sept. 1995). Fast Similarity Search in the Presence of Noise, Scaling and Translation in Time-Series Databases. Proc. of VLDB, Zurich, Switzerland.
- Babu, S. and J. Widom (2001). "Continuous Queries over Data Streams." SIGMOD Record 30(3): 109-120.
- Breunig, M. M., H.-P. Kriegel, et al. (2000). LOF: Identifying Density-Based Local Outliers. SIGMOD Conference, Dallas, TX.
- Berry, Michael: http://www.cs.utk.edu/~lsi/

References

- Ciaccia, P., M. Patella, et al. (1997). M-tree: An Efficient Access Method for Similarity Search in Metric Spaces. VLDB.
- Foltz, P. W. and S. T. Dumais (Dec. 1992). "Personalized Information Delivery: An Analysis of Information Filtering Methods." Comm. of ACM (CACM) 35(12): 51-60.
- Guttman, A. (June 1984). R-Trees: A Dynamic Index Structure for Spatial Searching. Proc. ACM SIGMOD, Boston, Mass.

References

- Gaede, V. and O. Guenther (1998). "Multidimensional Access Methods." Computing Surveys 30(2): 170-231.
- Gehrke, J. E., F. Korn, et al. (May 2001). On Computing Correlated Aggregates Over Continual Data Streams. ACM Sigmod, Santa Barbara, California.

References

- Gunopulos, D. and G. Das (2001). Time Series Similarity Measures and Time Series Indexing. SIGMOD Conference, Santa Barbara, CA.
- Eamonn J. Keogh, Themis Palpanas, Victor B. Zordan, Dimitrios Gunopulos, Marc Cardle: Indexing Large Human-Motion Databases. VLDB 2004: 780-791

References

- Hatonen, K., M. Klemettinen, et al. (1996). Knowledge Discovery from Telecommunication Network Alarm Databases. ICDE, New Orleans, Louisiana.
- Jolliffe, I. T. (1986). Principal Component Analysis, Springer Verlag.

References

- Keogh, E. J., K. Chakrabarti, et al. (2001). Locally Adaptive Dimensionality Reduction for Indexing Large Time Series Databases. SIGMOD Conference, Santa Barbara, CA.
- Kobla, V., D. S. Doermann, et al. (Nov. 1997). VideoTrails: Representing and Visualizing Structure in Video Sequences. ACM Multimedia 97, Seattle, WA.

References

- Oppenheim, I. J., A. Jain, et al. (March 2002). A MEMS Ultrasonic Transducer for Resident Monitoring of Steel Structures. SPIE Smart Structures Conference SS05, San Diego.
- Papadimitriou, C. H., P. Raghavan, et al. (1998). Latent Semantic Indexing: A Probabilistic Analysis. PODS, Seattle, WA.
- Rabiner, L. and B.-H. Juang (1993). Fundamentals of Speech Recognition, Prentice Hall.

Kumamoto U CMU CS

References

- Traina, C., A. Traina, et al. (October 2000). Fast feature selection using the fractal dimension,. XV Brazilian Symposium on Databases (SBBD), Paraiba, Brazil.

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 109

Kumamoto U CMU CS

References

- Dennis Shasha and Yunyue Zhu *High Performance Discovery in Time Series: Techniques and Case Studies* Springer 2004
- Yunyue Zhu, Dennis Shasha ``*StatStream: Statistical Monitoring of Thousands of Data Streams in Real Time*'' VLDB, August, 2002. pp. 358-369.
- Samuel R. Madden, Michael J. Franklin, Joseph M. Hellerstein, and Wei Hong. *The Design of an Acquisitional Query Processor for Sensor Networks*. SIGMOD, June 2003, San Diego, CA.

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 110

Kumamoto U CMU CS

References

- Lawrence Saul & Sam Roweis. *An Introduction to Locally Linear Embedding* (draft)
- Sam Roweis & Lawrence Saul. *Nonlinear dimensionality reduction by locally linear embedding*. Science, v.290 no.5500 , Dec.22, 2000. pp.2323--2326.
- B. Shaw and T. Jebara. "*Minimum Volume Embedding*" . Artificial Intelligence and Statistics, AISTATS, March 2007.

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos #111

Kumamoto U CMU CS

References

- Josh Tenenbaum, Vin de Silva and John Langford. *A Global Geometric Framework for Nonlinear dimensionality Reduction*. Science 290, pp. 2319-2323, 2000.

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos #112

Kumamoto U CMU CS

Roadmap

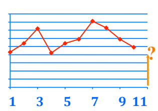
- Motivation
- Similarity Search and Indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

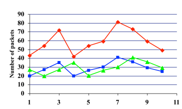
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 113

Kumamoto U CMU CS

Wish list

- Problem 1: find patterns/**rules**
- Problem 2: **forecast**
  - Problem 2': **similarity** search
- Problem 3: find patterns/rules/forecast, with **many** time sequences





http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 114

Kumamoto U | CMU CS

Forecasting

"Prediction is very difficult, especially about the future." - Niels Bohr

http://www.hfac.uh.edu/MediaFutures/thoughts.html

Kumamoto U | CMU CS

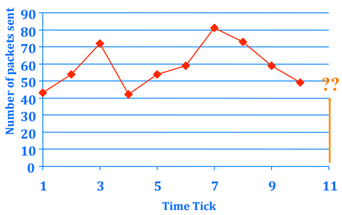
Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
- Linear forecasting
  - Auto-regression: Least Squares; RLS
  - Co-evolving time sequences
- Streaming pattern discovery
- Automatic mining

Kumamoto U | CMU CS

Problem: Forecasting

- Example: give x_{t-1}, x_{t-2}, …, forecast x_t



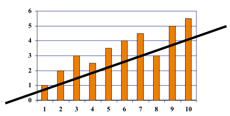
Kumamoto U | CMU CS

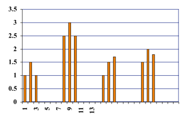
Forecasting: Preprocessing

MANUALLY:

remove trends

spot periodicities





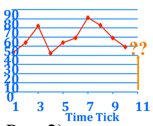
Kumamoto U | CMU CS

Problem: Forecast

- Solution: try to express
  x_t
  as a linear function of the past: x_{t-2}, x_{t-2}, …,
  (up to a window of w)

Formally:

$$\boxed{x_t \approx a_1 x_{t-1} + \ldots + a_w x_{t-w} + noise}$$

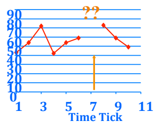


(if we **know** it is a non-linear model, see Part 2)

Kumamoto U | CMU CS

(Problem: Back-cast; interpolate)

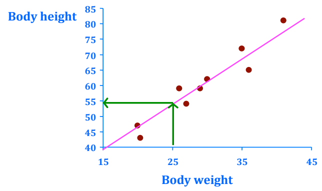
- Solution - interpolate: try to express
  x_t
  as a linear function of the past AND the future:
  x_{t+1}, x_{t+2}, … $x_{t+wfuture;}$ x_{t-1}, … $x_{t-wpast}$
  (up to windows of w_{past} , w_{future})
- EXACTLY the same algo's



Kumamoto U — CMU CS

Linear Regression: idea

patient	weight	height
1	27	43
2	43	54
3	54	72
…	…	…
N	25	??



- express what we don't know (= 'dependent variable')
- as a linear function of what we know (= 'indep. variable(s)')

Kumamoto U — CMU CS

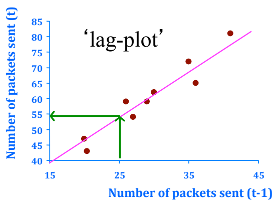
Linear Auto Regression:

Time	Packets Sent(t)
1	43
2	54
3	72
…	…
N	??

Kumamoto U — CMU CS

Linear Auto Regression:

Time	Packets Sent (t-1)	Packets Sent(t)
1	-	43
2	43	54
3	54	72
…	…	…
N	25	??

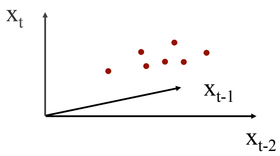


- lag w=1
- Dependent variable = # of packets sent (S [t])
- Independent variable = # of packets sent (S[t-1])

Kumamoto U — CMU CS

More details:

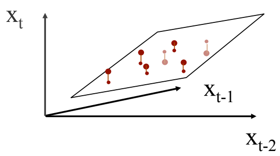
- Q1: Can it work with window w>1?
- A1: YES!



Kumamoto U — CMU CS

More details:

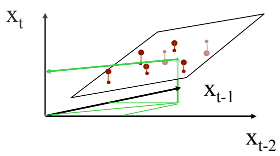
- Q1: Can it work with window w>1?
- A1: YES! (we'll fit a hyper-plane, then!)



Kumamoto U — CMU CS

More details:

- Q1: Can it work with window w>1?
- A1: YES! (we'll fit a hyper-plane, then!)



Kumamoto U

More details: DETAILS

- Q1: Can it work with window w>1?
- A1: YES! The problem becomes:

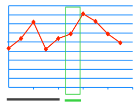
$$\mathbf{X}_{[N \times w]} \times \mathbf{a}_{[w \times 1]} = \mathbf{y}_{[N \times 1]}$$

- OVER-CONSTRAINED
  - **a** is the vector of the regression coefficients
  - **X** has the N values of the w indep. variables
  - **y** has the N values of the dependent variable

Kumamoto U

More details: DETAILS

- $\mathbf{X}_{[N \times w]} \times \mathbf{a}_{[w \times 1]} = \mathbf{y}_{[N \times 1]}$



Ind-var1 Ind-var-w

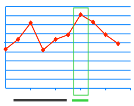
time

$$\begin{bmatrix} X_{11}, X_{12}, \cdots, X_{1w} \\ X_{21}, X_{22}, \ldots, X_{2w} \\ \vdots \\ \vdots \\ \vdots \\ X_{N1}, X_{N2}, \ldots, X_{Nw} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_w \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ \vdots \\ y_N \end{bmatrix}$$

Kumamoto U

More details: DETAILS

- $\mathbf{X}_{[N \times w]} \times \mathbf{a}_{[w \times 1]} = \mathbf{y}_{[N \times 1]}$



Ind-var1 Ind-var-w

time

$$\begin{bmatrix} X_{11}, X_{12}, \cdots, X_{1w} \\ X_{21}, X_{22}, \ldots, X_{2w} \\ \vdots \\ \vdots \\ \vdots \\ X_{N1}, X_{N2}, \ldots, X_{Nw} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_w \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ \vdots \\ y_N \end{bmatrix}$$

Kumamoto U

More details DETAILS

- Q2: How to estimate $a_1, a_2, \ldots a_w = \mathbf{a}$?
- A2: with Least Squares fit

$$\boxed{\mathbf{a} = (\mathbf{X}^T \times \mathbf{X})^{-1} \times (\mathbf{X}^T \times \mathbf{y})}$$

- (Moore-Penrose pseudo-inverse)
- **a** is the vector that minimizes the RMSE from **y**

Kumamoto U

Even more details DETAILS

- Q3: Can we estimate **a** incrementally?
- A3: Yes, with the brilliant, classic method of 'Recursive Least Squares' (RLS) (see, e.g., [Yi+00], for details) - pictorially:

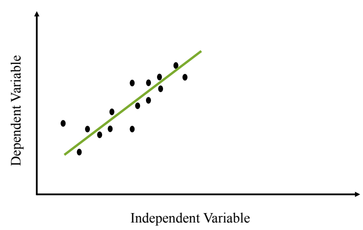
[Yi+00] Byoung-Kee Yi et al.: *Online Data Mining for Co-Evolving Time Sequences*, ICDE 2000.

Kumamoto U

CMU CS

Even more details

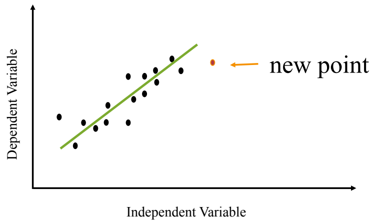
- Given:



Kumamoto U CMU CS

Even more details

- Given:

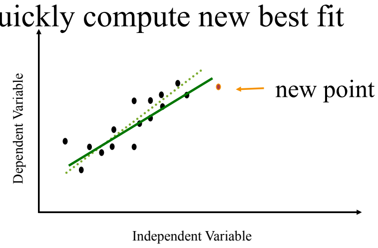


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 133

Kumamoto U CMU CS

Even more details

Recursive Least Squares (RLS): quickly compute new best fit



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 134

Kumamoto U CMU CS

Even more details

- Straightforward Least Squares
  - Needs huge matrix (**growing** in size) $O(N \times w)$
  - Costly matrix operation $O(N \times w^2)$
- Recursive LS
  - Need much smaller, fixed size matrix $O(w \times w)$
  - Fast, incremental computation $O(1 \times w^2)$

49,000,000 ⟷ 49

$N = 10^6$, w = 1-100

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 135

Kumamoto U CMU CS

Even more details

- Straightforward Least Squares
  - Needs huge matrix (**growing** in size) $O(N \times$
  - Costly matrix opera
  $O(N \times w^2)$
- Recursi
  - aller, fixed
  x $O(w \times w)$
  - , incremental computation $O(1 \times w^2)$

49,0 ⟷ 49

$N = 10^6$, w = 1-100

RLS: GREAT for streams

http://www amoto-u.ac.jp/~yasuko/TA KS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 136

Kumamoto U

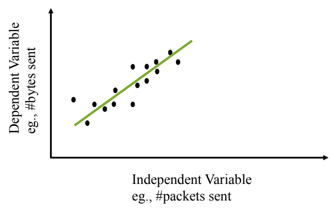
Even more detail DETAILS

- Q4: can we 'forget' the older samples?
- A4: Yes - RLS can easily handle that [Yi +00]:

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 137

Kumamoto U

Adaptability - 'forgetting' DETAILS

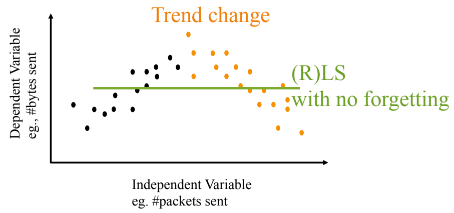


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 138

Kumamoto U

Adaptability - 'forgetting'

DETAILS

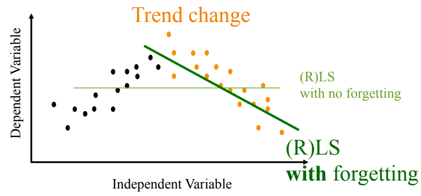


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 139

Kumamoto U

Adaptability - 'forgetting'

DETAILS



- RLS: can *trivially* handle 'forgetting'

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 140

Kumamoto U CMU CS

How to choose 'w' ?

- Quick & dirty answer: w=1 or w=2
- Better answer: Model selection (say, with BIC or MDL – see later)
- Even better answer: **multi-scale windows** [Papadimitriou+, vldb2003]

Spiros Papadimitriou, Anthony Brockwell and Christos Faloutsos *Adaptive, Hands-Off Stream Mining* VLDB 2003, Berlin, Germany, Sept. 2003

Kumamoto U CMU CS

How to choose 'w' ?

- goal: capture arbitrary periodicities
- with NO human intervention
- on a semi-infinite stream

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 142

Kumamoto U CMU CS

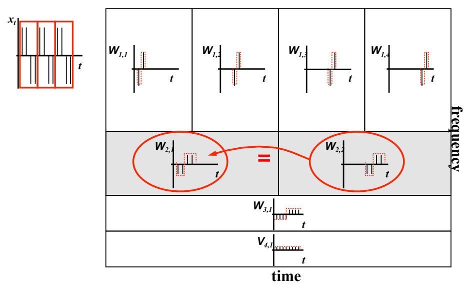
Answer:

- 'AWSOM' (Arbitrary Window Stream fOrecasting Method) [Papadimitriou+, vldb2003]
- idea: do AR on each wavelet level
- in detail:

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 143

Kumamoto U CMU CS

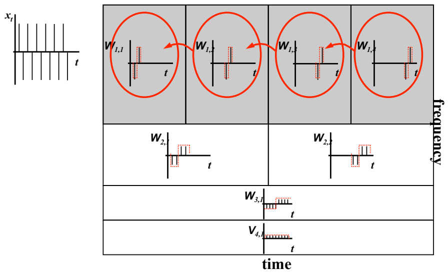
AWSOM



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 144

Kumamoto U CMU CS

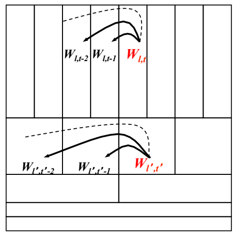
AWSOM



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 145

Kumamoto U CMU CS

AWSOM - idea



$W_{l,t} = \beta_{l,1} W_{l,t-1} + \beta_{l,2} W_{l,t-2} + \ldots$

$W_{l',t'} = \beta_{l',1} W_{l',t'-1} + \beta_{l',2} W_{l',t'-2} + \ldots$

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 146

Kumamoto U CMU CS

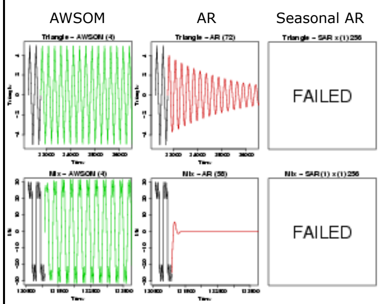
More details…

- Update of wavelet coefficients (incremental)
- Update of linear models (incremental; RLS)
- Feature selection (single-pass)
  - Not all correlations are significant
  - Throw away the insignificant ones ("noise")

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 147

Kumamoto U CMU CS

Results - Synthetic data

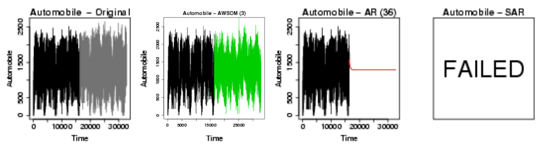


- Triangle pulse
- Mix (sine + square)
- AR captures wrong trend (or none)
- Seasonal AR estimation fails

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 148

Kumamoto U CMU CS

Results - Real data

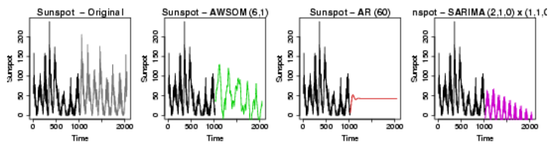


- Automobile traffic
  - Daily periodicity
  - Bursty "noise" at smaller scales
- AR fails to capture any trend
- Seasonal AR estimation fails

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 149

Kumamoto U CMU CS

Results - real data



- Sunspot intensity
  - Slightly time-varying "period"
- AR captures wrong trend
- Seasonal ARIMA
  - wrong downward trend, despite help by human!

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 150

Kumamoto U CMU CS

Complexity

Skip

- Model update

  Space: $O(lgN + mk^2) \approx O(lgN)$

  Time: $O(k^2) \approx O(1)$

- Where
  - N: number of points (so far)
  - k: number of regression coefficients; fixed
  - m: number of linear models; $O(lgN)$

Kumamoto U CMU CS

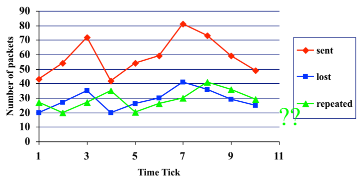
Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
- Streaming pattern discovery
- Linear forecasting
  - Auto-regression: Least Squares; RLS
  - Co-evolving time sequences
- Automatic mining

Kumamoto U CMU CS

Co-Evolving Time Sequences

- Given: A set of **correlated** time sequences
- Forecast '**Repeated(t)**'



Kumamoto U CMU CS

Solution:

Q: what should we do?

Kumamoto U CMU CS

Solution:

Least Squares, with

- Dep. Variable: Repeated(t)
- Indep. Variables: Sent(t-1) … Sent(t-w); Lost(t-1) …Lost(t-w); Repeated(t-1), ...
- (named: 'MUSCLES' [Yi+00])

Kumamoto U CMU CS

Practitioner's guide

- AR(IMA) methodology: prevailing method for linear forecasting
- Brilliant method of Recursive Least Squares for fast, incremental estimation.
- See [Box-Jenkins]

Resources: software and urls

- MUSCLES: Prof. Byoung-Kee Yi: **http://www.postech.ac.kr/~bkyi/** or **christos@cs.cmu.edu**
- free-ware: 'R' for stat. analysis (clone of Splus) **http://cran.r-project.org/**

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 157

Books

- George E.P. Box and Gwilym M. Jenkins and Gregory C. Reinsel, *Time Series Analysis: Forecasting and Control*, Prentice Hall, 1994 (the classic book on ARIMA, 3rd ed.)
- Brockwell, P. J. and R. A. Davis (1987). Time Series: Theory and Methods. New York, Springer Verlag.

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 158

Additional Reading

- [Papadimitriou+ vldb2003] Spiros Papadimitriou, Anthony Brockwell and Christos Faloutsos *Adaptive, Hands-Off Stream Mining* VLDB 2003, Berlin, Germany, Sept. 2003
- [Yi+00] Byoung-Kee Yi et al.: *Online Data Mining for Co-Evolving Time Sequences*, ICDE 2000. (Describes MUSCLES and Recursive Least Squares)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 159

Part 1

Similarity search, pattern discovery and summarization

Yasushi Sakurai (Kumamoto University)
Yasuko Matsubara (Kumamoto University)
Christos Faloutsos (Carnegie Mellon University)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 160

Roadmap

- Motivation
- Similarity Search and Indexing
- Feature extraction
  - DFT, DWT, DCT (data independent)
  - SVD, ICA (data independent)
  - (MDS, FastMap)
- Streaming pattern discovery

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 161

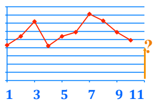
Roadmap

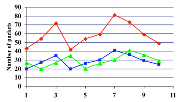
- DFT
  - Definition of DFT and properties
  - how to read the DFT spectrum
- DWT
  - Definition of DWT and properties
  - how to read the DWT scalogram

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 162

Wish list

- Problem 1: find patterns/**rules**
- ✔Problem 2: **forecast**
  - ✔ Problem 2': **similarity** search
- Problem 3: find patterns/rules/forecast, with **many** time sequences



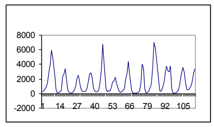


Introduction - Problem#1

Goal: given a signal (eg., packets over time)

Find: patterns and/or compress

count



lynx caught per year
(packets per day;
automobiles per hour)

year

DFT: definition **DETAILS**

- For a sequence $x_0, x_1, \dots x_{n-1}$
- the (**n-point**) Discrete Fourier Transform is
- $X_0, X_1, \dots X_{n-1}$:

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j 2\pi t f / n) \qquad f = 0, \dots, n-1$$

$$(j = \sqrt{-1})$$

inverse DFT

$$x_t = 1/\sqrt{n} \sum_{t=0}^{n-1} X_f * \exp(+j 2\pi t f / n)$$

DFT: definition

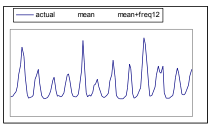
- **Good** news: Available in **all** symbolic math packages, eg., in 'mathematica'

```
x = [1,2,1,2];
X = Fourier[x];
Plot[ Abs[X] ];
```


DFT: Amplitude spectrum

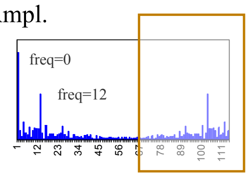
Amplitude: $A_f^2 = \mathrm{Re}^2(X_f) + \mathrm{Im}^2(X_f)$

count



year

Ampl.



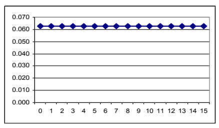
Freq.

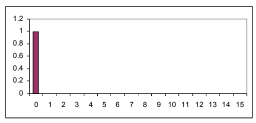
DFT: examples

Skip

flat

Amplitude





time

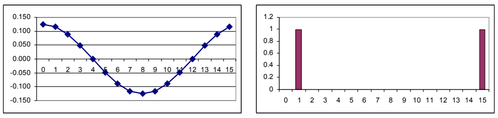
freq

Kumamoto U CMU CS

DFT: examples

Skip

Low frequency sinusoid

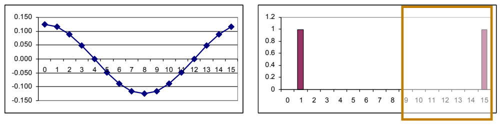


Kumamoto U CMU CS

DFT: examples

Skip

- Sinusoid - symmetry property: $X_f = X^*_{n-f}$

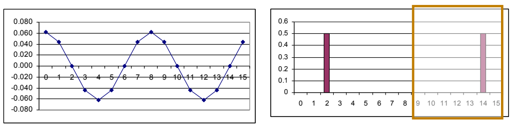


Kumamoto U CMU CS

DFT: examples

Skip

- Higher freq. sinusoid

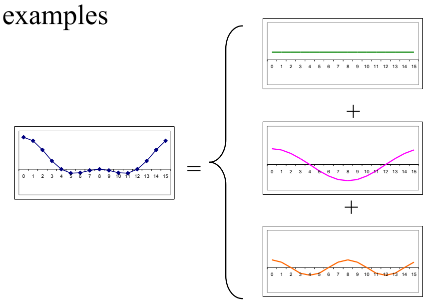


Kumamoto U CMU CS

DFT: examples

Skip

examples

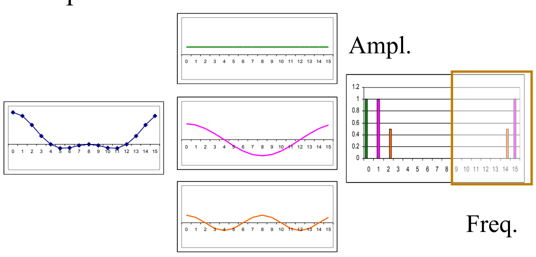


Kumamoto U CMU CS

DFT: examples

Skip

examples



Kumamoto U CMU CS

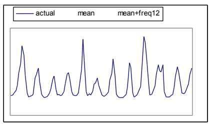
Roadmap

- DFT
  - Definition of DFT and properties
  - how to read the DFT spectrum
- DWT
  - Definition of DWT and properties
  - how to read the DWT scalogram

DFT: Amplitude spectrum

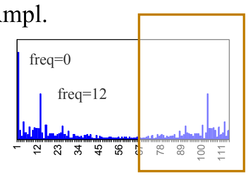
Amplitude: $A_f^{\,2} = \mathrm{Re}^2(X_f) + \mathrm{Im}^2(X_f)$

count



year

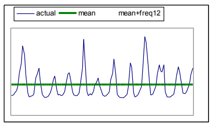
Ampl.



Freq.

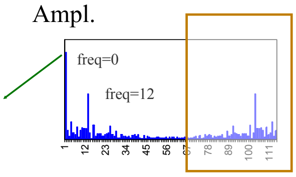
DFT: Amplitude spectrum

count



year

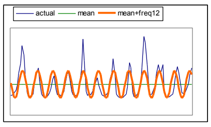
Ampl.



Freq.

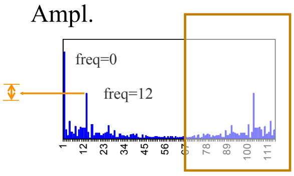
DFT: Amplitude spectrum

count



year

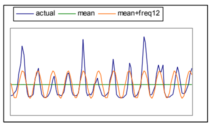
Ampl.

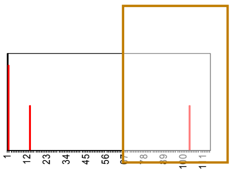


Freq.

DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?

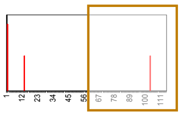




Freq.

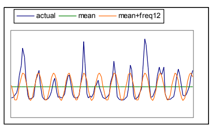
DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: **(lossy) compression**
- A2: pattern discovery



DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: (lossy) compression
- A2: **pattern discovery**



Kumamoto U CMU CS

DFT - Conclusions

- It spots periodicities (with the '**amplitude spectrum**')
- can be quickly computed (O(n log n)), thanks to the FFT algorithm.
- **standard** tool in signal processing (speech, image etc signals)
- (closely related to DCT and JPEG)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 181

Kumamoto U CMU CS

Roadmap

- DFT
  - Definition of DFT and properties
  - how to read the DFT spectrum
- DWT
  - Definition of DWT and properties
  - how to read the DWT scalogram

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 182

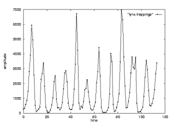
Kumamoto U CMU CS

Problem #1:

Goal: given a signal (eg., #packets over time)

Find: patterns, periodicities, and/or **compress**

count



year

lynx caught per year
(packets per day;
virus infections per month)

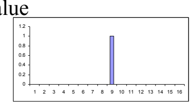
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 183

Kumamoto U CMU CS

Wavelets - DWT

- DFT is great - but, how about compressing a spike?

value



time

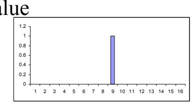
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 184

Kumamoto U CMU CS

Wavelets - DWT

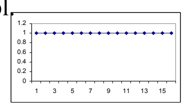
- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value



time

Ampl.



Freq.

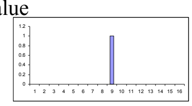
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 185

Kumamoto U CMU CS

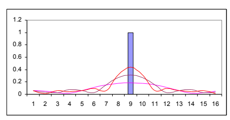
Wavelets - DWT

- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value



time

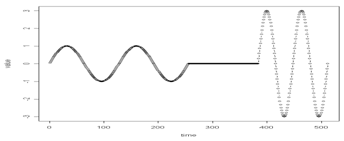


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 186

Wavelets - DWT

- Similarly, DFT suffers on short-duration waves (eg., baritone, silence, soprano)

value



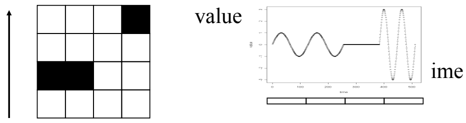
time

Wavelets - DWT

- Solution#1: Short window Fourier transform (SWFT)
- But: how short should be the window?

freq

value



ime

time

Wavelets - DWT

- Answer: **multiple** window sizes! -> DWT

`**Multi-scale windows**': brilliant idea that we'll see several times in this tutorial (BRAID, TriMine, etc)

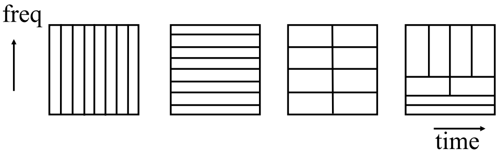
Wavelets - DWT

- Answer: **multiple** window sizes! -> DWT

Multi-scale windows

Time domain DFT SWFT DWT

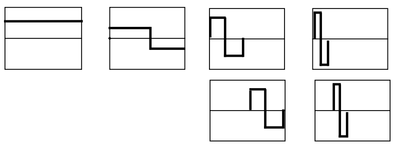
freq



time

Haar Wavelets

- subtract sum of left half from right half
- repeat recursively for quarters, eight-ths, ...

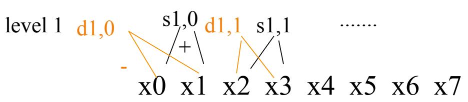


Wavelets - construc DETAILS

x0 x1 x2 x3 x4 x5 x6 x7

Kumamoto U

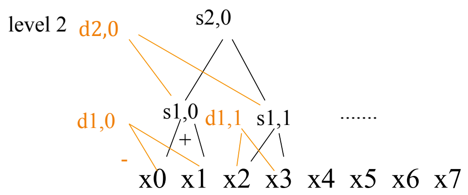
Wavelets - construction DETAILS



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 193

Kumamoto U

Wavelets - construction DETAILS

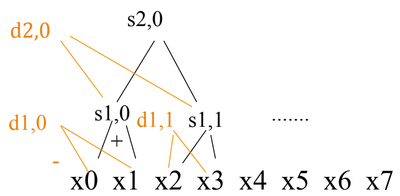


http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 194

Kumamoto U

Wavelets - construction DETAILS

etc ...



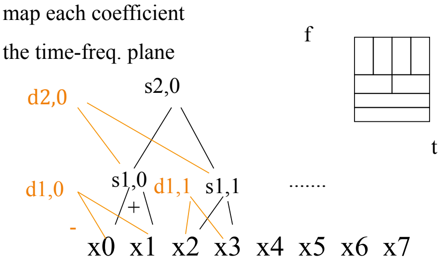
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 195

Kumamoto U

Wavelets - construction DETAILS

Q: map each coefficient

on the time-freq. plane



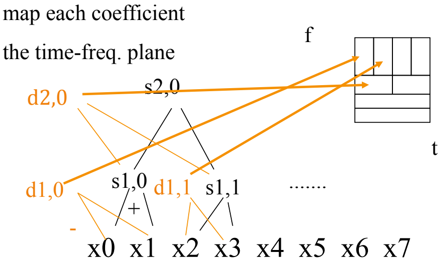
http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 196

Kumamoto U

Wavelets - construction DETAILS

Q: map each coefficient

on the time-freq. plane



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 197

Kumamoto U

Haar wavelets - code

CMU CS

```
#!/usr/bin/perl5
# expects a file with numbers
# and prints the dwt transform
# The number of time-ticks should be a power of 2
# USAGE
#    haar.pl <fname>

my @vals=();
my @smooth; # the smooth component of the signal
my @diff;   # the high-freq. component

# collect the values into the array @val
while(<>){
        @vals = ( @vals ,  split );
}
```

```
my $len = scalar(@vals);
my $half = int($len/2);
while($half >= 1 ){
  for(my $i=0; $i< $half; $i++){
        $diff [$i] = ($vals[2*$i] - $vals[2*$i + 1] )/ sqrt(2);
        print "\t", $diff[$i];
        $smooth [$i] = ($vals[2*$i] + $vals[2*$i + 1] )/ sqrt(2);
  }
  print "\n";
  @vals = @smooth;
  $half = int($half/2);
}
print "\t", $vals[0], "\n" ;     # the final, smooth component
```

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 198

Kumamoto U

Wavelets - construc DETAILS

Observation1:

'+' can be some weighted addition

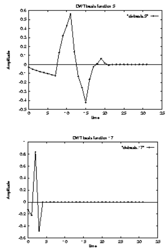
'-' is the corresponding weighted difference ('Quadrature mirror filters')

Observation2: unlike DFT/DCT,

there are *many* wavelet bases: Haar, Daubechies-4, Daubechies-6, Coifman, Morlet, Gabor, ...

Kumamoto U CMU CS

Wavelets - how do they look like?

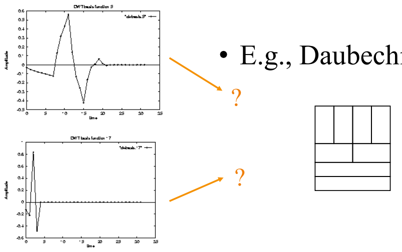


- E.g., Daubechies-4

Kumamoto U CMU CS

Wavelets - how do they look like?

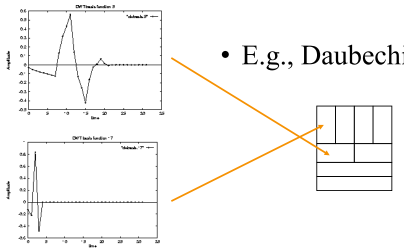
- E.g., Daubechies-4



Kumamoto U CMU CS

Wavelets - how do they look like?

- E.g., Daubechies-4



Kumamoto U CMU CS

Roadmap

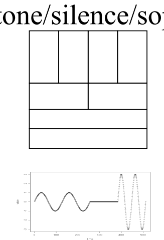
- DFT
  - Definition of DFT and properties
  - how to read the DFT spectrum
- DWT
  - Definition of DWT and properties
  - how to read the DWT scalogram

Kumamoto U CMU CS

Wavelets - Drill#1:

- Q: baritone/silence/soprano - DWT?

f



t

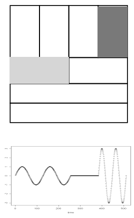
value

time

Kumamoto U — CMU CS

Wavelets - Drill#1:

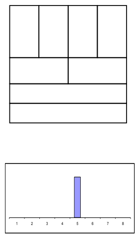
- Q: baritone/silence/soprano - DWT?



Kumamoto U — CMU CS

Wavelets - Drill#2:

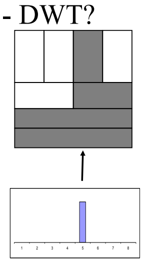
- Q: spike - DWT?



Kumamoto U — CMU CS

Wavelets - Drill#2:

- Q: spike - DWT?



0.00 0.00 **0.71** 0.00

0.00 **0.50**

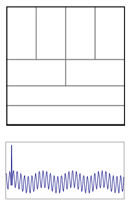
-0.35

0.35

Kumamoto U — CMU CS

Wavelets - Drill#3:

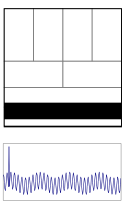
- Q: weekly + daily periodicity, + spike - DWT?



Kumamoto U — CMU CS

Wavelets - Drill#3:

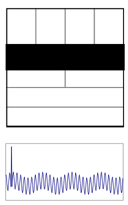
- Q: **weekly** + daily periodicity, + spike - DWT?



Kumamoto U — CMU CS

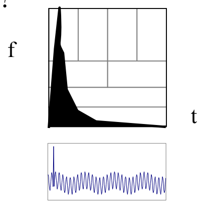
Wavelets - Drill#3:

- Q: weekly + **daily** periodicity, + spike - DWT?



Wavelets - Drill#3:

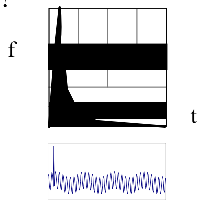
- Q: weekly + daily periodicity, + **spike** - DWT?



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 211

Wavelets - Drill#3:

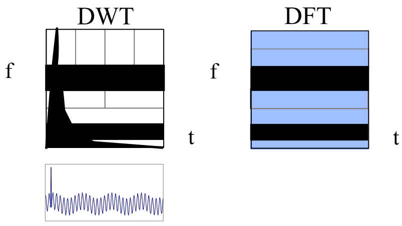
- Q: weekly + daily periodicity, + spike - DWT?



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 212

Wavelets - Drill#3:

- Q: DFT?



http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 213

Advantages of Wavelets

- Better compression (better RMSE with same number of coefficients - used in JPEG-2000)
- fast to compute (usually: O(*n*)!)
- very good for 'spikes'
- mammalian eye and ear: Gabor wavelets

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 214

Conclusions

- DFT, DCT spot periodicities
- **DWT** : multi-resolution - matches processing of mammalian ear/eye better
- All three: powerful tools for **compression**, **pattern detection** in real signals
- All three: included in math packages
  - (matlab, 'R', mathematica, … - often in spreadsheets!)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 215

Conclusions

- DWT : very suitable for self-similar traffic
- DWT: used for summarization of streams [Gilbert+01], db histograms etc

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ 216

Kumamoto U

CMU CS

Part 1 - Roadmap

- Motivation
- Sim. Search and Indexing
- Feature extraction
- Linear forecasting
- Streaming pattern discovery
- Automatic mining

Euclidean/DTW + Feature extraction + R-trees

DFT, DWT (SVD, ICA)

AR, RLS

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 217

Kumamoto U

CMU CS

Resources - software and urls

- http://www.dsptutor.freeuk.com/jsanalyser/FFTSpectrumAnalyser.html : Nice java applets for FFT
- http://www.relisoft.com/freeware/freq.html voice frequency analyzer (needs microphone)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 218

Kumamoto U

CMU CS

Resources: software and urls

- *xwpl:* open source wavelet package from Yale, with excellent GUI
- http://monet.me.ic.ac.uk/people/gavin/java/waveletDemos.html : wavelets and scalograms

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 219

Kumamoto U

CMU CS

Books

- William H. Press, Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery: *Numerical Recipes in C*, Cambridge University Press, 1992, 2nd Edition. (Great description, intuition and code for DFT, DWT)
- C. Faloutsos: *Searching Multimedia Databases by Content*, Kluwer Academic Press, 1996 (introduction to DFT, DWT)

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 220

Kumamoto U

CMU CS

Additional Reading

- [Gilbert+01] Anna C. Gilbert, Yannis Kotidis and S. Muthukrishnan and Martin Strauss, *Surfing Wavelets on Streams: One-Pass Summaries for Approximate Aggregate Queries*, VLDB 2001

http://www.cs.kumamoto-u.ac.jp/~yasuko/TALKS/15-SIGMOD-tut/ © 2015 Sakurai, Matsubara & Faloutsos 221