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BEN-GURION UNIVERSITY OF THE NEGEV FACULTY OF ENGINEERING SCIENCES DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Parametric Estimation of the Orientation of Textured Planar Surfaces

Haim Permuter and Prof. Yossi Francos

Presentation for the AI Lab at MIT

June 2003

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Problem Definition

- Estimate the orientation in space of a planar textured surface from a single observed image of it.
- In its own coordinate system the surface texture is homogeneous.
- The 2-D Wold decomposition implies that the deterministic component of any homogeneous texture field can be approximated by a sum of 2-D sinusoids.

$$t(x_s, y_s) = \sum_{l=1}^{L} A_l \cos(x_s\omega_l + y_s\nu_l + \varphi_l)$$

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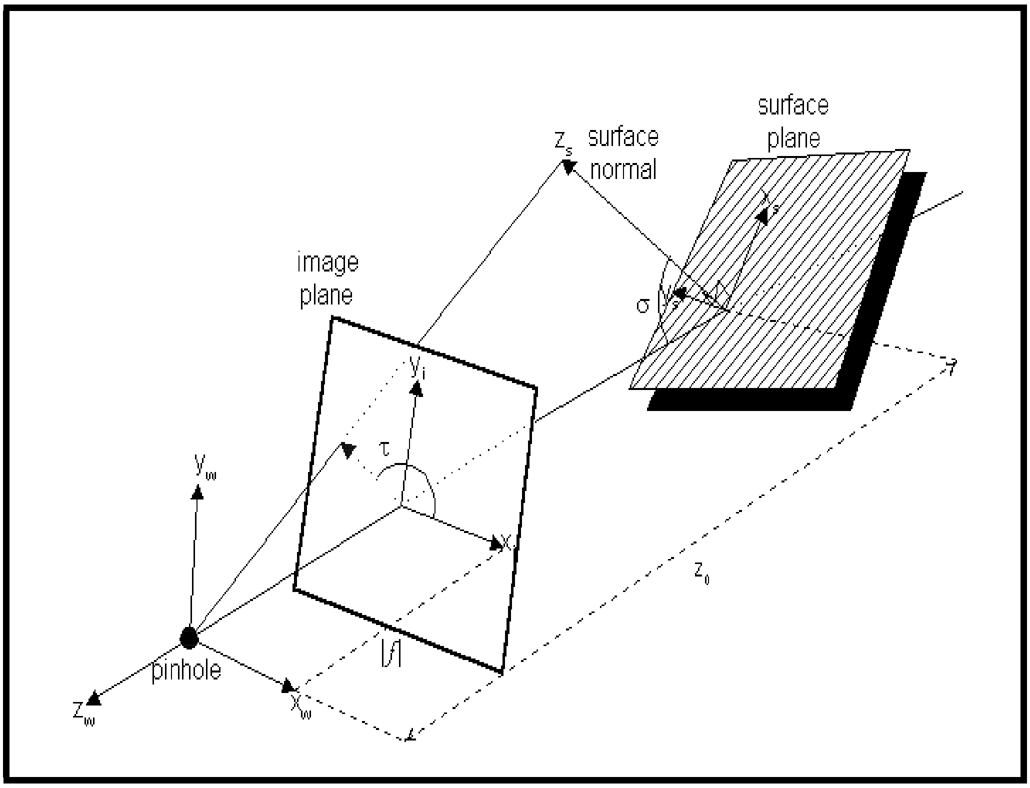
surface
plane
z_s surface
normal
image
plane
σ
y_i
τ
y_w
x_i
z_0
$|f|$
pinhole
z_w
x_w

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Projection of the Texture

The Phase $\Phi_s(x_s, y_s) = ux_s + vy_s + \varphi$ is transformed by the perspective projection

$$\Phi_l(x_i, y_i) = \frac{\frac{x_i}{f}\frac{\left(\widetilde{u}_l \cos\tau - \widetilde{v}_l \cos\sigma \sin\tau\right)}{\cos\sigma} + \frac{y_i}{f}\frac{\left(\widetilde{u}_l \sin\tau + \widetilde{v}_l \cos\sigma \cos\tau\right)}{\cos\sigma}}{\tan\sigma(\frac{x_i}{f}\cos\tau + \frac{y_i}{f}\sin\tau) + 1} + \varphi_l$$

$\widetilde{u} = uz_0$, $\widetilde{v} = vz_0$.

- The spatial frequencies of the harmonic components are now functions of the location !!!
- In the case of a planar surface the functional dependence of the sinusoid phase in location is **uniquely** determined by the tilt and slant angles of the surface.

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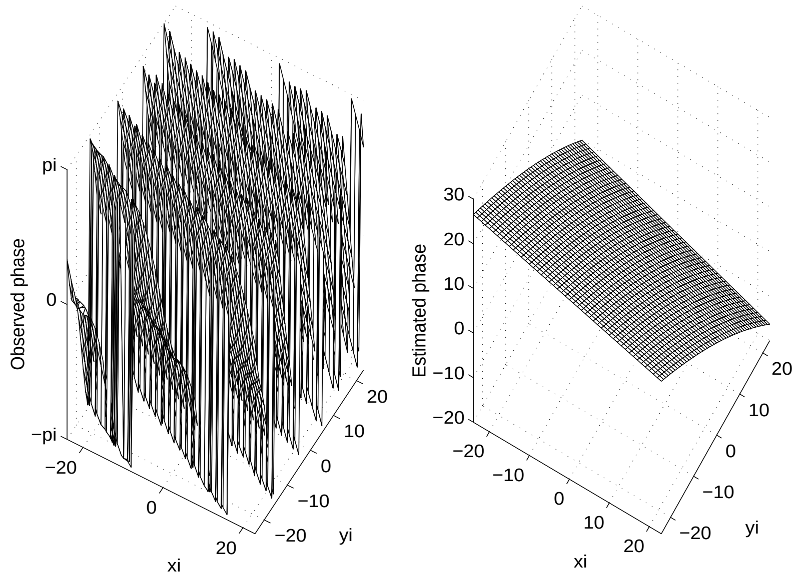
The Observed Phase and the Estimated (Unwrapped) phase



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The Parametric Phase Model

- Continuous functions can be approximated by polynomials.
- The parametric phase model is given by

$$\phi(x_i, y_i) = \sum_{\{0 \le k, \ell : 0 \le k+\ell \le Q\}} c(k, \ell) x_i^k y_i^\ell$$

- Since the assumption of phase smoothness is implicit to our model, no *explicit* phase unwrapping is required.

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Phase differencing algorithm

let $\{v(x,y)\}$ be a discrete 2-D constant amplitude polynomial phase signal of *total-degree* $Q+1$ *i.e.*,

$$v(x,y) = A\exp\{j\phi_{Q+1}(x,y)\}\ , x = 0,1,\ldots,N-1, y = 0,1,\ldots,M-1$$

Definition 1: Let τ_y and τ_x be some positive constants. Define

$$\mathrm{PD}_{y^{(0)}}[v(x,y)] = v(x,y)$$

and in general

$$\mathrm{PD}_{y^{(q)}}[v(x,y)] = \mathrm{PD}_{y^{(q-1)}}[v(x,y)]\left(\mathrm{PD}_{y^{(q-1)}}[v(x,y+\tau_y)]\right)^*$$

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Phase differencing algorithm (Continue)

The signal $\mathrm{PD}_{x^{(P)},y^{(Q-P)}}[v(x,y)]$ is a 2-D exponential given by

$$\mathrm{PD}_{x^{(P)},y^{(Q-P)}}[v(x,y)] = \exp\left\{ j[\omega_Q x + \nu_Q y + \gamma_Q(\tau_x, \tau_y] \right\}$$

where

$$\omega_Q = (-1)^Q c(P+1, Q-P)(P+1)!(Q-P)!\tau_x^P \tau_y^{Q-P} \ ,$$

$$\nu_Q = (-1)^Q c(P, Q+1-P)P!(Q+1-P)!\tau_x^P \tau_y^{Q-P} \ ,$$

and $\gamma_Q(\tau_x, \tau_y)$ is not a function of x nor y.

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Extraction of a Monocomponent Complex Signal

- **Motivation**: The PD algorithm is designed to work with complex valued constant amplitude polynomial phase monocomponent signals.
- Similarly to the 1-D case the way to define without ambiguity the instantaneous amplitude and phase of a real signal $d_q(x_i, y_i)$ is to associate it with its *analytic signal*

  $$z_q(x_i, y_i) = a_q(x_i, y_i) \exp(j\Phi_q(x_i, y_i))$$

  through the 2-D Hilbert transform.
- The analytic signal $z_q(x_i, y_i)$ of the real signal $d_q(x_i, y_i)$ is obtained by applying the operator

  $$M[\cdot] = (1 + jH[\cdot])$$

  $H[\cdot]$ denote the 2-D Hilbert transform operator

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Linear least squares estimation

$$\Phi_l(x_i, y_i) = \frac{\frac{x_i}{f}\frac{(\widetilde{u}_l \cos\tau - \widetilde{v}_l \cos\sigma \sin\tau)}{\cos\sigma} + \frac{y_i}{f}\frac{(\widetilde{u}_l \sin\tau + \widetilde{v}_l \cos\sigma \cos\tau)}{\cos\sigma}}{\tan\sigma(\frac{x_i}{f}\cos\tau + \frac{y_i}{f}\sin\tau) + 1} + \varphi_l$$

can be written in the *linear* form

$$\Phi(x_i, y_i) = \frac{x_i}{f}\beta_2 + \frac{y_i}{f}\delta_2 - \frac{x_i}{f}\Phi(x_i, y_i)l_1 - \frac{y_i}{f}\Phi(x_i, y_i)l_2 + \varphi \ .$$

$$\beta_1 = \frac{(\widetilde{u}\cos\tau - \widetilde{v}\cos\sigma\sin\tau)}{\cos\sigma}$$

$$\delta_1 = \frac{(\widetilde{u}\sin\tau + \widetilde{v}\cos\sigma\cos\tau)}{\cos\sigma}$$

$$l_1 = \tan\sigma\cos\tau \quad l_2 = \tan\sigma\sin\tau$$

$$\beta_2 = \beta_1 + \varphi l_1 \quad \delta_2 = \delta_1 + \varphi l_2$$

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Linear least squares estimation (cont.)

- Since the observed surface texture is homogeneous the variables $\widetilde{u}$, $\widetilde{v}$, φ are independent of (x_i, y_i).
- The unknown parameters β_2 δ_2 l_1 l_2 φ are obtained by least squares solution.

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Tilt and Slant Estimation Using the Unwrapped Phase

$$\psi(x_i, y_i) = 2\pi \cdot \mathrm{ROUND}\left(\frac{\hat{\phi}(x_i, y_i) - \phi_{PV}(x_i, y_i)}{2\pi}\right) + \phi_{PV}(x_i, y_i) \ .$$

$\hat{\phi}(x_i, y_i)$	estimated phase obtained using the PD algorithm
$\phi_{PV}(x_i, y_i)$	principle value of the observed phase
$\psi(x_i, y_i)$	unwrapped phase

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The CRB on the error variance

Homogeneous surface texture in the presence of zero mean, white Gaussian noise, whose variance is ρ^2

$$t(x_i, y_i) = \sum_{l=1}^{L} A_l \cos(\Phi_l(x_i, y_i)) + n(x_i, y_i)$$

The unknown parameters:

$$\boldsymbol{\theta} = [\sigma, \tau, \widetilde{u}_1 ..., \widetilde{u}_{L,}\widetilde{v}_1 ... \widetilde{v}_L, \varphi_1 ... \varphi_L, A_1, A_2, \ldots, A_L]^T \ ,$$

The CRB is simply the inverse of the FIM

$$\mathbf{F}_{ij} = -E\left\{ \frac{\partial^2 \Lambda}{\partial \boldsymbol{\theta}_i \partial \boldsymbol{\theta}_j} \right\} \ ,$$

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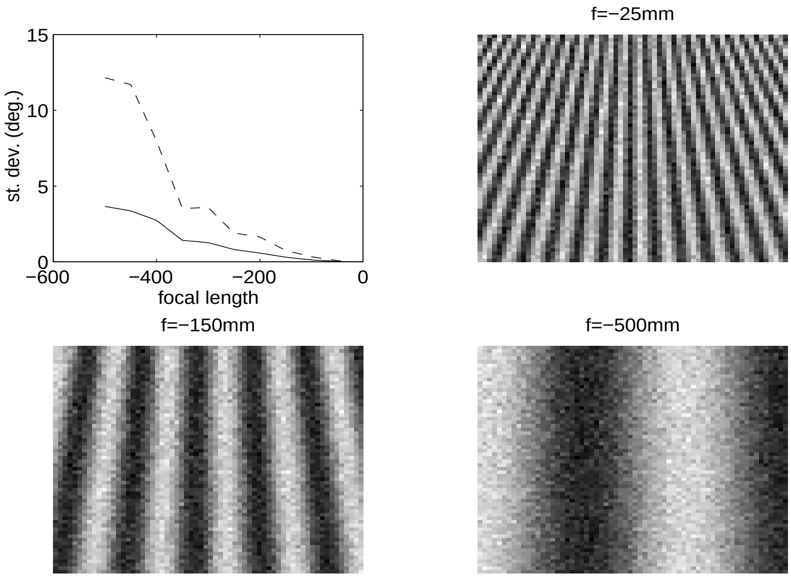
The Dependence of the CRB on the Camera Parameters



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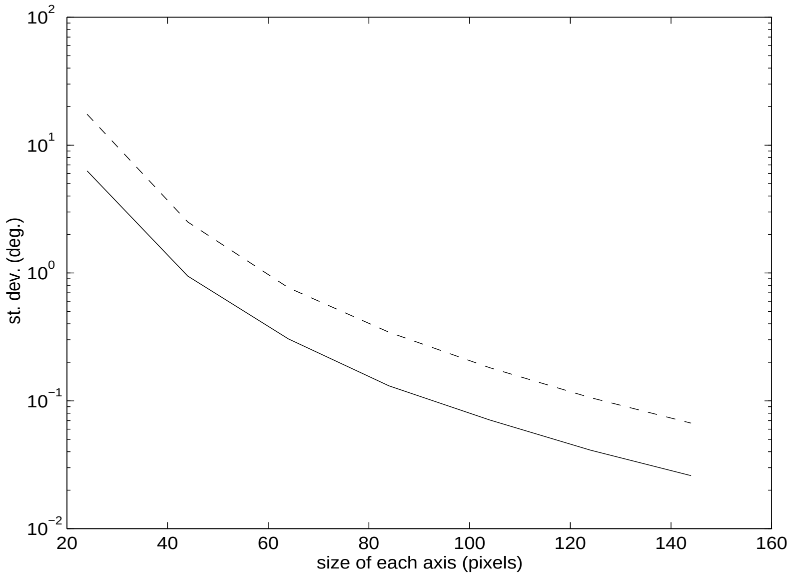
The Dependence on the CRB on the Scene Setting



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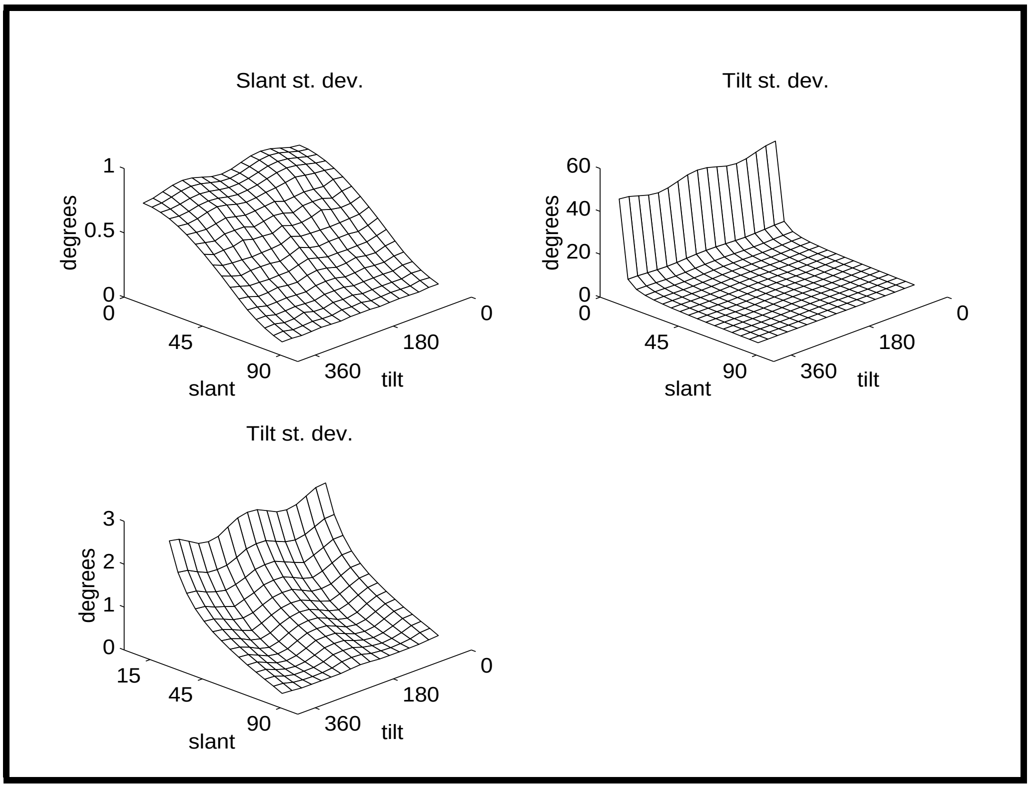
Slant st. dev.
degrees
1
0.5
0
0
45
90
slant
360
180
0
tilt
Tilt st. dev.
degrees
60
40
20
0
0
45
90
slant
360
180
0
tilt
Tilt st. dev.
degrees
3
2
1
0
15
45
90
slant
360
180
0
tilt

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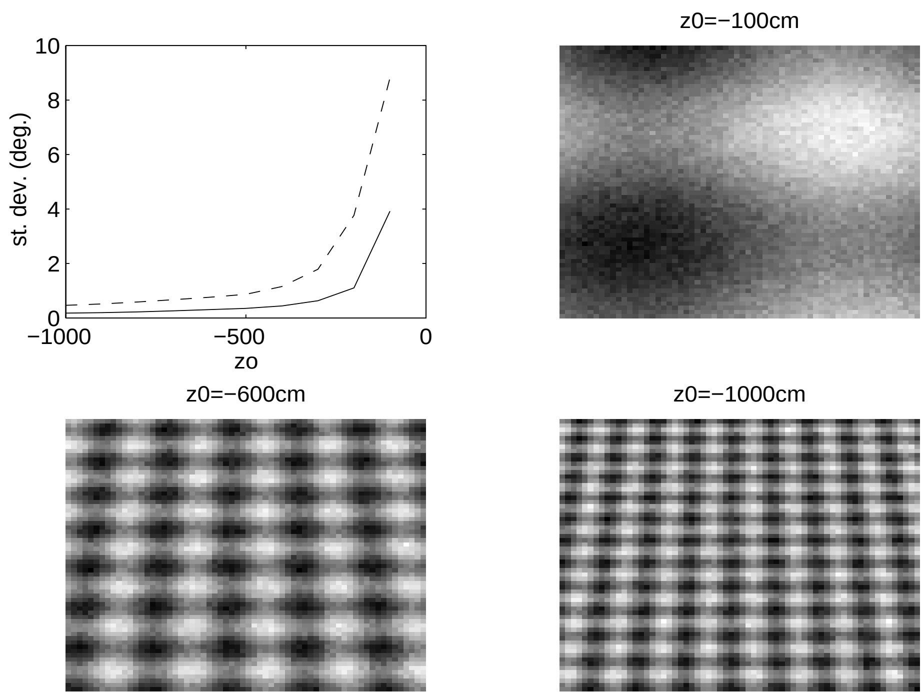

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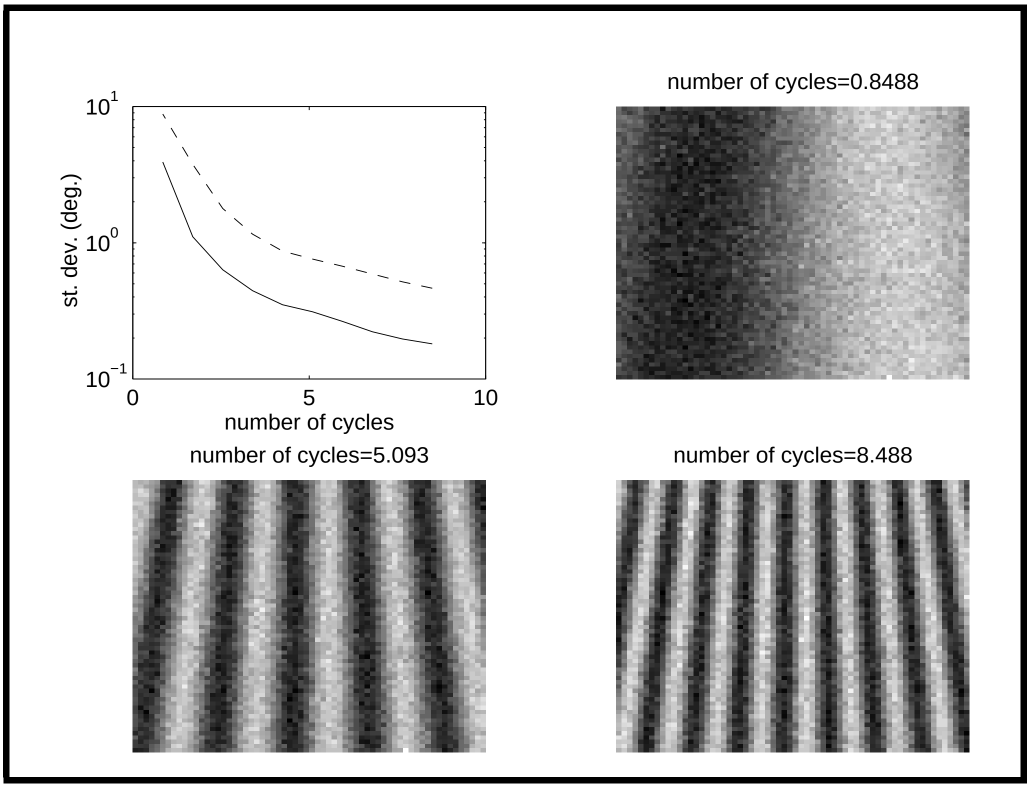

10^1
10^0
10^-1
st. dev. (deg.)
0
5
10
number of cycles
number of cycles=0.8488
number of cycles=5.093
number of cycles=8.488

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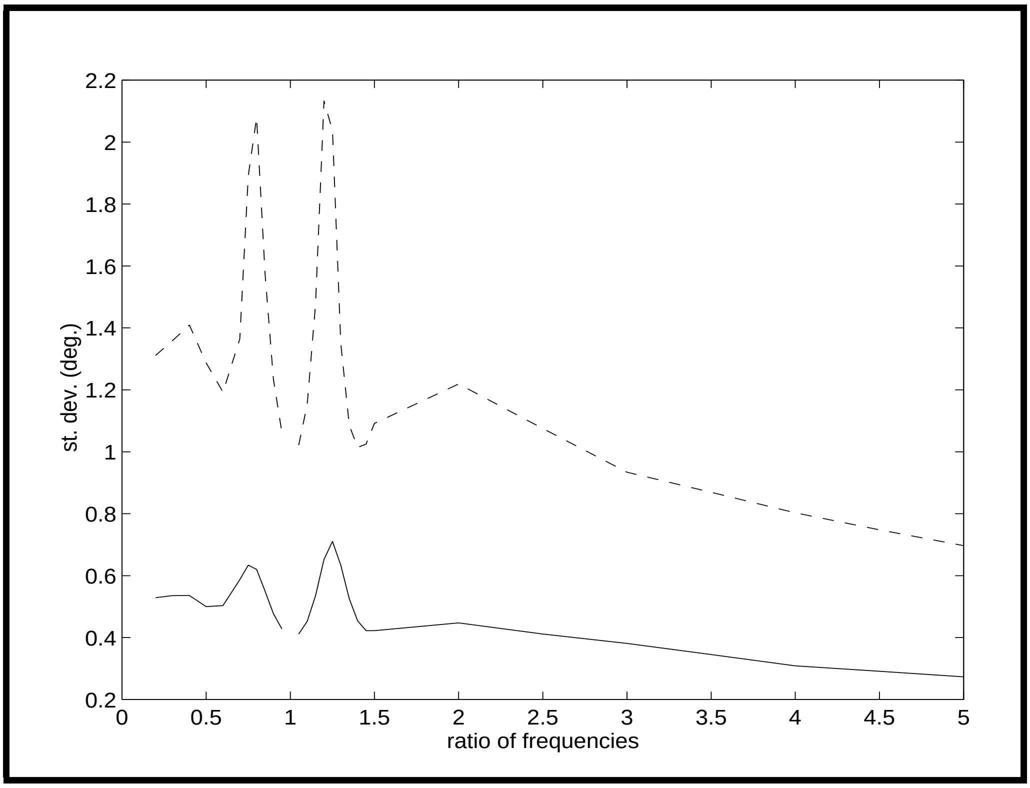
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
st. dev. (deg.)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
ratio of frequencies

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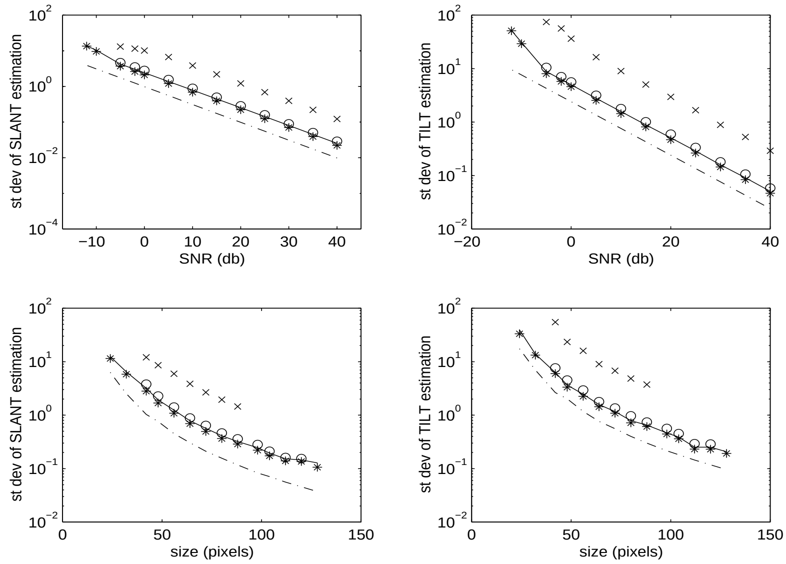
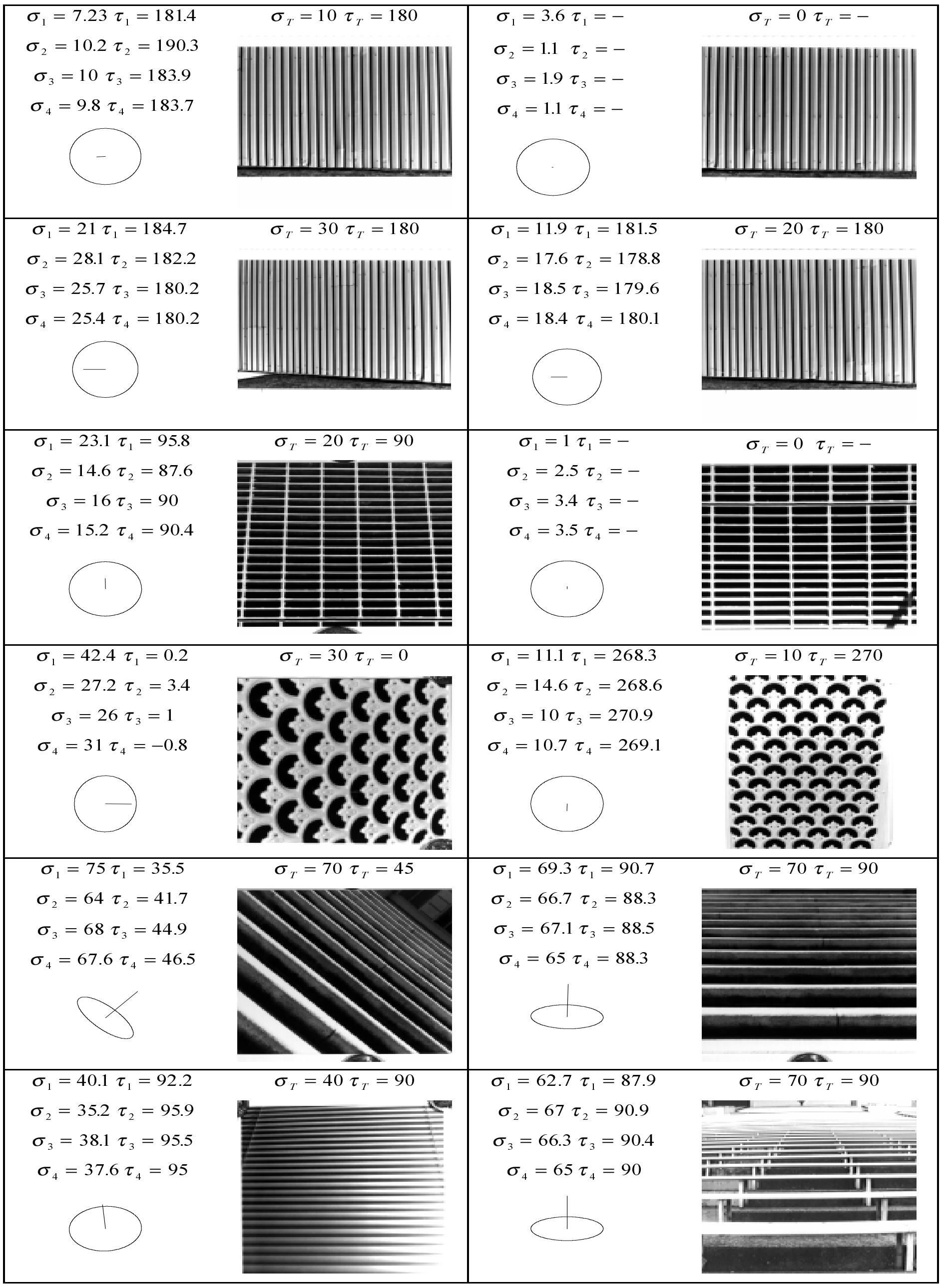
Statistical performance analysis of the estimation algorithms




$\sigma_1 = 7.23$ $\tau_1 = 181.4$
$\sigma_2 = 10.2$ $\tau_2 = 190.3$
$\sigma_3 = 10$ $\tau_3 = 183.9$
$\sigma_4 = 9.8$ $\tau_4 = 183.7$
$\sigma_T = 10$ $\tau_T = 180$
$\sigma_1 = 3.6$ $\tau_1 = -$
$\sigma_2 = 1.1$ $\tau_2 = -$
$\sigma_3 = 1.9$ $\tau_3 = -$
$\sigma_4 = 1.1$ $\tau_4 = -$
$\sigma_T = 0$ $\tau_T = -$
$\sigma_1 = 21$ $\tau_1 = 184.7$
$\sigma_2 = 28.1$ $\tau_2 = 182.2$
$\sigma_3 = 25.7$ $\tau_3 = 180.2$
$\sigma_4 = 25.4$ $\tau_4 = 180.2$
$\sigma_T = 30$ $\tau_T = 180$
$\sigma_1 = 11.9$ $\tau_1 = 181.5$
$\sigma_2 = 17.6$ $\tau_2 = 178.8$
$\sigma_3 = 18.5$ $\tau_3 = 179.6$
$\sigma_4 = 18.4$ $\tau_4 = 180.1$
$\sigma_T = 20$ $\tau_T = 180$
$\sigma_1 = 23.1$ $\tau_1 = 95.8$
$\sigma_2 = 14.6$ $\tau_2 = 87.6$
$\sigma_3 = 16$ $\tau_3 = 90$
$\sigma_4 = 15.2$ $\tau_4 = 90.4$
$\sigma_T = 20$ $\tau_T = 90$
$\sigma_1 = 1$ $\tau_1 = -$
$\sigma_2 = 2.5$ $\tau_2 = -$
$\sigma_3 = 3.4$ $\tau_3 = -$
$\sigma_4 = 3.5$ $\tau_4 = -$
$\sigma_T = 0$ $\tau_T = -$
$\sigma_1 = 42.4$ $\tau_1 = 0.2$
$\sigma_2 = 27.2$ $\tau_2 = 3.4$
$\sigma_3 = 26$ $\tau_3 = 1$
$\sigma_4 = 31$ $\tau_4 = -0.8$
$\sigma_T = 30$ $\tau_T = 0$
$\sigma_1 = 11.1$ $\tau_1 = 268.3$
$\sigma_2 = 14.6$ $\tau_2 = 268.6$
$\sigma_3 = 10$ $\tau_3 = 270.9$
$\sigma_4 = 10.7$ $\tau_4 = 269.1$
$\sigma_T = 10$ $\tau_T = 270$
$\sigma_1 = 75$ $\tau_1 = 35.5$
$\sigma_2 = 64$ $\tau_2 = 41.7$
$\sigma_3 = 68$ $\tau_3 = 44.9$
$\sigma_4 = 67.6$ $\tau_4 = 46.5$
$\sigma_T = 70$ $\tau_T = 45$
$\sigma_1 = 69.3$ $\tau_1 = 90.7$
$\sigma_2 = 66.7$ $\tau_2 = 88.3$
$\sigma_3 = 67.1$ $\tau_3 = 88.5$
$\sigma_4 = 65$ $\tau_4 = 88.3$
$\sigma_T = 70$ $\tau_T = 90$
$\sigma_1 = 40.1$ $\tau_1 = 92.2$
$\sigma_2 = 35.2$ $\tau_2 = 95.9$
$\sigma_3 = 38.1$ $\tau_3 = 95.5$
$\sigma_4 = 37.6$ $\tau_4 = 95$
$\sigma_T = 40$ $\tau_T = 90$
$\sigma_1 = 62.7$ $\tau_1 = 87.9$
$\sigma_2 = 67$ $\tau_2 = 90.9$
$\sigma_3 = 66.3$ $\tau_3 = 90.4$
$\sigma_4 = 65$ $\tau_4 = 90$
$\sigma_T = 70$ $\tau_T = 90$

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Orthogonalization of a Perspective Viewed Image

- Using the inverse coordinate transformation, find the coordinates of the image boundaries
- Uniformly sample the surface coordinate system.
- Evaluate the image coordinate $\mathbf{x}_i$ that corresponds to each $\mathbf{x}_s$ on the surface sampling grid.
- For each of the RGB planes, the gray level of each sample in the surface coordinate system is set to the gray level of the corresponding observed image sample $\mathbf{x}_i$ (using interpolation since in general the resulting x_i and y_i are not integers).

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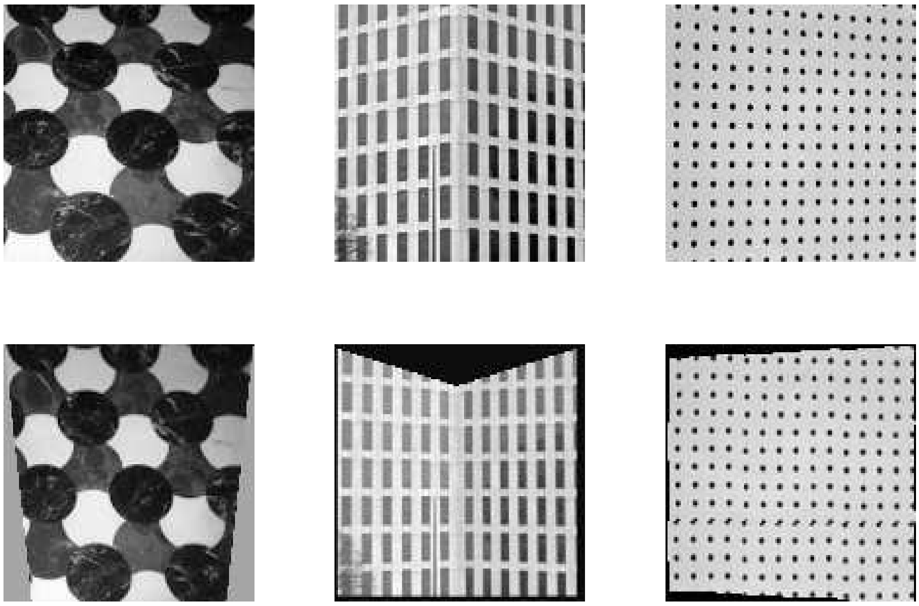

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