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A Heterogeneous Linguistic MAGDM Framework to Classroom Teaching Quality Evaluation

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ABSTRACT

Focusing on multi-attribute group decision making (MAGDM) regarding classroom teaching quality evaluation, this article aims to devise a novel evaluation framework based on heterogeneous linguistic information. In this framework, a four-level evaluation process of classroom teaching quality is established. Then, the weights of the sub-attributes are estimated objectively by integrating a newly proposed score function of interval linguistic 2-tuples and optimization models which consider the realistic situation that alternatives are not equally weighted. Subsequently, the exploitation process is implemented by two branches: taking the possibility measurement to rank teachers with respect to different attributes and extending the technique for order preference by similarity to ideal solution (TOPSIS) method to assess the overall performance of teachers. Finally, a simulated case is furnished to illustrate how to apply the presented framework to realistic classroom teaching quality evaluation problems. Hopefully, this work would be beneficial to the improvement of classroom teaching quality.

Keywords: classroom teaching quality evaluation, MAGDM, heterogeneous linguistic information, optimization models, possibility measurement

INTRODUCTION

With the promoting of higher education reform in China, the main theme of the development of higher education has shifted from the scale expansion to comprehensively strengthen the connotation construction. This requires universities to effectively change the concept of development and make talent training as a fundamental task and primary responsibility. Frankly speaking, classroom teaching is the main channel and key position of talent training in universities as well as the important determinant of the cultivation quality of good teachers. Classroom teaching quality evaluation is conducted for the purpose of checking the achievements of classroom teaching activities and finding problems in classroom teaching. Without doubt, scientific and effective evaluation of classroom teaching quality is in favor of producing a positive incentive and guidance role to improve service and management of universities, stimulating the enthusiasm of teachers, enhancing the teacher's teaching ability and improving the

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State of the literature

- Being the common activity in universities, the evaluation of classroom teaching quality plays a significant role in improving the service of the teacher and strengthening the management of the university.
- The weights of the sub-attributes can be estimated by various ways. However, most literatures do not pay attention to the realistic situation that alternatives are not equally weighted.
- A systematical framework is in need of being developed to tackle classroom teaching quality evaluation problems with heterogeneous linguistic information.

Contribution of this paper to the literature

- A weight generation method which has an advantage that different teachers are distinctively weighted is proposed.
- A new possibility formula and a definition of Euclidean distance for multi-granular interval linguistic 2-tuples which are suitable for realistic decision environment are introduced in the exploitation process.
- A novel evaluation framework which considers the partially rankings as well as the overall ranking of selective teachers is given. This may be beneficial to the improvement of classroom teaching quality evaluation.

quality of talent training. Thus, it is crucial to use a scientific and systematical method to evaluate the classroom teaching quality.

In the process of classroom teaching quality evaluation, teachers are assessed by professional people with respect to different attributes, which can be regarded as a multi-attribute group decision making (MAGDM) problem. Generally, the attribute values of teachers can be expressed by various forms, which can be roughly classified into two branches: quantitative variables (Nguyen, 2015; Wan & Dong, 2015; Zhou, Wang & Zhang, 2016; Li & Wang, 2017) and qualitative variables (Liu & Yu, 2014; Dutta & Guha, 2015; Yu, Wang & Wang, 2016; Wang, Wang, Zhang & Chen, 2017; Nie, Wang & Li, 2017; Zhang, Peng, Wang & Wang, 2017). Actually, compared with quantitative numbers, qualitative values are more flexible to express assessments of alternatives and more accurate to reflect the fuzziness of human thinking. Ulteriorly, owing to distinctive characteristics of attributes, it is necessary for decision-makers to express their judgments by means of linguistic variables denoted by different linguistic term sets. This difference is mainly reflected from three aspects: the information represented by the linguistic terms, the granularity of the linguistic term set or both. Up to now, some researchers have already studied the second case, where different decision makers employ different linguistic term sets to express opinions (Zhang, 2012; Liu, Chan & Ran, 2013; Zhang, 2013; Dong, Li, Xu & Gu, 2015; Dong, Zhang & Herrera-Viedma, 2016; Zhang & Wang, 2017).

To evaluate the teaching performance, different approaches have already been proposed. For instance, based on the works (Dong & Dai, 2009; He, Zhu, Zhou, Lu & Liu, 2010), Chen et al. (Chen, Hsieh & Do, 2015) introduced a new framework to evaluate teaching performance by combining fuzzy AHP and comprehensive evaluation method. Chang and Wang (Chang & Wang, 2016) introduced a cloud model for evaluating teachers in higher education. However, these researches only consider the overall evaluation of teaching quality, but do not take individual assessments (evaluations of teachers over different attributes) into account.

Additionally, in evaluating teaching quality, a practical and significant issue is that the weights of attributes ought to be furnished or determined. Indeed, it is always a challenge for decision-makers to directly provide a crisp weight vector for attributes. Thus, there is a trend for researchers to investigate how to develop rational models to determine the weights of attributes. Until now, group decision making with incomplete weight information has been studied extensively. To sum up, the methods or models for obtaining attribute weights include: TOPSIS-based (technique for order preference by similarity to ideal solution) optimization model (Wei, 2010; Zhang & Guo, 2012), interactive method (Xu & Chen, 2007), linear programming model (Wang, Li & Wang, 2009; Xue, You, Lai & Liu, 2016; Düğenci, 2016), maximizing deviation method (Sahin & Liu, 2015; Zhang, Xu & Wang, 2015), GRA-based optimization model (Wei, 2010), range-based linear inequalities (Kim, Choi & Kim, 1999), mathematical programming model (Zhang, Zhu, Liu & Chen, 2016), etc.

To scientifically evaluate the classroom teaching quality, a systematical framework ought to be designed. Bearing this in mind, this paper devotes to design a novel evaluation framework containing the following contents, which are considered as the novelties of this paper.

- Facing with the complexity of classroom teaching quality evaluation, this paper considers the aforementioned situation where the meanings of the linguistic terms as well as the granularities of the linguistic term sets are not identical. To the best of our knowledge, this has not been extensively studied yet. Additionally, the fact that decision-makers may not be able to give judgments by certain linguistic values should be sufficiently considered. From the above analyses, this paper deals with the evaluation problem of classroom teaching quality based on heterogeneous linguistic information that can be roughly divided into two levels: 1. The original decision information furnished by one decision-maker may comprise linguistic values and uncertain linguistic values; 2. The linguistic labels used for representing assessments over different objectives originate from different linguistic term sets.
- Considering an MAGDM problem, a reasonable index system of evaluating classroom teaching quality should be established. Extracting from the practical evaluation system of classroom teaching quality and learning from the research in (Chen et al., 2015), we build up an evaluation index system of classroom teaching quality, which consists of 5 attributes and 15 sub-attributes. The sub-attribute values of each teacher are denoted by heterogeneous linguistic information as afore-discussed.
- Given that the direct computation of heterogeneous information always performs complicatedly, normalizing the heterogeneous decision information is a foundational demand. Bearing this in mind, we introduce transformation functions between two interval-valued linguistic 2-tuples which are assessed in linguistic term sets with different granularities.
- The weights of sub-attributes are determined by optimization models for the purpose of reducing subjectivity. Compared with most literatures which have defaulted that the alternatives' weights are identical, these optimization models have an advantage that different teachers are not equally weighted, which is in line with the practical decision environments.
- Being a critical process of decision making, the comparison of alternatives ought to be an everlasting topic. Differing from the 2-rank selection process (Zhang, Dong & Chen, 2017) which originates from the idea that creating the ranking of a subset of alternatives over another one, we divide the exploitation process of classroom teaching quality evaluation into two branches which aim to deriving rankings of all alternatives from two perspectives. One is to evaluate teachers over different attributes, which is accomplished by a new possibility measurement for the sake that the assessments under one attribute are homogeneous. The other is the overall evaluation executed by establishing a distance measure, which calculates the distance between two heterogeneous interval 2-tuple linguistic vectors and is embedded into the TOPSIS method. As the TOPSIS method was initiated on the basis of the distances from the positive and negative solutions, it is suitable for cautious professors (Opricovic & Tzeng, 2004; Opricovic & Tzeng, 2007) and is applicable to the classroom teaching quality evaluation which requires to be carefully handled. Besides, the TOPSIS method has been extensively extended (Zhang & Xu, 2015; Zyoud, Kaufmann, Shaheen, Samban & Fuchs-Hanusch, 2016; Onu, Quan, Xu, Orji & Onu, 2017), which reveals its high applicability in handling real-world decision problems.

The remainder of this article is arranged as follows. Section 2 reviews concepts related to linguistic variables adopted in the expression of evaluating information in classroom teaching quality. An evaluation index system of classroom teaching quality is developed in Section 3. Subsequently, a systematical and practical framework for handling classroom teaching quality evaluation problem with heterogeneous linguistic information is designed in Section 4. Section 5 presents a case study to demonstrate the feasibility and usefulness of our proposed framework in evaluating teaching quality. Section 6 draws the conclusions of this paper.

BASIC CONCEPTS REVIEW

This section presents some basic concepts related to the theme of this paper.

Linguistic 2-Tuple

Generally, a linguistic term set can be denoted by various ways (Herrera & Martínez, 2000; Bordogna, Fedrizzi & Passi, 1997; Levrat, Voisin, Bombardier & Bremont, 1997; Xu, 2005). According to the problem to be solved, we express decision-makers' opinions by the notations $S = \{s_0, s_1, \dots, s_g\}$ (Herrera & Martínez, 2000), where $s_\propto (\propto = 0,1,\dots,g)$ stands for the $\propto^{th}$ linguistic term in S and $g+1$ refers to the granularity of S. For example, a linguistic label set containing 5 terms and another one including 7 terms are respectively given as $S^{(1)}$ and $S^{(2)}$ shown below.

$$S^{(1)} = \{s_0^{(1)} = very\ poor, s_1^{(1)} = poor, s_2^{(1)} = medium, s_3^{(1)} = good, s_4^{(1)} = very\ good\}$$

$$S^{(2)} = \{s_0^{(2)} = very\ poor, s_1^{(2)} = poor, s_2^{(2)} = slightly\ poor, s_3^{(2)} = medium, s_4^{(2)} = slightly\ good, s_5^{(2)} = good, s_6^{(2)} = very\ good\}$$

Evidently, the larger the granularity of a linguistic term set is, i.e., the more labels a linguistic term set includes, such that the less vagueness and uncertainty of information the linguistic term set represents.

Besides, the following characteristics are required to be satisfied (Herrera & Martínez, 2000):

(1) $s_{\propto_1} < s_{\propto_2}$, iff $\propto_1 < \propto_2$;

(2) There is a negation operator: $Neg(s_{\propto_1}) = s_{\propto_2}$, so that $\propto_2 = g - \propto_1$;

(3) $\min(s_{\propto_1}, s_{\propto_2}) = s_{\propto_1}$, iff $s_{\propto_1} \le s_{\propto_2}$; and

(4) $\max(s_{\propto_1}, s_{\propto_2}) = s_{\propto_1}$, iff $s_{\propto_1} \ge s_{\propto_2}$.

Until now, various fuzzy linguistic approaches have been researched (Rodríguez, Labella, & Martínez, 2016). As to the issue which we concerned: the subscripts of linguistic terms are all crisp values that the information expressed by linguistic terms is discrete; we elicit two different notions: the linguistic 2-tuple (Herrera & Martínez, 2000) and the virtual linguistic label (Xu et al., 2005). By comparison, the virtual linguistic label has a drawback that it can only appear in the operation of linguistic variables. For this, we employ the concept of linguistic 2-tuple, which is denoted as $(s_\propto, \beta)$, where $s_\alpha (s_\alpha \in S = \{s_0, s_1, \dots, s_g\})$ represents a linguistic term center of the decision information and $\beta (\beta \in [-0.5\ 0.5))$ indicates the deviation of a crisp number δ (the result of a symbolic aggregation operation) from the closest linguistic label $\alpha = round(\delta) (\delta \in [0, \mathrm{g}])$.

For the sake that linguistic 2-tuples could not be computed directly, different computational techniques for linguistic 2-tuple models are established (Herrera & Martínez, 2000; Tai & Chen, 2009; Dong, Xu & Yu, 2009; Martínez & Herrera, 2012; Dong, Li, & Herrera, 2016). Regarding the evaluation problem which is going to be solved in this paper, we review and compare two different transformation functions between a linguistic 2-tuple and a crisp number.

Definition 1 (Herrera & Martínez, 2000). Let $S = \{s_\alpha \mid \alpha = 0, 1, \dots, \mathrm{g}\}$ be a linguistic term set and $\delta \in [0, \mathrm{g}]$ be the result of a symbolic aggregation operation. A linguistic 2-tuple, (s_α, β), which represents identical information to δ, can be generated by the following function:

$$\begin{cases} \Delta : [0, g] \to S \times [-0.5, 0.5) \\ \Delta(\delta) = (s_\alpha, \beta),\ with \begin{cases} s_\alpha, & \alpha = round(\delta) \\ \beta = \delta - \alpha, & \beta \in [-0.5, 0.5) \end{cases} \end{cases} \tag{1}$$

where $round$ is the common rounding operation.

On the contrary, (s_α, β) can be converted into a corresponding crisp value δ by a converse function:

$$\begin{cases} \Delta^{-1}: S \times [-0.5, 0.5) \to [0, g] \\ \Delta^{-1}(s_\alpha, \beta) = \alpha + \beta = \delta \end{cases}. \tag{2}$$

Based on the above definition, the conversion of a linguistic term to a linguistic 2-tuple can be accomplished by adding a value 0 as symbolic translation. That is,

$$\Delta(s_\alpha) = (s_\alpha, 0),\ s_\alpha \in S. \tag{3}$$

Definition 2 (Tai & Chen, 2009). Let $S = \{s_\alpha \mid \alpha = 0, 1, \ldots, g\}$ be a linguistic term set and $\delta \in [0, 1]$ be the result of a symbolic aggregation operation, then the following function is utilized to convert a crisp value δ into an equivalent linguistic 2-tuple (s_α, β):

$$\begin{cases} \Lambda : [0,1] \to [-0.5/g, 0.5/g) \\ \Lambda(\delta) = (s_\alpha, \beta),\ with \begin{cases} s_\alpha, & \alpha = round(\delta \cdot g) \\ \beta = \delta - \dfrac{\alpha}{g}, & \alpha \in [-0.5/g, 0.5/g) \end{cases} \end{cases}, \tag{4}$$

where $round$ is the common rounding operation.

Conversely, a linguistic 2-tuple (s_α, β) can be translated into a crisp value through the following function:

$$\begin{cases} \Lambda^{-1}: S \times [-0.5/g, 0.5/g) \to [0,1] \\ \Lambda^{-1}(s_\alpha, \beta) = \dfrac{\alpha}{g} + \beta = \delta \end{cases}. \tag{5}$$

By comparison, the transformation functions in **Definition 2** may be regarded as generalized forms of those in **Definition 1**. On one hand, by making use of Eq. (4) and varying the value of g, a crisp number can be converted into different linguistic 2-tuples which are denoted by multi-granular linguistic term sets. Of course, these multi-granular linguistic 2-tuples are identical in expressing numerical information. Meanwhile, the value of g has no influence on the output of Eq. (1). In other words, the linguistic 2-tuple obtained by Eq. (1) cannot reflect the difference of several seemingly same linguistic 2-tuples which may be actually denoted by linguistic term sets with different granularities. On the other hand, by employing Eq. (5), multi-granular linguistic 2-tuples can be turned into numbers all lying in the unit interval [0, 1]. However, through Eq. (2), multi-granular linguistic 2-tuples are transformed into numbers in different ranges decided by the granularities of linguistic term sets. In other words, the numbers cannot be computed only if a further normalization is implemented.

Interval-Valued Linguistic 2-Tuple

As to interval linguistic 2-tuples, there are also many computational models (Zhang, 2012; Dong, Zhang, Hong & Yu, 2013; Dong & Herrera-Viedma, 2015). By extending **Definition 2**, Zhang (2012) put forward transformation functions between an interval value and an interval-valued linguistic 2-tuple as follows.

Definition 3 (Zhang, 2012). Let $S = \{s_\alpha \mid \alpha = 0, 1, \ldots, g\}$ be a linguistic term set and $[\delta^-, \delta^+]$ be an interval number, then an interval-valued linguistic 2-tuple, $[(s_{\alpha^-}, \beta^-), (s_{\alpha^+}, \beta^+)]$, representing identical information to $[\delta^-, \delta^+]$, can be generated with:

$$\Lambda([\delta^-,\delta^+]) = [(s_{\alpha^-},\beta^-),(s_{\alpha^+},\beta^+)] \text{ with } \begin{cases} s_{\alpha^-}, & \alpha^- = round(\delta^- \cdot g) \\ s_{\alpha^+}, & \alpha^+ = round(\delta^+ \cdot g) \\ \beta^- = \delta^- - \dfrac{\alpha^-}{g}, & \beta^- \in [-0.5/g, 0.5/g) \\ \beta^+ = \delta^+ - \dfrac{\alpha^+}{g}, & \beta^+ \in [-0.5/g, 0.5/g) \end{cases} \cdot \tag{6}$$

In turn, an equivalent interval number $[\delta^-,\delta^+]$ can be returned from an interval-valued linguistic 2-tuple $[(s_{\alpha^-},\beta^-),(s_{\alpha^+},\beta^+)]$ by a converse function Λ^{-1} defined as:

$$\Lambda^{-1}\left([(s_{\alpha^-},\beta^-),(s_{\alpha^+},\beta^+)]\right) = \left[\frac{\alpha^-}{g}+\beta^-, \frac{\alpha^+}{g}+\beta^+\right] = [\delta^-,\delta^+]. \tag{7}$$

In order to make a comparison between two interval-valued linguistic 2-tuples, Xu (2004) proposed a formula to measure the possibility of one over another.

Definition 4 (Xu 2004). Let $S = \{s_\alpha \mid \alpha = 0,1,\dots,g\}$ be a linguistic term set, $\nu_1 = [(s_{\alpha_1^-},\beta_1^-),(s_{\alpha_1^+},\beta_1^+)]$ and $\nu_2 = [(s_{\alpha_2^-},\beta_2^-),(s_{\alpha_2^+},\beta_2^+)]$ be two interval-valued linguistic 2-tuples, then the possibility of $\nu_1 \geq \nu_2$ is calculated with

$$\kappa(\nu_1 \geq \nu_2) = \frac{\max\left\{0, len(\nu_1)+len(\nu_2)-\max\left(\left(\Delta^{-1}(s_{\alpha_2^+},\beta_2^+)-\Delta^{-1}(s_{\alpha_1^-},\beta_1^-)\right),0\right)\right\}}{len(\nu_1)+len(\nu_2)}, \tag{8}$$

where $len(\nu_1) = \Delta^{-1}(s_{\alpha_1^+},\beta_1^+) - \Delta^{-1}(s_{\alpha_1^-},\beta_1^-)$ and $len(\nu_2) = \Delta^{-1}(s_{\alpha_2^+},\beta_2^+) - \Delta^{-1}(s_{\alpha_2^-},\beta_2^-)$.

Obviously, if $\kappa(\nu_1 \geq \nu_2) < 0.5$, then $\nu_1 < \nu_2$; if $\kappa(\nu_1 \geq \nu_2) = 0.5$, then $\nu_1 = \nu_2$; if $\kappa(\nu_1 \geq \nu_2) > 0.5$, then $\nu_1 > \nu_2$. Particularly, $\kappa(\nu_1 \geq \nu_2) = 0.5$. In addition, the properties of $\kappa(\nu_1 \geq \nu_2)$ are concluded as follows.

Theorem 1 (Xu 2004). Let $\nu_1 = [(s_{\alpha_1^-},\beta_1^-),(s_{\alpha_1^+},\beta_1^+)]$ and $\nu_2 = [(s_{\alpha_2^-},\beta_2^-),(s_{\alpha_2^+},\beta_2^+)]$ be two interval-valued linguistic 2-tuples, then

(1) $\kappa(\nu_1 \geq \nu_2) \in [0,1]$;

(2) $\kappa(\nu_1 \geq \nu_2) = 1$, iff $\Delta^{-1}(s_{\alpha_2^+},\beta_2^+) \leq \Delta^{-1}(s_{\alpha_1^-},\beta_1^-)$;

(3) $\kappa(\nu_1 \geq \nu_2) = 0$, iff $\Delta^{-1}(s_{\alpha_2^-},\beta_2^-) \geq \Delta^{-1}(s_{\alpha_1^+},\beta_1^+)$; and

(4) $\kappa(\nu_1 \geq \nu_2) + \kappa(\nu_2 \geq \nu_1) = 1$.

Remark 1. In terms of **Definition 4**, there is a point should be noticed: Eq. (8) is established on the basis of Eq. (2), which implies that Eq. (8) is not applicable to the situations where two interval-valued linguistic 2-tuples are denoted by linguistic term sets with different granularities. For instance, assume $\nu_1 = [(s_2^{(1)},0),(s_4^{(1)},0)]$ and $\nu_2 = [(s_2^{(2)},0),(s_4^{(2)},0)]$, then by Eq. (8), $\kappa(\nu_1 \geq \nu_2) = 0.5$, say, $\nu_1 = \nu_2$. Indeed, ν_1 is distinctly superior to ν_2. In this case, Eq. (8) makes no sense. Therefore, only the multi-granular interval linguistic 2-tuples have been uniformed, can Eq. (8) be used to calculate the possibility of one interval 2-tuple over another.

Another significant issue in multi-attribute group decision making is aggregation. It includes two parts: the aggregation of individual assessments and the integration of attribute values of one alternative. For sake of tractability, we employ the following aggregation method to accomplish our aggregation process.

Definition 5 (Zhang, 2012). Let $S = \{s_\alpha \mid \alpha = 0, 1, \dots, g\}$ be a predefined linguistic term set, and $V = \{v = [(s_{\alpha^-}, \beta^-), (s_{\alpha^+}, \beta^+)] | s_{\alpha^-}, s_{\beta^+} \in S = \{s_0, s_1, \dots, s_g\}, \beta^-, \beta^+ \in [-0.5/g, 0.5/g)\}$ be a set of interval-valued linguistic 2-tuples. An IVTWA operator of dimension n is a function with the following form:

$$
\begin{aligned}
&IVTWA(V)^n \to V \\
&IVTWA(v_1, v_2, \dots, v_n) = [v^-, v^+] \\
&v^- = \Lambda\left[\sum\nolimits_{l=1}^{n} \omega_l \Lambda^{-1}(s_{\alpha_i^-}, \beta_i^-)\right], \quad v^+ = \Lambda\left[\sum\nolimits_{l=1}^{n} \omega_l \Lambda^{-1}(s_{\alpha_i^+}, \beta_i^+)\right],
\end{aligned} \tag{9}
$$

where $\varpi = (\omega_1, \omega_2, \dots, \omega_n)$ is the weighting vector of the given n linguistic 2-tuples and satisfies $\omega_1 \in [0, 1]$ and $\sum_{l=1}^{n} w_l = 1$.

EVALUATION INDEX SYSTEM OF CLASSROOM TEACHING QUALITY

Being a comprehensive concept, classroom teaching quality is a unification of the teaching from teachers and the studying from students. It is not only a subjective evaluation of the teaching capacity of teachers, but also an objective evaluation of application of teaching method and teaching performance. In the proceeding of evaluating classroom teaching quality, it is of great requirement and essential to devise a reasonable evaluation index system, so as to assure the validity and scientific nature of the evaluation result. By drawing on the study in (Chen et al., 2015) and extracting realistic evaluating indices, this section constructs and describes an evaluation index system of classroom teaching quality including 5 attributes $a_j (j = 1, 2, 3, 4, 5)$ and 15 sub-attributes $a_{j_k} (j = 1, 2, 3, 4, 5; k = 1, 2, 3)$ shown in **Table 1**.

In the following, we make a description of each attribute briefly.

Teaching attitude, is a psychological tendency of teachers towards students, teaching process and relative teaching phenomenon. During the teaching process, the teaching attitude of a teacher may influence even alter students' mentality and behaviors. Since attitude is the basis of doing anything, it decides whether the teacher could build up a good relationship with students and whether the development of students' personality can be promoted healthily. A good teaching attitude can be reflected from various perspectives: good teaching planning, well prepared lessons, clean and tidy clothing, decent deportment, optimistic emotion, full spirit and so on. Thus, the better the teaching attitude, the better the teaching performance, such that the higher the quality of classroom teaching.

Table 1. Evaluation system of classroom teaching quality

Attribute	Sub-attribute
Teaching attitude (a_1)	Planning and preparation (a_{l_1}) Dressing and behavior (a_{l_2}) Emotion and spirit (a_{l_3})
Teaching capacity (a_2)	Writing and oral language (a_{2_1}) Communication and interaction (a_{2_2}) Dealing with teaching materials (a_{2_3})
Teaching content (a_3)	Clear goals and enough information (a_{3_1}) Schedule (a_{3_2}) Introduction of relative frontier research (a_{3_3})
Teaching method (a_4)	Using various media/approaches (a_{4_1}) Discussion (a_{4_2}) Combining theory with practice (a_{4_3})
Teaching effect (a_5)	Enhancing students' responsibility and self-management (a_{5_1}) Feedback (a_{5_2}) Students' attendance rate (a_{5_3})

Teaching capacity, in general, is a collection of various abilities that the teacher ought to be equipped. It is a necessary guarantee of high-quality education and high-level development of universities. This attribute is made up of several aspects:

First, being tools for teachers to impart knowledge and cultivate humanity, the writing and oral language of a teacher are very important. The writing on the blackboard should be highly summarized because it has a significant role in assisting students to review and consolidate lessons. The oral language directly decides whether the teacher could play the leading role in the class and affects the improvement of the language and thoughts of students.

Second, communication and interaction are imperative abilities in promoting teaching process. Where there is teaching proceeding, where there is communication and interaction. By communicating and interacting with the student, the teacher could get the characteristic, interest and curiosity of different students, such that he/she could teach students in coincidence with their aptitude. Good communicative and interactive skills assist the teacher to adapt to the teaching environment and enhance teaching effect.

Third, handling teaching materials, is the ability to understand teaching materials and make the emphases and difficulties clear. With this ability, the teacher may organize the teaching contents scientifically and attract students' attentions successfully.

Teaching content, is a core attribute in evaluating teaching quality. It is the main information being transmitted during the interaction of teachers and students. Teaching content not only refers to the knowledge and skills that teachers impart to students, but also means the thoughts and viewpoints that teachers infuse to students as well as the habits and behaviors that teachers affects students. The more abundant and innovative the teaching content is, the more plentiful knowledge the students acquire, such that the better classroom teaching quality the teacher provides. A rich teaching content probably contains several distinctive but interactive aspects, such as crisp teaching goals, rational teaching schedules, substantial content of lessons, proper quantities of information, frontier researches and so on.

Teaching method, is a collection of teaching approaches being employed by teachers who are aiming to achieve teaching objectives and complete teaching tasks. Generally, there are different teaching methods such as using various media/approaches, discussing with students and combining theory with practice. A suitable teaching method can reach the unification of the teaching from teachers and the studying from students perfectly, and thus improve the quality and effect of classroom teaching.

Teaching effect, is a concentrated reflection of classroom teaching quality. It is used to measure whether the teaching goals have been achieved, whether the students have participated in the teaching process actively, and whether the teacher has devoted to the development of his/her students. Good teaching effect may be reflected by enhanced responsibility and self-management of students, high students' attendance rate, good feedback and assessments from students, etc.

FRAMEWORK FOR CLASSROOM TEACHING QUALITY EVALUATION

To perform a systematical evaluation process of classroom teaching quality, this section puts forward a novel framework comprising several methods and models flowed in **Figure 1**.

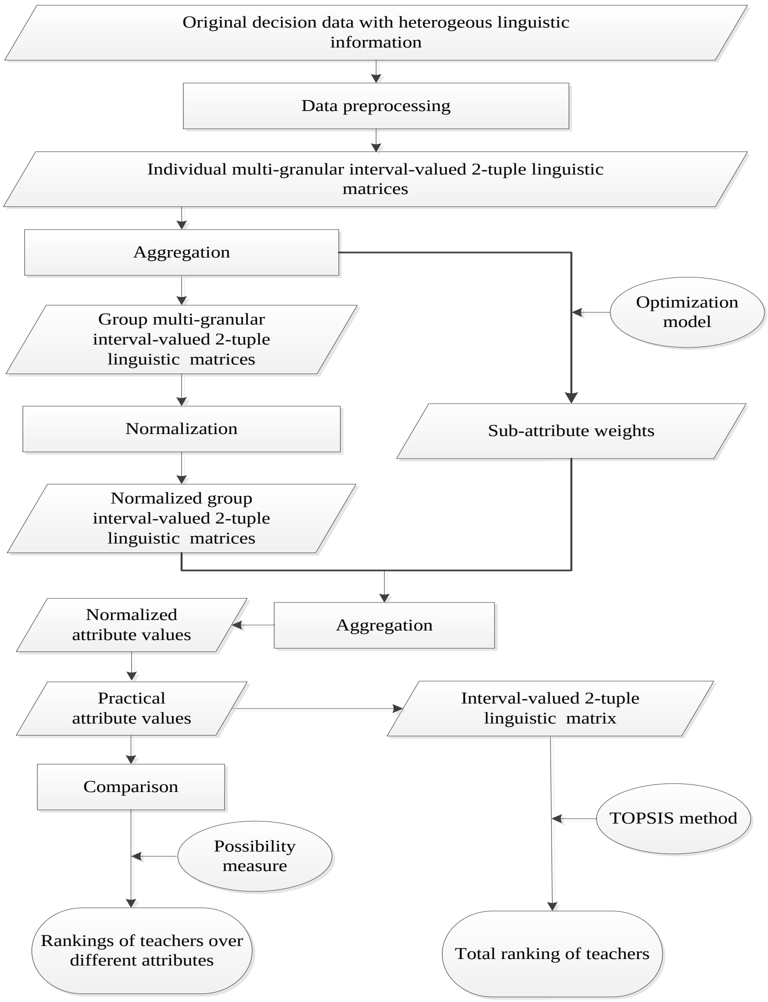

Figure 1. Framework of classroom teaching quality evaluation

Hierarchical Structure of Classroom Teaching Quality Evaluation

In order to intuitively understand the MAGDM problem about classroom teaching quality evaluation, the structure of a four-level evaluation process is built up and shown in Figure 2. Concretely, it is a typical hierarchical structure for MAGDM problems and consists of four levels from top to bottom: the objective, a fixed set of attributes $A = \{a_j \mid j = 1, 2, \ldots, m\}$, the corresponding sub-attribute sets to the attributes $a_j = \{a_{j_k} \mid k = 1, 2, \ldots, q\}$ $(j = 1, 2, \ldots, m)$, and a finite set of optional teachers $X = \{x_i \mid i = 1, 2, \ldots, n\}$.

Establishment of Evaluation Standard for Sub-Attributes

In accordance with the characteristics of predefined sub-attributes, different linguistic term sets $S^{j_k} = \{s_1^{j_k}, s_2^{j_k}, \ldots, s_{g_{j_k}}^{j_k}\}(j = 1, 2, \ldots, m; k = 1, 2, \ldots, q)$ are defined to express sub-attribute values for each teacher. All decision-makers should employ the same standard to assess alternatives (teachers).

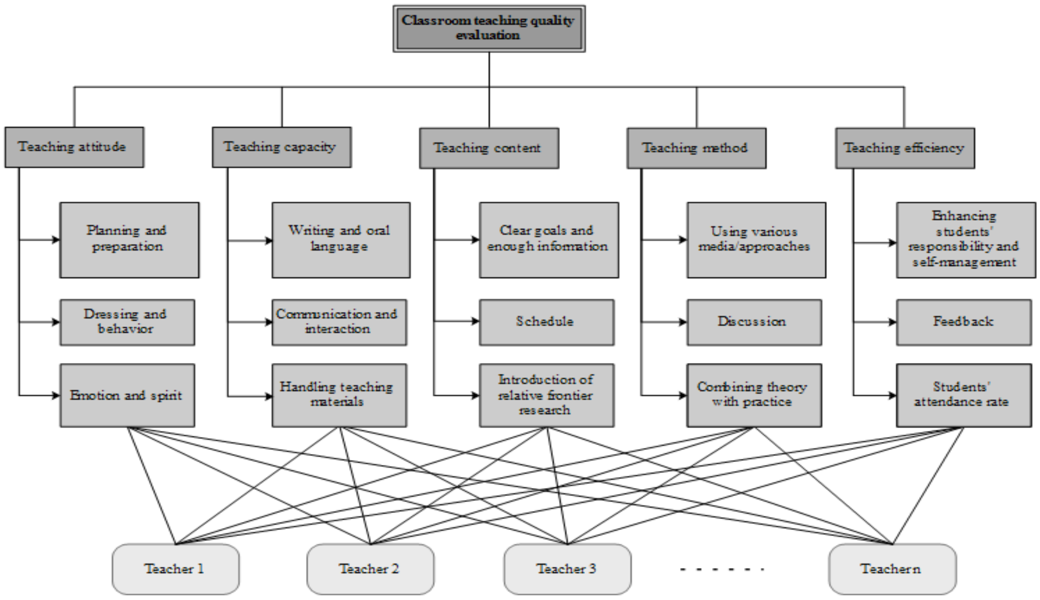

Figure 2. Hierarchical structure of classroom teaching quality

Determination of a Committee of Decision-Makers

To form a rational and appropriate committee of decision-makers for objectively assessing the classroom teaching quality of the teacher, professors who possess distinctive background and personal expertise ought to be considered. Assume p professors $D = \{d^1, d^2, \dots, d^p\}$ are selected and invited to evaluate the classroom teaching quality of n teachers from different colleges which are denoted as $X = \{x_1, x_2, \dots, x_n\}$. The weight vector of professors is given as $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_p)^T$, satisfying $\sum_{k=1}^{q} \lambda_k = 1$ and $0 \le \lambda_k \le 1$.

Generation and Preprocessing of Decision Data

Indeed, professors, who share commonalities as well as distinctive knowledge and experiences, need to solve a decision problem jointly but cannot afford to provide all sub-attribute values for alternatives with certain linguistic terms; due to their limited knowledge of the attribute or the teacher. In this situation, it is appropriate and preferable for professors to give uncertain linguistic values. Taking this into consideration, the primitive decision data furnished by professors is presented by heterogeneous decision matrices which contain multi-granular linguistic and uncertain linguistic information. Facing with this, measures for unifying the original data and making it computable should be conducted.

Easily, a linguistic value can be equally represented by an uncertain linguistic value where the lower and upper bounds of the interval are identical. For instance, s_5 is equivalent to an uncertain linguistic value $[s_5, s_5]$. Consequently, the primitive decision data are unified into interval-valued linguistic matrices $R_{(t)}^{a_j} = (r_{tij_k})_{n\times q} = \left(s^{j_k}_{\alpha^-_{tij_k}}, s^{j_k}_{\alpha^+_{tij_k}}\right)_{n\times q}$ $(t = 1, 2, \dots, p; j = 1, 2, \dots, m)$. Furthermore, linguistic terms are discrete that they are not suitable to express the synthesized results which are likely to be continuous. Therefore, the given linguistic values are in need of being transformed into a continuous form such as linguistic 2-tuples. Obviously, this transformation can be implemented via Eq. (3). By this proceeding, interval-valued 2-tuple linguistic matrices $\tilde{R}_{(t)}^{a_j} = (\tilde{r}_{tij_k})_{n\times q} = \left(\left[\left(s^{j_k}_{\tilde{\alpha}^-_{tij_k}}, \tilde{\beta}^-_{tij_k}\right), \left(s^{j_k}_{\tilde{\alpha}^+_{tij_k}}, \tilde{\beta}^+_{tij_k}\right)\right]\right)_{n\times q}$ $(t = 1, 2, \dots, p; j = 1, 2, \dots, m)$ are constructed.

Aggregation of Individual Judgments

To obtain a comprehensive assessment for teachers on different sub-attributes, individual opinions should be integrated. Due to the feature of multi-granular linguistic information in our case, we make a little adjustment of the IVTWA operator as

$$\tilde{r}_{ij_k} = [(s^{j_k}_{\tilde{\alpha}^-_{ij_k}}, \tilde{\beta}^-_{ij_k}), (s^{j_k}_{\tilde{\alpha}^+_{ij_k}}, \tilde{\beta}^+_{ij_k})]$$

$$= \left[\begin{array}{l} \left(s^{j_k}_{round\left(g_{j_k} \cdot \left(\sum_{t=1}^{p} \lambda_t \cdot \left(\frac{\alpha^-_{tij_k}}{g_{j_k}} + \beta^-_{tij_k} \right) \right) \right)}, \sum_{t=1}^{p} \lambda_t \cdot \left(\frac{\alpha^-_{tij_k}}{g_{j_k}} + \beta^-_{tij_k} \right) - \frac{round\left(g_{j_k} \cdot \left(\sum_{t=1}^{p} \lambda_t \cdot \left(\frac{\alpha^-_{tij_k}}{g_{j_k}} + \beta^-_{tij_k} \right) \right) \right)}{g_{j_k}} \right), \\ \left(s^{j_k}_{round\left(g_{j_k} \cdot \left(\sum_{t=1}^{p} \lambda_t \cdot \left(\frac{\alpha^+_{tij_k}}{g_{j_k}} + \beta^+_{tij_k} \right) \right) \right)}, \sum_{t=1}^{p} \lambda_t \cdot \left(\frac{\alpha^+_{tij_k}}{g_{j_k}} + \beta^+_{tij_k} \right) - \frac{round\left(g_{j_k} \cdot \left(\sum_{t=1}^{p} \lambda_t \cdot \left(\frac{\alpha^+_{tij_k}}{g_{j_k}} + \beta^+_{tij_k} \right) \right) \right)}{g_{j_k}} \right) \end{array}\right] \quad (10)$$

Repeatedly, making use of Eq. (10), one can derive collective interval-valued 2-tuple linguistic matrices $\tilde{R}^{a_j} = (\tilde{r}_{ij_k})_{n\times q} = \left(\left[\left(s^{j_k}_{\tilde{\alpha}^-_{ij_k}}, \tilde{\beta}^-_{ij_k}\right), \left(s^{j_k}_{\tilde{\alpha}^+_{ij_k}}, \tilde{\beta}^+_{ij_k}\right)\right]\right)_{n\times q}$ $(j = 1, 2, \dots, m)$ which refer to the group assessments for alternatives over sub-attributes.

Optimization Models for Determining Sub-Attribute Weights

Naturally, there are differences between different sub-attributes which belong to one attribute, and this can be reflected by distributing different importance weights to different sub-attributes. Actually, owing to the increasing complication of real decision situations, professors may not have the ability to provide exact weights values for sub-attributes. Thus, it is significant to make a research on incomplete sub-attribute weights. Herein, the weight information of sub-attributes is totally unknown and decided objectively. We denote $w = (w_{j_1}, w_{j_2}, \dots, w_{j_q})^T$ as the weighting vector of the sub-attributes which fulfills $\sum_{k=1}^{q} w_{j_k} = 1$ and $w_{j_k} \in [0, 1]$. Generally, there are five forms of weight information (Kim & Han, 1999), which are listed as follows.

(1) A weak ranking: $\left\{w_{j_k} \geq w_{j_{k'}}\right\}, k \neq k'$;

(2) A strict ranking: $\left\{w_{j_k} \geq w_{j_{k'}} \geq \sigma_{j_k}(> 0)\right\}, k \neq k'$;

(3) A ranking with multiples: $\left\{w_{j_k} \geq \sigma_{j_k} w_{j_{k'}}\right\}, 0 < \sigma_{j_k} < 1, k \neq k'$;

(4) An interval form: $\{\sigma_{j_k} \leq w_{j_k} \leq \sigma_{j_k} + \varepsilon_{j_k}\}, 0 \leq \sigma_{j_k} < \sigma_{j_k} + \varepsilon_{j_k} \leq 1$; and

(5) A ranking of differences: $\left\{w_{j_k} - w_{j_{k'}} \geq w_{j_{k''}} - w_{j_{k'''}}\right\}, for\ k \neq k' \neq k'' \neq k'''$.

In the following, some optimization models for deriving sub-attribute weights in group decision making with interval-valued linguistic 2-tuples are presented.

At first, to adapt the interval 2-tuples to optimization models, we put forward a score function of interval 2-tuples.

Definition 6. Let $S=\{s_\alpha \mid \alpha=0,1,\dots,g\}$ be a linguistic term set and $R=(r_{ij})_{n\times n}=\left(\left[\left(s_{\alpha_{ij}^-},\beta_{ij}^-\right),\left(s_{\alpha_{ij}^+},\beta_{ij}^+\right)\right]\right)_{n\times n}$ be an interval-valued 2-tuple linguistic matrix, then we call $B=(b_{ij})_{n\times n}$ the score matrix of R, where

$$b_{ij}=\frac{1}{2}\left[\Lambda^{-1}(s_{\alpha_{ij}^-},\beta_{ij}^-)+\Lambda^{-1}(s_{\alpha_{ij}^+},\beta_{ij}^+)\right],\qquad i=1,2,\dots,n;\ j=1,2,\dots,n \tag{11}$$

Based on Eq. (11), scores of sub-attribute values can be computed by

$$b_{ij_k}=\frac{1}{2}\left[\left(\frac{\tilde{\alpha}_{ij_k}^-}{g_{j_k}}+\tilde{\beta}_{ij_k}^-\right)+\left(\frac{\tilde{\alpha}_{ij_k}^+}{g_{j_k}}+\tilde{\beta}_{ij_k}^+\right)\right],\qquad i=1,2,\dots,n;\ j=1,2,\dots,m;k=1,2,\dots,q. \tag{12}$$

Then, the comprehensive score of each alternative over different attributes can be figured out by incorporating the weights of sub-attributes and sub-attribute scores by

$$b_{ij}(w)=\sum\nolimits_{k=1}^{q} w_{j_k} b_{ij_k},\qquad i=1,2,\dots,n \tag{13}$$

Evidently, the larger the value of $b_{ij}(w)$ is, the better alternative x_i on attribute a_j. In order to maximize $b_{ij}(w)$, reasonable weight vectors of sub-attributes which are only related to alternative x_i, should be decided. Thus, we establish an optimization model as

$$\begin{aligned}&\max b_{ij}(w)=\sum\nolimits_{k=1}^{q} w_{j_k} b_{ij_k}\\ &s.t.\begin{cases} w_{j_k}\geq 0,\ j=1,2,\dots,m,\\ \sum\nolimits_{k=1}^{q} w_{j_k}=1,\\ w_j=(w_{j_1},w_{j_2},\dots,w_{j_q})^T\in H\end{cases}\end{aligned} \tag{\textbf{M-1}}$$

By solving model (**M-1**), one can obtain the optimal weight vector of sub-attributes corresponding to alternative x_i as $w^i=(w_{j_1}^i,w_{j_2}^i,\dots,w_{j_q}^i)^T$. Nonetheless, it is demanded to consider all the alternatives $x_i(i=1,2,\dots,n)$ as one in the proceeding of determining weights of sub-attributes. For this reason, a combined weight vector is constructed as:

$$w_j=\omega_j^1 w_j^1+\omega_j^2 w_j^2+\cdots+\omega_j^n w_j^n=\begin{bmatrix} w_{j_1}\\ w_{j_2}\\ \vdots\\ w_{j_q}\end{bmatrix}=W_j\varpi_j, \tag{14}$$

where

$$W_j=\begin{bmatrix} w_{j_1}^1 & w_{j_1}^2 & \cdots & w_{j_1}^n\\ w_{j_2}^1 & w_{j_2}^2 & \cdots & w_{j_2}^n\\ \vdots & \vdots & \vdots & \vdots\\ w_{j_q}^1 & w_{j_q}^2 & \cdots & w_{j_q}^n\end{bmatrix}, \tag{15}$$

and $\varpi_j=(\omega_j^1,\omega_j^2,\dots,\omega_j^n)^T$ is an undetermined nonnegative vector, referring to the weights of alternatives under different attributes and satisfying:

$$(\varpi_j)^T \varpi_j = 1 \tag{16}$$

Next, let $\bar{b}_{ij} = (b_{ij_1}, b_{ij_2}, \dots, b_{ij_q})^T$ stand for the score vector of alternative x_i over attribute a_j, then the score matrix of all alternatives over attribute a_j can be formed and expressed as $B_j = (\bar{b}_{1j}, \bar{b}_{2j}, \dots, \bar{b}_{nj})^T$.

Plug Eq. (14) into Eq. (13), we have

$$b_{ij}(w) = \sum_{k=1}^{q} w_{j_k} b_{ij_k} = w_j^T \overline{b_{ij}} = (W_j \varpi_j)^T \overline{b_{ij}} \tag{17}$$

Since the greater the value of $b_{ij}(w)$, the better alternative x_i over attribute a_j, all the comprehensive scores of alternatives should be maximized so as to determine the combined weight vector. Thus, a multi-objective optimization model is established as:

$$\begin{aligned} &\max b_j(w) = (b_{1j}(w), b_{2j}(w), \dots, b_{nj}(w)) \\ &s.t.\ \ (\varpi_j)^T \varpi_j = 1 \end{aligned} \tag{M-2}$$

By the equal weighted summation method, model (**M-2**) can be converted to a single objective optimization model as:

$$\begin{aligned} &\max b_j(w)^T b_j(w) \\ &s.t.\ \ (\varpi_j)^T \varpi_j = 1 \end{aligned} \tag{M-3}$$

Let $F_j(w) = b_j(w)^T b_j(w)$, by Eq. (17), we get

$$F_j(w) = b_j(w)^T b_j(w) = \varpi_j^T (B_j W_j)^T (B_j W_j) \varpi_j \tag{18}$$

Let $\Omega_j = (B_j W_j)^T (B_j W_j)$, then $\Omega_j^T = ((B_j W_j)^T (B_j W_j))^T = (B_j W_j)^T (B_j W_j) = \Omega_j$. That is to say, Ω_j is a real symmetrical matrix. Obviously, $\Omega_j \geq 0$. Therefore, Ω_j is a nonnegative definite matrix.

In order to obtain the combined weights of attributes, we introduce two theorems below.

Theorem 2 (Xu, 2005). Let $Y = (y_{ij})_{n\times n}$be a real symmetric matrix, i.e., $Y^T = Y$, then

$$\max \frac{\alpha^T Y \alpha}{\alpha^T \alpha} = \lambda_{\max} \tag{19}$$

where α is a nonzero vector, and $\lambda_{\max}$ is the largest eigenvalue of Y.

Theorem 3 (Xu, 2005). Let $Y = (y_{ij})_{n\times n}$ be a real irreducible nonnegative matrix, then

(1) There is a largest eigenvalue as well as a unique eigenvalue of Y, denoted as $\lambda_{\max}$;

(2) Assume $\beta = (\beta_1, \beta_2, \dots, \beta_n)^T$ is the corresponding eigenvector of $\lambda_{\max}$, then all $\beta_j > 0$ $(j = 1, 2, \dots, n)$, which indicates that the eigenvector β is positive.

As per **Theorems 2** and **3**, it is easy to understand that $\max F_j(w)$ is not only the largest value of $F_j(w)$, but also the largest eigenvalue $\lambda_j^{\max}$ of Ω_j. Besides, the eigenvector of $\lambda_j^{\max}$ is $\varpi = (\omega_1, \omega_2, \dots, \omega_n)^T$, where $\lambda_j^{\max}$ is unique and ϖ is positive. Subsequently, normalize $\varpi_j = (\omega_j^1, \omega_j^2, \dots, \omega_j^n)^T$, and then plug the unified vector into Eq. (14), one can obtain the weight vectors of sub-attributes.

Conclusively, the process of generating sub-attribute weighting vectors can be summarized by the following steps:

Step 1: Make use of Eq. (11) to compute the scores of each element in $\tilde{R}^{a_j}(j = 1, 2, \dots, m)$, and thus make up score matrices $B_j = \left(b_{ij_k}\right)_{n\times q}(j = 1, 2, \dots, m; k = 1, 2, \dots, q)$ for $\tilde{R}^{a_j}(j = 1, 2, \dots, m)$.

Step 2: By applying model (**M-1**), one can obtain the optimal solutions $w_j^i = \left(w_{j_1}^i, w_{j_2}^i, \dots, w_{j_q}^i\right)^T$ $(i = 1, 2, \dots, n; j = 1, 2, \dots, m)$ relative to alternatives $x_i (i = 1,2, \dots, n)$, respectively. Then, weight matrices $W_j = (w_j^1, w_j^2, \dots, w_j^n)$ are constructed.

Step 3: Calculate the normalized eigenvector $\varpi_j = (\omega_j^1, \omega_j^2, \dots, \omega_j^n)^T$ of matrix $\left(B_j W_j\right)^T \left(B_j W_j\right)$.

Step 4: Use Eq. (14) to obtain the weight vectors of sub-attributes $w_j = (w_{j_1}, w_{j_2}, \dots, w_{j_m})^T$ with respect to different attributes $a_j (j = 1,2, \dots, m)$.

Normalization of Multi-Granular Linguistic Sub-Attribute Values

To obtain comprehensive attribute values for one teacher, multi-granular interval-valued information should be normalized. In the following, we put forward a function to accomplish the interactive transformation between two interval-valued linguistic 2-tuples which are assessed in linguistic term sets with different granularities.

Definition 7. Let $S^h = \{s_\alpha^h \mid \alpha = 0, 1, \dots, g_h\} (h = 1, 2)$ be two linguistic term sets with $g_1 \neq g_2$ and $v_h = (s_{\alpha_h^+}^h, \beta_h^+) (h = 1, 2)$ be two linguistic 2-tuples where $s_{\alpha_h}^h \in S^h$ and $\beta_h \in [-0.5/g_h, 0.5/g_h)$. Then the conversion of v_1 to v_2 can be achieved by the following functions:

$$\begin{cases} \mathrm{T}: v_1 \to v_2 \\ T\left([(s_{\alpha_1^-}^1, \beta_1^-), (s_{\alpha_1^+}^1, \beta_1^+)] \right) = [(s_{\alpha_2^-}^2, \beta_2^-), (s_{\alpha_2^+}^2, \beta_2^+)], with \begin{cases} s_{\alpha_2^-}^2, & \alpha_2^- = round\left(g_2 \cdot (\dfrac{\alpha_1^-}{g_1} + \beta_1^-) \right) \\ s_{\alpha_2^+}^2, & \alpha_2^+ = round\left(g_2 \cdot (\dfrac{\alpha_1^+}{g_1} + \beta_1^+) \right) \\ \beta_2^- = (\dfrac{\alpha_1^-}{g_1} + \beta_1^-) - \dfrac{\alpha_2^-}{g_2}, & \beta_2^- \in [-0.5/g_2, 0.5/g_2) \\ \beta_2^+ = (\dfrac{\alpha_1^+}{g_1} + \beta_1^+) - \dfrac{\alpha_2^+}{g_2}, & \beta_2^+ \in [-0.5/g_2, 0.5/g_2). \end{cases} \end{cases} \tag{20}$$

where $round$ is the common rounding operation.

By employing **Definition 7**, the sub-attribute values which are denoted by multi-granular linguistic scales but exist in the same row of one decision matrix can be unified. To be concrete, we utilize the following expression to complete this unification.

$$\begin{cases} \mathrm{T}: \tilde{r}_{tij_k} \to \bar{r}_{tij_k} \\ T\left([(s_{\tilde{\alpha}_{tij_k}^-}^{j_k}, \tilde{\beta}_{tij_k}^-), (s_{\tilde{\alpha}_{tij_k}^+}^{j_k}, \tilde{\beta}_{tij_k}^+)] \right) = [(s_{\bar{\alpha}_{tij_k}^-}^{j_*}, \bar{\beta}_{tij_k}^-), (s_{\bar{\alpha}_{tij_k}^+}^{j_*}, \bar{\beta}_{tij_k}^+)], with \begin{cases} s_{\bar{\alpha}_{tij_k}^-}^{j_*}, & \bar{\alpha}_{tij_k}^- = round\left(g_{j_*} \cdot \left(\dfrac{\tilde{\alpha}_{tij_k}^-}{g_{j_k}} + \tilde{\beta}_{tij_k}^- \right) \right) \\ s_{\bar{\alpha}_{tij_k}^+}^{j_*}, & \bar{\alpha}_{tij_k}^+ = round\left(g_{j_*} \cdot \left(\dfrac{\tilde{\alpha}_{tij_k}^+}{g_{j_k}} + \tilde{\beta}_{tij_k}^+ \right) \right) \\ \bar{\beta}_{tij_k}^- = \left(\dfrac{\tilde{\alpha}_{tij_k}^-}{g_{j_k}} + \tilde{\beta}_{tij_k}^- \right) - \dfrac{\bar{\alpha}_{tij_k}^-}{g_{j_*}}, & \bar{\beta}_{tij_k}^- \in \left[-\dfrac{0.5}{g_{j_*}}, \dfrac{0.5}{g_{j_*}} \right) \\ \bar{\beta}_{tij_k}^+ = \left(\dfrac{\tilde{\alpha}_{tij_k}^+}{g_{j_k}} + \tilde{\beta}_{tij_k}^+ \right) - \dfrac{\bar{\alpha}_{tij_k}^+}{g_{j_*}}, & \bar{\beta}_{tij_k}^+ \in \left[-\dfrac{0.5}{g_{j_*}}, \dfrac{0.5}{g_{j_*}} \right). \end{cases} \end{cases} \tag{21}$$

where $s_{\bar{\alpha}_{tij_k}^-}^{j*}, s_{\bar{\alpha}_{tij_k}^+}^{j*} \in S^{j*} = \{s_0^{j*}, s_1^{j*}, \dots, s_{g_{j*}}^{j*}\}$ with $g_{j*} = \max\{g_{j_1}, g_{j_2}, \dots, g_{j_q}\}$.

Consequently, normalized interval-valued 2-tuple linguistic matrices including different sub-attribute values of teachers are constructed and denoted as $\bar{R}^{a_j} = (\bar{r}_{ij_k})_{n\times q} = \left([(s^{j_*}_{\bar{\alpha}^-_{ij_k}}, \bar{\beta}^-_{ij_k}), (s^{j_*}_{\bar{\alpha}^+_{ij_k}}, \bar{\beta}^+_{ij_k})]\right)_{n\times q}$ $(j = 1,2,\dots,m)$.

Aggregation of Normalized Sub-Attribute Values

Utilize the IVTWA operator to integrate sub-attribute values under each attribute and generate attribute values for teachers. That is,

$$\bar{r}_{ij} = [(s^{j_*}_{\bar{\alpha}^-_{ij}}, \bar{\beta}^-_{ij}), (s^{j_*}_{\bar{\alpha}^+_{ij}}, \bar{\beta}^+_{ij})]$$

$$= \left[\begin{array}{l} \left(s^{j_*}_{round\left(g_{j_*}\cdot\left(\sum_{k=1}^{q} w_{j_k}\cdot\left(\frac{\bar{\alpha}^-_{ij_k}}{g_{j_*}}+\bar{\beta}^-_{ij_k}\right)\right)\right)}, \sum_{k=1}^{q} w_{j_k}\cdot\left(\frac{\bar{\alpha}^-_{ij_k}}{g_{j_*}}+\bar{\beta}^-_{ij_k}\right) - \frac{round\left(g_{j_*}\cdot\left(\sum_{k=1}^{q} w_{j_k}\cdot\left(\frac{\bar{\alpha}^-_{ij_k}}{g_{j_*}}+\bar{\beta}^-_{ij_k}\right)\right)\right)}{g_{j_*}} \right), \\ \left(s^{j_*}_{round\left(g_{j_*}\cdot\left(\sum_{k=1}^{q} w_{j_k}\cdot\left(\frac{\bar{\alpha}^+_{ij_k}}{g_{j_*}}+\bar{\beta}^+_{ij_k}\right)\right)\right)}, \sum_{k=1}^{q} w_{j_k}\cdot\left(\frac{\bar{\alpha}^+_{ij_k}}{g_{j_*}}+\bar{\beta}^+_{ij_k}\right) - \frac{round\left(g_{j_*}\cdot\left(\sum_{k=1}^{q} w_{j_k}\cdot\left(\frac{\bar{\alpha}^+_{ij_k}}{g_{j_*}}+\bar{\beta}^+_{ij_k}\right)\right)\right)}{g_{j_*}} \right) \end{array}\right] \quad (22)$$

By the above aggregation process, the sub-attribute values are synthesized and the results can be regarded as attribute values for alternatives. Let $\bar{r}_j = (\bar{r}_{1j}, \bar{r}_{2j}, \dots, \bar{r}_{nj})^T$ refer to the vector of attribute values for alternative x_i, then the matrix containing all attribute values is formed as $\bar{R} = (\bar{r}_i)^T = (\bar{r}_{ij})_{n\times m} = \left([(s^{j_*}_{\bar{\alpha}^-_{ij}}, \bar{\beta}^-_{ij}), (s^{j_*}_{\bar{\alpha}^+_{ij}}, \bar{\beta}^+_{ij})]\right)_{n\times m}$.

Generation of Practical Attribute Values

According to real-life evaluation environments, when decision-makers evaluate alternatives over one attribute by means of linguistic variables, there always exists a conventional evaluation description. However, the afore-obtained attribute values, which are assessed in a linguistic term set whose granularity is the largest among all linguistic term sets for expressing sub-attribute values, may be different from the realistic linguistic scale from the perspective of meaning or granularity. Thus, it is imperative to transform the generated attribute values $\bar{r}_{ij} (i = 1,2,\dots,n, j = 1,2,\dots,m)$ into practical attribute values $r_{ij} (i = 1,2,\dots,n, j = 1,2,\dots,m)$. That is,

$$\begin{cases} \mathrm{T}: \bar{r}_{ij} \to r_{ij} \\ T\left([(s^{j_*}_{\bar{\alpha}^-_{ij}}, \bar{\beta}^-_{ij}), (s^{j_*}_{\bar{\alpha}^+_{ij}}, \bar{\beta}^+_{ij})]\right) = [(s^{j_*}_{\alpha^-_{ij}}, \beta^-_{ij}), (s^{j_*}_{\alpha^+_{ij}}, \beta^+_{ij})], with \begin{cases} s^{j_*}_{\alpha^-_{ij}}, & \alpha^-_{ij} = round\left(g_j\cdot\left(\frac{\bar{\alpha}^-_{ij}}{g_{j_*}}+\bar{\beta}^-_{ij}\right)\right) \\ s^{j_*}_{\alpha^+_{ij}}, & \alpha^+_{ij} = round\left(g_j\cdot\left(\frac{\bar{\alpha}^+_{ij}}{g_{j_*}}+\bar{\beta}^+_{ij}\right)\right) \\ \beta^-_{ij} = \left(\frac{\bar{\alpha}^-_{ij}}{g_{j_*}}+\bar{\beta}^-_{ij}\right) - \frac{\alpha^-_{ij}}{g_j}, & \beta^-_{ij} \in \left[-\frac{0.5}{g_j}, \frac{0.5}{g_j}\right) \\ \beta^+_1 = \left(\frac{\bar{\alpha}^+_{ij}}{g_{j_*}}+\bar{\beta}^+_{ij}\right) - \frac{\alpha^+_{ij}}{g_j}, & \beta^-_{ij} \in \left[-\frac{0.5}{g_j}, \frac{0.5}{g_j}\right). \end{cases} \end{cases} \quad (23)$$

where $S^j = \{s^j_\alpha \mid \alpha = 0,1,\dots,g_j\} (j = 1,2,\dots,m)$ denotes the real-life descriptions of the given attributes.

By this conversion, the generated attribute values construct an interval-valued 2-tuple linguistic matrix $R = (r_{ij})_{n\times m} = \left([(s^j_{\alpha^-_{ij}}, \beta^-_{ij}), (s^j_{\alpha^+_{ij}}, \beta^+_{ij})]\right)_{n\times m}$, in which the elements have been endowed with realistic meanings.

Rankings of Alternatives Over Different Attributes

Commonly, the teachers should also be partially evaluated to check which aspect(s) of teaching he/she should pay attention. This subsection puts forward a possibility formula to compare attribute values of different teachers, i.e., each column of elements in R.

To be in line with the functions we employed in this paper, we make an extension of Eq. (8) and introduce the following generated possibility formula to yield the possibility matrix related to each attribute.

Definition 8. Let $S = \{s_\alpha \mid \alpha = 0,1,\dots,g\}$ be a linguistic term set, $v_1 = [(s_{\alpha^-_1}, \beta^-_1), (s_{\alpha^+_1}, \beta^+_1)]$ and $v_2 = [(s_{\alpha^-_2}, \beta^-_2), (s_{\alpha^+_2}, \beta^+_2)]$ be two interval-valued linguistic 2-tuples, then the possibility of $v_1 \geq v_2$ is calculated with

$$\widehat{\kappa}(v_1 \geq v_2) = \frac{\max\left\{0, len(v_1) + len(v_2) - \max\left(\left(\Lambda^{-1}(s_{\alpha^+_2}, \beta^+_2) - \Lambda^{-1}(s_{\alpha^-_1}, \beta^-_1)\right), 0\right)\right\}}{len(v_1) + len(v_2)} \tag{24}$$

where $len(v_1) = \Lambda^{-1}(s_{\alpha^+_1}, \beta^+_1) - \Lambda^{-1}(s_{\alpha^-_1}, \beta^-_1)$ and $len(v_2) = \Lambda^{-1}(s_{\alpha^+_2}, \beta^+_2) - \Lambda^{-1}(s_{\alpha^-_2}, \beta^-_2)$.

Thereby, the possibility degrees of one teacher over others over different attributes can be obtained by the following expression:

$$\widehat{\kappa}(r_{1j} \geq r_{2j}) = \frac{\max\left\{0, len(r_{1j}) + len(r_{2j}) - \max\left(\left(\Lambda^{-1}(s_{\alpha^+_{2j}}, \beta^+_{2j}) - \Lambda^{-1}(s_{\alpha^-_{1j}}, \beta^-_{1j})\right), 0\right)\right\}}{len(r_{1j}) + len(r_{2j})} \tag{25}$$

where $len(r_{1j}) = \Lambda^{-1}\left(s_{\alpha^+_{1j}}, \beta^+_{1j}\right) - \Lambda^{-1}\left(s_{\alpha^-_{1j}}, \beta^-_{1j}\right)$ and $len(r_{2j}) = \Lambda^{-1}\left(s_{\alpha^+_{2j}}, \beta^+_{2j}\right) - \Lambda^{-1}\left(s_{\alpha^-_{2j}}, \beta^-_{2j}\right)$.

This operation results in possibility matrices representing the degrees that one teacher precedes the others on different attributes, according to which the ranking of teachers on each attribute can be inferred.

TOPSIS-Based Evaluation Process

To produce the overall evaluation result and make a comparison of the teachers, we extend the traditional TOPSIS technique to select the teacher whose comprehensive performance is the best. This selection mainly contains the following four steps:

Step 1: Determination of ideal solutions

According to the real-life descriptions of different attributes, decide the positive solution $Z = ([z^-_1, z^+_1], [z^-_2, z^+_2], \dots, [z^-_m, z^+_m])$ and the negative ideal solution $F = ([f^-_1, f^+_1], [f^-_2, f^+_2], \dots, [f^-_m, f^+_m])$, respectively.

Step 2: Calculation of distances

Use the following extended Euclidean distance formulae to separately measure how close an alternative approach to the positive ideal solution and how far an alternative is away from the negative ideal one.

$$d^+ = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left(\left(\left(\frac{s^j_{\alpha^-_{ij}}}{g_j} + \beta^-_{ij}\right) - \left(\frac{s^j_{\alpha^{z-}_j}}{g_j} + \beta^{z-}_j\right)\right)^2 + \left(\left(\left(\frac{s^j_{\alpha^+_{ij}}}{g_j} + \beta^+_{ij}\right) - \left(\frac{s^j_{\alpha^{z+}_j}}{g_j} + \beta^{z+}_j\right)\right)\right)^2\right)}\ , \tag{26}$$

$$d^{-}=\sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left(\left(\left(\frac{s^{j}_{\alpha_{ij}^{-}}}{g_j}+\beta_{ij}^{-}\right)-\left(\frac{s^{j}_{\alpha_j^{f-}}}{g_j}+\beta_j^{f-}\right)\right)^2+\left(\left(\left(\frac{s^{j}_{\alpha_{ij}^{+}}}{g_j}+\beta_{ij}^{+}\right)-\left(\frac{s^{j}_{\alpha_j^{f+}}}{g_j}+\beta_j^{f+}\right)\right)\right)^2\right)} \quad . \tag{27}$$

Step 3: Computation of closeness coefficients

Employ the following equation to count the closeness coefficient of each teacher.

$$CC_i=\frac{d^-}{d^-+d^+} \tag{28}$$

The larger the closeness coefficient of a teacher, the better he/she is evaluated. Oppositely, the smaller the closeness coefficient of a teacher, the worse assessment he/she obtains. Especially, if the closeness coefficient of a teacher equals to 1 (0), the attribute values of this teacher is the positive (negative) ideal solution.

Step 4: Evaluation

As per the closeness coefficients, make a conclusive evaluation by ranking the optional teachers.

ILLUSTRATION AND DISCUSSION

This section aims to present an example to illustrate the applicability of the proposed framework and provide a discussion of the framework based on the results of the example.

Illustrative Example

To illustrate how to apply our framework for evaluating classroom teaching quality, a simulated example is shown below.

In this case, three professors $d_t(t=1,2,3)$ are selected to evaluate four teachers $x_i(i=1,2,3,4)$ over the 15 sub-attributes $a_{j_k}(j=1,2,3,4,5;k=1,2,3)$ depicted in **Section 3**. The weight vector of professors is $\lambda=(0.35,0.25,0.4)^T$. As per different linguistic term sets $S^{j_k}=\left\{s_\alpha^{j_k}\middle|\alpha=0,1,\dots,g_{j_k}\right\}(j=1,2,3,4,5;k=1,2,3)$, each professor is required to evaluate alternatives over different sub-attributes and provides assessments by means of multi-granular linguistic matrices $R_t^{a_j}=\left(r_{tij_k}^{a_j}\right)_{4\times3}(t=1,2,3;j=1,2,3,4,5)$. The predefined linguistic term sets are listed in **Table 2**, followed by the original heterogeneous linguistic information.

Table 2. Evaluation standard of sub-attributes

Sub-attribute	Linguistic term set
a_{1_1}	$S^{1_1}=\{s_0^{1_1}=very\ poor, s_1^{1_1}=poor, s_2^{1_1}=medium, s_3^{1_1}=rich, s_4^{1_1}=very\ rich\}$
a_{1_2}	$S^{1_2}=\{s_0^{1_2}=very\ untidy, s_1^{1_2}=untidy, s_2^{1_2}=medium, s_3^{1_2}=tidy, s_4^{1_2}=very\ tidy\}$
a_{1_3}	$S^{1_3}=\{s_0^{1_3}=very\ negative, s_1^{1_3}=negative, s_2^{1_3}=slightly\ negative, s_3^{1_3}=medium,$ $s_4^{1_3}=slightly\ positive, s_5^{1_3}=positive, s_6^{1_3}=very\ positive\}$
a_{2_1}	$S^{2_1}=\{s_0^{2_1}=very\ not\ skillful, s_1^{2_1}=not\ skillful, s_2^{2_1}=medium, s_3^{2_1}=skillful,$ $s_4^{2_1}=very\ skillful\}$
a_{2_2}	$S^{2_2}=\{s_0^{2_2}=extremely\ bad, s_1^{2_2}=very\ bad, s_2^{2_2}=bad, s_3^{2_2}=slightly\ bad, s_4^{2_2}=$ $medium, s_5^{2_2}=slightly\ good, s_6^{2_2}=good, s_7^{2_2}=very\ good, s_8^{2_2}=extremely\ good\}$
a_{2_3}	$S^{2_3}=\{s_0^{2_3}=very\ improper, s_1^{2_3}=improper, s_2^{2_3}=slightly\ improper, s_3^{2_3}=$ $medium, s_4^{2_3}=slightly\ proper, s_5^{2_3}=proper, s_6^{2_3}=very\ proper\}$
a_{3_1}	$S^{3_1}=\{s_0^{3_1}=very\ bad, s_1^{3_1}=bad, s_2^{3_1}=medium, s_3^{3_1}=good, s_4^{3_1}=very\ \mathrm{good}\}$
a_{3_2}	$S^{3_2}=\{s_0^{3_2}=very\ inappropriate, s_1^{3_2}=inappropriate, s_2^{3_2}=slightly$ $inappropriate, s_3^{3_2}=medium, s_4^{3_2}=slightly\ appropriate, s_5^{3_2}=appropriate,$ $s_6^{3_2}=very\ appropriate\}$
a_{3_3}	$S^{3_3}=\{s_0^{3_3}=very\ insufficient, s_1^{3_3}=insufficient, s_2^{3_3}=medium,$ $s_3^{3_3}=sufficient, s_4^{3_3}=very\ sufficient\}$
a_{4_1}	$S^{4_1}=\{s_0^{4_1}=very\ unsuitable, s_1^{4_1}=unsuitable, s_2^{4_1}=medium, s_3^{4_1}=suitable,$ $s_4^{4_1}=very\ suitable\}$
a_{4_2}	$S^{4_2}=\{s_0^{4_2}=very\ insufficient, s_1^{4_2}=insufficient, s_2^{4_2}=slightly\ insufficient, s_3^{4_2}=$ $medium, s_4^{4_2}=slightly\ sufficient, s_5^{4_2}=sufficient, s_6^{4_2}=very\ sufficient\}$
a_{4_3}	$S^{4_3}=\{s_0^{4_3}=extremely\ bad, s_1^{4_3}=very\ bad, s_2^{4_3}=bad, s_3^{4_3}=slightly\ bad, s_4^{4_3}=$ $medium, s_5^{4_3}=slightly\ good, s_6^{4_3}=good, s_7^{4_3}=very\ good, s_8^{4_3}=extremely\ good\}$
a_{5_1}	$S^{5_1}=\{s_0^{5_1}=very\ little, s_1^{5_1}=little, s_2^{5_1}=slightly\ little, s_3^{5_1}=medium, s_4^{5_1}=$ $slightly\ much, s_5^{5_1}=much, s_6^{5_1}=very\ much\}$
a_{5_2}	$S^{5_2}=\{s_0^{5_2}=extremely\ bad, s_1^{5_2}=very\ bad, s_2^{5_2}=bad, s_3^{5_2}=slightly\ bad, s_4^{5_2}=$ $medium, s_5^{5_2}=slightly\ good, s_6^{5_2}=good, s_7^{5_2}=very\ good, s_8^{5_2}=extremely\ good\}$
a_{5_3}	$S^{5_3}=\{s_0^{5_3}=very\ low, s_1^{5_3}=low, s_2^{5_3}=slightly\ low, s_3^{5_3}=medium,$ $s_4^{5_3}=slightly\ high, s_5^{5_3}=high, s_6^{5_3}=very\ high\}$

The original heterogeneous linguistic matrices $R_t^{a_j} (t = 1,2,3; j = 1,2,3,4,5)$ are:

$$R_1^{a_1} = \begin{bmatrix} [s_2^{1_1}, s_3^{1_1}] & s_3^{1_2} & s_2^{1_3} \\ s_3^{1_1} & s_4^{1_2} & [s_5^{1_3}, s_6^{1_3}] \\ [s_1^{1_1}, s_2^{1_1}] & s_2^{1_2} & s_4^{1_3} \\ s_4^{1_1} & [s_3^{1_2}, s_4^{1_2}] & [s_3^{1_3}, s_5^{1_3}] \end{bmatrix}, \quad R_2^{a_1} = \begin{bmatrix} s_3^{1_1} & s_3^{1_2} & [s_4^{1_3}, s_5^{1_3}] \\ [s_2^{1_1}, s_4^{1_1}] & [s_2^{1_2}, s_4^{1_2}] & s_6^{1_3} \\ [s_2^{1_1}, s_3^{1_1}] & s_1^{1_2} & [s_3^{1_3}, s_5^{1_3}] \\ s_3^{1_1} & [s_3^{1_2}, s_4^{1_2}] & s_5^{1_3} \end{bmatrix},$$

$$R_3^{a_1} = \begin{bmatrix} s_2^{1_1} & [s_3^{1_2}, s_4^{1_2}] & s_5^{1_3} \\ [s_3^{1_1}, s_4^{1_1}] & s_3^{1_2} & [s_4^{1_3}, s_6^{1_3}] \\ s_2^{1_1} & s_4^{1_2} & [s_2^{1_3}, s_3^{1_3}] \\ s_4^{1_1} & s_4^{1_2} & s_1^{1_3} \end{bmatrix}, \quad R_1^{a_2} = \begin{bmatrix} [s_3^{2_1}, s_4^{2_1}] & s_4^{2_2} & s_4^{2_3} \\ s_3^{2_1} & [s_7^{2_2}, s_8^{2_2}] & [s_1^{2_3}, s_3^{2_3}] \\ [s_2^{2_1}, s_4^{2_1}] & [s_3^{2_2}, s_4^{2_2}] & s_5^{2_3} \\ s_4^{2_1} & s_6^{2_2} & s_6^{2_3} \end{bmatrix},$$

$$R_2^{a_2} = \begin{bmatrix} s_3^{2_1} & s_7^{2_2} & [s_4^{2_3}, s_6^{2_3}] \\ [s_2^{2_1}, s_4^{2_1}] & [s_6^{2_2}, s_8^{2_2}] & s_5^{2_3} \\ s_4^{2_1} & s_4^{2_2} & [s_4^{2_3}, s_5^{2_1}] \\ s_4^{2_1} & [s_5^{2_2}, s_7^{2_2}] & s_6^{2_3} \end{bmatrix}, \quad R_3^{a_2} = \begin{bmatrix} s_4^{2_1} & s_6^{2_2} & s_4^{2_3} \\ s_3^{2_1} & [s_5^{2_2}, s_6^{2_2}] & [s_4^{2_3}, s_5^{2_3}] \\ [s_1^{2_1}, s_2^{2_1}] & [s_4^{2_2}, s_8^{2_2}] & s_6^{2_3} \\ s_3^{2_1} & s_7^{2_2} & [s_5^{2_3}, s_6^{2_3}] \end{bmatrix},$$

$$R_1^{a_3} = \begin{bmatrix} s_2^{3_1} & [s_5^{3_2}, s_6^{3_2}] & s_0^{3_3} \\ [s_3^{3_1}, s_4^{3_1}] & s_4^{3_2} & s_3^{3_3} \\ [s_1^{3_1}, s_2^{3_1}] & [s_3^{3_2}, s_5^{3_2}] & [s_1^{3_3}, s_2^{3_3}] \\ s_4^{3_1} & s_6^{3_2} & s_2^{3_3} \end{bmatrix}, \quad R_2^{a_3} = \begin{bmatrix} [s_0^{3_1}, s_1^{3_1}] & s_4^{3_2} & s_2^{3_3} \\ [s_0^{3_1}, s_1^{3_1}] & [s_1^{3_2}, s_3^{3_2}] & s_3^{3_3} \\ [s_2^{3_1}, s_3^{3_1}] & s_6^{3_2} & [s_3^{3_3}, s_4^{3_3}] \\ s_4^{3_1} & [s_3^{3_2}, s_6^{3_2}] & s_3^{3_3} \end{bmatrix},$$

$$R_3^{a_3} = \begin{bmatrix} s_4^{3_1} & [s_0^{3_2}, s_2^{3_2}] & s_2^{3_3} \\ [s_1^{3_1}, s_2^{3_1}] & [s_3^{3_2}, s_5^{3_2}] & s_3^{3_3} \\ [s_3^{3_1}, s_4^{3_1}] & s_6^{3_2} & [s_2^{3_3}, s_4^{3_3}] \\ s_4^{3_1} & s_4^{3_2} & s_3^{3_3} \end{bmatrix}, \quad R_1^{a_4} = \begin{bmatrix} [s_2^{4_1}, s_3^{4_1}] & s_5^{4_2} & [s_4^{4_3}, s_8^{4_3}] \\ s_3^{4_1} & [s_3^{4_2}, s_5^{4_2}] & s_7^{4_3} \\ [s_1^{4_1}, s_2^{4_1}] & [s_1^{4_2}, s_2^{4_2}] & s_8^{4_3} \\ s_4^{4_1} & s_6^{4_2} & [s_0^{4_3}, s_4^{4_3}] \end{bmatrix},$$

$$R_2^{a_4} = \begin{bmatrix} s_4^{4_1} & s_6^{4_2} & [s_1^{4_3}, s_4^{4_3}] \\ [s_2^{4_1}, s_4^{4_1}] & [s_3^{4_2}, s_5^{4_2}] & s_5^{4_3} \\ s_2^{4_1} & s_4^{4_2} & [s_6^{4_3}, s_8^{4_3}] \\ [s_3^{4_1}, s_4^{4_1}] & [s_1^{4_2}, s_3^{4_1}] & s_7^{4_3} \end{bmatrix}, \quad R_3^{a_4} = \begin{bmatrix} s_0^{4_1} & s_6^{4_2} & [s_6^{4_3}, s_8^{4_3}] \\ [s_2^{4_1}, s_3^{4_1}] & [s_0^{4_2}, s_3^{4_2}] & s_5^{4_3} \\ [s_0^{4_1}, s_1^{4_1}] & [s_3^{4_2}, s_6^{4_2}] & [s_4^{4_3}, s_7^{4_3}] \\ s_3^{4_1} & s_5^{4_2} & s_4^{4_3} \end{bmatrix},$$

$$R_1^{a_5}=\begin{bmatrix} s_6^{5_1} & s_5^{5_2} & s_5^{5_3} \\ [s_4^{5_1},s_6^{5_1}] & s_7^{5_2} & [s_4^{5_3},s_5^{5_3}] \\ s_2^{5_1} & [s_7^{5_2},s_8^{5_2}] & [s_0^{5_3},s_3^{5_3}] \\ [s_3^{5_1},s_6^{5_1}] & s_4^{5_2} & s_6^{5_3} \end{bmatrix},\quad R_2^{a_5}=\begin{bmatrix} s_0^{5_1} & [s_3^{5_2},s_4^{5_2}] & s_2^{5_3} \\ [s_5^{5_1},s_6^{5_1}] & s_5^{5_2} & [s_1^{5_3},s_3^{5_3}] \\ s_5^{5_1} & [s_6^{5_2},s_8^{5_2}] & s_4^{5_3} \\ s_6^{5_1} & s_4^{5_2} & [s_3^{5_3},s_6^{5_3}] \end{bmatrix},$$

$$R_3^{a_5}=\begin{bmatrix} s_2^{5_1} & s_6^{5_2} & s_4^{5_3} \\ s_5^{5_1} & [s_1^{5_2},s_4^{5_2}] & s_4^{5_3} \\ s_5^{5_1} & s_7^{5_2} & [s_3^{5_3},s_4^{5_2}] \\ [s_4^{5_1},s_6^{5_1}] & s_0^{5_2} & s_6^{5_3} \end{bmatrix}.$$

Take the first element $r_{111_1}^{a_1}$ in $R_1^{a_1}$ as example, the meaning of each cell in the matrices can be interpreted as follows. 1) Matrix $R_1^{a_1}$ denotes the assessments of different alternatives over the three sub-attributes under attribute a_1 given by professor d_1. 2) The element $r_{111_1}^{a_1}$ refers to professor d_1's evaluation result of teacher x_1 on sub-attribute a_{1_1}. 3) The result is presented by an uncertain linguistic value $[s_2^{1_1}, s_3^{1_1}]$, which implies that the performance of teacher x_1 on sub-attribute a_{1_1} is linguistically described between "medium" and "rich".

By expressing the linguistic values with interval forms, and then utilizing Eq. (3), we have the following 15 multi-granular interval-valued 2-tuple linguistic matrices.

$$\tilde{R}_1^{a_1}=\begin{bmatrix} [(s_2^{1_1},0),[s_3^{1_1},0)] & [(s_3^{1_2},0),(s_3^{1_2},0)] & [(s_2^{1_3},0),(s_2^{1_3},0)] \\ [(s_3^{1_1},0),(s_3^{1_1},0)] & [(s_4^{1_2},0),(s_4^{1_2},0)] & [(s_5^{1_3},0),(s_6^{1_3},0)] \\ [(s_1^{1_1},0),(s_2^{1_1},0)] & [(s_2^{1_2},0),(s_2^{1_2},0)] & [(s_4^{1_3},0),(s_4^{1_3},0)] \\ [(s_4^{1_1},0),(s_4^{1_1},0)] & [(s_3^{1_2},0),(s_4^{1_2},0)] & [(s_3^{1_3},0),(s_5^{1_3},0)] \end{bmatrix},\quad \tilde{R}_2^{a_1}=\begin{bmatrix} [(s_3^{1_1},0),(s_3^{1_1},0)] & [(s_3^{1_2},0),(s_3^{1_2},0)] & [(s_4^{1_3},0),(s_5^{1_3},0)] \\ [(s_2^{1_1},0),(s_4^{1_1},0)] & [(s_2^{1_2},0),(s_4^{1_2},0)] & [(s_6^{1_3},0),(s_6^{1_3},0)] \\ [(s_2^{1_1},0),(s_3^{1_1},0)] & [(s_1^{1_2},0),(s_1^{1_2},0)] & [(s_3^{1_3},0),(s_5^{1_3},0)] \\ [(s_3^{1_1},0),(s_3^{1_1},0)] & [(s_3^{1_2},0),(s_4^{1_2},0)] & [(s_5^{1_3},0),(s_5^{1_3},0)] \end{bmatrix},$$

$$\tilde{R}_3^{a_1}=\begin{bmatrix} [(s_2^{1_1},0),(s_2^{1_1},0)] & [(s_3^{1_2},0),(s_4^{1_2},0)] & [(s_5^{1_3},0),(s_5^{1_3},0)] \\ [(s_3^{1_1},0),(s_4^{1_1},0)] & [(s_3^{1_2},0),(s_3^{1_2},0)] & [(s_4^{1_3},0),(s_6^{1_3},0)] \\ [(s_2^{1_1},0),(s_2^{1_1},0)] & [(s_4^{1_2},0),(s_4^{1_2},0)] & [(s_2^{1_3},0),(s_3^{1_3},0)] \\ [(s_4^{1_1},0),(s_4^{1_1},0)] & [(s_4^{1_2},0),(s_4^{1_2},0)] & [(s_1^{1_3},0),(s_1^{1_3},0)] \end{bmatrix},\quad \tilde{R}_1^{a_2}=\begin{bmatrix} [(s_3^{2_1},0),(s_4^{2_1},0)] & [(s_4^{2_2},0),(s_4^{2_2},0)] & [(s_4^{2_3},0),(s_4^{2_3},0)] \\ [(s_3^{2_1},0),(s_3^{2_1},0)] & [(s_7^{2_2},0),(s_8^{2_2},0)] & [(s_1^{2_3},0),(s_3^{2_3},0)] \\ [(s_2^{2_1},0),(s_4^{2_1},0)] & [(s_3^{2_2},0),(s_4^{2_2},0)] & [(s_5^{2_3},0),(s_5^{2_3},0)] \\ [(s_4^{2_1},0),(s_4^{2_1},0)] & [(s_6^{2_2},0),(s_6^{2_2},0)] & [(s_6^{2_3},0),(s_6^{2_3},0)] \end{bmatrix},$$

$$\tilde{R}_2^{a_2}=\begin{bmatrix} [(s_3^{2_1},0),(s_3^{2_1},0)] & [(s_7^{2_2},0),(s_7^{2_2},0)] & [(s_4^{2_3},0),(s_6^{2_3},0)] \\ [(s_2^{2_1},0),(s_4^{2_1},0)] & [(s_6^{2_2},0),(s_8^{2_2},0)] & [(s_5^{2_3},0),(s_5^{2_3},0)] \\ [(s_4^{2_1},0),(s_4^{2_1},0)] & [(s_4^{2_2},0),(s_4^{2_2},0)] & [(s_4^{2_3},0),(s_5^{2_1},0)] \\ [(s_4^{2_1},0),(s_4^{2_1},0)] & [(s_5^{2_2},0),(s_7^{2_2},0)] & [(s_6^{2_3},0),(s_6^{2_3},0)] \end{bmatrix},\quad \tilde{R}_3^{a_2}=\begin{bmatrix} [(s_4^{2_1},0),(s_4^{2_1},0)] & [(s_6^{2_2},0),(s_6^{2_2},0)] & [(s_4^{2_3},0),(s_4^{2_3},0)] \\ [(s_3^{2_1},0),(s_3^{2_1},0)] & [(s_5^{2_2},0),(s_6^{2_2},0)] & [(s_4^{2_3},0),(s_5^{2_3},0)] \\ [(s_1^{2_1},0),(s_2^{2_1},0)] & [(s_4^{2_2},0),(s_8^{2_2},0)] & [(s_6^{2_3},0),(s_6^{2_3},0)] \\ [(s_3^{2_1},0),(s_3^{2_1},0)] & [(s_7^{2_2},0),(s_7^{2_2},0)] & [(s_5^{2_3},0),(s_6^{2_3},0)] \end{bmatrix},$$

$$\tilde{R}_1^{a_3}=\begin{bmatrix} [(s_2^{3_1},0),(s_2^{3_1},0)] & [(s_5^{3_2},0),(s_6^{3_2},0)] & [(s_0^{3_3},0),(s_0^{3_3},0)] \\ [(s_3^{3_1},0),(s_4^{3_1},0)] & [(s_4^{3_2},0),(s_4^{3_2},0)] & [(s_3^{3_3},0),(s_3^{3_3},0)] \\ [(s_1^{3_1},0),(s_2^{3_1},0)] & [(s_3^{3_2},0),(s_5^{3_2},0)] & [(s_1^{3_3},0),(s_2^{3_3},0)] \\ [(s_4^{3_1},0),(s_4^{3_1},0)] & [(s_6^{3_2},0),(s_6^{3_2},0)] & [(s_2^{3_3},0),(s_2^{3_3},0)] \end{bmatrix},\quad \tilde{R}_2^{a_3}=\begin{bmatrix} [(s_0^{3_1},0),(s_1^{3_1},0)] & [(s_4^{3_2},0),(s_4^{3_2},0)] & [(s_2^{3_3},0),(s_2^{3_3},0)] \\ [(s_0^{3_1},0),(s_1^{3_1},0)] & [(s_1^{3_2},0),(s_3^{3_2},0)] & [(s_3^{3_3},0),(s_3^{3_3},0)] \\ [(s_2^{3_1},0),(s_3^{3_1},0)] & [(s_6^{3_2},0),(s_6^{3_2},0)] & [(s_3^{3_3},0),(s_4^{3_3},0)] \\ [(s_4^{3_1},0),(s_4^{3_1},0)] & [(s_3^{3_2},0),(s_6^{3_2},0)] & [(s_3^{3_3},0),(s_3^{3_3},0)] \end{bmatrix},$$

$$\tilde{R}_3^{a_3}=\begin{bmatrix} [(s_4^{3_1},0),(s_4^{3_1},0)] & [(s_0^{3_2},0),(s_2^{3_2},0)] & [(s_2^{3_3},0),(s_2^{3_3},0)] \\ [(s_1^{3_1},0),(s_2^{3_1},0)] & [(s_3^{3_2},0),(s_5^{3_2},0)] & [(s_3^{3_3},0),(s_3^{3_3},0)] \\ [(s_3^{3_1},0),(s_4^{3_1},0)] & [(s_6^{3_2},0),(s_6^{3_2},0)] & [(s_2^{3_3},0),(s_4^{3_3},0)] \\ [(s_4^{3_1},0),(s_4^{3_1},0)] & [(s_4^{3_2},0),(s_4^{3_2},0)] & [(s_3^{3_3},0),(s_3^{3_3},0)] \end{bmatrix},\quad \tilde{R}_1^{a_4}=\begin{bmatrix} [(s_2^{4_1},0),(s_3^{4_1},0)] & [(s_5^{4_2},0),(s_5^{4_2},0)] & [(s_4^{4_3},0),(s_8^{4_3},0)] \\ [(s_3^{4_1},0),(s_3^{4_1},0)] & [(s_3^{4_2},0),(s_5^{4_2},0)] & [(s_7^{4_3},0),(s_7^{4_3},0)] \\ [(s_1^{4_1},0),(s_2^{4_1},0)] & [(s_1^{4_2},0),(s_2^{4_2},0)] & [(s_8^{4_3},0),(s_8^{4_3},0)] \\ [(s_4^{4_1},0),(s_4^{4_1},0)] & [(s_6^{4_2},0),(s_6^{4_2},0)] & [(s_0^{4_3},0),(s_4^{4_3},0)] \end{bmatrix},$$

$$\tilde{R}_2^{a_4}=\begin{bmatrix}[(s_4^{4_1},0),(s_4^{4_1},0)] & [(s_6^{4_2},0),(s_6^{4_2},0)] & [(s_1^{4_3},0),(s_4^{4_3},0)]\\ [(s_2^{4_1},0),(s_4^{4_1},0)] & [(s_3^{4_2},0),(s_5^{4_2},0)] & [(s_5^{4_3},0),(s_5^{4_3},0)]\\ [(s_2^{4_1},0),(s_2^{4_1},0)] & [(s_4^{4_2},0),(s_4^{4_2},0)] & [(s_6^{4_3},0),(s_8^{4_3},0)]\\ [(s_3^{4_1},0),(s_4^{4_1},0)] & [(s_1^{4_2},0),(s_3^{4_1},0)] & [(s_7^{4_3},0),(s_7^{4_3},0)]\end{bmatrix},\ \tilde{R}_3^{a_4}=\begin{bmatrix}[(s_0^{4_1},0),(s_0^{4_1},0)] & [(s_6^{4_2},0),(s_6^{4_2},0)] & [(s_6^{4_3},0),(s_8^{4_3},0)]\\ [(s_2^{4_1},0),(s_3^{4_1},0)] & [(s_0^{4_2},0),(s_3^{4_2},0)] & [(s_5^{4_3},0),(s_5^{4_3},0)]\\ [(s_0^{4_1},0),(s_1^{4_1},0)] & [(s_3^{4_2},0),(s_6^{4_2},0)] & [(s_4^{4_3},0),(s_7^{4_3},0)]\\ [(s_3^{4_1},0),(s_3^{4_1},0)] & [(s_5^{4_2},0),(s_5^{4_2},0)] & [(s_4^{4_3},0),(s_4^{4_3},0)]\end{bmatrix},$$

$$\tilde{R}_1^{a_5}=\begin{bmatrix}[(s_6^{5_1},0),(s_6^{5_1},0)] & [(s_5^{5_2},0),(s_5^{5_2},0)] & [(s_5^{5_3},0),(s_5^{5_3},0)]\\ [(s_4^{5_1},0),(s_6^{5_1},0)] & [(s_7^{5_2},0),(s_7^{5_2},0)] & [(s_4^{5_3},0),(s_5^{5_3},0)]\\ [(s_2^{5_1},0),(s_2^{5_1},0)] & [(s_7^{5_2},0),(s_8^{5_2},0)] & [(s_0^{5_3},0),(s_3^{5_3},0)]\\ [(s_3^{5_1},0),(s_6^{5_1},0)] & [(s_4^{5_2},0),(s_4^{5_2},0)] & [(s_6^{5_3},0),(s_6^{5_3},0)]\end{bmatrix},\ \tilde{R}_2^{a_5}=\begin{bmatrix}[(s_0^{5_1},0),(s_0^{5_1},0)] & [(s_3^{5_2},0),(s_4^{5_2},0)] & [(s_2^{5_3},0),(s_2^{5_3},0)]\\ [(s_5^{5_1},0),(s_6^{5_1},0)] & [(s_5^{5_2},0),(s_5^{5_2},0)] & [(s_1^{5_3},0),(s_3^{5_3},0)]\\ [(s_5^{5_1},0),(s_5^{5_1},0)] & [(s_6^{5_2},0),(s_8^{5_2},0)] & [(s_4^{5_3},0),(s_4^{5_3},0)]\\ [(s_6^{5_1},0),(s_6^{5_1},0)] & [(s_4^{5_2},0),(s_4^{5_2},0)] & [(s_3^{5_3},0),(s_6^{5_3},0)]\end{bmatrix},$$

$$\tilde{R}_3^{a_5}=\begin{bmatrix}[(s_2^{5_1},0),(s_2^{5_1},0)] & [(s_6^{5_2},0),(s_6^{5_2},0)] & [(s_4^{5_3},0),(s_4^{5_3},0)]\\ [(s_5^{5_1},0),(s_5^{5_1},0)] & [(s_1^{5_2},0),(s_4^{5_2},0)] & [(s_4^{5_3},0),(s_4^{5_3},0)]\\ [(s_5^{5_1},0),(s_5^{5_1},0)] & [(s_7^{5_2},0),(s_7^{5_2},0)] & [(s_3^{5_3},0),(s_4^{5_2},0)]\\ [(s_4^{5_1},0),(s_6^{5_1},0)] & [(s_0^{5_2},0),(s_0^{5_2},0)] & [(s_6^{5_3},0),(s_6^{5_3},0)]\end{bmatrix}.$$

Evidently, the first level of heterogeneous information contained in the original decision data is unified and identically replaced by interval 2-tuple linguistic matrices with multi-granular linguistic information.

Then, make use of Eq. (10) to fuse individual multi-granular interval-valued linguistic 2-tuples into a collective linguistic 2-tuple, which represents the comprehensive opinions of professors over different sub-attributes. As a result, 5 collective multi-granular interval-valued 2-tuple linguistic matrices are formed as:

$$\tilde{R}^{a_1}=\begin{bmatrix}[(s_2^{1_1},0.0625),(s_3^{1_1},-0.1)] & [(s_3^{1_2},0),(s_3^{1_2},0.1)] & [(s_4^{1_3},-0.05),(s_4^{1_3},-0.0084)]\\ [(s_3^{1_1},-0.0625),(s_4^{1_1},-0.0875)] & [(s_3^{1_2},0.025),(s_4^{1_2},-0.1)] & [(s_5^{1_3},-0.025),(s_6^{1_3},0)]\\ [(s_2^{1_1},-0.0875),(s_2^{1_1},0.0625)] & [(s_3^{1_2},-0.1125),(s_3^{1_2},-0.1125)] & [(s_3^{1_3},-0.0083),(s_4^{1_3},-0.025)]\\ [(s_4^{1_1},-0.0625),(s_4^{1_1},-0.0625)] & [(s_3^{1_2},0.1),(s_4^{1_2},0)] & [(s_3^{1_3},-0.05),(s_3^{1_3},0.0667)]\end{bmatrix},$$

$$\tilde{R}^{a_2}=\begin{bmatrix}[(s_3^{2_1},0.1),(s_4^{2_1},-0.0625)] & [(s_6^{2_2},-0.0563),(s_6^{2_2},-0.0563)] & [(s_4^{2_3},0),(s_5^{2_3},-0.0833)]\\ [(s_3^{2_1},-0.0625),(s_3^{2_1},0.0625)] & [(s_6^{2_2},-0.0063),(s_7^{2_2},0.025)] & [(s_3^{2_3},0.0333),(s_4^{2_3},0.05)]\\ [(s_2^{2_1},0.025),(s_3^{2_1},0.05)] & [(s_4^{2_2},-0.0438),(s_6^{2_2},-0.05)] & [(s_5^{2_3},0.025),(s_5^{2_3},0.0667)]\\ [(s_4^{2_1},-0.1),(s_4^{2_1},-0.1)] & [(s_6^{2_2},0.0188),(s_7^{2_2},-0.0438)] & [(s_6^{2_3},-0.0667),(s_6^{2_3},0)]\end{bmatrix},$$

$$\tilde{R}^{a_3}=\begin{bmatrix}[(s_2^{3_1},0.075),(s_3^{3_1},-0.1125)] & [(s_3^{3_2},-0.0417),(s_4^{3_2},-0.0167)] & [(s_1^{3_3},0.075),(s_1^{3_3},0.075)]\\ [(s_1^{3_1},0.1125),(s_2^{3_1},0.1125)] & [(s_3^{3_2},-0.025),(s_4^{3_2},0.025)] & [(s_3^{3_3},0),(s_3^{3_3},0)]\\ [(s_2^{3_1},0.0125),(s_3^{3_1},0.0125)] & [(s_5^{3_2},-0.0083),(s_6^{3_2},-0.0583)] & [(s_2^{3_3},-0.025),(s_3^{3_3},0.075)]\\ [(s_4^{3_1},0),(s_4^{3_1},0)] & [(s_4^{3_2},0.075),(s_5^{3_2},0.0334)] & [(s_3^{3_3},-0.0875),(s_3^{3_3},-0.0875)]\end{bmatrix},$$

$$\tilde{R}^{a_4}=\begin{bmatrix}[(s_2^{4_1},-0.075),(s_2^{4_1},0.0125)] & [(s_6^{4_2},-0.0583),(s_6^{4_2},-0.0583)] & [(s_4^{4_3},0.0063),(s_7^{4_3},0)]\\ [(s_2^{4_1},0.0875),(s_3^{4_1},0.0625)] & [(s_2^{4_2},-0.0333),(s_4^{4_2},0.0333)] & [(s_6^{4_3},-0.0375),(s_6^{4_3},-0.0375)]\\ [(s_1^{4_1},-0.0375),(s_2^{4_1},-0.1)] & [(s_3^{4_2},-0.075),(s_4^{4_2},0.0166)] & [(s_6^{4_3},-0.0125),(s_8^{4_3},-0.05)]\\ [(s_3^{4_1},0.0875),(s_4^{4_1},-0.1)] & [(s_4^{4_2},0.0583),(s_5^{4_2},-0.025)] & [(s_3^{4_3},0.0438),(s_5^{4_3},-0.0313)]\end{bmatrix},$$

$$\tilde{R}^{a_5}=\begin{bmatrix}[(s_3^{5_1},-0.0167),(s_3^{5_1},-0.0167)] & [(s_5^{5_2},-0.0125),(s_5^{5_2},0.0188)] & [(s_4^{5_3},-0.025),(s_4^{5_3},-0.025)]\\ [(s_5^{5_1},-0.0583),(s_6^{5_1},-0.0667)] & [(s_4^{5_2},0.0125),(s_5^{5_2},0.0375)] & [(s_3^{5_3},0.0417),(s_4^{5_3},0.0166)]\\ [(s_4^{5_1},-0.0084),(s_4^{5_1},-0.0084)] & [(s_7^{5_2},-0.0313),(s_8^{5_2},-0.05)] & [(s_2^{5_3},0.0334),(s_4^{5_3},-0.0584)]\\ [(s_4^{5_1},0.025),(s_6^{5_1},0)] & [(s_2^{5_2},0.05),(s_2^{5_2},0.05)] & [(s_5^{5_3},0.0417),(s_6^{5_3},0)]\end{bmatrix}.$$

Next, to integrate the sub-attribute values, the weights of sub-attributes which belong to different attributes should be decided. By employing Eq. (12), score matrices $B_j = \left(b_{ij_k}\right)_{4\times 3} (j = 1,2,\dots,5)$ which are in one-to-one correspondence with interval-valued 2-tuple linguistic matrices $\tilde{R}^{a_j} (j = 1,2,\dots,5)$ can be formed as follows.

$$B_1 = \begin{bmatrix} 0.6063 & 0.8 & 0.6375 \\ 0.8 & 0.8375 & 0.9042 \\ 0.4875 & 0.6375 & 0.5667 \\ 0.9375 & 0.925 & 0.5084 \end{bmatrix}, B_2 = \begin{bmatrix} 0.8938 & 0.6937 & 0.7084 \\ 0.75 & 0.8219 & 0.625 \\ 0.6625 & 0.5781 & 0.8792 \\ 0.9 & 0.8 & 0.9667 \end{bmatrix}, B_3 = \begin{bmatrix} 0.6063 & 0.5542 & 0.325 \\ 0.4875 & 0.5834 & 0.75 \\ 0.6375 & 0.8834 & 0.65 \\ 1 & 0.8042 & 0.6625 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} 0.4688 & 0.9417 & 0.6907 \\ 0.7 & 0.5 & 0.7125 \\ 0.3063 & 0.5542 & 0.8438 \\ 0.8688 & 0.7667 & 0.5063 \end{bmatrix}, B_5 = \begin{bmatrix} 0.4833 & 0.6282 & 0.6417 \\ 0.8542 & 0.5875 & 0.6125 \\ 0.6583 & 0.8969 & 0.4875 \\ 0.8459 & 0.3 & 0.9375 \end{bmatrix}.$$

Following the steps in Subsection 4.6, and employ the optimization Modelling Software Lingo 11, five weight vectors which respectively contains weights of different sub-attributes under the five attributes are generated as

$$w_1 = (0.2418, 0.5388, 0.2194)^T, \quad w_2 = (0.2567, 0.2307, 0.5126)^T,$$
$$w_3 = (0.5172, 0.2626, 0.2202)^T, \quad w_4 = (0.2198, 0.2620, 0.5182)^T,$$
$$w_5 = (0.2701, 0.2187, 0.5112)^T.$$

Next, we successively substitute the sub-attribute values from the collective interval-valued multi-granular linguistic matrices $\tilde{R}^{a_j} (j = 1,2,3,4,5)$ into Eq. (21), and thus construct the normalized forms of $\tilde{R}^{a_j} (j = 1,2,3,4,5)$ as $\bar{R}^{a_j} = (\bar{r}_{ij_k})_{4\times 3} = \left(\left[\left(s^{j_*}_{\bar{\alpha}^-_{ij_k}}, \bar{\beta}^-_{ij_k}\right), \left(s^{j_*}_{\bar{\alpha}^+_{ij_k}}, \bar{\beta}^+_{ij_k}\right)\right]\right)_{4\times 3} (j = 1,2,3,4,5)$:

$$\bar{R}^{a_1} = \begin{bmatrix} [(s^{1_*}_3, 0.0625), (s^{1_*}_4, -0.0167)] & [(s^{1_*}_5, -0.0833), (s^{1_*}_5, 0.0167)] & [(s^{1_*}_4, -0.05), (s^{1_*}_4, -0.0084)] \\ [(s^{1_*}_4, 0.0208), (s^{1_*}_5, 0.0792)] & [(s^{1_*}_5, -0.0583), (s^{1_*}_5, 0.0667)] & [(s^{1_*}_5, -0.025), (s^{1_*}_6, 0)] \\ [(s^{1_*}_2, 0.0792), (s^{1_*}_3, 0.0625)] & [(s^{1_*}_4, -0.0292), (s^{1_*}_4, -0.0292)] & [(s^{1_*}_3, -0.0083), (s^{1_*}_4, -0.025)] \\ [(s^{1_*}_6, -0.0625), (s^{1_*}_6, -0.0625)] & [(s^{1_*}_5, 0.0167), (s^{1_*}_6, 0)] & [(s^{1_*}_3, -0.05), (s^{1_*}_3, 0.0667)] \end{bmatrix},$$

$$\bar{R}^{a_2} = \begin{bmatrix} [(s^{2_*}_7, -0.025), (s^{2_*}_8, -0.0625)] & [(s^{2_*}_6, -0.0563), (s^{2_*}_6, -0.0563)] & [(s^{2_*}_5, 0.0417), (s^{2_*}_6, 0)] \\ [(s^{2_*}_6, -0.0625), (s^{2_*}_7, -0.0625)] & [(s^{2_*}_6, -0.0063), (s^{2_*}_7, 0.025)] & [(s^{2_*}_4, 0.0333), (s^{2_*}_6, -0.0333)] \\ [(s^{2_*}_4, 0.025), (s^{2_*}_6, 0.05)] & [(s^{2_*}_4, -0.0438), (s^{2_*}_6, -0.05)] & [(s^{2_*}_7, -0.0167), (s^{2_*}_7, 0.025)] \\ [(s^{2_*}_7, 0.025), (s^{2_*}_7, 0.025)] & [(s^{2_*}_6, 0.0188), (s^{2_*}_7, -0.0438)] & [(s^{2_*}_7, 0.0583), (s^{2_*}_8, 0)] \end{bmatrix},$$

$$\bar{R}^{a_3} = \begin{bmatrix} [(s^{3_*}_3, 0.075), (s^{3_*}_4, -0.0292)] & [(s^{3_*}_3, -0.0417), (s^{3_*}_4, -0.0167)] & [(s^{3_*}_2, -0.0083), (s^{3_*}_2, -0.0083)] \\ [(s^{3_*}_2, 0.0292), (s^{3_*}_4, -0.0542)] & [(s^{3_*}_3, -0.025), (s^{3_*}_4, 0.025)] & [(s^{3_*}_5, -0.0833), (s^{3_*}_5, -0.0833)] \\ [(s^{3_*}_3, 0.0125), (s^{3_*}_5, -0.0708)] & [(s^{3_*}_5, -0.0083), (s^{3_*}_6, -0.0583)] & [(s^{3_*}_3, -0.025), (s^{3_*}_5, -0.0083)] \\ [(s^{3_*}_6, 0), (s^{3_*}_6, 0)] & [(s^{3_*}_4, 0.075), (s^{3_*}_5, 0.0334)] & [(s^{3_*}_4, -0.0042), (s^{3_*}_4, -0.0042)] \end{bmatrix},$$

$$\bar{R}^{a_4} = \begin{bmatrix} [(s_3^{4_*}, 0.05), (s_4^{4_*}, 0.0125)] & [(s_8^{4_*}, -0.0583), (s_8^{4_*}, -0.0583)] & [(s_4^{4_*}, 0.0063), (s_7^{4_*}, 0)] \\ [(s_5^{4_*}, -0.0375), (s_7^{4_*}, -0.0625)] & [(s_2^{4_*}, 0.05), (s_6^{4_*}, -0.05)] & [(s_6^{4_*}, -0.0375), (s_6^{4_*}, -0.0375)] \\ [(s_2^{4_*}, -0.0375), (s_3^{4_*}, 0.025)] & [(s_3^{4_*}, 0.05), (s_5^{4_*}, 0.0583)] & [(s_6^{4_*}, -0.0125), (s_8^{4_*}, -0.05)] \\ [(s_7^{4_*}, -0.0375), (s_7^{4_*}, 0.025)] & [(s_6^{4_*}, -0.025), (s_6^{4_*}, 0.0583)] & [(s_3^{4_*}, 0.0438), (s_5^{4_*}, -0.0313)] \end{bmatrix},$$

$$\bar{R}^{a_5} = \begin{bmatrix} [(s_4^{5_*}, -0.0167), (s_4^{5_*}, -0.0167)] & [(s_5^{5_*}, -0.0125), (s_5^{5_*}, 0.0188)] & [(s_5^{5_*}, 0.0167), (s_5^{5_*}, 0.0167)] \\ [(s_6^{5_*}, 0.025), (s_7^{5_*}, 0.0583)] & [(s_4^{5_*}, 0.0125), (s_5^{5_*}, 0.0375)] & [(s_4^{5_*}, 0.0417), (s_5^{5_*}, 0.0583)] \\ [(s_5^{5_*}, 0.0333), (s_5^{5_*}, 0.0333)] & [(s_7^{5_*}, -0.0313), (s_8^{5_*}, -0.05)] & [(s_3^{5_*}, -0.0083), (s_5^{5_*}, -0.0167)] \\ [(s_6^{5_*}, -0.0583), (s_8^{5_*}, 0)] & [(s_2^{5_*}, 0.05), (s_2^{5_*}, 0.05)] & [(s_7^{5_*}, 0), (s_8^{5_*}, 0)] \end{bmatrix},$$

where $S^{j_*} = \left\{ s_\alpha^{j_*} \mid \alpha = 0,1,\ldots,g_{j_*} \right\} (j = 1,3)$ with $g_{1_*} = \mathrm{g}_{1_3} = \mathrm{g}_{3_*} = \mathrm{g}_{3_2} = 6$ and $S^{j_*} = \left\{ s_\alpha^{j_*} \mid \alpha = 0,1,\ldots,8 \right\} (j = 2,4,5)$ with $g_{2_*} = g_{2_2} = g_{4_*} = g_{4_3} = g_{5_*} = g_{5_2} = 8$. To be noticed, the linguistic terms in the normalized linguistic term sets do not make actual sense.

Plug sub-attribute values from $\bar{R}^{a_j}$ and sub-attribute weights from w_j into Eq. (22), and yields attribute values that compose an interval-valued multi-granular 2-tuple linguistic matrix:

$$\bar{R} = \begin{bmatrix} [(s_4^{1_*}, 0.0087), (s_5^{1_*}, -0.0737)] & [(s_6^{2_*}, -0.03), (s_6^{2_*}, 0.0351)] & [(s_3^{3_*}, -0.0107), (s_3^{3_*}, 0.072)] & [(s_5^{4_*}, -0.0225), (s_7^{4_*}, -0.0622)] & [(s_5^{5_*}, -0.0325), (s_5^{5_*}, -0.0256)] \\ [(s_5^{1_*}, -0.0722), (s_6^{*}, -0.075)] & [(s_5^{2_*}, -0.0036), (s_6^{2_*}, 0.0336)] & [(s_3^{3_*}, -0.0226), (s_4^{3_*}, -0.0031)] & [(s_5^{4_*}, -0.048), (s_6^{4_*}, -0.0188)] & [(s_5^{5_*}, -0.0267), (s_6^{5_*}, -0.0037)] \\ [(s_3^{1_*}, 0.0511), (s_4^{1_*}, -0.0464)] & [(s_5^{2_*}, 0.055), (s_7^{2_*}, -0.0468)] & [(s_4^{3_*}, -0.0804), (s_5^{3_*}, -0.0100)] & [(s_4^{4_*}, 0.0402), (s_6^{4_*}, 0.0092)] & [(s_4^{5_*}, 0.0498), (s_6^{5_*}, -0.0535)] \\ [(s_5^{1_*}, -0.0499), (s_5^{1_*}, 0.0565)] & [(s_7^{2_*}, 0.0118), (s_7^{2_*}, 0.0604)] & [(s_5^{3_*}, 0.0246), (s_5^{3_*}, 0.0574)] & [(s_5^{4_*}, -0.0339), (s_6^{4_*}, -0.0328)] & [(s_6^{5_*}, -0.0503), (s_7^{5_*}, -0.0281)] \end{bmatrix}.$$

Herein, each column of matrix $\bar{R}$ refers to the vector involving the comprehensive assessments of teachers on the j^{th} attribute.

For the sake that the components in $S^{j_*}(j = 1,2,3,4,5)$ is of no significance, it is imperative to endow the attribute values with realistic meanings. To do this, an evaluation standard of the attributes, which is decided by taking consideration of the practical evaluation system of teachers, is shown in **Table 3**.

Table 3. Evaluation standard of attributes

Attribute	Linguistic term set
Teaching attitude (a_1)	$S^1 = \{s_0^1 = bad, s_1^1 = slightly\ bad, s_2^1 = medium, s_3^1 = slightly\ good, s_4^1 = good\}$
Teaching ability (a_2)	$S^2 = \{s_0^2 = weak, s_1^2 = slightly\ weak, s_2^2 = medium, s_3^2 = slightly\ strong, s_4^2 = strong\}$
Teaching content (a_3)	$S^3 = \{s_0^3 = extremely\ poor, s_1^3 = poor, s_2^3 = slightly\ poor, s_3^3 = medium, s_4^3 = slightly\ rich, s_5^3 = rich, s_6^3 = extremely\ rich\}$
Teaching method (a_4)	$S^4 = \{s_0^4 = not\ scientific, s_1^4 = slightly\ not\ scientific, s_2^4 = medium, s_3^4 = slightly\ scientific, s_4^4 = scientific\}$
Teaching effect (a_5)	$S^5 = \{s_0^5 = extremely\ bad, s_1^5 = slightly\ bad, s_2^5 = bad, s_3^5 = medium, s_4^5 = slightly\ good, s_5^5 = good, s_6^5 = extremely\ good\}$

Based on **Table 3**, we have the following practical attribute values via Eq. (23):

$$R=\begin{bmatrix} [(s_3^1,-0.0746),(s_3^1,0.0096)] & [(s_3^2,-0.03),(s_3^2,0.0351)] & [(s_3^3,-0.0107),(s_3^3,0.072)] & [(s_2^4,0.1025),(s_3^4,0.0628)] & [(s_4^5,-0.0742),(s_4^5,-0.0673)] \\ [(s_3^1,0.0111),(s_4^1,-0.075)] & [(s_2^2,0.1214),(s_3^2,0.0336)] & [(s_3^3,-0.0226),(s_4^3,-0.0031)] & [(s_2^4,0.077),(s_3^4,-0.0188)] & [(s_4^5,-0.0684),(s_4^5,0.0796)] \\ [(s_2^1,0.0511),(s_2^1,0.1203)] & [(s_3^2,-0.07),(s_3^2,0.0782)] & [(s_4^3,-0.0804),(s_5^3,-0.01)] & [(s_2^4,0.0402),(s_3^4,0.0092)] & [(s_3^5,0.0498),(s_4^5,0.0298)] \\ [(s_3^1,0.0334),(s_4^1,-0.1102)] & [(s_4^2,-0.1132),(s_4^2,-0.0646)] & [(s_5^3,0.0246),(s_5^3,0.0574)] & [(s_2^4,0.0911),(s_3^4,-0.0328)] & [(s_4^5,0.033),(s_5^5,0.0136)] \end{bmatrix}.$$

In matrix R, the linguistic term indicates the comprehensive linguistic description of a teacher related to an attribute; meanwhile, the decimal shows the deviation between the linguistic term and the evaluation result of the teacher. Moreover, each row in R indicates the assessments of each teacher in the five aspects, while each column in R implies the evaluation results of different teachers over each attribute.

Through Eq. (25), the possibility matrices of a teacher is superior/inferior to others are generated as

$$K_1=\begin{bmatrix} 0.5 & 0 & 1 & 0 \\ 1 & 0.5 & 1 & 0.4761 \\ 0 & 0 & 0.5 & 0 \\ 1 & 0.5239 & 1 & 0.5 \end{bmatrix}, K_2=\begin{bmatrix} 0.5 & 0.2798 & 0.4927 & 0 \\ 0.7202 & 0.5 & 0.3338 & 0 \\ 0.5073 & 0.6662 & 0.5 & 0 \\ 1 & 1 & 1 & 0.5 \end{bmatrix}, K_3=\begin{bmatrix} 0.5 & 0.3518 & 0 & 0 \\ 0.6482 & 0.5 & 0.1827 & 0 \\ 1 & 0.8173 & 0.5 & 0 \\ 1 & 1 & 1 & 0.5 \end{bmatrix},$$

$$K_4=\begin{bmatrix} 0.5 & 0.6469 & 0.6350 & 0.6590 \\ 0.3531 & 0.5 & 0.5118 & 0.4998 \\ 0.3650 & 0.4882 & 0.5 & 0.4871 \\ 0.3410 & 0.5002 & 0.5129 & 0.5 \end{bmatrix}, K_5=\begin{bmatrix} 0.5 & 0.0071 & 0.3229 & 0 \\ 0.9929 & 0.5 & 0.6668 & 0.1579 \\ 0.6771 & 0.3332 & 0.5 & 0 \\ 1 & 0.8421 & 1 & 0.5 \end{bmatrix}.$$

The above five possibility matrices are used to point out the rankings of teachers related to different attributes. In the following, we apply the extended TOPSIS method to obtain the overall ranking of teachers.

Firstly, according to **Table 3**, determine the absolute multi-granular linguistic positive and negative ideal solutions as

$$Z=\left([(s_4^1,0),(s_4^1,0)],[(s_4^2,0),(s_4^2,0)],[(s_6^3,0),(s_6^3,0)],[(s_4^4,0),(s_4^4,0)],[(s_6^5,0),(s_6^5,0)]\right)$$
$$F=\left([(s_0^1,0),(s_0^1,0)],[(s_0^2,0),(s_0^2,0)],[(s_0^3,0),(s_0^3,0)],[(s_0^4,0),(s_0^4,0)],[(s_0^5,0),(s_0^5,0)]\right).$$

Secondly, the distances between each row of elements in matrix R which represent the attribute values of each teacher and the ideal solutions, as well as the closeness coefficients are calculated as shown in **Table 4.**

Consequently, as per the possibility matrices and **Table 4**, the ranking results are listed in **Table 5**.

From the above performed ranking results, several conclusions can be made. The fourth teacher x_4 is assessed as the teacher whose teaching quality is the best among all teachers. Specifically, in terms of the attitude, ability, content and effect, the fourth teacher performs superior than others. As to the aspect of teaching method, teacher x_1 wins the first prize; although there is little difference among the selected four teachers.

Table 4. Distances and closeness coefficient of teachers

Alternative	**Distance**		**Closeness coefficient**
	Positive d_i^+	**Negative d_i^-**	
Teacher x_1	0.3814	0.6427	0.6276
Teacher x_2	0.3720	0.6525	0.6369
Teacher x_3	0.3565	0.6688	0.6523
Teacher x_4	0.2184	0.8131	0.7883

Table 5. Rankings of alternative teachers

	Ranking
Teaching attitude	$x_4 \overset{52.39\%}{\succ} x_2 \overset{100\%}{\succ} x_1 \overset{100\%}{\succ} x_3$
Teaching ability	$x_4 \overset{100\%}{\succ} x_3 \overset{66.62\%}{\succ} x_2 \overset{72.02\%}{\succ} x_1$
Teaching content	$x_4 \overset{100\%}{\succ} x_3 \overset{81.73\%}{\succ} x_2 \overset{64.82\%}{\succ} x_1$
Teaching method	$x_1 \overset{65.90\%}{\succ} x_4 \overset{50.02\%}{\succ} x_2 \overset{51.18\%}{\succ} x_3$
Teaching effect	$x_4 \overset{84.21\%}{\succ} x_2 \overset{66.68\%}{\succ} x_3 \overset{67.71\%}{\succ} x_1$
Total ranking	$x_4 \succ x_3 \succ x_2 \succ x_1$

Practical Discussion

This simulated example illustrates how to apply the proposed framework to evaluate classroom teaching quality. By analyzing the results, the evaluation organizations, which are interested in the improvement of classroom teaching quality, can acquire two useful implications.

First, the weights of sub-attributes generated in this example are objectively determined under the condition that alternatives are not equally weighted. This is quite in accordance with real-life situations and supports the objectivity of the evaluation process. Therefore, the teachers are inclined to accreditation of the presented results, according to which they are willing to improve his/her classroom teaching quality.

Second, the exploitation process performs not only the overall ranking but also the partial rankings of teachers, which may aid teachers improve himself/herself more concretely. On one hand, the overall ranking helps the teacher recognize which level his/her teaching quality locates. On the other hand, the partial rankings illuminate two directions to the teacher: which aspect he/she should pay significant attention and which teacher he/she can imitate in improving the classroom teaching quality.

CONCLUSIONS

Classroom teaching quality evaluation is an imperative and effective instrument to maintain the good service and improve the competitiveness level of universities. This paper mainly tackles classroom teaching quality evaluation problems with heterogeneous linguistic information which exists in real-life decision environment. The contributions and advantages are summarized as below.

First, we put forward a practical and rational evaluation index system for classroom teaching quality, which may be directly applied to the realistic evaluation of teaching quality.

Second, the proposed transformations provide a new way to normalize multi-granular linguistic information which commonly exists in evaluation problems.

Third, the optimization models to determine the sub-attribute weights are objective. They could overcome the deficiency of most researches that alternatives are equally weighted and reduce the subjectivity and controversy in the evaluation process.

Fourth, the proposed framework has an advantage that it is in accordance with the real evaluation pattern for teachers. Not only the overall assessment, but also the evaluation related to different attributes for teachers is conducted. Based on these evaluation results, teachers can realize what aspect he/she is in need of improvement.

Finally, the illustrated example shows flexibility and applicability of solving practical evaluation problems where the decision information is heterogeneous and the weights of alternatives are not equal.

This study also has some limitations, which can be studied in the future.

- The proposed framework only considers the situation where the decision information is denoted by linguistic and uncertain linguistic values. When the decision information in the evaluation is given by various forms of linguistic information which could not entirely be transformed into interval-valued linguistic 2-tuples, other normalization methods should be considered, such as establishing transformation standards to uniform different types of linguistic information into fuzzy numbers. As this situation commonly exists in real life, a future research is worth being conducted.
- This paper only focuses on the case that the weight information about the sub-attributes is totally blank. For the situations where the relative importance degrees of different sub-attributes are given, how to allocate the weights to these sub-attributes is a noteworthy issue. To tackle this, generating weights by extending the fuzzy AHP method can be a good and effective choice.
- Practically speaking, this evaluation framework is in the interest of evaluating classroom teaching quality. By changing the four-level evaluation progress, it can be adjusted and further applied to other similar existing group decision-making problems, such as risk evaluation and supplier evaluation. It is of no doubt that these are widespread problems in real world, and thus it is noteworthy to make a further research on the application of the proposed framework.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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