Credit Market Frictions and Capital Structure Dynamics*

Julien Hugonnier[†] Semyon Malamud[‡] Erwan Morellec[§]

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Abstract

We study the implications of credit market frictions for the dynamics of corporate capital structure and the risk of default of corporations. To do so, we develop a dynamic capital structure model in which firms face uncertainty regarding their ability to raise funds in credit markets and have to search for investors when seeking to adjust their capital structure. We provide a general analysis of shareholders' dynamic financing and default decisions, show when rational expectations equilibria in financing and default barrier strategies exist, and when uniqueness can be achieved. We then use the model to generate a number of novel testable implications relating credit market frictions to target leverage, the pace and size of capital structure changes, creditor turnover, and the likelihood of default.

Keywords: Credit supply uncertainty; dynamic capital structure; default risk; search.

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[†]Swiss Finance Institute and EPFL. E-mail: julien.hugonnier@epfl.ch.

[‡]Swiss Finance Institute and EPFL. E-mail: semyon.malamud@epfl.ch.

[§]Swiss Finance Institute, EPFL, and CEPR. E-mail: erwan.morellec@epfl.ch.

1 Introduction

Since the famous irrelevance theorem of Modigliani and Miller (1958), financial economists have devoted much effort to understanding the effects of frictions, such as corporate taxes and bankruptcy costs, on corporate capital structure.[1] Although we have learned much from this work, virtually all existing models implicitly assume that a firm's capital structure is entirely determined by its demand for debt or equity. That is, the supply of capital is perfectly elastic in these models so that corporate behavior and capital availability depend solely on firm characteristics.

This demand-driven approach has recently been called into question by a number of large-sample empirical studies.[2] These studies show that firms often face uncertainty regarding their future access to credit markets and that credit supply conditions are very important in determining capital structure decisions. Using a different research approach, several surveys of corporate managers from around the globe have confirmed the findings of these large-sample studies. These surveys indicate that financing decisions are generally governed by the preferences of the suppliers of capital rather than by the demands of the users of capital and that capital supply has first order effects on firms' financing decisions (see Graham and Harvey, 2001, or Bancel and Mittoo, 2004).

Our purpose in this paper is therefore twofold. First, we want to examine the importance of credit supply frictions in capital structure choice. Second, we seek to characterize their effects on the dynamics of leverage ratios and the pricing of risky debt. To this end, we build a dynamic model of financing decisions in which the Modigliani and Miller assumption of infinitely elastic supply of credit is relaxed and firms may have to search for investors when seeking to adjust their capital structure. As in Fisher, Heinkel, and Zechner (1989) and Leland (1998), we consider a firm with assets in place that generate a continuous stream of cash flows. The firm is levered because debt allows it to shield part of its profits from taxation. Leverage, however, is limited because debt financing increases the likelihood of

[1] A partial list of such models includes Leland (1994a, 1998), Fisher, Heinkel, and Zechner (1989), Mello and Parsons (1992), Collin-Dufresne and Goldstein (2001), Duffie and Lando (2001), Morellec (2001), Strebulaev (2007), Tserlukevich (2008), Gomes and Schmid (2010), Gorbenko and Strebulaev (2010), Morellec and Schürhoff (2010), or Hackbarth and Mauer (2012).

[2] See Faulkender and Petersen (2006), Becker (2007), Leary (2009), Massa and Zhang (2009), Lemmon and Roberts (2010), Ivashina and Scharfstein (2010), Choi, Getmansky, Henderson, and Tookes (2010) or Becker and Ivashina (2011).

costly financial distress and is subject to credit supply frictions. In our model, these frictions include not only issuance costs, as in prior contributions, but also search frictions.

In the model, management acts in the best interests of shareholders and makes three types of decisions to maximize equity value. First, it selects the firm's initial debt level. Second, it selects the firm's restructuring policy (i.e. the pace and size of capital structure changes). Third, it selects its default policy. Because the default and restructuring policies are selected after debt has been issued, management may have incentives to deviate from the policies conjectured by creditors at the time of debt issuance. In the paper, we therefore focus on rational expectations equilibria in which the policies selected ex post by management coincide with those conjectured ex ante by creditors. We derive conditions under which such equilibria exist in barrier strategies, show when uniqueness can be achieved, and provide a full characterization of the associated financing and default decisions.

Our analysis emphasizes the role of credit supply frictions in affecting the time series of leverage ratios. In the model, firms are always on their optimal capital structure path but, due to refinancing costs and search frictions, they restructure infrequently and do not keep their leverage at the target at all times. As a result, leverage is best described not just by a number, the target, but by its entire distribution. The model also reflects the interaction between credit supply uncertainty and firm characteristics, allowing us to produce a number of new predictions relating capital supply in credit markets to target leverage, the frequency of capital structure changes, creditor turnover, and the likelihood of default.

Specifically, in our framework, debt provides a tax benefit so that firms that perform well may seek to re-leverage. Because the surplus from changing the firm's capital structure is uncertain and restructuring is costly, the optimal policy is to re-leverage only when the firm's taxable income exceeds an endogenously determined threshold. We show that with search frictions, the firm balances the opportunity cost of restructuring early with the risk of not finding creditors in the future. As search frictions increase, the opportunity cost of waiting increases, leading to a decrease in the value-maximizing restructuring threshold. We also show that even though credit supply uncertainty makes it more difficult for firms to restructure, it may actually increase the frequency of capital structure changes. Another result of the paper is that as credit supply weakens, firms issue more debt when restructuring their capital structure. That is, because weaker credit supply makes it more difficult to re-leverage if profitability improves, the firm takes on more debt whenever it refinances.

After solving for optimal policy choices of shareholders in the presence of search frictions, we allow the firm to bargain over the terms of new debt issues with current creditors. We assume that mobilizing dispersed creditors is costly and analyze shareholders' decision to restructure with current creditors or search for new creditors. We show that in this more general model shareholders follow a target capital structure policy and that the target does not depend on whether the firm restructures with new or existing creditors. We also show that the restructuring triggers and target leverage ratios implied by this bargaining process differ significantly from those of standard dynamic trade-off models. Finally, we relate creditor turnover to a number of firm and industry characteristics, such as cash flow volatility, credit supply, or refinancing costs.

The present paper relates to several contributions in the literature. Fisher, Heinkel, and Zechner (1989), Leland (1998), and Goldstein, Ju, and Leland (2001) are the first to develop dynamic capital structure models with refinancing costs. They show that refinancing costs imply that firms rebalance their capital structure infrequently and are most often away from their target leverage. Strebulaev (2007) simulates artificial data from a dynamic trade-off model to show that the financing behavior implied by this class of models is consistent with the data on financing decisions. Hackbarth, Miao, and Morellec (2006) and Bhamra, Kuehn, and Strebulaev (2010) extend these models to incorporate the effects of macroeconomic conditions on financing and default decisions. Morellec, Nikolov, and Schürhoff (2012) examine the effects of agency conflicts on dynamic capital structure choice. Glover (2012) uses a dynamic capital structure model to derive estimates of bankruptcy costs.

A growing literature examines the financing decisions of firms in models with roll-over debt structure. In these models, firms *costlessly* replace an exogenous fraction of their debt with newly issued debt at each point in time (see e.g. Leland (1994b), Leland and Toft (1996), Hilberink and Rogers (2002), Eom, Helwege, and Huang (2004), He and Xiong (2011), Cheng and Milbradt (2011), or Schroth, Suarez, and Taylor (2012)). In a recent contribution, Décamps and Villeneuve (2011) show that there exists a unique equilibrium in default barrier strategies in these models when the liquidation value of assets is zero.

Our paper also relates to the recent literature that examines the effects of market liquidity on the pricing of risky bonds (see e.g. Ericson and Renault (2006), He and Xiong (2012) or He and Milbradt (2012)). These models generally focus on the analysis of secondary market frictions on default risk and the pricing of risky bonds, given some exogenous financing and restructuring strategies. Our model considers instead the effects of funding liquidity, i.e. the ease with which funds can be raised from creditors, on the choice of corporate financing, restructuring, and default strategies. Finally, our paper relates to Manso (2011), which shows that when the cost of debt depends on the rating of a firm, the rating affects shareholders' default decision, which in turn affects the rating. As shown by Manso, these games of strategic complementarities generally have multiple equilibria.

Our paper advances the literature on dynamic capital structure choice in two important dimensions. First, the paper provides the first analysis of the effects of credit supply uncertainty on financing decisions. That is, although the Modigliani and Miller irrelevance is assumed not to hold on the demand side of the market in prior models, it is assumed to hold on the supply side. Second, our paper contributes to this literature by providing a general analysis of shareholders' dynamic optimization problem. In particular, while prior contributions always assumed the existence of an equilibrium, we provide the first formal analysis of rational expectations equilibria in barrier strategies, show when equilibria in financing, restructuring, and default strategies exist, and when uniqueness can be achieved.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes firms' financing and default policies in the presence of credit supply uncertainty. Sections 4 extends the model to allow firms to bargain over the terms of new debt issues with current debtholders. Section 5 develops the empirical implications of the model. Section 6 concludes. The proofs are gathered in the Appendix.

2 The Model

This section presents our model of dynamic capital structure choice with uncertain credit supply. The model closely follows Fisher, Heinkel, and Zechner (1989) and Leland (1998). Throughout the paper, assets are traded in complete and arbitrage-free markets. The default-free term structure is flat with an after-tax risk-free rate r, at which investors may lend and borrow freely. Time is continuous and uncertainty is modeled by a complete probability space $(\Omega, \mathscr{F}, \mathscr{Q}; \mathbb{F})$, where $\mathscr{Q}$ is the risk neutral probability measure and the filtration $\mathbb{F} = \{\mathscr{F}_t : t \geq 0\}$ satisfies the usual conditions. Management acts in the best interest of shareholders and maximizes shareholder wealth when making policy choices.

We consider an infinitely lived firm with assets that generates a cash flow X_t at time t as long as the firm is in operation. This operating cash flow is independent of financing choices and governed by the process:

$$dX_t = \mu X_t \, dt + \sigma X_t \, dB_t \,, \quad X_0 > 0,$$

where $\mu < r$ and $\sigma > 0$ are constant parameters and $(B_t)_{t \geq 0}$ is a standard Brownian motion under $\mathscr{Q}$. Operating cash flows are taxed at a constant rate $\tau < 1$. As a result, firms have an incentive to issue debt to shield profits from taxation. To stay in a simple time-homogeneous setting, we consider debt contracts that are characterized by a perpetual flow of coupon payments C and a principal P that shareholders have to repay in default (as in Duffie and Lando, 2001, or Manso, 2008). Debt is issued at par and callable at market value. The proceeds from the debt issue are distributed to shareholders at the time of issuance.

Firms whose conditions improve sufficiently may re-leverage by incurring a proportional cost q.[3] Firms whose conditions deteriorate sufficiently may default on their debt obligations. In the model, default leads to the liquidation of the firm. At the chosen time of default, a fraction $\omega \in (0, 1)$ of assets is lost as a frictional cost and the value of the remaining assets is assigned to debtholders. Under these assumptions, the firm's debt structure remains fixed until the firm goes into default or calls its debt and restructures with newly issued debt.

We are interested in building a model in which capital structure choices depend not only on firm characteristics but also on frictions in debt markets. Indeed, as documented by a series of recent empirical studies, credit supply conditions are very important in determining financing decisions. For example, Faulkender and Petersen (2006), Leary (2009), and Sufi (2009) provide evidence suggesting an important role for the supply of credit in determining leverage ratios. Lemmon and Roberts (2009) find that negative shocks to the supply of credit lead to large declines in debt issuance. Massa and Zhang (2009) find that the relative availability of bond and bank financing affect the firm's ability to borrow. Choi, Gemantsky, Henderson, and Tookes (2010) show that the issuance of convertible bonds is positively related to a number of capital supply measures. Finally, a number of surveys indicate that financing decisions are generally governed by the preferences of the suppliers of capital rather than by the demands of the users of capital (see Graham and Harvey, 2001).

[3] While in principle management can both increase and decrease future debt levels, Gilson (1997) finds that transaction costs discourage debt reductions outside of renegotiation.

These studies suggest that credit supply is a key determinant of financing decisions and that firms often face uncertainty regarding their access to credit markets. To capture this important feature of credit markets, we consider that it takes time for firms to secure debt financing and that credit supply is uncertain. In particular, we assume that if a firm decides to issue debt, then it has to search for creditors.[4] In the analysis below, we assume that creditors appear to firms with Poisson arrival rate λ. That is, conditional on searching, the probability of getting financing from new creditors over each time interval $[t, t + dt]$ is λdt and the expected financing lag is $1/\lambda$.[5]

In section 3, we start by formulating the dynamic capital structure problem of a firm that needs to search for outside debt investors when seeking to re-leverage and can only issue debt with these new investors. In section 4, we generalize the model to allow the firm to search for new creditors or to bargain over the terms of new debt issues with current debtholders. To highlight the effects of credit market frictions on debt dynamics, we follow previous contributions by assuming that the firm's shareholders are able to finance temporary cash shortfalls so that the firm defaults when equity value is zero.

3 Capital structure and credit supply uncertainty

3.1 Shareholders' optimization problem

When credit supply is uncertain, the firm raises new debt when two conditions are met. First, it must be optimal to re-leverage. Second, the firm must find creditors. The second constraint implies that the coupon payment C_t on the firm's debt evolves according to

$$dC_t = (a_t - 1)C_{t-}dN_t$$

where $a \geq 1$ is a predictable process that represents the relative change in the coupon at the time of a restructuring, and N is a Poisson process with intensity $\lambda > 0$ whose jumps represent the times at which the firm can restructure its debt.

[4] A growing body of literature argues that assets prices are more sensitive to supply shocks than standard asset pricing theory predicts. Search theory plays a key role in the formulation of models capturing this idea (see e.g. Duffie, Garleanu, and Perdersen, 2005, Duffie and Strulovici, 2011, or Vayanos and Weill, 2008).

[5] One possible interpretation for our assumption is that the firm cannot find a single creditor with deep pockets and has to rely on a syndicate of banks or a group of debt investors as in He and Xiong (2011b). The firm will then restructure once it has found sufficiently many creditors.

Denote by $D(X, C|(a, \delta))$ the value of the firm's *current debt* under the assumptions that the current coupon rate is C and that the firm follows the restructuring and default policies (a, δ), where δ is stopping time representing the default time selected by shareholders. This value can be written as:

$$D(X_0, C|(a, \delta)) = E_0\left[\int_0^{\iota\wedge\delta} e^{-rs}Cds + e^{-r\iota}1_{\{\iota\leq\delta\}}D(X_\iota, C|(a, \delta)) + e^{-r\delta}1_{\{\iota>\delta\}}(1-\omega)\phi_0 X_\delta\right],$$

where ι denotes the first restructuring time, $\iota \wedge \delta \equiv \inf\{\iota, \delta\}$ is the first time that the firm either defaults or restructures, and ϕ_0 is defined by

$$E_t\left[\int_t^\infty e^{-r(s-t)}(1-\tau)X_s ds\right] = \phi_0 X_t.$$

Since debt is callable at market value, its price is independent of the restructuring policy a selected by shareholders and only depends on the default policy δ of the firm. As a result, we can also write the value of debt as:

$$D(X_0, C|(a, \delta)) = D(X_0, C|\delta) = \frac{C}{r}(1 - E_0[e^{-r\delta}]) + \phi_0(1-\omega)E_0[e^{-r\delta}X_\delta]. \tag{1}$$

The first term on the right hand side of this equation captures the present value of the cash flows accruing to debtholders up to the default time. The second term represents their cash flow in default, i.e. the value of assets net of liquidation costs.

Because credit supply is uncertain, debtholders may be able to capture part of the restructuring surplus at refinancing dates. That is, we consider that once management and debt investors meet, they bargain over the terms of the new debt issue to determine the cost of debt or, equivalently, the proceeds from the debt issue. Specifically, denote firm value under the policy (a, δ) by $V(X, C|(a, \delta))$ and assume that shareholders may select a restructuring strategy a' that potentially differs from that anticipated by creditors at the time of debt issuance. We can then define the restructuring surplus $S(a', X, C|(a, \delta))$ by:

$$S(a', X, C|(a, \delta)) = V(X, a'C|(a, \delta)) - V(X, C|(a, \delta)) - q1_{\{a'>1\}}D(X, a'C|\delta). \tag{2}$$

where the last term on the right hand side represents the restructuring cost. Given a non-negative surplus, we consider below that the allocation of this surplus between shareholders and new creditors results from Nash bargaining. Denoting the bargaining power of creditors by $\eta \in (0, 1)$, the amount π^* that creditors can extract at the time of a restructuring satisfies

$$\pi^* = \underset{\pi \geq 0}{\operatorname{argmax}}\ \pi^\eta \left[S(a', X, C|(a, \delta)) - \pi\right]^{1-\eta} = \eta S(a', X, C|(a, \delta)).$$

This Nash bargaining solution determines uniquely the cost of new debt issues and allows us to write equity value under the restructuring and default policy (a, δ) as

$$E(X_0, C_0|(a, \delta)) = E_0 \left[\int_0^\delta e^{-rs} \left[(1 - \tau)(X_s - C_{s-})ds + H(a_s, X_s, C_{s-}|(a, \delta))dN_s\right] \right],$$

where

$$H(a', X, C|(a, \delta)) = (1 - q 1_{\{a'>1\}})D(X, a'C|\delta) - D(X, C|\delta) - \eta S(a', X, C|(a, \delta)),$$

represents the cash flow to shareholders following a change in the firm's coupon rate from C to $a'C$. This cash flow corresponds to the proceeds from the debt issue net of the flotation costs and of the surplus extraction by new debtholders at the time of debt issuance.

In our model, management maximizes equity value by choosing the firm's initial capital structure as well as its restructuring and default policies. Because the latter are selected after debt has been issued, management may have incentives to deviate from the policies (a, δ) conjectured by creditors when pricing corporate debt. Rational creditors anticipate this strategic behavior and finance the firm only if they expect the strategy (a, δ) to be implemented. An equilibrium for our dynamic capital structure model is therefore reached if the restructuring and default policies selected by management are the same as those conjectured ex ante by creditors. More formally, we will use the following definition.

Definition 1 *A rational expectations equilibrium is a policy* $(a^*, \delta^*) \in \mathbb{A} \times \mathbb{S}$ *such that*

$$(a^*, \delta^*) \in \underset{(a,\delta) \in \mathbb{A}(a^*,\delta^*)}{\operatorname{argmax}} E_0 \left[\int_0^\delta e^{-rs} \left[(1 - \tau)(X_s - C_{s-})ds + H(a_s, X_s, C_{s-}|(a^*, \delta^*))dN_s\right] \right], \quad (3)$$

where $\mathbb{A}(a^*, \delta^*)$ *denotes the set of default and restructuring strategies* (a, δ) *with* $a \geq 1$ *and*

$$E_0 \left[\int_0^\delta e^{-rs} (C_{s-} + \lambda |H(a_s, X_s, C_{s-}|(a^*, \delta^*))|) ds \right] < \infty,$$

and $\mathbb{S}$ denotes the set of stopping times.

Solving for equilibria in general strategy spaces is a priori not obvious since shareholders' optimal strategy may depend in very complicated ways on creditors beliefs. As a result, in all the existing literature on dynamic capital structure choice, it is assumed that shareholders follow a barrier default strategy whereby the firm defaults when the cash flow shock goes down to a constant barrier $X_d(C)$ and seek to restructure their capital structure towards a constant target $T = C/X$ whenever the current value cash flow shock exceeds a barrier $X_u(C) > X_d(C)$. In particular, these models characterize rational expectations equilibria in restructuring and default barrier strategies, defined as follows:

Definition 2 *A rational expectations equilibrium is in barrier strategies if*

$$\begin{aligned} \delta &= \inf\{t \geq 0 : X_t \leq X_d(C_{t-})\}, \\ a_t &= 1_{\{X_t < X_u(C_{t-})\}} + 1_{\{X_t \geq X_u(C_{t-})\}}(TX_t/C_{t-}) \end{aligned}$$

for linear functions $X_u(C) \geq X_d(C)$ with $X_u(0) = X_d(0) = 0$ and some nonnegative constant target $T \in [1/X_u(1), 1/X_d(1)]$.

In the Appendix, we establish necessary and sufficient conditions such that, even if debtholders have very complicated beliefs about the firm's default strategy, then the optimal response of shareholders to these beliefs is to default when the cash flow level falls below a given barrier. We also show that in any rational expectations equilibrium in which shareholders follow a default barrier strategy, the optimal restructuring policy is also always of barrier type. In particular, we establish the following result.

Proposition 1 *If creditors conjecture a default policy δ such that the function $D(X, C|\delta)$ is homogeneous of degree one in (X, C) and the function $G(X, C|\delta)$, defined by*

$$G(X, C|\delta) \equiv (1-\tau)(X-C) + \lambda \max_{a \geq 1} \{(1 - q1_{a>1}) D(X, aC|\delta) - D(X, C|\delta)\},$$

is decreasing in C, then the optimal default policy for problem (3) is of barrier type. If, in addition, the conjectured default policy is of barrier type, then both the optimal default and restructuring policies are of barrier type. In particular, all equilibria in which $D(X, C|\delta)$ is homogeneous of degree one and $G(X, C|\delta)$ is decreasing in C are in barrier strategies.

In our analysis below, we focus on rational expectations equilibria in barrier strategies and verify that if creditors conjecture that the firm follows a barrier strategy, then $G(X, C|\delta)$ is decreasing in C. To construct such equilibria, we first pick a default threshold $X_d(C) \equiv X_d(1)C$, where $X_d(1)$ is a positive constant, and solve the following equilibrium problem: Given that debt holders believe that the firm defaults when the cash flow shock reaches the barrier $X_d(C)$ and the firm has to follow this default policy, what is the equilibrium restructuring strategy for shareholders? Having constructed the solution to this artificial problem, we start varying the default threshold to find a value $X_d(1)$ such that it is indeed optimal for shareholders to default at $X_d(C)$. By Proposition 1, this algorithm allows to uncover all rational expectations equilibria in barrier strategies.

3.2 Equilibrium in barrier strategies

Fix a default threshold $X_d(C) \equiv X_d(1)C$ and denote by $\delta(X_d(1))$ the corresponding default time. Using equation (1) and standard derivations, it is immediate to show that the corresponding debt value function $D(X, C|\delta(X_d(1)))$ is given by:

$$D(X, C|\delta(X_d(1))) = \begin{cases} \frac{C}{r} - \left[\frac{C}{r} - (1-\omega)\phi_0 X_d(C)\right]\left(\frac{X}{X_d(C)}\right)^{\beta}, & \text{for } X > X_d(C), \\ (1-\omega)\phi_0 X, & \text{for } X \leq X_d(C), \end{cases}$$

where $\alpha > 0$ and $\beta < 0$ denote the roots of the characteristic equation

$$Q(x; r) = \mu x + \frac{\sigma^2}{2}x(x-1) - r = 0.$$

Thus, when $X > X_d(C)$, the value of corporate debt is equal to the value of risk-free debt minus the change in value that occurs at the time of default. When $X \leq X_d(C)$, it is equal to the liquidation value of assets (even if the firm has not defaulted yet).

Because the firm's default and restructuring strategies are interrelated, we need to know when restructuring may be optimal for shareholders in order to characterize the optimal default strategy. In the model, the value maximizing restructuring strategy results from a trade-off between the additional tax benefits of debt that the firm can get by raising its leverage ratio and the refinancing costs (including the potential surplus extraction by creditors). Proposition 2 below shows that when refinancing costs exceed potential tax savings, i.e. when $q \geq \tau$, it is never optimal for the firm to restructure its debt.

Proposition 2 *If $q \geq \tau$, then there exists a unique equilibrium in barrier strategies. In this equilibrium, restructuring never happens, default occurs at the threshold $X_d^S(C)$ given by*

$$X_d^S(C) = \frac{\beta}{\beta - 1} \frac{r - \mu}{r} C \equiv X_d^S(1)C,$$

and firm value satisfies for $X \geq X_d^S(C)$

$$V^S(X, C) = \phi_0 X + \frac{\tau C}{r} \left[1 - \left(\frac{X}{X_d^S(C)} \right)^\beta \right] - \omega \frac{(1-\tau)\beta C}{r(1-\beta)} \left(\frac{X}{X_d^S(C)} \right)^\beta .$$

If issuance costs satisfy $q < \tau$, then there will always be restructuring in equilibrium.

Assume now that $\tau > q$ so that restructuring the firm's capital structure may be optimal. Given the default policy $\delta(X_d(1))$, a rational expectation equilibrium with fixed default threshold $X_d(C)$ is a restructuring policy $a^* \in \mathbb{A}$ such that

$$\begin{aligned} a^* \in \underset{(a,\delta(X_d(1)))\in\mathbb{A}(a^*,\delta(X_d(1)))}{\operatorname{argmax}} \; & E_0 \left[\int_0^{\delta(X_d(1))} e^{-rs}(1-\tau)(X_s - C_{s-})ds \right] \\ & + E_0 \left[\int_0^{\delta(X_d(1))} e^{-rs} H(a_s, X_s, C_{s-} | (a^*, \delta(X_d(1)))) dN_s \right]. \end{aligned} \tag{4}$$

We shall now use the above representation of shareholders' optimization problem to derive equity value. Consider first the equity value function $E(X, C|(a, \delta(X_d(1))))$ associated with the (possibly suboptimal) barrier strategy $(a, \delta(X_d(1)))$. In the region where the firm does not default (i.e. for $X > X_d(C)$), shareholders receive a cash flow $(1-\tau)(X_t - C)dt$ over each time interval $[t, t+dt]$. As a result, equity value satisfies:

$$\begin{aligned} rE(X, C|(a, \delta(X_d(1)))) = \mu X \frac{\partial E(X, C|(a, \delta(X_d(1))))}{\partial X} + \frac{\sigma^2}{2} X^2 \frac{\partial^2 E(X, C|(a, \delta(X_d(1))))}{\partial X^2} \\ + (1-\tau)(X - C) + 1_{\{X > X_u(C)\}} \lambda (1-\eta) S\left(T, X, C|(a, \delta(X_d(1)))\right) \end{aligned}$$

where $S(T, X, C|(a, \delta(X_d(1))))$ is the restructuring surplus defined in equation (2) and the last term on the right hand side reflects the effects of credit supply uncertainty on equity value. This equation is solved subject to the following boundary conditions

$$\begin{aligned}
\lim_{X\downarrow X_d(C)} E(X,C|(a,\delta(X_d(1)))) &= 0,\\
\lim_{X\uparrow X_u(C)} E(X,C|(a,\delta(X_d(1)))) &= \lim_{X\downarrow X_u(C)} E(X,C|(a,\delta(X_d(1)))),\\
\lim_{X\uparrow X_u(C)} \frac{\partial}{\partial X}E(X,C|(a,\delta(X_d(1)))) &= \lim_{X\downarrow X_u(C)} \frac{\partial}{\partial X}E(X,C|(a,\delta(X_d(1)))),\\
\lim_{X\to\infty}(E(X,C|(a,\delta(X_d(1))))/X) &= \kappa,
\end{aligned}$$

where $\kappa > 0$ is a constant. The first boundary condition shows that the value of assets net of liquidation costs goes to debtholders in default. The second and third conditions are continuity and smoothness conditions satisfied by equity value at the restructuring threshold. The last condition is a standard no-bubbles condition.

To complete the characterization of equity value, we need to derive the optimality conditions corresponding to the restructuring strategy that solves problem (4). In the Appendix, we establish the following result.

Proposition 3 *Assume that issuance costs are such that $q < \tau$. Then for any constant $X_d(1) > 0$ there exists a unique barrier restructuring strategy $\mathbf{P} = \mathbf{P}(X_d(1)) = (a^*, \delta(X_d(1))$ that solves problem (4) and is characterized by*

$$V(X_u(C), C|\mathbf{P}) = \{E(X, TX|\mathbf{P}) + (1-q)\, D(X, TX|\mathbf{P})\}|_{X=X_u(C)},$$

and

$$T = \underset{T>0}{\operatorname{argmax}}\,\{E(X, TX|\mathbf{P}) + (1-q)\, D(X, TX|\mathbf{P})\}.$$

The first condition in Proposition 3 determines the value-maximizing threshold when seeking to restructure with new creditors. The second condition determines the target leverage ratio T. This target leverage ratio does not depend on X due to the homogeneity of the functions $E(X,C|\mathbf{P})$ and $D(X,C|\mathbf{P})$. Using Proposition 3, we can now solve for the equity value function associated with a fix default threshold $X_d(C)$ as

$$E(X,C|\mathbf{P}) = 1_{\{X>X_u(C)\}}V_s(X,C|\mathbf{P}) + 1_{\{X_d(C)<X\le X_u(C)\}}V_{ns}(X,C|\mathbf{P}) - 1_{\{X>X_d(C)\}}D(X,C|\mathbf{P})$$

where

$$V_s(X,C|\mathbf{P}) = \frac{1-\tau+\lambda^* A_1(\mathbf{P})}{r-\mu+\lambda^*}X + \frac{\tau C}{r+\lambda^*} + A_2(\mathbf{P})C\left(\frac{X}{X_u(C)}\right)^{\psi},$$
$$V_{ns}(X,C|\mathbf{P}) = \phi_0 X + \frac{\tau C}{r} + A_3(\mathbf{P})C\left(\frac{X}{X_d(C)}\right)^{\beta} + A_4(\mathbf{P})C\left(\frac{X}{X_u(C)}\right)^{\alpha}$$

where $\lambda^* = \lambda(1-\eta)$, the constant ψ is the negative root of the equation $Q(x; r+\lambda^*) = 0$ and the four constants $A_i(\mathbf{P})$, the target T, and the restructuring threshold $X_u(1)$ solve

$$\begin{aligned}
A_3(\mathbf{P}) &= V_{ns}(1,T|\mathbf{P}) - qD(1,T|\mathbf{P}),\\
V_{ns}(X_d(C),C|\mathbf{P}) &= (1-\omega)\phi_0 X_d(C),\\
V_{ns}(X_u(C),C|\mathbf{P}) &= V_s(X_u(C),C|\mathbf{P}),\\
V_s(X_u(C),C|\mathbf{P}) &= V_{ns}(X_u(C),TX_u(C)|\mathbf{P}) - qD(X_u(C),TX_u(C)|\mathbf{P})\\
\frac{\partial}{\partial X}V_{ns}(X,C|\mathbf{P})\Big|_{X=X_u(C)} &= \frac{\partial}{\partial X}V_s(X,C|\mathbf{P})\Big|_{X=X_u(C)}\\
\frac{\partial}{\partial C}V_{ns}(X,C|\mathbf{P})\Big|_{X=1,C=T} &= -q\,\frac{\partial}{\partial C}D(X,C|\mathbf{P})\Big|_{X=1,C=T}.
\end{aligned}$$

These equations show that equity value can be written as the difference between firm value and the value of outstanding debt. The value of the firm depends on whether the cash flow shock is in the restructuring region ($V_s(X,C|\mathbf{P})$ for $X > X_u(C)$) or in the inaction region ($V_{ns}(X,C|\mathbf{P})$ for $X < X_u(C)$). Firm value in the inaction region ($X < X_u(C)$) is given by the sum of the present value of cash flows from assets in place (first term) and tax savings (second term), plus the change in firm value occurring when the firm defaults (third term), plus the change in firm value occurring when the cash flow shock reaches the restructuring threshold (last term). Firm value in the restructuring region ($X > X_u(C)$) can be interpreted similarly. Finally, we also establish the following result in the Appendix:

Proposition 4 *For all $X_d(1) > X_d^S(1)$, the equity value satisfies*

$$\frac{\partial}{\partial X}E(X,C|\mathbf{P}(X_d(1)))\Big|_{X=X_d(C)} > 0, \tag{5}$$

so that defaulting slightly later leads to a local increase in equity value.

Proposition 4 shows that under the value-maximizing restructuring strategy, the continuation value of equity is larger when the firm has the option to change its capital structure in the future. This in turn implies that shareholders will always default later than in the static model in which no restructuring is allowed.

3.3 Existence of equilibria in barrier strategies

Having constructed equity value for a given default boundary, we now have to find a constant $X_d(1)$ that maximizes equity value. We expect such an $X_d(1)$ to satisfy

$$\frac{\partial}{\partial X} E(X, C|\mathbf{P}(X_d(1)))\bigg|_{X=X_d(C)} = 0. \tag{6}$$

This smooth pasting condition is clearly necessary. Indeed, if the derivative was positive (negative), shareholder could increase equity value in a small neighborhood of $X_d(C)$ by slightly decreasing (increasing) $X_d(C)$. Proposition 5 below shows that this condition is also sufficient and thus provides a complete characterization of equilibria in barrier strategies.

Proposition 5 *The policy* $\mathbf{P}(X_d(1)) = (a^*, \delta(X_d(1)))$ *is a rational expectations equilibrium if and only if condition* (6) *is satisfied.*

Proposition 5 shows that the existence/uniqueness of an equilibrium in barrier strategies is equivalent to the existence/uniqueness of a solution $X_d(1)$ to the smooth-pasting condition (6). As we show below, this equation does not always have a solution. In order to characterize rational expectations equilibria in barrier strategies, we therefore start by analyzing the special case in which it is costless to refinance and derive conditions under which an equilibrium in barrier strategies exists when $q = 0$. We then turn to the analysis of positive issuance costs and provide a general characterization of the firm's policy choices.

Assume first that issuing debt is costless, in that $q = 0$. In this case, it is optimal for shareholders to start searching for investors whenever the cash flow shock X reaches a new maximum and the restructuring threshold satisfies $X_u(1) = 1/T$. Define the constants $\kappa_1(\lambda)$ and $\kappa_2(\lambda)$ by

$$\begin{aligned}
\kappa_1(\lambda) &\equiv \frac{(1-\psi)\alpha\lambda(1-\eta)\mu - r(r-\mu+\lambda(1-\eta))(\alpha-\psi)}{(r-\mu)(r+\lambda(1-\eta))(1-\psi)(\alpha-\beta)}, \\
\kappa_2(\lambda) &\equiv \frac{1}{\alpha-1}\left[1+(1-\beta)\kappa_1(\lambda)\right].
\end{aligned}$$

A direct calculation shows that $\alpha\mu < r$ and therefore $\kappa_1(\lambda) < 0 < \kappa_2(\lambda)$. Hence, there exists a unique solution J_0 to the equation $f(y) = 0$ where

$$f(y) \equiv 1 + \kappa_1(\lambda)\, y^{-\beta} + \kappa_2(\lambda)\, y^{-\alpha}. \tag{7}$$

Denote by $J > J_0$ the unique solution to the equation $g(y) = 0$ where

$$g(y) \equiv \beta\omega - \frac{\beta\omega}{\tau} + (1-\beta)\left[1 + \kappa_1(\lambda)\, y^{-\beta}\right] + \left[(\beta-\alpha)\,\omega + 1 - \beta)\right]\kappa_2(\lambda)\, y^{-\alpha}. \tag{8}$$

We then have the following result.

Proposition 6 *When $q = 0$, an equilibrium in barrier strategies exists if and only if the corporate tax rate τ is below the critical tax rate τ^* defined by*

$$\tau^* = \frac{\beta}{\beta - \kappa_2(\lambda)(\alpha-\beta)\, J_0^{-\alpha}} \in (0,1).$$

In this case, there exists a unique equilibrium in barrier strategies and the corresponding default and restructuring thresholds satisfy

$$X^*_{u,0}(1) = \frac{1}{T^*_0} = \frac{1}{J X^*_{d,0}(1)} = \frac{r\omega\phi_0}{\rho J |f(J)|}$$

Having characterized optimal policy choices when $q = 0$, we now turn to the analysis of equilibria in barrier strategies with positive issuance costs. It follows from the proof of Proposition 6 that, when $\tau < \tau^*$, equity value satisfies

$$\frac{\partial}{\partial X} E(X, C|\mathbf{P}(X_d(1)))\bigg|_{q=0, X=X_d(C)} < 0,$$

for any constant $X_d(1) < X^*_{d,0}(1)$. Thus, for any $X_d(1) < X^*_{d,0}(1)$, equity value is negative for in a right neighborhood of $X_d(C)$ when $q = 0$. Since equity value is monotone decreasing in the issuance cost q, it follows that

$$E(X, C|\mathbf{P}(X_d(1))) \leq 0$$

for any $q > 0$ in a right neighborhood of $X_d(C)$. This in turn implies that for any $X_d(1) < X^*_{d,0}(1)$ and $q > 0$, we have

$$\frac{\partial}{\partial X} E(X, C|\mathbf{P}(X_d(1)))\Big|_{X=X_d(C)} < 0.$$

This fact, together with condition (5) and the intermediate value theorem, implies that the smooth pasting condition (6) has a solution in $\left(X^*_{d,0}(C), X^S_d(C)\right)$ for any $q > 0$ when $\tau < \tau^*$. Furthermore, standard implicit function type arguments imply that this solution is unique when q is sufficiently small. This leads to the main result of this section.

Theorem 1 *Assume that issuance costs are positive in that $q > 0$. If the corporate tax rate satisfies $\tau < \tau^*$, then there exists a rational expectations equilibrium in barrier strategies, which is unique when q is sufficiently small.*

Theorem 1 provides sufficient conditions for a rational expectations equilibrium in barrier strategies to exist. In such equilibria, shareholders default the first time that the cash flow shock decreases to an endogenous threshold $X^*_d(C) \equiv X^*_d(1)C$. In addition, they issue new debt the first time that the cash flow shock is above $X^*_u(C)$ and shareholders meet new debt investors. Even though Theorem 1 does not establish uniqueness of the equilibrium for large q, our extensive numerical simulations suggest that uniqueness does hold. If there were multiple equilibria, the equilibrium with the minimal default threshold $X_d(1)$ would be the most desirable outcome from the social welfare perspective. Indeed, when the cost q is sufficiently small, firm value with a fixed default policy, $V(X, C|\mathbf{P}(X_d(1)))$, is monotone decreasing in $X_d(1)$ and hence the minimal threshold equilibrium maximizes firm value.

In the Appendix, we also show that rational expectations equilibria in barrier strategies may fail to exist when tax benefits are very large. That is, we show that when $\tau > \tau^*$, there exists a cutoff level q^* for issuance costs satisfying

$$q^* \equiv \inf\left\{q > 0 : \inf_{X_d(1)>0} \frac{\partial}{\partial X} E(X, C|\mathbf{P}(X_d(1)))\Big|_{X=X_d(C)} < 0\right\},$$

such that a rational expectations in barrier strategies exists if and only if $q > q^*$. To aid in the intuition of this result, suppose that q is sufficiently small that issuing new debt is almost costless. Suppose also that debtholders believe that the firm will default very late, that is assume that $X_d(1)$ is very small. Let $\tilde{X}_d(1)$ be the default policy selected by shareholders (which is of barrier type by Proposition 1). Suppose now that we start decreasing $X_d(1)$ and make it small so that debtholders are willing to invest in bonds as if they were almost riskless. Then, because issuing debt is cheap, the firm will find it optimal to issue very large amounts of debt. This will increase the expected surplus from restructuring and significantly increase equity value when the tax benefits of debt are sufficiently high. This is turn will make it optimal to default even later, so that $\tilde{X}_d(1) < X_d(1)$. Thus, an equilibrium (that is, beliefs $X_d(1)$ for which $\tilde{X}_d(1) = X_d(1)$) will fail to exist. In section 5, we show that for the typical U.S. firm, τ^* is generally above 99% (that is, well beyond the statutory corporate tax rate). In the following, we thus focus on the case described in Theorem 1.

To conclude this section, note that when the firm needs to search for creditors, restructuring generally occurs at an inefficient time compared to an environment in which capital supply is infinite. A number of questions naturally arise in such a context. First, can the firm contact current debtholders to restructure its capital structure instead of looking for new investors? Second, what are the terms of the new debt contract if the firm restructures with current creditors? The next section answers these and other related questions.

4 Restructuring with existing creditors

In this section, we generalize our dynamic capital structure model to allow the firm to bargain over the terms of new debt issues with current debtholders. Because bonds are generally held dispersedly and many of the loans issued by large firms are syndicated, mobilizing current creditors to issue new debt may be costly, with the cost increasing in the amount of outstanding debt.[6] In the following, we model this market imperfection by assuming that contacting existing debt holders is costly and let ϵC_{t-} denote the cost of mobilizing these creditors, where $\epsilon > 0$ and C_{t-} is the total outstanding coupon when the firm contacts existing creditors.[7]

If shareholders can contact existing creditors every time they want to restructure, then the set of restructuring times is no longer restricted to the set $\mathscr{N}$ of jump times of the

[6] As documented by Rauh and Sufi (2010), debt heterogeneity is a common feature of the real world in that firms simultaneously use different types, sources, and priorities of debt. While our model does not capture this richness in capital structures, it reflects the potential costs associated with complex debt structures.

[7] The assumption of linear cost is made purely for convenience. The proof presented in the Appendix shows that our results hold for a large class of cost functions.

Poisson process N. In particular, the set of strategies available to shareholders now consists in pairs (b, γ), where γ is the default time and $b \geq 1$ is a process such that

$$b_t = 1 + \sum_{k=1}^{\infty} 1_{\{\tau_k = t\}} \xi_k$$

for some increasing sequence of restructuring times $\tau_k \in \mathbb{S}$ and some sequence of nonnegative random variables $\xi_k \in \mathscr{F}_{\tau_k}$ that represent the relative increase in the firm's coupon payment. The corresponding dynamics of the firm's coupon payment C_t are then given by

$$dC_t = (b_t - 1)C_{t-} = \sum_{k=1}^{\infty} 1_{\{\tau_k = t\}} \xi_k C_{t-},$$

where ξ_k corresponds to restructuring with new creditors when $\tau_k \in \mathscr{N}$.

Assume that debt holders anticipate that shareholders will use a strategy (b, γ) as above. Let $\theta \in [0, 1]$ denote the bargaining power of existing creditors. As in the search model, Nash bargaining implies that the part of the restructuring surplus that accrues to creditors at restructuring dates is

$$\pi^* = (\eta 1_{\{\tau_k \in \mathscr{N}\}} + \theta 1_{\{\tau_k \notin \mathscr{N}\}}) \left[S(b, X, C|(b, \gamma)) - \epsilon C 1_{\{\tau_k \notin \mathscr{N}\}} \right].$$

where S is defined as in equation (2) with $V_{\mathbf{b}}$ replacing V. For a given strategy (b, γ) the value of the firm's equity, denoted by $E_{\mathbf{b}}(X, C|(b, \gamma))$, is then defined by

$$E_{\mathbf{b}}(X, C|(b, \gamma)) = E_0 \left[\int_0^{\gamma} e^{-rt}(1 - \tau)(X_t - C_{t-})dt + \sum_{k=1}^{\infty} 1_{\{\tau_k \leq \gamma\}} e^{-r\tau_k} H_{\mathbf{b}}(b_{\tau_k}, X_{\tau_k}, C_{\tau_k -}|(b, \gamma)) \right],$$

where

$$\begin{aligned} H_{\mathbf{b}}(b', X, C|(b, \gamma)) &= (1 - q 1_{\{b' > 1\}}) D(X, b'C|(b, \gamma)) - D(X, C|(b, \gamma)) \\ &\quad - \eta S(b', X, C|(b, \gamma)) \mathbf{1}_{\tau_k \in \mathscr{N}} - \left[\theta S(b', X, C|(b, \gamma)) + (1 - \theta)\epsilon C \right] \mathbf{1}_{\tau_k \notin \mathscr{N}}. \end{aligned}$$

When $\theta = 0$ and $\epsilon = 0$, there is no surplus extraction by current creditors and no cost of collective action. In this case, our model reduces to the models of Fisher, Heinkel, and

Zechner (1989) and Leland (1998), in which the supply of credit is infinitely elastic at the correct price and the firm's capital structure is entirely determined by demand factors.

A rational expectation equilibrium for the model in which the firm can restructure with current creditors can be defined as follows:

Definition 3 *A strategy (b^*, γ^*) is a rational expectations equilibrium if*

$$
\begin{aligned}
(b^*, \gamma^*) \in \underset{(b,\gamma)\in\mathbb{B}(b^*,\gamma^*)}{\operatorname{argmax}} \; & E_0\left[\int_0^\gamma e^{-rt}(1-\tau)(X_t - C_{t-})dt\right] \\
& + E_0\left[\sum_{k=1}^\infty 1_{\{\tau_k \le \gamma\}} e^{-r\tau_k} H_{\mathbf{b}}(b_{\tau_k}, X_{\tau_k}, C_{\tau_k -}|(b^*, \gamma^*))\right],
\end{aligned}
$$

where $\mathbb{B}(b^*, \gamma^*)$ denotes the set of strategies (b, γ) such that

$$
E_0\left[\int_0^\gamma e^{-rt} C_{t-} dt + \sum_{k=1}^\infty 1_{\{\tau_k \le \gamma\}} e^{-r\tau_k} \left|H_{\mathbf{b}}(b_{\tau_k}, X_{\tau_k}, C_{\tau_k -}|(b^*, \gamma^*))\right|\right] < \infty.
$$

In the current model, the firm can either search for new investors or renegotiate outstanding debt with existing creditors when seeking to adjust its capital structure. Each strategy is associated with a different cost, i.e. a monetary cost when negotiating with current creditors and a waiting cost when searching for new creditors. Therefore, we expect the equity value-maximizing restructuring strategy to be characterized by two thresholds, $X_{\mathbf{b}u}(C)$ and $\bar{X}_{\mathbf{b}u}(C) \ge X_{\mathbf{b}u}(C)$, such that the optimal policy is (i) to search for new creditors (and restructure with them if they are found) when $X \ge X_{\mathbf{b}u}(C)$; (ii) to immediately contact current creditors if $X \ge \bar{X}_{\mathbf{b}u}(C)$. In the latter case, restructuring occurs exactly at $\bar{X}_{\mathbf{b}u}(C)$. In the former case, restructuring may not occur at $X_{\mathbf{b}u}(C)$ since the firm needs to find debt investors before changing its capital structure. More formally, barrier strategies are defined as follows when the firm can restructure with existing creditors:

Definition 4 *A rational expectations equilibrium is in barrier strategies if*

$$
\begin{aligned}
\gamma &= \inf\{t \ge 0 : X_t \le X_{\mathbf{b}d}(C_{t-})\}, \\
b_t &= 1_{\{X_t < X_{\mathbf{b}u}(C_{t-})\}} + 1_{\{X_t \ge X_{\mathbf{b}u}(C_{t-}), t\in\mathscr{N}\}\cup\{X_t \ge \bar{X}_{\mathbf{b}u}(C_{t-})\}}(T_{\mathbf{b}} X_t / C_{t-})
\end{aligned}
$$

for linear functions $\bar{X}_{\mathbf{b}u}(C) \ge X_{\mathbf{b}u}(C) \ge X_{\mathbf{b}d}(C)$ with $\bar{X}_{\mathbf{b}u}(0) = X_{\mathbf{b}u}(0) = X_{\mathbf{b}d} = 0$ and some nonnegative constant target $T_{\mathbf{b}} \in [1/\bar{X}_{\mathbf{b}u}(1), 1/X_{\mathbf{b}d}(1)]$.

To characterize rational expectations equilibria in barrier strategies, we proceed as in the model with search. We first fix a default policy $X_{\mathbf{b}d}(C) \equiv X_{\mathbf{b}d}(1)C$ for some constant $X_{\mathbf{b}d}(1) > 0$ and then find the associated equilibrium restructuring policy b. Because the optimal stopping/impulse control problem of shareholders is much more complex than before, none of the standard techniques can be directly applied to solve for optimal policies. To do so, we thus develop a new approach that can be described as follows. We first assume that shareholders can only contact current creditors at the jump times of a Poisson process with some fixed intensity and solve for the corresponding policy. We then let the intensity tend to infinity. Going through these steps allows us to establish the following result.

Proposition 7 *For any $X_{\mathbf{b}d}(1) > 0$ there exists a rational expectations equilibrium with fixed default policy that is given by some barrier strategy $\mathbf{P} = \mathbf{P}(X_{\mathbf{b}d}(1))$. The strategy $\mathbf{P}(X_{\mathbf{b}d}(1))$ is a rational expectations equilibrium if and only if it satisfies*

$$\frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}(X_{\mathbf{b}d}(1)))\bigg|_{X=X_{\mathbf{b}d}(C)} = 0\,. \tag{9}$$

Using Proposition 7, we can now derive equity value under the value-maximizing default and restructuring strategy. In the region $(X_{\mathbf{b}d}\,(C)), \bar{X}_{\mathbf{b}u}(C))$, shareholders receive a cash flow $(1-\tau)(X_t - C)dt$ over each time interval $[t, t+dt]$. As a result, the equity value function associated with the barrier strategy $\mathbf{P} = \mathbf{P}(X_{\mathbf{b}d}(1))$ satisfies:

$$\begin{aligned} rE_{\mathbf{b}}(X, C|\mathbf{P}) &= \mu X \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}) + \frac{\sigma^2}{2} X^2 \frac{\partial^2}{\partial X^2} E_{\mathbf{b}}(X, C|\mathbf{P}) + (1-\tau)(X - C) \\ &\quad + 1_{\{X \geq X_{\mathbf{b}u}(C)\}} \lambda^* \left[V_{\mathbf{b}}(X, T_{\mathbf{b}} X|\mathbf{P}) - V_{\mathbf{b}}(X, C|\mathbf{P}) - qD(X, T_{\mathbf{b}} X|\mathbf{P})\right] \end{aligned}$$

where $V_{\mathbf{b}}(X, C|\mathbf{P}) = E_{\mathbf{b}}(X, C|\mathbf{P}) + D(X, C|\mathbf{P})$ denotes firm value and the last term on the right hand side reflects the effects of credit supply uncertainty on equity value. This equation is solved subject to the following conditions at the default and restructuring thresholds:

$$\begin{aligned} E_{\mathbf{b}}(X_{\mathbf{b}d}\,(C)\,, C|\mathbf{P}) &= 0, \\ V_{\mathbf{b}}(\bar{X}_{\mathbf{b}u}\,(C)\,, C|\mathbf{P}) &= \left\{V_{\mathbf{b}}(X, T_{\mathbf{b}} X|\mathbf{P}) - qD(X, T_{\mathbf{b}} X|\mathbf{P}) - \epsilon C\right\}\big|_{X=\bar{X}_{\mathbf{b}u}(C)}\,, \\ \frac{\partial}{\partial X} V_{\mathbf{b}}(X, C|\mathbf{P})\bigg|_{X=\bar{X}_{\mathbf{b}u}(C)} &= \frac{\partial}{\partial X}\left\{V_{\mathbf{b}}(X, T_{\mathbf{b}} X|\mathbf{P}) - qD(X, T_{\mathbf{b}} X|\mathbf{P}) - \epsilon C\right\}\bigg|_{X=\bar{X}_{\mathbf{b}u}(C)}\,. \end{aligned}$$

The first and second boundary conditions are the value-matching and smooth-pasting conditions that apply to equity value at the default threshold. The third and fourth boundary conditions are value-matching and smooth-pasting conditions that apply when restructuring with existing creditors at $\bar{X}_{\mathbf{b}u}$. The value-matching condition at $\bar{X}_{\mathbf{b}u}$ implies that there is no surplus at the time of a restructuring. This in turn implies that the bargaining power of existing creditors θ has no bearing on the equilibrium.

In addition to these boundary conditions, equity value satisfies:

$$\begin{aligned}
V_{\mathbf{b}}(X_{\mathbf{b}u}(C), C|\mathbf{P}) &= (V_{\mathbf{b}}(X, T_{\mathbf{b}}X|\mathbf{P}) - qD(X, T_{\mathbf{b}}X|\mathbf{P}))|_{X=X_{\mathbf{b}u}(C)} \\
\lim_{X\uparrow X_{\mathbf{b}u}(C)} E_{\mathbf{b}}(X, C|\mathbf{P}) &= \lim_{X\downarrow X_{\mathbf{b}u}(C)} E_{\mathbf{b}}(X, C|\mathbf{P}), \\
\lim_{X\uparrow X_{\mathbf{b}u}(C)} \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}) &= \lim_{X\downarrow X_{\mathbf{b}u}(C)} \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}),
\end{aligned}$$

at the search threshold. The first condition determines the value-maximizing threshold when seeking to restructure with new creditors. The last two conditions are continuity and smoothness conditions. In these conditions, the target leverage ratio $T_{\mathbf{b}}$ satisfies

$$T_{\mathbf{b}} = \underset{T_{\mathbf{b}}>0}{\operatorname{argmax}} \left\{ E_{\mathbf{b}}(X, T_{\mathbf{b}}X|\mathbf{P}) + (1-q)D(X, T_{\mathbf{b}}X|\mathbf{P}) \right\}.$$

and the default threshold satisfies the smooth pasting condition (9).

As in the model with search, we need to solve the smooth pasting condition (9) for $X_{\mathbf{b}d}(1)$ to prove existence/uniqueness of a rational expectations equilibrium. To this end, we will use monotonicity arguments, similar to those used in the proof of Theorem 1. In the model, equity value is monotone decreasing in q and ϵ. Therefore, an equilibrium exists as soon as it exists when $q = 0$ and $\epsilon = 0$. Since the latter case corresponds to the search model with an infinite intensity, we define

$$\begin{aligned}
\kappa_1(\infty) &\equiv \lim_{\lambda\to\infty} \kappa_1(\lambda) = \frac{\alpha\mu - r}{(r-\mu)(\alpha-\beta)}, \\
\kappa_2(\infty) &\equiv \lim_{\lambda\to\infty} \kappa_2(\lambda) = \frac{1 + (1-\beta)\kappa_1(\infty)}{\alpha - 1}.
\end{aligned}$$

We can now define the unique solution $\overline{J}_0$ to the equation

$$1 + \kappa_1(\infty)\overline{J}_0^{-\beta} + \kappa_2(\infty)\overline{J}_0^{-\alpha} = 0,$$

and the critical tax rate $\tau^*(\infty) < \tau^*$ by

$$\tau^*(\infty) = \frac{-\beta}{-\beta + (\alpha - \beta)\kappa_2(\infty)\,\overline{J}_0^{-\alpha}} \in (0, 1).$$

We then have the following result:

Theorem 2 *Assume that $q > 0$ and that shareholders can raise new debt from existing creditors by paying a cost of collective action ϵC_{t-}. If the corporate tax rate satisfies $\tau < \tau^*(\infty)$, then there exists a rational expectations equilibrium in barrier strategies. For generic parameter values, this equilibrium is unique if ϵ and q are sufficiently small.*

Theorem 2 provides sufficient conditions for a rational expectations equilibrium in barrier strategies to exist when the firm can restructure with existing creditors. As in the search model, the level of the corporate tax rate is key in determining whether an equilibrium exists. While in our base case environment, the corporate tax rate does satisfy the restriction $\tau < \tau^*(\infty)$ of Theorem 2, it is also interesting to characterize firm behavior when this restriction is not satisfied. In the Appendix, we show that if the corporate tax rate satisfies $\tau > \tau^*(\infty)$, then there exists a threshold $q^*_{\mathbf{b}}$ satisfying

$$q^*_{\mathbf{b}} \equiv \inf\left\{ q > 0 : \inf_{X_{\mathbf{b}d}(1)>0} \frac{\partial}{\partial X} E_{\mathbf{b}}(X, C|\mathbf{P}(X_{\mathbf{b}d}(1)))\Big|_{X = X_{\mathbf{b}d}(C)} < 0 \right\},$$

such that a rational expectations equilibrium in barrier strategies exists if and only if $q > q^*_{\mathbf{b}}$. This threshold is monotone increasing in the meeting intensity λ, decreasing in the bargaining in the bargaining power η, and satisfies $q^*_{\mathbf{b}} \geq q^*$. This implies again that there may not exist a rational expectations equilibrium for the model when tax benefits of debt are very large.

5 Corporate financing with credit supply frictions

5.1 Calibration of parameter values

This section examines the empirical predictions of the model for financing policies, creditor turnover, and the decision to default. To do so, we need to select parameter values for the initial value of the firm's cash flow X_0, the risk free interest rate r, the tax advantage of debt

τ, liquidation costs ω, the physical and risk neutral growth rates of the firm's income m and μ, the volatility of the cash flow shock σ, refinancing costs q, the cost of collective action ϵ, and the bargaining power of outside creditors η. In what follows, we select parameter values that roughly reflect a typical U.S. firm. These parameter values are reported in Table 1.

Insert Table 1 Here

Consider first the parameters governing operating cash flows. We set the initial value of cash flows to $X_0 = 1$. This is without loss of generality since the homogeneity of our model implies that the quantities of interest do not depend on X_0. The main parameters describing the cash flow dynamics are (m, μ, σ). Morellec, Nikolov, and Schuerhoff (2012) construct estimates for these variables using data from Compustat, CRSP, and the Institutional Brokers' Estimate System (IBES). They find that for the average firm in their sample, the risk-neutral growth rate, the physical growth rate, and the volatility of the cash flow process are respectively given by $\mu = 0.67\%$, $m = 8.24\%$, and $\sigma = 28.86\%$.

The risk-free rate $r = 4.2\%$ is calibrated to the one-year treasury rate. The tax advantage of debt captures corporate and personal taxes and is set equal to $\tau = 15\%$. This corresponds to a tax environment in which the corporate tax rate is set at the highest possible marginal tax rate of 35% and the tax rates on dividends and interest income are set to 11.6% and 29.3%, consistent with the estimates in Graham (1996). Liquidation costs are defined as the firm's going concern value minus its liquidation value, divided by its going concern value. We base the calibration of liquidation costs on Glover (2012) and set $\omega = 45\%$.

Several empirical studies provide estimates of issuance costs as a function of the amount of debt being issued. The model, however, is written in terms of debt issuance cost q as a fraction of total debt outstanding. We set the cost of debt issuance to $q = 1\%$. This produces a cost of debt issuance representing 2% of the issue size at the search threshold, consistent with the values found in the empirical literature (see Altinkilic and Hansen, 2000, and Kim, Palia, and Saunders, 2008). We also assume that $\epsilon = 2.5\%$, which implies that the cost of collective action represents 25% of the total restructuring cost when the firm restructures with current creditors. Finally, we set the bargaining power of new creditors to $\eta = 50\%$ and check robustness by varying η. These input parameter values imply that $\tau^* = 99.34\%$ and $\tau^*(\infty) = 81.16\%$.

5.2 Optimal restructuring strategies

We start by analyzing the effects of credit supply uncertainty on shareholders' restructuring strategy and creditor turnover. In the model, restructuring is endogenous and occurs the first time the cash flow process reaches the region $[X_{\mathbf{b}u}(C), \infty)$ and the firm finds new debt investors or upon reaching $\bar{X}_{\mathbf{b}u}(C)$, in which case the firm restructures with existing creditors. Credit supply uncertainty therefore affects dynamic capital structure choice through its effects on $X_{\mathbf{b}u}(C)$ and $\bar{X}_{\mathbf{b}u}(C)$. Credit supply uncertainty also implies that, in contrast with standard dynamic capital structure models, there exists some time series variation in the size of capital structure changes in our model.

To illustrate these effects, Figure 1 plots the value-maximizing restructuring triggers as functions of the arrival rate of creditors λ, the share η of the restructuring surplus captured by new debtholders, the volatility of the cash flow shock σ, the corporate tax rate τ, liquidation costs ω, and refinancing costs q. In the figure, the dashed line represents the model with credit supply uncertainty and without bargaining with current creditors, i.e. $X_u(C)$, the solid line represents the search threshold in the model with credit supply uncertainty and bargaining with current creditors, i.e. $X_{\mathbf{b}u}(C)$, and the dotted line represents the restructuring threshold with current creditors, i.e. $\bar{X}_{\mathbf{b}u}(C)$. Finally, the dot-dashed line represents the probability of restructuring with current creditors when $\epsilon = 0$, i.e. in models like those of Fisher, Heinkel and Zechner (1989) or Leland (1998).

Insert Figure 1 Here

Figure 1 shows that the search (and restructuring) threshold in the model with bargaining, $X_{\mathbf{b}u}(C)$, is always above the search threshold in the model without bargaining, $X_u(C)$. This is due to the fact that when the firm can contact current creditors to restructure, the value of the option to look for new creditors is lower, so that shareholders exercise this option later. The figure also shows that the search threshold in the model with bargaining is lower than the restructuring threshold $\bar{X}_{\mathbf{b}u}(C)$ since shareholders need to pay a cost of collective action when restructuring with current creditors. Accordingly, the wedge between these two thresholds increases as ϵ increases. As shown by the figure, this wedge also increases as λ increases since the option value of restructuring with existing creditors decreases with the strength of credit supply.

Another result illustrated by the figure is that the search and restructuring thresholds increase with credit supply λ and decrease with the share of the restructuring surplus that new creditors can capture η. In particular, as the arrival rate of investors decreases, the opportunity cost of waiting to restructure increases (as the likelihood of finding investors decreases), so that the selected search threshold decreases. That is, when the firm has to find investors to restructure its debt, it balances the opportunity cost of early restructuring with the opportunity cost of waiting (the risk of not finding creditors to refinance). In addition, as η increases, the selected restructuring threshold decreases, reflecting shareholders's incentives to reduce the potential cash flow transfers towards new creditors at restructuring dates.

To illustrate the effects of credit market frictions on the frequency of capital structure changes, Figure 2 plots the probability of a restructuring over a 3-year horizon (panel A) as well as the unconditional probability of a restructuring (panel B) as functions of the arrival rate of creditors λ, the cost of collective action ϵ, and issuance costs q. In the figure, the dotted line represents the probability of restructuring with current creditors, the dashed line represents the probability of restructuring with new creditors, and the solid line represents the probability of restructuring with new or existing creditors. Finally, the dot-dashed line represents the probability of restructuring with current creditors when $\epsilon = 0$. The Appendix shows how to compute these probabilities.

Insert Figure 2 Here

In the model, credit supply uncertainty has two opposing effects on the likelihood of capital structure changes. First, for any given restructuring threshold, an increase in λ makes it easier for the firm to find creditors and to restructure. Second, an increase in λ leads to an increase in the selected restructuring threshold. Figure 2 shows that in our base case environment the first effect dominates so that the frequency of restructurings increases with λ. The figure also shows that as the cost of collective action increases, the overall probability of restructuring over a finite horizon first decreases and then increases. This is due to two opposing effects. On the one hand, an increase in ϵ leads to a decrease in the likelihood of a restructuring with existing creditors. On the other hand, an increase in ϵ leads to a decrease in the search threshold $X_{\mathbf{b}u}(C)$ and, hence, to an increase in the likelihood of a restructuring with new creditors. Another result illustrated by Figure 2 is that the frequency of capital structure changes decreases with issuance costs q.

Figure 2, Panel B, also shows that the frequency of creditor changes – i.e. the creditor turnover – depends on the various frictions faced by firms when seeking to restructure. In particular, the figure shows that the creditor turnover increases with the cost of collective action ϵ and with credit supply λ. It also shows that the cost of financing q increases the unconditional probability of refinancing with existing creditors since the cost of collective action becomes a smaller fraction of the total cost of financing as issuance costs increase. The quantitative effects of ϵ and λ on creditor turnover are large. Notably, the unconditional creditor turnover increases from 30.66% to 81.77% when ϵ is raised from 1% to 5%, while it increases from 26.63% to 78.07% when λ increases from 1 to 5.

5.3 Optimal capital structure and default decisions

We now turn to the analysis of leverage and default decisions. In the following, we focus on financing decisions made by shareholders at refinancing points (i.e. optimal leverage). When making such decisions, the objective of shareholders is to maximize the value of equity after the issuance of corporate debt (i.e. $E(X, XT|\mathbf{P})$) plus the proceeds from the debt issue. Figure 3 Panel A plots the value-maximizing leverage ratio as a function of the arrival rate of investors λ, issuance costs q, and the cost of collective action ϵ. Input parameter values for Figure 3 are set as in our base case and the leverage ratio is defined by:

$$L^* = \frac{D(1, T|\mathbf{P})}{D(1, T|\mathbf{P}) + E(1, T|\mathbf{P})},$$

The figure shows that, as credit supply weakens, firms issue more debt when restructuring their capital structure. This result is consistent with the idea that, with weaker credit supply, it will be more difficult to restructure if economic conditions improve so that the firm needs to take on more debt ex ante. It implies that the reduction in search costs due to the higher debt level outweights the additional default costs borne by the firm. Another result illustrated by Figure 3 is that the optimal leverage ratio increases with the cost of collective action ϵ. However, the quantitative effect is very small. Finally, as in prior studies (see Strebulaev, 2007, or Morellec, Nikolov, and Schuerhoff, 2012), the leverage ratio decreases with bankruptcy costs and cash flow volatility (not shown in the figure).

Interestingly, the target leverage ratio first increases and then decreases with restructuring costs. That is, firms with low and high refinancing costs prefer lower leverage than those with intermediate costs. This non-monotonicity is due to two opposing effects. First, as restructuring costs increase, shareholders find it optimal to delay changes in capital structure, making it optimal to issue more debt at restructuring dates. Second, as restructuring costs increase, the cost of issuing new debt increases, making it optimal to issue less debt at restructuring dates. The figure shows that the first effect dominates for low values of q while the second effect dominates for large values of q.

Insert Figure 3 Here

The leverage levels reported in the figure are consistent with those of prior studies. In our base case environment, the optimal leverage at refinancing points in the model with search is 27.36%. This value lies between the optimal leverage ratio in a dynamic model with perfectly liquid bond markets (in which firms issue debt conservatively as the supply of credit is certain) and the optimal leverage ratio in a static model (in which firms issue debt aggressively as they will not be able to issue additional debt in the future).

Panel A of Figure 3 investigates the effects of credit market frictions on shareholders' default decision. In the model, the default policy that maximizes equity value balances the present value of the cash flows that shareholders receive in continuation with the cash flow that they receive in liquidation. By reducing the value of shareholder's restructuring options (and therefore the continuation value of equity), credit supply uncertainty leads shareholders to default earlier. The quantitative effect is however very small as the value of the restructuring options is small when the firm is close to default.[8]

6 Conclusion

Following Modigliani and Miller (1958), extant theoretical research in corporate finance generally assumes that capital markets are frictionless so that corporate behavior and capital availability depend solely on firm characteristics. This demand-driven approach has recently been called into question by a large number of empirical studies. These studies document the central role of supply conditions in credit markets in explaining corporate policy choices

[8] By contrast, He and Xiong (2012) and He and Milbradt (2012) develop models in which liquidity shocks in the secondary market for corporate bonds affect shareholders' default decision by increasing the cost of debt but have no bearing on restructuring strategies and optimal leverage, which are fixed exogenously.

and highlight the need for an improved understanding of the precise role of supply in firms' financing decisions.

This paper takes a first step in constructing a dynamic model of financing decisions with capital supply effects by considering a setup in which firms face uncertainty regarding their future access to credit markets. The model provides an explicit characterization of the optimal default and financing policies for a firm acting in the best interests of incumbent shareholders and shows that credit market frictions have first-order effects on corporate policy choices. The analysis in the paper yields a wide range of empirical implications relating supply conditions in the credit markets to firms' default risk, dynamic financing policies, the timing of security issues, creditor turnover, and the role of firm characteristics in shaping corporate policies. Overall, the analysis demonstrates that accounting for both demand and supply factors is critical to understanding firms' capital structure decisions.

A Equilibria with a fixed default threshold

Fix a default threshold $X_d(1) = 1/Z_d$ and denote by τ_d the corresponding default time. The scale invariance of the geometric Brownian motion and the linearity of the payoffs to the firm's stakeholders imply that the equilibrium value functions are homogeneous of degree one with respect to the firm's current coupon and cash flows. That is, there are functions $d(Z|Z_d), e(Z|Z_d), v(Z|Z_d)$ such that

$$D(X, C|\mathbf{P}) = Xd(Z|Z_d),$$
$$E(X, C|\mathbf{P}) = Xe(Z|Z_d),$$

and

$$V(X, C|\mathbf{P}) = E(X, C|\mathbf{P}) + D(X, C|\mathbf{P}) = Xv(Z|Z_d)$$

where $Z = C/X$. A direct calculation using well-known properties of geometric Brownian motion shows that the reduced form debt value is given by

$$d(Z|Z_d) = (Z/r) - (Z_d/r)(1 - r\phi/Z_d)(Z_d/Z)^\beta.$$

With a fixed default policy, maximizing the equity value is equivalent to maximizing firm value. It turns out that, for technical reasons, firm value maximization problem is easier to deal with. For this reason, whenever the fixed default policy case is considered in the Appendix, we will always study the latter problem. By definition, the equilibrium firm value is the solution to the stochastic control problem defined by

$$V(X, C|\mathbf{P}) = \sup_{a \in \mathbb{A}} E\left[\int_0^{\tau_d} e^{-rs}((1-\tau)X_s + \tau C_{s-})ds + \tilde{H}(a_s, X_s, C_{s-}|\mathbf{P})dN_s) + e^{-r\tau_d}\phi X_{\tau_d}\right] \tag{10}$$

where we have set

$$\tilde{H}(a, X, C|\mathbf{P}) = X\tilde{h}(a, Z|Z_d) = -\eta(V(X, aC|\mathbf{P}) - V(X, C|\mathbf{P})) - (1-\eta)qD(X, aC|\mathbf{P}).$$

The following result follows by direct calculation and allows to recast the equilibrium problem in terms of a single state variable.

Lemma 1 *The process $Z_t = C_t/X_t$ evolves according to*

$$dZ_t = -\mu Z_{t-}dt - \sigma Z_{t-}d\hat{B}_t + Z_{t-}(a_t - 1)dN_t \tag{11}$$

where the process $\hat{B}_t$ is a standard one dimensional Brownian motion under the equivalent *probability measure defined by*

$$\hat{P}(A) = E\left[e^{-\mu t}\frac{X_t}{X_0}1_{\{A\}}\right], \qquad \forall A \subseteq \mathscr{F}_t$$

Consequently, (10) *is equivalent to*

$$v(Z|Z_d) = \sup_{a\in\mathbb{A}} \hat{E}\left[\int_0^{\tau_d} e^{-(r-\mu)s}((1-\tau+\tau Z_{s-})ds + h(a_s, Z_{s-}|Z_d)dN_s) + e^{-(r-\mu)\tau_d}\phi\right]$$

and any solution to this equation satisfies the inequalities

$$0 \leq v(Z|Z_d) \leq \frac{1-\tau(1-Z_d)}{r-\mu}.$$

Now, standard dynamic programming arguments imply

Lemma 2 *Let τ_N denote the first jump of the Poisson process and define*

$$\mathscr{P}(v)(Z) = \max_{a\geq 1}(1-\eta)(v(aZ) - qd(aZ|Z_d)) + \eta v(Z).$$

Then the function $v(Z|Z_d)$ is the unique Borel-measurable, bounded function satisfying

$$v(Z|Z_d) = \hat{E}\bigg[\int_0^{\tau_d\wedge\tau_N} e^{-(r-\mu)t}\left(1-\tau+\tau Z_t\right)dt + 1_{\{\tau_d\leq\tau_N\}}e^{-(r-\mu)\tau_d}\phi \\ + 1_{\{\tau_d>\tau_N\}}e^{-(r-\mu)\tau_N}\mathscr{P}(v(\cdot|Z_d))(Z_{\tau_N-})\bigg] \tag{12}$$

We then have the following result.

Lemma 3 *The transformation that maps a function v into the right-hand side of equation* (12) *is a contraction in the space $L_\infty[0, Z_d]$ of essentially bounded measurable functions and has a unique fixed point that belongs to the space $C[0, Z_d]$ of continuous functions on $[0, Z_d]$. Consequently, $v(Z|Z_d)$ is this unique fixed point.*

Proof. Let $A(v)$ denote the operator in the statement. Using the fact that τ_N is independent of the Brownian motion and exponentially distributed with parameter λ we obtain

$$A(v)(Z) = \hat{E}\bigg[\int_0^{\tau_d} e^{-(r-\mu+\lambda)t}\left(1-\tau+\tau Z_t^0 + \lambda\mathscr{P}(v)(Z_s^0)\right)dt + e^{-(r-\mu+\lambda)\tau_d}\phi\bigg].$$

where the nonnegative process Z_t^0 evolves according to equation (11) with $a \equiv 1$. For any pair of continuous functions (v_1, v_2) and any $z \in [0, Z_d)$, let

$$a_i = \arg\max_{a\geq 1}\left(v_i(az) - qd(az)1_{\{a>1\}}\right)$$

and assume for simplicity that $a_i > 1$. Then, we have

$$\begin{aligned}
\mathscr{P}(v_1)(z) - \mathscr{P}(v_2)(z) &= (1-\eta)(v_1(a_1 z) - qd(a_1 z)) \\
&- (1-\eta)(v_2(a_2 z) - qd(a_2 z)) + \eta(v_1(z) - v_2(z)) \\
&\le (1-\eta)(v_1(a_1 z) - qd(a_1 z)) \\
&- (1-\eta)(v_2(a_1 z) - qd(a_1 z)) + \eta\|v_1 - v_2\|_{C[0,z_d]} \\
&= (1-\eta)(v_1(a_1 z) - v_2(a_1 z)) + \eta\|v_1 - v_2\|_{C[0,z_d]} \\
&\le \|v_1 - v_2\|_{C[0,z_d]}.
\end{aligned}$$

and interchanging the roles of v_1 and v_2 we get that

$$|\mathscr{P}(v_1)(z) - \mathscr{P}(v_2)(z)| \le \|v_1 - v_2\|_{C[0,Z_d]}.$$

This immediately implies that

$$\|A(v_1) - A(v_2)\|_{C[0,Z_d]} \le \frac{\lambda\|v_1 - v_2\|_{C[0,Z_d]}}{r - \mu + \lambda}.$$

and the desired result now follows from the fact that $r - \mu > 0$ by assumption. □

Lemma 4 *The map $A(v)$ is monotone increasing in v, and monotone decreasing in r, η, ω and q for any nonnegative function v.*

Proof. To prove monotonicity in v it suffices to show that $\mathscr{P}$ is increasing in v. This is obvious because $\eta \in (0,1)$. Monotonicity in r and q is also clear. To prove monotonicity in η, it suffices to show that the operator $\mathscr{P}$ is monotone decreasing in η. Fix $z \ge 0$ and consider

$$G(\eta) = \max_{a \ge 1} \left\{ \eta v(z) + (1-\eta)(v(az) - qd(az)1_{\{a>1\}}) \right\}$$

If the maximum is attained for some $a > 1$ then we clearly must have $v(az) - qd(az) > v(z)$. This in turn implies that we have

$$G(\eta) = \max\{v(z), k(\eta)\}$$

with

$$k(\eta) = \max_{a>1} \left\{ \eta v(z) + (1-\eta)(v(az) - qd(az))1_{\{v(az)-qd(az)>v(z)\}} \right\}$$

and the desired result follows by noting that $k(\eta)$ is monotone decreasing. □

Lemma 5 *The firm value function $v(Z)$ is monotone increasing in λ, μ and monotone decreasing in q, η, r and ω.*

Proof. Pick an arbitrary bounded function $v_0 \in C[0, Z_d]$ and denote by A^n the n-th iteration of A so that $v = \lim_n A^n(v_0)$. Let α be a parameter with respect to which the operator A is increasing in the sense that

$$A(v_0; \alpha_1) < A(v_0; \alpha_2), \qquad \forall \alpha_1 < \alpha_2 \ , \ \forall v_0 \in C[0, Z_d]$$

Since A is increasing in v, a simple induction argument implies that we have

$$A^n(v_0; \alpha_1) < A^n(v_0; \alpha_2), \qquad \forall \alpha_1 < \alpha_2, n \geq 1.$$

Sending $n \to \infty$ shows that v is increasing in α and monotonicity in all parameters except λ now follows from Lemma 4. Finally, monotonicity in λ follows from that in η because the firm value function depends on λ only through $\lambda(1 - \eta)$. $\square$

Lemma 3 constructs the value function as a continuous fixed point of a non-linear map. We will now show that it is in fact in $C^2[0, Z_d]$ and solves the corresponding HJB equation. To this end, we will first need the following lemma. Let

$$\mathscr{L} f(Z) \equiv -\mu Z f'(Z) + \frac{1}{2}\sigma^2 Z^2 f''(Z)$$

be the continuous part of the $\hat{P}-$generator of the state variable and denote by $\psi < 0 < 1 < \psi_1$ the solutions to the quadratic equation $Q(x; r + \lambda(1 - \eta)) = 0$

Lemma 6 *Let $f(Z)$ be a bounded and Borel measurable function. The unique bounded solution to*

$$(r - \mu + \lambda)Y(Z) = \mathscr{L}Y(Z) + f(Z), \qquad Z \in [0, Z_d),$$

such that $Y(Z_d) = \phi$ is explicitly given by

$$Y(Z) = y_1 \, Z^{1-\psi} - \frac{Z^{1-\psi_1}}{\sigma^2/2} \int_0^Z \frac{f(x)}{\psi_1 - \psi} x^{\psi_1 - 2} dx - \frac{Z^{1-\psi}}{\sigma^2/2} \int_Z^{Z_d} \frac{f(x)}{\psi_1 - \psi} x^{\psi - 2} \, dx$$

with

$$y_1 = Z_d^{\psi-1} \left(\frac{Z_d^{1-\psi_1}}{\sigma^2/2} \int_0^{Z_d} \frac{f(x)}{\psi_1 - \psi} x^{\psi_1 - 2} dx - \phi \right)$$

In particular, the derivative $Y'(Z_d)$ depends continuously on Z_d and $f(Z)$ in the $L_\infty[0, Z_d]-$topology.

This existence and uniqueness result immediately allows us to establish the required regularity of the firm value function.

Lemma 7 *Let*

$$\mathscr{O}(v)(Z) = \max_{a\geq 1}\left(v(aZ) - qd(aZ)1_{\{a>1\}} - v(Z)\right) \tag{13}$$

For a fixed default threshold the equilibrium firm value function $v(Z|Z_d)$ *is the unique* $C^2[0, Z_d]$ *solution to the HJB equation*

$$(r-\mu)v(Z) = \mathscr{L}v(Z) + 1 - \tau + \tau Z + \lambda(1-\eta)\mathscr{O}(v)(Z) \tag{14}$$

with boundary condition $v(Z_d|Z_d) = \phi$.

Proof. Let $Y \in C^2[0, Z_d]$ be the unique bounded solution to

$$(r-\mu+\lambda)Y(Z) = \mathscr{L}Y(Z) + 1 - \tau + \tau Z + \lambda\mathscr{P}(v(\cdot|Z_d))(Z), \tag{15}$$

such that $Y(Z_d) = \phi$ whose existence is provided by Lemma 6. Then, standard arguments based on Itô's lemma imply that

$$Y(Z) = E_0\left[\int_0^{\tau_d} e^{-(r-\mu+\lambda)t}\left(1-\tau+\tau Z_t^0 + \lambda\mathscr{P}(v)(Z_{t-}^0)\right)dt + e^{-(r-\mu+\lambda)\tau_d}\phi\right]$$

and it now follows from Lemma 2 that $Y(Z) = v(Z|Z_d)$. Given this identity a direct calculation implies that (15) is equivalent to (14). $\square$

Lemma 8 *Suppose that* $\mathscr{A} : C[0, Z_d] \to C[0, Z_d]$ *is a contraction mapping which is monotone in the sense that* $v_1 \leq v_2$ *implies* $\mathscr{A}v_1 \leq \mathscr{A}v_2$*, and denote by* $v \in C[0, Z_d]$ *its unique fixed point. If* $w \in C[0, Z_d]$ *is such that* $w \leq \mathscr{A}w$ *then* $w \leq v$*. Similarly, if* $w \geq \mathscr{A}w$ *then* $w \geq v$.

Proof. Monotonicity of A together with $w \geq Aw$ implies $w \geq A^n w$ for any $n \geq 1$. Therefore, $w \geq \lim_{n\to\infty} A^n w = v$ and the claim follows. $\square$

The following lemma directly implies the results of Propositions 2 and 4 and will be of repeated use in what follows.

Let

$$Z_d^{\mathrm{nr}} \equiv \frac{\beta-1}{\beta}\frac{r}{r-\mu}.$$

Lemma 9 *Let* $\hat{v}(Z|Z_d)$ *be the value of a firm that never restructures its debt and defaults at the stopping time* τ_d*. Then* $v(Z|Z_d) \geq \hat{v}(Z|Z_d) > \phi$ *for all* $Z \in [0, Z_d)$ *and we have*

1. *If* $q \geq \tau$ *then* $v(Z|Z_d) \equiv \hat{v}(Z|Z_d)$*, the unique equilibrium threshold is given by* Z_d^{nr} *and the equity value function satisfies* $e_Z(Z_d|Z_d) > 0$ *for all* $Z_d > Z_d^{\mathrm{nr}}$;

2. *If* $q < \tau$ *then* $v(Z|Z_d^{\rm nr}) \not\equiv \hat{v}(Z|Z_d^{\rm nr})$ *and the equity value function satisfies* $e_Z(Z_d|Z_d) < 0$ *for all* $Z_d < Z_d^{\rm nr}$*. In particular, we have* $Z_d \geq Z_d^{\rm nr} > 1$ *in any equilibrium.*

Proof. Since $v(Z|Z_d)$ is the value function of a firm following an optimal policy, it dominates the sub-optimal policy of never restructuring. The value of such a firm is

$$\hat{v}(Z|Z_d) = \phi_0 + \frac{\tau Z}{r} - (\phi_0\omega + \tau Z_d/r)\,(Z/Z_d)^{1-\beta} \tag{16}$$

and satisfies

$$\hat{v}(Z|Z_d) - \phi = \omega\phi_0\left[1 - (Z/Z_d)^{1-\beta}\right] + \frac{\tau}{r}Z\left[1 - (Z/Z_d)^{-\beta}\right] > 0\,. \tag{17}$$

for all $Z < Z_d$ since $\beta < 0$. Let $\tilde{v}(Z|Z_d) \equiv \hat{v}(Z|Z_d) - qd(Z|Z_d)$ so that

$$\tilde{v}'(Z|Z_d) = \frac{\tau - q}{r}\left[1 - (1-\beta)(Z_d/Z)^{\beta}\right] - \phi_0(\omega + q(1-\omega))(1-\beta)Z_d^{-1}(Z_d/Z)^{\beta}. \tag{18}$$

and assume first that $q \geq \tau$. To prove that $v(Z|Z_d) \equiv \hat{v}(Z|Z_d)$ we need to show that

$$\mathscr{O}(\hat{v}(\cdot|Z_d)) \equiv 0. \tag{19}$$

We have $\tilde{v}'(0|Z_d) = (\tau - q)/r \leq 0$ and since

$$\tilde{v}''(Z|Z_d) = \beta(\beta-1)Z_d^{\beta-1}Z^{-\beta-1}\left(\frac{q-\tau}{r}Z_d - \phi_0(\omega + q(1-\omega))\right)$$

does not change sign we have that the function $\tilde{v}(Z|Z_d)$ is either convex, or concave and decreasing. If $\tilde{v}(Z|Z_d)$ is decreasing then (19) clearly holds. On the other hand, if $\tilde{v}(Z|Z_d)$ is convex then (17) implies that we have

$$\max_{y\in(Z,Z_d]} \tilde{v}(y|Z_d) = \max\{\tilde{v}(Z|Z_d)\,,\,\tilde{v}(Z_d|Z_d)\} = \max\{\tilde{v}(Z|Z_d)\,,\,(1-q)\phi\} < \hat{v}(Z|Z_d)$$

and (19) follows. To complete the proof of the first part, set $q = 1$ in (18) to obtain

$$e_Z(Z_d|Z_d) = \frac{\tau - 1}{r}\beta - (1-\beta)Z_d^{-1}\phi_0.$$

This shows that $e_Z(Z_d|Z_d)$ is positive for $Z_d > Z_d^{\rm nr}$ and negative for $Z_d < Z_d^{\rm nr}$ and implies that the desired result. Consequently, $e(Z|Z_d^{\rm nr})$ is C^1 and satisfies the HJB equation

$$\max\{-(r-\mu)e(Z|Z_d^{\rm nr}) + \mathscr{L}e(Z|Z_d^{\rm nr}) + (1-\tau)(1-Z) + \lambda\,(1-\eta)\mathscr{O}(e(Z|Z_d^{\rm nr}) + d(Z|Z_d^{\rm nr})), -e(Z|Z_d^{\rm nr})\} \;=\; 0\,.$$

Standard verification results for optimal stopping problems (see, e.g., Dayanik and Karatzas

(2003)) combined with Lemma 7 implies that $Z_d^{\rm nr}$ is indeed the optimal default boundary.

Let now $q < \tau$ and suppose on the contrary that $v(Z|Z_d) \equiv \hat{v}(Z|Z_d)$. To reach a contradiction, it suffices to show that $\mathscr{O}(\hat{v}) \not\equiv 0$. By (16) we have that

$$\tilde{v}(Z|Z_d) = \phi_0 + \frac{\tau - q}{r} - \tilde{a} Z^{1-\beta}\,, \tag{20}$$

where

$$\tilde{a} = -a - (q/r)(1 - r\phi/Z_d)\, Z_d^{\beta} = \frac{\tau - q}{r} Z_d^{\beta} + \phi_0(\omega + q(1-\omega)) Z_d^{\beta-1} > 0.$$

It follows that the function $\tilde{v}(Z|Z_d)$ is concave and therefore attains a global maximum at the unique point Z_o such that

$$\tilde{v}_Z(Z_o|Z_d) = 0 \Longleftrightarrow \frac{(\tau - q)Z_o}{r} = \tilde{a}\,(1-\beta)\, Z_o^{1-\beta}\,.$$

Substituting this identity into (20) gives

$$\max_{y \ge 0} \tilde{v}(y|Z_d) = \tilde{v}(Z_o|Z_d) = \phi_0 + \frac{(\tau - q)Z_o}{r} - \frac{(\tau - q)Z_o}{r(1-\beta)} > \phi_0 = \hat{v}(0|Z_d)$$

and it follows that $\mathscr{O}(\hat{v})(0) > 0$ which is a contradiction. To prove the remaining claims in the statement we will use the fact that by definition $e(Z|Z_d) \ge \hat{e}(Z|Z_d)$. By the first part of the statement we have $\hat{e}_Z(Z|Z_d) < 0$ for all $Z_d < Z_d^{\rm nr}$ and it follows that

$$e(Z|Z_d) > \hat{e}(Z|Z_d) > \epsilon(Z_d - Z)$$

for some $\epsilon > 0$ in a left neighborhood of $Z_d < Z_d^{\rm nr}$ since $e(Z_d|Z_d) = 0$. This immediately implies that $e_Z(Z_d|Z_d) < 0$ for all $Z_d < Z_d^{\rm nr}$ which is what had to be proved. □

In order to prove existence of rational expectations equilibria, we will need an auxiliary construction. Let $\tilde{e}(Z|\bar{Z}_d)$ be the equity value for a firm whose debt-holders price the debt using $\tilde{d}(Z)$, whereas the firm actually defaults when $Z = \bar{Z}_d$.

We will prove several properties of this function. The first is provided in the following lemma.

Lemma 10 *$\tilde{e}(Z|\bar{Z}_d)$ is C^2 in Z on $[0, \bar{Z}_d]$ and $e_Z(\bar{Z}_d|\bar{Z}_d)$ is continuous in $Z_d, \bar{Z}_d$.*

Proof. The claim follows from Lemma 6 is complete analogy with the proof of Lemma 7. □

Lemma 11 *We have $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) < 0$ for $\bar{Z}_d \le Z_d^{\rm nr}$.*

Proof. Since the optimal equity value function dominates the equity value value of a firm that never restructures, we have that $\tilde{e}(Z|\bar{Z}_d) >$ is positive for all $Z \le \bar{Z}_d \le Z_d^{\mathrm{nr}}$ and therefore $\tilde{e}_Z(Z_d|Z_d) < 0$ for all $Z_d < Z_d^{\mathrm{nr}}$. $\square$

Lemma 12 *If $\bar{Z}_d^1 > \bar{Z}_d^2$ and $\tilde{e}(\bar{Z}_d^2|\bar{Z}_d^1) \ge 0$. Then, $\tilde{e}(Z|\bar{Z}_d^1) \ge \tilde{e}(Z|\bar{Z}_d^2)$ for all $Z \le \bar{Z}_d^2$.*

Proof. Let $\tilde{d}(Z)$ be the function that the debt-holders use to value debt. Denote by $\tau_{d,i}$ the first time that the process Z_t hits $\bar{Z}_d^i$ and observe that $\tau_{d,1} > \tau_{d,2}$. Then, it follows directly by standard dynamic programming arguments that

$$\begin{aligned}
e(Z|Z_d^1) &= \hat{E}\bigg[\int_0^{\tau_{d,2}\wedge\tau_N} e^{-\rho t}(1-\tau)(1-Z_t)dt + 1_{\{\tau_{d,2}\le\tau_N\}}e^{-\rho\tau_{d,2}}\tilde{e}(\bar{Z}_d^2|\bar{Z}_d^1) \\
&\quad + 1_{\{\tau_{d,2}>\tau_N\}} \max_{a\in[1,\bar{Z}_d^1/Z_{\tau_N -}]} e^{-\rho\tau_N}\big[(1-\eta)\big(\tilde{v}(aZ_{\tau_N -}|Z_d^1) - q1_{\{a>1\}}\tilde{d}(aZ_{\tau_N -}) - \tilde{d}(Z_{\tau_N -})\big) \\
&\quad + \eta e(Z_{\tau_N -}|Z_d^1)\big)\bigg] \\
&\ge \hat{E}\bigg[\int_0^{\tau_{d,2}\wedge\tau_N} e^{-\rho t}(1-\tau)(1-Z_t)dt \\
&\quad + 1_{\{\tau_{d,2}>\tau_N\}} \max_{a\in[1,\bar{Z}_d^2/Z_{\tau_N -}]} e^{-\rho\tau_N}\big((1-\eta)\big(\tilde{v}(aZ_{\tau_N -}|Z_d^1) - q1_{\{a>1\}}\tilde{d}(aZ_{\tau_N -}) - \tilde{d}(Z_{\tau_N -})\big) \\
&\quad + \eta e(Z_{\tau_N -}|Z_d^1)\big)\bigg]
\end{aligned} \tag{21}$$

for all $Z \in [0, \bar{Z}_d^2]$ where $\rho = r - \mu$ and $\tilde{v}(z|Z_d) = \tilde{e}(z, |Z_d) - \tilde{d}(z)$ denotes the corresponding firm value function. Note that we only take the maximum over $a \in [1, \bar{Z}_d^1/Z_{\tau_N -}]$ because, by assumption, the firm always defaults when $Z \ge \bar{Z}_d^1$. Denote the map on the right-hand side of (21) by $\mathscr{A}$. The same arguments as in the proof of Lemma 3 imply that the operator $\mathscr{A}$ is a monotone contraction and the required assertion now follows from Lemma 8 since we have $\tilde{e}(Z|\bar{Z}_d^1) \ge \mathscr{A}\tilde{e}(Z|\bar{Z}_d^1)$, and $\tilde{e}(Z|\bar{Z}_d^2) = \mathscr{A}\tilde{e}(Z|\bar{Z}_d^2)$ by the above. $\square$

Lemma 13 *Fix an arbitrary $\bar{Z}_d > 0$ and suppose that $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) < 0$. Then, $\tilde{e}(Z|\bar{Z}_d)$ is monotone increasing in $\bar{Z}_d$ for $\bar{Z}_d$ in a left neighborhood of $\bar{Z}_d$. Similarly, if $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) > 0$ then $\tilde{e}(Z|\bar{Z}_d)$ is monotone decreasing in $\bar{Z}_d$ for $\bar{Z}_d$ in a left neighborhood of $\bar{Z}_d$.*

Proof. The first claim follows directly from Lemma 12 because, by assumption $\tilde{e}(Z_d^2|Z_d^1) > \tilde{e}(Z_d^1|Z_d^1) = 0$ for any $Z_d^1 > Z_d^2$ that are sufficiently close to $\bar{Z}_d$. The proof of the second claim is analogous. $\square$

The following result is a direct consequence of Lemma 13.

Lemma 14 *If $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) < 0$ for all $\bar{Z}_d > 0$ let $Z_d^* = \infty$. Otherwise, set*

$$Z_d^* \equiv \min\{Z_d > Z_d^{\text{nr}} : \tilde{e}_Z(Z_d|Z_d) = 0\}$$

Then, $\tilde{e}(Z|Z_d^*) > 0$ for all $Z \le Z_d^*$.

Proof. By Lemma 10, $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d)$ is continuous and therefore, by Lemma 11, $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) < 0$ for all $\bar{Z}_d < Z_d^*$. By Lemma 13, $\tilde{e}(Z|\bar{Z}_d)$ is monotone increasing in $\bar{Z}_d \in [0, Z_d^*)$ and therefore $\tilde{e}(Z|Z_d^*) > 0$ for all $Z < Z_d^*$ and $e_Z(Z_d^*|Z_d^*) = 0$. □

Lemma 15 *We have*

$$(1-\tau)(1-Z) + \max_{a\ge 1}\left((1-q\mathbf{1}_{a>1})\tilde{d}(aZ) - \tilde{d}(Z)\right) \;<\; 0$$

for all $Z > Z_d^*$.

Proof. We have

$$\begin{aligned}
0.5\sigma^2\tilde{e}_{ZZ}(Z_d^*|Z_d^*) \;&=\; (r-\mu+\lambda(1-\eta))\tilde{e}(Z_d^*|Z_d^*) + \mu Z_d^*\tilde{e}_Z(Z_d^*|Z_d^*) - (1-\tau)(1-Z_d^*) \\
&- \max_{a\ge1}\left(\tilde{e}(aZ_d^*|Z_d^*) + (1-q\mathbf{1}_{a>1})\tilde{d}(aZ_d^*) - \tilde{d}(Z_d^*)\right) \\
&=\; -\left((1-\tau)(1-Z_d^*) + \max_{a\ge1}\left((1-q\mathbf{1}_{a>1})\tilde{d}(aZ_d^*) - \tilde{d}(Z_d^*)\right)\right)
\end{aligned} \tag{22}$$

because $\tilde{e}(aZ_d^*|Z_d^*) = 0$ for all $a \ge 1$. Since $\tilde{e}(Z|Z_d^*) > 0$ for all $Z \in [0, Z_d^*)$ and $\tilde{e}(Z_d^*|Z_d^*) = \tilde{e}_Z(Z_d^*|Z_d^*) = 0$, we get that $\tilde{e}_{ZZ}(Z_d^*|Z_d^*) \ge 0$. Consequently,

$$(1-\tau)(1-Z_d^*) + \max_{a\ge1}\left((1-q\mathbf{1}_{a>1})\tilde{d}(aZ_d^*) - \tilde{d}(Z_d^*)\right) \;\le\; 0$$

and the claim follows from the fact that, by assumption, $(1-\tau)(1-Z) + \max_{a\ge1}((1-q\mathbf{1}_{a>1})\tilde{d}(aZ) - \tilde{d}(Z))$ is monotone decreasing in Z. □

Proof of Proposition 1. To prove the result, it suffices to show that $\tilde{e} = \tilde{e}(Z|Z_d^*)$ is the value function of thee firm. Standard verification results for optimal stopping (see, e.g., Dayanik and Karatzas (2003)) combined with the arguments from the proof of Lemma 7 imply that it suffices to verify that $\tilde{e}(Z|Z_d^*)$ satisfies the HJB equation

$$\begin{aligned}
&\max\{-(r-\mu+\lambda(1-\eta))\tilde{e}(Z) + \mathscr{L}\tilde{e}(Z) + (1-\tau)(1-Z) \\
&+ \lambda\,(1-\eta)\max_{a\ge1}\left(\tilde{e}(aZ) + (1-q\mathbf{1}_{a>1})\tilde{d}(aZ) - \tilde{d}(Z)\right), -\tilde{e}(Z)\} \;=\; 0\,.
\end{aligned}$$

The arguments from the proof of Lemma 7 imply that

$$-(r-\mu+\lambda(1-\eta))\tilde{e}(Z)+\mathscr{L}\tilde{e}(Z)+(1-\tau)(1-Z)+\lambda\,(1-\eta)\max_{a\ge1}\left(\tilde{e}(aZ)+(1-q\mathbf{1}_{a>1})\tilde{d}(aZ)-\tilde{d}(Z)\right) \;=\; 0$$

for all $Z \in [0, Z_d^*]$ and, by Lemma 14, $\tilde{e} \geq 0$. For $Z > Z_d^*$, $\tilde{e}(Z) = 0$ and hence, by Lemma 15,

$$\begin{aligned}
&-(r-\mu+\lambda(1-\eta))\tilde{e}(Z) + \mathscr{L}\tilde{e}(Z) + (1-\tau)(1-Z) \\
&+ \lambda\,(1-\eta)\max_{a\geq 1}\,(\tilde{e}(aZ) + (1-q\mathbf{1}_{a>1})\tilde{d}(aZ) - \tilde{d}(Z)) \\
&= \;(1-\tau)(1-Z) + \lambda\,(1-\eta)\max_{a\geq 1}\,(\tilde{e}(aZ) + (1-q\mathbf{1}_{a>1})\tilde{d}(aZ) - \tilde{d}(Z)) < 0\,.
\end{aligned}$$

The proof is complete. □

Lemma 16 *We have $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) < 0$ for all $\bar{Z}_d > Z_d^*$. Hence, Z_d^* is the unique solution $\bar{Z}_d$ to $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) = 0$.*

Proof. Suppose the contrary. Then, there exists a $\bar{Z}_d > Z_d^*$ such that $\tilde{e}_Z(\bar{Z}_d|\bar{Z}_d) = 0$. By Lemma 15 and (22), we have

$$0.5\sigma^2\tilde{e}_{ZZ}(\bar{Z}_d|\bar{Z}_d) \;=\; -((1-\tau)(1-\bar{Z}_d) + \max_{a\geq 1}\,((1-q\mathbf{1}_{a>1})\tilde{d}(a\bar{Z}_d) - d(\bar{Z}_d))) \;>\; 0\,.$$

Therefore, $\tilde{e}(Z|\bar{Z}_d) > 0 = \tilde{e}(Z|Z_d^*)$ for Z sufficiently close to $\bar{Z}_d$. But this is impossible because, by Proposition 1, defaulting at Z_d^* is optimal. □

Finally, the next result shows that the debt function for a barrier default policy does satisfy the required monotonicity condition.

Lemma 17 *The function*

$$(1-\tau)(1-Z) + \max_{a\geq 1}\,((1-q\mathbf{1}_{a>1})d(aZ|Z_d) - d(Z|Z_d))$$

is strictly monotone decreasing in Z.

Proof. Clearly, it suffices to show that $\max_{a\geq 1}((1-q\mathbf{1}_{a>1})d(aZ|Z_d) - d(Z|Z_d))$ is non-increasing in Z. A direct calculation shows that $d(Z|Z_d)$ is either increasing in Z or is convex and attains a maximum at some $Z_m < Z_d$. In the first case,

$$\max_{a\geq 1}\,((1-q\mathbf{1}_{a>1})d(aZ|Z_d) - d(Z|Z_d)) \;=\; \max\{0, (1-q)d(Z_d|Z_d) - d(Z|Z_d)\}$$

is obviously non-increasing. In the second case,

$$\max_{a\geq 1}\,((1-q\mathbf{1}_{a>1})d(aZ|Z_d) - d(Z|Z_d)) \;=\; \max\{0, (1-q)d(Z_m|Z_d) - d(Z|Z_d)\}$$

is also non-increasing. □

The next result proves Proposition 3. Namely, it shows that, given a fixed barrier default policy, the optimal restructuring policy is of barrier type.

Lemma 18 *Either restructuring is not optimal or there exist* $0 < Z_u \leq Z_o < Z_d$ *such that*

$$Z_o = \underset{Z \in [Z_u, Z_d]}{\operatorname{argmax}} \{v(Z|Z_d) - q\, d(Z|Z_d)\}$$

and

$$\mathscr{O}(v(\cdot|Z_d))(Z) = 1_{\{Z<Z_u\}} \{v(Z_o|Z_d) - qd(Z_o|Z_d) - v(Z|Z_d)\} \; .$$

Proof. Assume that restructuring is optimal and let $q > 0$. The case $q = 0$ can be treated similarly. To simplify the notation we fix the default threshold Z_d and write $v(Z) = v(Z|Z_d)$ and $d(Z) = d(Z|Z_d)$. Since it is optimal for the firm to restructure its capital structure at some point we know that the operator $\mathscr{O}(v)$ is not zero and it follows that

$$Z_u \equiv \max\{Z \geq 0 : \mathscr{O}(v)(Z) > 0\}$$

is well-defined and smaller or equal to Z_d. Furthermore, by continuity we have

$$\mathscr{O}(v)(Z_u) = 0 \Longleftrightarrow v(Z_u) = \max_{y \geq Z_u} \{v(y) - qd(y)\} > 0.$$

Now consider the higher threshold defined by

$$Z_o = \min\{y \geq Z_u : v(Z_u) = v(y) - qd(y)\}. \tag{23}$$

By Lemma 9 we have $v(Z_d) = \phi < v(Z_u) = v(Z_o) - qd(Z_o) < v(Z_o)$ and therefore $Z_u < Z_o < Z_d$ since issuance costs are strictly positive. This in turn implies that the point Z_o is a local maximum of the function $\bar{v}(Z) = v(Z) - qd(Z)$ and since $v(Z_u) \leq v(Z)$ for $Z \in [Z_u, Z_o]$ by definition of the threshold Z_u we necessarily have that $v'(Z_u) \geq 0$.

Before carrying on with the rest of the proof we start by proving that $\bar{v}(Z)$ does not have admit any local maximum such that $\bar{v}(Z) > v(Z_u)$ on the interval $[0, Z_o]$. Suppose the contrary and let $\bar{Z}_n < Z_o$ denote the location of the largest such local maximum. Since the point Z_o is a local maximum of $\bar{v}(Z)$ this implies, as illustrated by the left panel of Figure A, that the function $\bar{v}(Z)$ achieves a local minimum at some point $\bar{Z}_m \in [\bar{Z}_n, Z_o]$ such that

$$\bar{v}(\bar{Z}_m) < \bar{v}(Z_o) = v(Z_u) < \bar{v}(\bar{Z}_n). \tag{24}$$

This in turn implies that we have $\mathscr{O}(v)(\bar{Z}_n) = 0$ and combining this with the fact that the functions d and v solve

$$(r-\mu)d(Z) = \mathscr{L}d(Z) + Z$$
$$(r-\mu)v(Z) = \mathscr{L}v(Z) + 1 - \tau(1-Z) + \lambda(1-\eta)\mathscr{O}(v)(Z)$$

we finally obtain

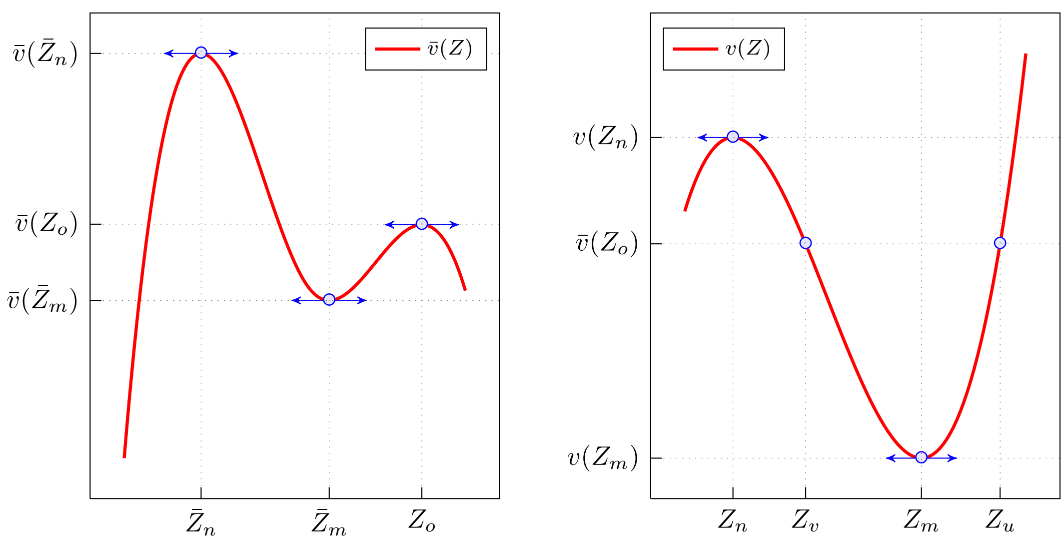
$$\begin{aligned}(r-\mu)\bar{v}(Z_o) &> (r-\mu)\bar{v}(\bar{Z}_m) \\ &= \mathscr{L}v(\bar{Z}_m) + 1 - \tau + (\tau-q)\bar{Z}_m + \lambda(1-\eta)\mathscr{O}(v)(\bar{Z}_m) \\ &\geq \mathscr{L}v(\bar{Z}_m) + 1 - \tau + (\tau-q)\bar{Z}_m \\ &\geq 1 - \tau + (\tau-q)\bar{Z}_m > 1 - \tau + (\tau-q)\bar{Z}_n \\ &\geq \mathscr{L}v(\bar{Z}_n) + 1 - \tau + (\tau-q)\bar{Z}_n \\ &= \mathscr{L}v(\bar{Z}_n) + 1 - \tau + (\tau-q)\bar{Z}_n + \lambda(1-\eta)\mathscr{O}(v)(\bar{Z}_n) = (r-\mu)\bar{v}(Z_n)\end{aligned}$$

where the second inequality follows from the nonnegativity of $\mathscr{O}$, and the third and fifth inequalities follow from the fact that $v'(Z) = 0$ and $v''(Z) \leq 0$ (resp. ≥ 0) at a local maximum (resp. local minimum). This contradicts equation (24) and therefore establishes our claim regarding the local maxima of the function $\bar{v}(Z)$. To complete the proof we now need to establish that $v(Z) \leq v(Z_u)$ on $[0, Z_u]$. Suppose that this is not the case, let

$$Z_v = \max\{Z \leq Z_u : v(Z) = v(Z_u) = v(Z_o) - qd(Z_o)\}$$

and assume for simplicity that $Z_v < Z_u$ so that the function $v(Z)$ reaches a local minimum

Figure A: Shape of the functions $\bar{v}(Z)$ and $v(Z)$ in the proof of Lemma 18



at some point $Z_m \in [Z_v, Z_u]$.[9] As a first step towards a contradiction we claim that the function $v(Z)$ is monotone decreasing on $[0, Z_v]$. If not then as illustrated by the right panel of Figure A there is a point $Z_n \in [0, Z_v]$ at which the function $v(Z)$ achieves a local maximum such that

$$v(Z_n) > v(Z_v) = v(Z_u) = v(Z_o) - qd(Z_o) = \max_{y \geq Z_n} \bar{v}(y)$$

where the last equality follows from the first part of the proof. This immediately implies that we have $\mathscr{O}(v)(Z_n) = 0$ and combining this property with the same arguments as in the first part of the proof then gives

$$\begin{aligned}
(r-\mu)v(Z_m) &= \mathscr{L}v(Z_m) + 1 - \tau(1 - Z_m) + \lambda(1-\eta)\mathscr{O}(v)(Z_m) \\
&\geq \mathscr{L}v(Z_m) + 1 - \tau(1 - Z_m) \\
&\geq 1 - \tau(1 - Z_m) \\
&> \ 1 - \tau(1 - Z_n) \\
&\geq \mathscr{L}v(Z_n) + 1 - \tau(1 - Z_n) \\
&= \mathscr{L}v(Z_n) + 1 - \tau(1 - Z_n) + \lambda(1-\eta)\mathscr{O}(v)(Z_n) = (r-\mu)v(Z_n)
\end{aligned}$$

which contradicts equation (**??**) and therefore establishes our claim regarding the monotonicity of $v(Z)$ on the interval $[0, Z_v]$. Combining this property with the fact that $v(0) = \phi_0$ we immediately get that $\phi_0 > v(Z)$ on $(0, Z_o)$ but this is impossible since $v(Z) \geq \hat{v}(Z) \geq \phi_0$ in a right neighborhood of zero by Lemma 9. ☐

Proof of Proposition 5. Proposition 5 follows directly from Lemmas 18 and 16. ☐

B The case $q = 0$ and the general existence result

Proof of Proposition 6. Let $\lambda^* = \lambda(1-\eta)$. It follows from equation (14) and Lemma 18 that there are constants a_1, a_3, a_4 such that

$$v(Z|Z_d) = v_s(Z, Z_o, a_1, a_3, a_4; q) \equiv \frac{1 - \tau + \lambda^*(v(Z_o|Z_d) - qd(Z_o|Z_d))}{r - \mu + \lambda^*} + \frac{\tau Z}{r + \lambda^*} + a_1 Z^{1-\psi}$$

for all $Z \in [0, Z_u]$, and

$$v(Z|Z_d) = v_{ns}(Z, a_3, a_4) \equiv \phi_0 + \frac{\tau Z}{r} + a_3 Z^{1-\beta} + a_4 Z^{1-\alpha}$$

[9] When the point Z_u is a local minimum of the function $v(Z)$ we have $Z_v = Z_u$. This case is completely analogous, up to small modifications.

for all $Z \in [Z_u, Z_d)$. Since $q = 0$ it follows immediately from (23) that we have $Z_o = Z_u$. Evaluating the first of the above identities at the point $Z = Z_o$ gives

$$v(Z_o|Z_d) = \frac{1 - \tau + \lambda^* v(Z_o)}{r - \mu + \lambda^*} + \frac{\tau Z_o}{r + \lambda^*} + a_1 Z_o^{1-\psi}$$

and solving this equation for $v(Z_o)$ we obtain

$$v(Z_o|Z_d) = \frac{r - \mu + \lambda^*}{r - \mu} \left(\frac{1 - \tau}{r - \mu + \lambda^*} + \frac{\tau Z_o}{r + \lambda^*} + a_1 Z_o^{1-\psi} \right).$$

Therefore, the value matching condition at the point Z_o can be written as

$$\phi_0 + \frac{\tau Z_o}{r} + a_3 Z_o^{1-\beta} + a_4 Z_o^{1-\alpha} = \frac{r - \mu + \lambda^*}{r - \mu} \left(\frac{1 - \tau}{r - \mu + \lambda^*} + \frac{\tau Z_o}{r + \lambda^*} + a_1 Z_o^{1-\psi} \right).$$

which is equivalent to

$$\frac{r - \mu + \lambda^*}{r - \mu} \left(\frac{\tau \mu \lambda^* Z_o}{r(r + \lambda^*)(r - \mu + \lambda^*)} + a_1 Z_o^{1-\psi} \right) = a_3 Z_o^{1-\beta} + a_4 Z_o^{1-\alpha}.$$

The facts that $Z_o = Z_u$ and that $v(Z|Z_d)$ is continuously differentiable with $v(Z_d) - \phi = v'(Z_o) = 0$ jointly imply the remaining free constants are determined by

$$\begin{aligned} \frac{\tau Z_o}{r + \lambda^*} + a_1 (1 - \psi) Z_o^{1-\psi} &= 0, \\ \frac{\tau Z_o}{r} + a_3 (1 - \beta) Z_o^{1-\beta} + a_4 (1 - \alpha) Z_o^{1-\alpha} &= 0, \\ \omega \phi_0 + \frac{\tau Z_d}{r} + a_3 Z_d^{1-\beta} + a_4 Z_d^{1-\alpha} &= 0. \end{aligned}$$

Combining these equations shows that we have

$$\tau / r = \frac{a_3 Z_o^{-\beta}}{\kappa_1(\lambda)} = \frac{a_4 Z_o^{-\alpha}}{\kappa_2(\lambda)} \tag{25}$$

$$f(J) = 1 + \kappa_1(\lambda) J^{-\beta} + \kappa_2(\lambda) J^{-\alpha} = -(r \omega \phi_0 / \tau) Z_d^{-1} \tag{26}$$

where the function f is defined as in the text and we have set $J = Z_d / Z_o$. Note that since $\kappa_1(\lambda) < 0 < \kappa_2(\lambda)$ we have that $a_3 \leq 0$ and $a_4 \geq 0$. In order to calculate the equilibrium, it remains to impose the smooth pasting condition which now takes the form

$$\tau + \tau(1 - \beta) \kappa_1(\lambda) J^{-\beta} + \tau(1 - \alpha) \kappa_2(\lambda) J^{-\alpha} = \beta + r(1 - \beta)(1 - \omega) \phi_0 / Z_d.$$

Substituting the value for Z_d we get the required equation (8) for J. Thus, if an equilibrium exists, it is given by the expressions from Proposition 3. It remains to show that the corresponding equation has a solution if and only if $\tau < \tau^*$ and that this solution is unique.

Since $\kappa_1(\lambda) < 0 < \kappa_2(\lambda)$ we have that f is decreasing. Therefore, the default threshold in (27) is positive if and only if we have $J > J_0$ where J_0 is the unique solution to (7). Let g be as in the text. A direct calculation shows that the function g diverges to $-\infty$ at ∞ and that

$$g(J_0) - \beta/r > 0 \Longleftrightarrow \tau < \tau^*$$

Thus, existence follows from the intermediate value theorem. To prove uniqueness, it suffices to show that g is decreasing for $J \geq J_0$. This is obvious if $1-\beta+\omega(\beta-\alpha) > 0$. Otherwise, g increases up to the point J_* where its derivative vanishes and decreases afterwards. Therefore, it suffices to show that we have $g(J_*) > 0$ but this follows from the fact that

$$g(J_*) = 1 + \left(1 + \frac{\alpha(1-\beta-\alpha\omega)(\alpha-\beta)}{\beta(\beta-1)}\right)\kappa_2(\lambda)J_*^{-\alpha} > 0$$

since $\alpha\omega + \beta < \alpha + \beta = 1 - 2\mu/\sigma^2 < 1$. □

In order to prove the general existence result of Theorem 1 for the model with search we will also need the following standard lemma.

Lemma 19 *Suppose that the function $f(Z,x)$ is continuous, monotone decreasing in x and satisfies $f(Z_1,x) > 0 > f(Z_2,x)$. Then, the minimal solution $Z_0(x) \in [Z_1, Z_2]$ to the equation $f(Z,x) = 0$ is monotone decreasing in x.*

Proof. Let $A(x) \equiv \{Z \in [Z_1, Z_2] : f(Z,x) \leq 0\}$. Then, the set $A(x)$ is a compact set and it is monotone increasing in x in the inclusion order. Therefore, $Z_0(x) = \min\{A(x)\}$ is monotone decreasing. □

Now, to prove uniqueness we will need the following generic non-degeneracy result. Let

$$\mathscr{C} = (\omega, \lambda, \eta, \mu, r-\mu, \tau, \theta) \in \mathbb{R}_+^6$$

denote the vector of model parameters and say that the vector $\mathscr{C}$ is admissible if its components are such that $\tau < \tau^*$ and $r > \mu$.

Lemma 20 *Consider the system*

$$\begin{aligned}
&F_1(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \equiv v_s(Z_u, Z_o, a_1, a_3, a_4; q) - v_{ns}(Z_u, a_3, a_4) = 0,\\
&F_2(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \equiv v_s'(Z_u, Z_o, a_1, a_3, a_4; q) - v_{ns}'(Z_u, a_3, a_4) = 0,\\
&F_3(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \equiv v_{ns}'(Z_o, a_3, a_4) - q\, d'(Z_o|Z_d) = 0,\\
&F_4(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \equiv v_s(Z_u, Z_o, a_1, a_3, a_4; q) - v_{ns}(Z_o, a_3, a_4) + qd(Z_o|Z_d) = 0,\\
&F_5(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \equiv v_{ns}(Z_d, a_3, a_4) - d(Z_d|Z_d) = 0,\\
&F_6(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \equiv v_{ns}'(Z_d, a_3, a_4) - d'(Z_d|Z_d) = 0.
\end{aligned}$$

Denote by $J(\mathscr{C})$ the unique solution to (8), define $z_o(\mathscr{C})$ and $z_d(\mathscr{C})$ by (26), (27) and let

$$\begin{aligned}
&\tilde{a}_1(\mathscr{C}) = -\tau z_o(\mathscr{C})/((r+\lambda^*)(1-\psi)),\\
&\tilde{a}_3(\mathscr{C}) = (\tau/r)\kappa_1(\lambda) z_o(\mathscr{C})^\beta,\\
&\tilde{a}_4(\mathscr{C}) = (\tau/r)\kappa_2(\lambda) z_o(\mathscr{C})^\alpha.
\end{aligned}$$

Suppose that there exists an admissible $\mathscr{C}$ such that

$$\mathscr{J} F(z_o(\mathscr{C}), z_o(\mathscr{C}), z_d(\mathscr{C}), \tilde{a}_1(\mathscr{C}), \tilde{a}_3(\mathscr{C}), \tilde{a}_4(\mathscr{C}); 0) \neq 0\,.$$

where $\mathscr{J}$ denotes the Jacobian operator. Then, for Lebesque almost every admissible $\mathscr{C}$ there exists an open neighborhood

$$\mathscr{B} \supseteq (z_o(\mathscr{C}), z_o(\mathscr{C}), z_d(\mathscr{C}), \tilde{a}_1(\mathscr{C}), \tilde{a}_3(\mathscr{C}), \tilde{a}_4(\mathscr{C}))$$

and an $\epsilon > 0$ such that, for all $q \in [0, \epsilon)$, there exists a unique rational expectations equilibrium in barrier strategies whose parameters satisfy $(Z_u, Z_o, Z_d, a_1, a_3, a_4; q) \in \mathscr{B}$.

Proof. The function $\mathscr{J} F(z_o(\mathscr{C}), z_o(\mathscr{C}), z_d(\mathscr{C}), \tilde{a}_1(\mathscr{C}), \tilde{a}_3(\mathscr{C}), \tilde{a}_4(\mathscr{C}); 0)$ is clearly real analytic in $\mathscr{C}$. Therefore, if it is not identically zero, it is non-zero for almost every $\mathscr{C}$. The last claim follows then from the implicit function theorem. $\square$

Proof of Theorem 1. Uniqueness of equilibrium in a small neighborhood of the $q = 0$ equilibrium for the case when q is small follows from Lemma 20. Continuous dependence of the equity value and its derivative on all model parameters follow from Lemma 6.

Suppose that $\tau < \tau^*$. In this case it follows from Proposition 6 and its proof that the exists a unique equilibrium default barrier $Z^*_{d,0}$ such that

$$e_Z(Z^*_{d,0}|Z^*_{d,0})\Big|_{q=0} = 0 < e_Z(Z_d|Z_d)\Big|_{q=0}, \qquad \forall Z_d > Z^*_{d,0}.$$

Consequently, $e(Z|Z_d)$ is negative for $Z < Z_d$ in a left neighborhood of Z_d when $q = 0$ and since equity value decreases with the issuance cost parameter we conclude that $e(Z|Z_d)$ is negative for $Z < Z_d$ in a left neighborhood of Z_d and all $q > 0$. Since $e(Z_d|Z_d) = 0$ this in turn implies

$$e_Z(Z_d|Z_d) > 0, \qquad \forall Z > Z^*_{d,0}.$$

On the other hand, Lemma 9 shows that

$$e_Z(Z_d|Z_d) < 0, \qquad \forall Z_d < Z^{\mathrm{nr}}_d,$$

and it now follows from the intermediate value theorem that there exists at least one

$Z_d^* \in [Z_d^{\mathrm{nr}}, Z_{d,0}^*]$ for which $e_Z(Z_d|Z_d) = 0$. The proof of Proposition 5 implies that Z_d^* is an equilibrium default strategy.

Since the required monotonicity follows from Lemmas 5 and 19 it now only remains to show that when $\tau < \tau^*$ there exists an $\epsilon > 0$ such that for all $q < \epsilon$ there exists a unique equilibrium in barrier strategies. Suppose the contrary. Then, there exists a sequence $q_n \downarrow 0$ such that for each n there exist at least two equilibria $Z_d^1(n) < Z_d^2(n)$. By Lemma 20, we cannot have that both $Z_d^1(n)$ and $Z_d^2(n)$ converge to $Z_{d,0}^*$ and the above argument show that $Z_d^1(n), Z_d^2(n) \leq Z_{d,0}^*$. Therefore, there exists a subsequence of equilibrium default thresholds that converges to some constant $Z_d^c < Z_{d,0}^*$ and by continuity (see Lemma 6) we have $e_Z(Z_d^c|Z_d^c)|_{q=0} = 0$ which is impossible because there exists a unique equilibrium when $q = 0$. $\square$

Existence for the case $\tau > \tau^*$. In this case we have $e_Z(Z_d|Z_d)|_{q=0} < 0$ for all $Z_d > 0$ by the proof of Proposition 6. On the other hand, Lemma 9 guarantees that

$$e_Z(Z_d|Z_d)\Big|_{q=\tau} > 0, \qquad \forall Z_d > Z_d^{\mathrm{nr}},$$

and by continuity the same is true for q sufficiently close to τ. Therefore, the cutoff level of the issuance costs parameter defined by

$$q^* \equiv \inf\left\{q > 0 : \sup_{Z_d>0} e_Z(Z_d|Z_d) > 0\right\}$$

satisfies $q^* < \tau$ and the fact that an equilibrium exists if and only if $q > q^*$ now follows by the same argument as in the proof of Theorem 1. $\square$

C Restructuring with existing creditors

In this appendix we study the model in which the firm can raise funds by contacting either outside or inside creditors. Instead of assuming as in the main text that the cost of collective action is proportional to the firm's coupon level prior to restructuring we allow here for more general costs given by $X\,\nu(C/X)$ for some function $\nu(Z)$ that satisfies the following condition:

Assumption 1 *The function*

$$\mathscr{L}\nu(Z) - (r - \mu + \lambda^*)\nu(Z) - \tau Z$$

is monotone decreasing in Z.

As we show below, this assumption guarantees that barrier restructuring strategies are optimal. This assumption trivially holds if $\nu(Z) = \epsilon Z$ as in the text but many other cases can also be considered. In particular, we note that no monotonicity conditions on the function $\nu(Z)$ itself need to be imposed for the validity of this assumption.

Fix an arbitrary default threshold $X_{\mathbf{b}d}(1) = 1/Z_d$ and denote by $\mathbf{P} = \mathbf{P}(X_{\mathbf{b}d}(1))$ the associated equilibrium strategy. With this notation we have that the corresponding equilibrium firm value is given by

$$V_{\mathbf{b}}(X, C|\mathbf{P}) = \sup_{b \in \mathbb{B}(\mathbf{P})} E\left[\int_0^{\tau_d} e^{-rt}((1-\tau)X_t + \tau C_{t-})dt + e^{-(r-\mu)\tau_d}\phi X_{\tau_d}\right] + E\left[\sum_{k=1}^{\infty} 1_{\{\tau_k \le \tau_d\}} e^{-r\tau_k} H_{\mathbf{b}}(\tau_k, b_{\tau_k}, X_{\tau_k}, C_{\tau_k -}|\mathbf{P})\right]$$

where

$$\bar{H}_{\mathbf{b}}(\tau, b', X, C|\mathbf{P}) = -qD(X, b'C|\mathbf{P}) - 1_{\{\tau \in \mathscr{N}\}}\eta S(b', X, C|\mathbf{P}) - 1_{\{\tau \notin \mathscr{N}\}}[(1-\theta)X\nu(X/C) + \theta S(b', X, C|\mathbf{P})]$$

represents the cash flow from restructuring and $\mathscr{N}$ is the set of jump times of the Poisson process that governs meetings between the firm and outside investors. Let also

$$\mathscr{I}(v)(Z) = \max_{a \ge 1}\{v(aZ) - qd(aZ|Z_d) - v(Z)\}$$
$$\mathscr{E}(v)(Z) = \max_{a \ge 1}\{v(aZ) - qd(aZ|Z_d) - v(Z) - \nu(Z)\}$$

Our first result in this section follows from standard dynamic programming arguments.

Lemma 21 *If $v^{\mathbf{b}}(Z|Z_d)$ is a bounded and Borel measurable function such that*

$$v_{\mathbf{b}}(Z|Z_d) = \sup_{\tau \in \mathbb{S}} \hat{E}\Bigg[\int_0^{\tau \wedge \tau^N \wedge \tau_d} e^{-(r-\mu)s}(1 - \tau + \tau Z_{s-})ds + 1_{\{\tau_d \le \tau \wedge \tau^N\}} e^{-(r-\mu)\tau_d}\phi + 1_{\{\tau < \tau^N \wedge \tau^d\}} e^{-(r-\mu)\tau}((1-\theta)\mathscr{E}(v_{\mathbf{b}}(\cdot|Z_d))(aZ_{\tau-}) + \theta v^{\mathbf{b}}(Z_{\tau-})) + 1_{\{\tau^N < \tau \wedge \tau^d\}} e^{-(r-\mu)\tau^N}((1-\eta)\mathscr{I}(v_{\mathbf{b}}(\cdot|Z_d))(aZ_{\tau^N -}) + \eta v^{\mathbf{b}}(Z_{\tau^N -}))\Bigg].$$

then $V_{\mathbf{b}}(X, C|Z_d) = Xv_{\mathbf{b}}(C/X|Z_d)$.

As a result of Lemma 21, our problem reduces to that of finding a bounded solution to the dynamic programming equation. Note that it is a priori not obvious that such a solution exists. In particular, the contraction mapping techniques that we used in the model with search cannot be directly applied here due to the possibility of contacting creditors at all times, and so new methods need to be developed. We start with a standard lemma for solving optimal stopping problems.

Lemma 22 *Let $\varphi(Z) \in C[0, Z_d]$ be a bounded function and $\xi(Z)$ a bounded, Borel measurable function. Suppose that a bounded function $y(Z)$ on $[0, Z_d]$ with $y(Z_d) = \phi$ is such that there exists a threshold $\bar{Z}_{\mathbf{b}u} < Z_d$ with the following properties*

1. *The function $y(Z)$ is C^1 and piecewise C^2 on $[0, Z_d)$.*
2. *On $[\bar{Z}_{\mathbf{b}u}, Z_d]$ the function $y(Z)$ satisfies*
$$(r - \mu + \lambda^*) y(Z) = \mathscr{L} y(Z) + \xi(Z).$$
3. *On $[0, Z_d)$ the function $y(Z)$ satisfies $y(Z) \geq \varphi(Z)$.*
4. *On $[0, \bar{Z}_{\mathbf{b}u}]$ the function $y(Z)$ satisfies $y(Z) = \varphi(Z)$ and*
$$(r - \mu + \lambda^*) y(Z) \geq \mathscr{L} y(Z) + \xi(Z).$$

Then the function $y(Z)$ is given by

$$y(Z) = \sup_{\tau \in \mathbb{S}} \hat{E} \left[\int_0^{\tau \wedge \tau_d} e^{-(r-\mu+\lambda^*)s} \xi(Z_s^0) ds + e^{-(r-\mu+\lambda^*)\tau \wedge \tau_d} \left(1_{\{\tau_d \leq \tau\}} \phi + 1_{\{\tau < \tau_d\}} \varphi(Z_\tau^0) \right) \right]$$

where the process Z_t^0 evolves according to (11) with $a \equiv 1$.

To find a solution to our problem, we will approximate the optimal stopping problem by a problem in which the firm can only contact existing creditors at the jump times of an independent Poisson process with intensity $\Lambda > 0$ and then let this intensity increase to infinity. The following proposition describes this auxiliary problem.

Proposition 8 *Fix a default threshold $Z_d > Z_d^{\text{nr}}$ and let $\rho(\Lambda) \equiv r - \mu + \lambda + \Lambda$. Then the dynamic programming equation:*

$$\begin{aligned} v^\Lambda(Z) = \hat{E}\bigg[\int_0^{\tau_d} e^{-\rho(\Lambda)t} \big(& (1 - \tau + \tau Z_t^0) \\ & + \Lambda((1-\theta)\mathscr{E}(v^\Lambda)(aZ_t^0) + \theta v^\Lambda(Z_t^0)) \\ & + \lambda((1-\eta)\mathscr{I}(v^\Lambda)(aZ_t^0) + \eta v^\Lambda(Z_t^0) \big) dt + e^{-\rho(\Lambda)\tau_d} \phi \bigg]. \end{aligned} \tag{27}$$

admits a unique solution that belongs to $C^2[0, Z_d]$ and the corresponding optimal restructuring policy is a barrier policy that is characterized by thresholds $\bar{Z}_{\mathbf{b}u}(\Lambda) < Z_{\mathbf{b}u}(\Lambda) < Z_{\mathbf{b}o}(\Lambda) < Z_d$.

The proof of the above proposition will be based on a sequence of lemmas. The same argument as in the model with search implies that the following is true.

Lemma 23 *The unique solution to* (28) *is* $C^2[0, Z_d]$ *and satisfies*

$$(r-\mu)v^{\Lambda}(Z) = \mathscr{L}v^{\Lambda}(Z) + 1 - \tau + \tau Z + \lambda(1-\eta)\mathscr{O}(v^{\Lambda})(Z) + \Lambda(1-\theta)\mathscr{O}_{\mathbf{b}}(v^{\Lambda})(Z)$$

where the operators on the right are defined by

$$\mathscr{O}_{\mathbf{b}}(v)(Z) \equiv \max_{a\geq 1}\big(v(aZ) - q1_{\{a>1\}}d(aZ|Z_d) - \nu(Z) - v(Z)\big)^+$$

and equation (13).

Lemma 24 *There are thresholds* $Z_{\mathbf{b}u}(\Lambda) < Z_{\mathbf{b}o}(\Lambda) < Z_d$ *such that*

$$\mathscr{O}(v^{\Lambda})(Z) = 1_{\{Z\leq Z_{\mathbf{b}u}\}}\big(v^{\Lambda}(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d) - v^{\Lambda}(Z)\big)$$

Proof. Assume for simplicity that restructuring with new creditors is optimal and that $q > 0$. It follows that

$$Z_{\mathbf{b}u} \equiv \max\{Z : \mathscr{O}(v^{\Lambda}) > 0\} < Z_d\,.$$

is well-defined and is smaller than or equal to Z_d. Furthermore, by continuity, we have

$$\mathscr{O}(v^{\Lambda})(Z_{\mathbf{b}u}) = 0 \;\Leftrightarrow\; v^{\Lambda}(Z_{\mathbf{b}u}) = \max_{y\geq Z_{\mathbf{b}u}}(v^{\Lambda}(y) - qd(y))\,.$$

Now consider the higher threshold defined by

$$Z_{\mathbf{b}o} \equiv \min\{y \geq Z_{\mathbf{b}u} :\; v^{\Lambda}(Z_{\mathbf{b}u}) = v^{\Lambda}(y) - qd(y)\}\,.$$

By the same argument as in the proof of Lemma 9, we have $v^{\Lambda}(\phi) < v^{\Lambda}(Z_{\mathbf{b}u}) = v^{\Lambda}(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d) < v^{\Lambda}(Z_{\mathbf{b}o})$ and therefore $Z_{\mathbf{b}u} < Z_{\mathbf{b}o} < Z_d$ since issuance costs are strictly positive. This in turn implies that the point $Z_{\mathbf{b}o}$ is a local maximum of the function $v^{\Lambda}(y) - qd(y)$.

To complete the proof, we need to show that for $Z \leq Z_{\mathbf{b}u}$, we have $v^{\Lambda}(Z) \leq v^{\Lambda}(Z_{\mathbf{b}u})$. Indeed, in that case,

$$\max_{y\geq Z}(v^{\Lambda}(y) - qd(y)) \;\leq\; \max_{y\geq Z} v^{\Lambda}(y) \;\leq\; v^{\Lambda}(Z_{\mathbf{b}u}) = v^{\Lambda}(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o})$$

and, consequently,

$$\mathscr{O}(v^{\Lambda})(Z) = 1_{\{Z\leq Z_{\mathbf{b}u}\}}\big(v^{\Lambda}(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d) - v^{\Lambda}(Z)\big)\,.$$

Suppose that this is not the case. Let

$$Z_v = \max\{Z \leq Z_{\mathbf{b}u} : v^{\Lambda}(Z) = v^{\Lambda}(Z_{\mathbf{b}u}) = v(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o})\}$$

and assume for simplicity that $Z_v < Z_{\mathbf{b}u}$ so that the function $v^\Lambda(Z)$ reaches a local minimum at some point $Z_m \in [Z_v, Z_{\mathbf{b}u}]$.[10] As a first step towards a contradiction we claim that the function $v^\Lambda(Z)$ is monotone decreasing on $[0, Z_v]$. If not then as illustrated by the right panel of Figure A there is a point $Z_n \in [0, Z_v]$ at which the function $v^\Lambda(Z)$ achieves a local maximum such that

$$v^\Lambda(Z_n) > v^\Lambda(Z_v) = v^\Lambda(Z_{\mathbf{b}u}) = v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}) \;=\; \max_{y \geq Z_v}(v^\Lambda(y) - qd(y))\,. \tag{28}$$

Furthermore, by the definition of Z_n, $v^\Lambda(Z_n)$ is monotone decreasing on $[Z_n, Z_v]$ and hence, for all $y \in [Z_n, Z_v]$, we have

$$v^\Lambda(Z_n) \geq v^\Lambda(y) \geq v^\Lambda(y) - qd(y)\,. \tag{29}$$

Combining (29) and (30), we get

$$v^\Lambda(Z_n) \;\geq\; \max_{y \geq Z_n}(v^\Lambda(y) - qd(y))\,.$$

This immediately implies that we have $\mathscr{O}(v^\Lambda)(Z_n) = 0$. Furthermore, by definition,

$$0 \leq \;\mathscr{O}^\Lambda(v^\Lambda) \;\leq\; \mathscr{O}(v^\Lambda)$$

and hence $\mathscr{O}^\Lambda(v^\Lambda)(Z_n) = 0$. Therefore,

$$\begin{aligned}(r-\mu)v^\Lambda(Z_m) &= \mathscr{L}v^\Lambda(Z_m) + 1 - \tau(1-Z_m) + \lambda(1-\eta)\mathscr{O}(v)(Z_m) + \Lambda(1-\theta)\mathscr{O}(v^\Lambda)(Z_m)\\
&\geq \mathscr{L}v^\Lambda(Z_m) + 1 - \tau(1-Z_m)\\
&\geq 1 - \tau(1-Z_m)\\
&> \;1 - \tau(1-Z_n)\\
&\geq \mathscr{L}v^\Lambda(Z_n) + 1 - \tau(1-Z_n)\\
&= \mathscr{L}v^\Lambda(Z_n) + 1 - \tau(1-Z_n) = (r-\mu)v^\Lambda(Z_n)\end{aligned}$$

which contradicts equation (29) and therefore establishes our claim regarding the monotonicity of the function $v^\Lambda(Z)$ in the interval $[0, Z_v]$. Therefore, (29) and the same argument as in (30) implies that this property with the fact that $\mathscr{O}(v)(Z) = 0$ for all $Z \leq Z_v$. Consequently,

$$(r-\mu)v^\Lambda(Z) = \mathscr{L}v^\Lambda(Z) + 1 - \tau(1-Z)$$

[10] When the point $Z_{\mathbf{b}u}$ is a local minimum of the function $v^\Lambda(Z)$ we have $Z_v = Z_{\mathbf{b}u}$. This case is completely analogous, up to small modifications.

on $[0, Z_v]$ and therefore

$$v^\Lambda(Z) \;=\; \frac{1-\tau}{r-\mu} + \frac{\tau Z}{r} + a_1 Z^{1-\beta} + a_2 Z^{1-\alpha}$$

for some $a_1, a_2 \in \mathbb{R}$. Since v^Λ is bounded, we have $a_2 = 0$ and therefore $v^\Lambda(0) = \phi_0$. Since $v^\Lambda(Z)$ is decreasing on $[0, Z_v]$, we immediately get that $\phi_0 > v^\Lambda(Z)$ on that interval. But this is impossible since

$$v^\Lambda(Z) \geq \hat{v}^\Lambda(Z) \geq \phi_0$$

in a right neighborhood of zero by Lemma 9. □

Lemma 25 *There is a threshold* $\bar{Z}_{\mathbf{b}u}(\Lambda) < Z_{\mathbf{b}u}(\Lambda)$ *such that*

$$\mathscr{O}_{\mathbf{b}}(v^\Lambda)(Z) = 1_{\{Z \leq \bar{Z}_{\mathbf{b}u}\}} \big(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d) - v^\Lambda(Z) - \nu(Z)\big)$$

Proof. Since $\mathscr{O}_{\mathbf{b}}(v^\Lambda)(Z) < \mathscr{O}(v^\Lambda)(Z)$, we have that the threshold

$$\bar{Z}_{\mathbf{b}u} \equiv \sup\{Z > 0 : \mathscr{O}_{\mathbf{b}}(v^\Lambda)(Z) > 0\}$$

is well defined and satisfies $\bar{Z}_{\mathbf{b}u} < Z_{\mathbf{b}u}$. By continuity, we have

$$v^\Lambda(\bar{Z}_{\mathbf{b}u}) + \nu(\bar{Z}_{\mathbf{b}u}) \;=\; v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)\,.$$

Suppose that the claim of the statement is not true. Then, there exists a $Z_v < \bar{Z}_{\mathbf{b}u}$ such that $v^\Lambda(Z_v) + \nu(Z_v) = v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)$. Let us show that $W(Z) \equiv v^\Lambda(Z) + \nu(Z)$ is monotone decreasing on $[0, Z_v]$. Indeed, suppose the contrary. Let Z_n be the largest local maximum of $W(Z)$ on $[0, Z_v]$. Let also

$$\lambda^* \equiv \lambda(1-\eta), \Lambda^* \equiv \Lambda(1-\theta)\,.$$

Then, defining

$$\zeta(Z) \equiv \mathscr{L}\nu(Z) - (r - \mu + \lambda^*)\nu(Z),$$

we have

$$(r-\mu+\lambda^*)W(Z) = -\zeta(Z) + \mathscr{L}W(Z) + 1 - \tau + \tau Z + \lambda^*(\mathscr{O}(v^\Lambda) + v^\Lambda) + \Lambda^* \mathscr{O}_{\mathbf{b}}(v^\Lambda).$$

Since $W(Z_v) = W(\bar{Z}_{\mathbf{b}u}) > W(Z)$ for all $Z \in (Z_v, \bar{Z}_{\mathbf{b}u})$, $W(Z)$ also has the largest local minimum at some $Z_m \in (Z_v, \bar{Z}_{\mathbf{b}u})$. Therefore, using the fact that, by assumption, $\tau Z - \zeta(Z)$

is monotone increasing, we get

$$\begin{aligned}(r-\mu+\lambda^*)W(Z_m) &= \mathscr{L}W(Z_m) - \zeta(Z_m) + 1 - \tau(1-Z_m) \\ &\quad + \lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)) + \Lambda^* \mathscr{O}^\Lambda(v^\Lambda)(Z_m) \\ &\geq -\zeta(Z_m) + 1 - \tau(1-Z_m) + \lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)) + \Lambda^* \mathscr{O}^\Lambda(v^\Lambda)(Z_m) \\ \\ &\geq -\zeta(Z_m) + 1 - \tau(1-Z_m) + \lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)) \\ &> -\zeta(Z_n) + 1 - \tau(1-Z_n) + \lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)) \\ &\geq \mathscr{L}W(Z_n) - \zeta(Z_n) + 1 - \tau(1-Z_n) + \lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)) \\ \\ &= (r-\mu+\lambda^*)W(Z_n)\end{aligned}$$

which is a contradiction. Thus, it has to be that $W(Z)$ is monotone decreasing on $[0, Z_v]$ and still has a local minimum at Z_m, so that

$$W(0) \geq W(Z_m) \geq \frac{-\zeta(Z_m) + (1-\tau+\tau Z_m) + \lambda^* \left(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)\right)}{r-\mu+\lambda^*}.$$

Since $\mathscr{O}_{\mathbf{b}}(v^\Lambda) = 0$ for $Z \leq Z_v$, we have

$$\frac{1}{2}\sigma^2 Z^2\, v^\Lambda_{ZZ}(Z) - \mu Z_n v^\Lambda_Z(Z) + (1-\tau+\tau Z) + \lambda^* \left(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)\right) = (r-\mu+\lambda^*)v^\Lambda(Z)$$

in that interval. A direct calculation implies that

$$v^\Lambda(0) = \frac{1-\tau+\lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d))}{r-\mu+\lambda^*}$$

Therefore,

$$\begin{aligned}W(0) &= \nu(0) + \frac{1-\tau+\lambda^*(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d))}{r-\mu+\lambda^*} \\ &\leq \frac{-\zeta(Z_m) + (1-\tau+\tau Z_m) + \lambda^* \left(v^\Lambda(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d)\right)}{r-\mu+\lambda^*} \leq W(Z_m)\end{aligned}$$

because

$$(r-\mu+\lambda^*)\,\nu(0) = \tau\,\cdot 0 - \zeta(0) \leq \tau Z_m - \zeta(Z_m)$$

since, by assumption, $\tau Z - \zeta(Z)$ is increasing. This is a contradiction, and the claim follows. □

Lemma 26 *As $\Lambda \to \infty$ the thresholds $\bar{Z}_{\mathbf{b}u}$, $Z_{\mathbf{b}u}$, $Z_{\mathbf{b}o}$ converge to some finite limits and*

$v^\Lambda(Z)$ converges uniformly to a function $v_{\mathbf{b}}(Z|Z_d)$ which satisfies (21). In particular, the optimal stopping time is the first time that the state variable is enter the interval $[0, \bar{Z}_{\mathbf{b}u}]$.

Proof. To prove this result we will show that $v^\Lambda(Z)$ converges to a function $v^{\mathbf{b}}(Z)$ that satisfies the conditions of Lemma 22. Let us first discuss convergence. Since the interval of interest $[0, Z_d]$ is compact, we can always pick a subsequence Λ_n such that

$$(\bar{Z}_{\mathbf{b}u}(\Lambda_n), Z_{\mathbf{b}u}(\Lambda_n), Z_{\mathbf{b}o}(\Lambda_n)) \longrightarrow (\bar{Z}_{\mathbf{b}u}, Z_{\mathbf{b}u}, Z_{\mathbf{b}o})$$

for some constants

$$\bar{Z}_{\mathbf{b}u} \le Z_{\mathbf{b}u} \le Z_{\mathbf{b}o} \le Z_d.$$

The same arguments as in the proof of Lemma 5 imply that $v^\Lambda(Z)$ is increasing as a function of Λ. In particular, $\max_{y \ge 0}(v^\Lambda(y) - qd(y|Z_d))$ is increasing in Λ and it follows that $Z_{\mathbf{b}o}(\Lambda_n)$ cannot converge to the exogenously fixed default threshold Z_d.

The fact that the function $v^\Lambda(Z)$ converges on the interval $[\bar{Z}_{\mathbf{b}u}, Z_d]$ to a limit $v_{\mathbf{b}}(Z|Z_d)$ that solves the same equation as the function $v(Z|Z_d)$ follows directly from Lemma 6 and the fact that the function $v^\Lambda(Z)$ is increasing in Λ and bounded from above. A direct calculation based on Lemma 6 implies that only the function but also its derivative converges. On the interval $[0, \bar{Z}_{\mathbf{b}u}]$ we define the limiting function by

$$v_{\mathbf{b}}(Z) = \lim_{n \to \infty} (v^{\Lambda_n}(Z_{\mathbf{b}o}(\Lambda_n)) - qd(Z_{\mathbf{b}o}(\Lambda_n)|Z_d)) - \nu(Z)\,.$$

By definition of the threshold $\bar{Z}_{\mathbf{b}u}$ we have that $v_{\mathbf{b}}(Z)$ is continuous at the point $\bar{Z}_{\mathbf{b}u}$ and to complete the proof we will now sequentially verify that the limiting functions satisfies the conditions of Lemma 22 with

$$\xi(Z) = 1 - \tau(1 - Z) + \lambda^*(v_{\mathbf{b}}(Z) + \mathscr{O}(v_{\mathbf{b}})(Z))$$

and

$$\varphi(Z) = \max_{a \ge 1}(v_{\mathbf{b}}(aZ) - qd(aZ|Z_d)1_{\{a>1\}} - \nu(Z)) = v_{\mathbf{b}}(Z) + \mathscr{O}_{\mathbf{b}}(v_{\mathbf{b}})(Z)$$

To prove that the limiting function is C^1 consider the function

$$W(Z) = W^\Lambda(Z) \equiv v^\Lambda(Z) + \nu(Z)$$

and observe that since the derivative of $v^\Lambda(Z)$ converges to that of $v_{\mathbf{b}}(Z)$ for $Z \ge \bar{Z}_{\mathbf{b}u}$ by the above it suffices to prove that

$$\lim_{\Lambda \to \infty} W_Z^\Lambda(\bar{Z}_{\mathbf{b}u}(\Lambda)) = 0. \tag{30}$$

By definition of the restructuring threshold $\bar{Z}_{\mathbf{b}u}(\Lambda)$ we have

$$\begin{aligned}\mathscr{O}_{\mathbf{b}}(v^\Lambda)(Z) = \mathscr{O}(v^\Lambda)(Z) - \nu(Z) &= v(Z_{\mathbf{b}o}) - qd(Z_{\mathbf{b}o}|Z_d) - v^\Lambda(Z) - \nu(Z)\\ &= W(\bar{Z}_{\mathbf{b}u}(\Lambda)) - W(Z)\end{aligned}$$

for all $Z \leq \bar{Z}_{\mathbf{b}u}(\Lambda)$ and it now follows from Lemma 23 that over this region the function solves the ordinary differential equation

$$(r - \mu + \Lambda^*)W(Z) = \mathscr{L}W(Z) - \vartheta(Z) + 1 - \tau + \Lambda^* W(\bar{Z}_{\mathbf{b}u}(\Lambda))$$

where

$$\vartheta(Z) = \mathscr{L}\nu(Z) - (r - \mu + \lambda^*)\nu(Z) - \tau Z$$

is a decreasing function by Assumption 1 and $\Lambda^* = \lambda^* + \Lambda(1-\theta)$. Define $\gamma < 0 < 1 < \gamma_1$ to be the solutions to $Q(x, r + \Lambda^*) = 0$. With this notation it follows from a slight modification of Lemma 6 that the solution is explicitly given by

$$W(Z) = W(0) + y_1 Z^{1-\gamma} + y_2 Z^{1-\gamma_1} + \frac{2}{\sigma^2 Z}\int_Z^{\bar{Z}_{\mathbf{b}u}(\Lambda)} \left[\left(\frac{x}{Z}\right)^{\gamma-2} - \left(\frac{x}{Z}\right)^{\gamma_1-2}\right]\frac{\vartheta(x)dx}{\gamma_1 - \gamma}$$

for some constants y_1 and y_2 where

$$W(0) = \frac{1 - \tau - \vartheta(0) + \Lambda^* W(\bar{Z}_{\mathbf{b}u}(\Lambda))}{r - \mu + \Lambda^*}.$$

Let us first determine the constant y_2 using the fact the function is bounded at the origin. Since the function $\vartheta(Z)$ is decreasing we have

$$\left|\frac{2}{\sigma^2 Z}\int_Z^{\bar{Z}_{\mathbf{b}u}(\Lambda)} \left(\frac{x}{Z}\right)^{\gamma-2}\frac{\vartheta(x)dx}{\gamma_1-\gamma}\right| \leq \frac{K_0}{Z}\int_Z^{\bar{Z}_{\mathbf{b}u}(\Lambda)}\left(\frac{x}{Z}\right)^{\gamma-2} dx \leq K_1$$

for some constants K_0 and $K_1 > 0$. Thus, we only need to take care of the terms with negative exponent and it follows that

$$y_2 = \frac{2}{\sigma^2}\int_0^{\bar{Z}_{\mathbf{b}u}(\Lambda)} \frac{x^{\gamma_1-2}\vartheta(x)dx}{\gamma_1-\gamma}$$

where the integral does not explode at $x = 0$ because by definition $\gamma_1 > 1$. Using this constant we can rewrite the solution as

$$W(Z) = W(0) + y_1 Z^{1-\gamma} + \frac{2}{\sigma^2 Z}\left[\int_0^Z \left(\frac{x}{Z}\right)^{\gamma_1-2}\frac{\vartheta(x)dx}{\gamma_1-\gamma} + \int_Z^{\bar{Z}_{\mathbf{b}u}(\Lambda)}\left(\frac{x}{Z}\right)^{\gamma-2}\frac{\vartheta(x)dx}{\gamma_1-\gamma}\right]. \quad (31)$$

and the remaining constant is now determined by requiring that the solution be continuous at the upper boundary point:

$$W(\bar{Z}_{\mathbf{b}u}(\Lambda)) = W(0) + y_1 \bar{Z}_{\mathbf{b}u}(\Lambda)^{1-\gamma} + \frac{2}{\sigma^2 \bar{Z}_{\mathbf{b}u}(\Lambda)} \int_0^{\bar{Z}_{\mathbf{b}u}(\Lambda)} \left(\frac{x}{\bar{Z}_{\mathbf{b}u}(\Lambda)} \right)^{\gamma_1 - 2} \frac{\vartheta(x)dx}{\gamma_1 - \gamma}.$$

Solving this equation, substituting the solution into (32) and differentiating the resulting expression at the upper boundary point gives

$$W_Z(\bar{Z}_{\mathbf{b}u}(\Lambda)) = (1-\gamma)\frac{W(\bar{Z}_{\mathbf{b}u}(\Lambda)) - W(0)}{\bar{Z}_{\mathbf{b}u}(\Lambda)} - \frac{2}{(\sigma \bar{Z}_{\mathbf{b}u}(\Lambda))^2} \int_0^{\bar{Z}_{\mathbf{b}u}(\Lambda)} \left(\frac{x}{\bar{Z}_{\mathbf{b}u}(\Lambda)} \right)^{\gamma_1 - 2} \vartheta(x)dx.$$

A direct calculation shows that the constants $(1-\gamma)/(r-\mu+\Lambda^*)$ converge to zero as Λ^* goes to infinity, Therefore, since $W(Z) = W^\Lambda(Z)$ is bounded as a function of Λ and the restructuring threshold converges to a finite number we obtain

$$\lim_{\Lambda\to\infty} (1-\gamma)\frac{W(\bar{Z}_{\mathbf{b}u}(\Lambda)) - W(0)}{\bar{Z}_{\mathbf{b}u}(\Lambda)} = \lim_{\Lambda\to\infty} \frac{(1-\gamma)(\tau - 1 + \vartheta(0) + (r-\mu)W(\bar{Z}_{\mathbf{b}u}(\Lambda)))}{(r-\mu+\Lambda^*)\bar{Z}_{\mathbf{b}u}(\Lambda)} = 0.$$

On the other hand, since γ_1 diverges to infinity as Λ increases we have that $(x/Z)^{\gamma_1 - 2}$ converges to zero for all $x < Z$ and it now follows from the dominated convergence theorem that

$$\lim_{\Lambda\to\infty} \frac{2}{(\sigma \bar{Z}_{\mathbf{b}u}(\Lambda))^2} \int_0^{\bar{Z}_{\mathbf{b}u}(\Lambda)} \left(\frac{x}{\bar{Z}_{\mathbf{b}u}(\Lambda)} \right)^{\gamma_1 - 2} \vartheta(x)dx = 0.$$

This shows that (31) holds and completes the verification of condition 1. The validity of conditions 2 and 3 follows directly from the above arguments. To establish the validity of condition 4 we need to show that the quantity

$$C(Z) = \mathscr{L}\varphi(Z) - (r-\mu+\lambda^*)\varphi(Z) + \xi(Z) = 1 - \tau - \vartheta(Z) - (r-\mu)W(\bar{Z}_{\mathbf{b}u})$$

is non positive for all $Z \le \bar{Z}_{\mathbf{b}u}$ and since $\vartheta(Z)$ is decreasing it suffices to check that this property holds at the upper boundary point. The above result implies that $W'(\bar{Z}_{\mathbf{b}u}) = 0$ and since the function $W(Z)$ cannot be decreasing to the left of $\bar{Z}_{\mathbf{b}u}$ we have

$$\mathscr{L}W(\bar{Z}_{\mathbf{b}u}) = \frac{1}{2}(\bar{Z}_{\mathbf{b}u})^2 W''(\bar{Z}_{\mathbf{b}u}) \ge 0$$

Combining this with the definition of the function $W(Z)$ and the fact that

$$(r-\mu)v_{\mathbf{b}}(\bar{Z}_{\mathbf{b}u}) = \mathscr{L}v_{\mathbf{b}}(\bar{Z}_{\mathbf{b}u}) + 1 - \tau(1 - \bar{Z}_{\mathbf{b}u}) + \lambda^* \mathscr{O}(v_{\mathbf{b}})(\bar{Z}_{\mathbf{b}u})$$

then gives

$$
\begin{aligned}
C(\bar{Z}_{\mathbf{b}u}) &= 1-\tau(1-\bar{Z}_{\mathbf{b}u})-\vartheta(\bar{Z}_{\mathbf{b}u})-(r-\mu)W(\bar{Z}_{\mathbf{b}u})\\
&\le 1-\tau(1-\bar{Z}_{\mathbf{b}u})-\vartheta(\bar{Z}_{\mathbf{b}u})-(r-\mu)W(\bar{Z}_{\mathbf{b}u})+\mathscr{L}W(\bar{Z}_{\mathbf{b}u})=0
\end{aligned}
$$

and completes the proof. $\square$

Recall that the functions v_{ns} and v_s are defined by

$$
\begin{aligned}
v_{ns}(Z) &= v_{ns}(Z,a_3,a_4)\equiv a_3Z^{1-\beta}+a_4Z^{1-\alpha}+\phi_0+\frac{\tau Z}{r}\\
v_s(Z) &= v_s(Z,Z_{\mathbf{b}o},a_1,a_2,a_3,a_4;q)\\
&\equiv a_1Z^{1-\psi}+a_2Z^{1-\psi_1}+\frac{\tau Z}{r}+\frac{1-\tau+\lambda^*(v_{ns}(Z_{\mathbf{b}o})-qd(Z_{\mathbf{b}o}|Z_d))}{r-\mu+\lambda^*}.
\end{aligned}
$$

Solving

$$
v_s(Z_{\mathbf{b}u})-v_{ns}(Z_{\mathbf{b}u})=v_s'(Z_{\mathbf{b}u})-v_{ns}'(Z_{\mathbf{b}u})=0
$$

for a_1, a_2 gives

$$
a_1 \;=\; A_1(Z_{\mathbf{b}u},a_3,a_4;q)\;,\;\; a_2 \;=\; A_2(Z_{\mathbf{b}u},a_3,a_4;q)\,.
$$

The following lemma establishes the local uniqueness of the rational expectations equilibrium in barrier strategies and constitutes the direct counterpart of Lemma 20 for the model in which the firm can issue debt to inside creditors.

Lemma 27 *Let $\nu(Z)=\epsilon Z$. Consider the following system*

$$
\begin{aligned}
&F_{1\mathbf{b}}(\bar{Z}_{\mathbf{b}u},Z_{\mathbf{b}u},Z_{\mathbf{b}o},Z_{\mathbf{b}d},a_3,a_4;q)\equiv v_{ns}(Z_d)-d(Z_d|Z_d)=0\\
&F_{2\mathbf{b}}(\bar{Z}_{\mathbf{b}u},Z_{\mathbf{b}u},Z_{\mathbf{b}o},Z_{\mathbf{b}d},a_3,a_4;q)\equiv v_{ns}'(Z_d)-d'(Z_d|Z_d)=0\\
&F_{3\mathbf{b}}(\bar{Z}_{\mathbf{b}u},Z_{\mathbf{b}u},Z_{\mathbf{b}o},Z_{\mathbf{b}d},a_3,a_4;q)\equiv v_{ns}'(Z_{\mathbf{b}o})-qd'(Z_{\mathbf{b}o}|Z_d)=0\\
&F_{4\mathbf{b}}(\bar{Z}_{\mathbf{b}u},Z_{\mathbf{b}u},Z_{\mathbf{b}o},Z_{\mathbf{b}d},a_3,a_4;q)\equiv v_{ns}(Z_{\mathbf{b}o})-qd(Z_{\mathbf{b}o}|Z_d)-v_{ns}(Z_{\mathbf{b}u})=0\\
&F_{7\mathbf{b}}(\bar{Z}_{\mathbf{b}u},Z_{\mathbf{b}u},Z_{\mathbf{b}o},Z_{\mathbf{b}d},a_3,a_4;q)\equiv v_s(\bar{Z}_{\mathbf{b}u})+\nu(\bar{Z}_{\mathbf{b}u})-(v_{ns}(Z_{\mathbf{b}o})-qd(Z_{\mathbf{b}o}|Z_d))=0\\
&F_{8\mathbf{b}}(\bar{Z}_{\mathbf{b}u},Z_{\mathbf{b}u},Z_{\mathbf{b}o},Z_{\mathbf{b}d},a_1,a_2,a_3,a_4;q)\equiv v_s'(\bar{Z}_{\mathbf{b}u})+\nu'(\bar{Z}_{\mathbf{b}u})=0.
\end{aligned}
$$

Denote by $J(\mathscr{C})$ the unique solution to (8), define $z_o(\mathscr{C})$ and $z_d(\mathscr{C})$ by (26), (27) and let

$$
\begin{aligned}
\tilde{a}_3(\mathscr{C}) &= (\tau/r)\kappa_1(\infty)z_o(\mathscr{C})^{\beta},\\
\tilde{a}_4(\mathscr{C}) &= (\tau/r)\kappa_2(\infty)z_o(\mathscr{C})^{\alpha}.
\end{aligned}
$$

Suppose that there exists an admissible $\mathscr{C}$ *such that*

$$\mathscr{J} F_{\mathbf{b}}(z_o(\mathscr{C}), z_o(\mathscr{C}), z_o(\mathscr{C}), z_d(\mathscr{C}), \tilde{a}_3(\mathscr{C}), \tilde{a}_4(\mathscr{C}); 0) \neq 0 \,.$$

where $\mathscr{J}$ *denotes the Jacobian operator. Then, for Lebesque almost every admissible* $\mathscr{C}$ *there exists an open neighborhood*

$$\mathscr{B}_{\mathbf{b}} \supseteq (z_o(\mathscr{C}), z_o(\mathscr{C}), z_o(\mathscr{C}), z_d(\mathscr{C}), \tilde{a}_1(\mathscr{C}), \tilde{a}_2(\mathscr{C}), \tilde{a}_3(\mathscr{C}), \tilde{a}_4(\mathscr{C}))$$

and a $\delta > 0$ *such that, for all* $q, \epsilon \in [0, \delta)$*, there exists a unique rational expectations equilibrium in barrier strategies whose parameters satisfy* $(\bar{Z}_{\mathbf{b}u}, Z_{\mathbf{b}u}, Z_{\mathbf{b}o}, Z_{\mathbf{b}d}, a_3, a_4) \in \mathscr{B}_{\mathbf{b}}$*.*

Proof of Theorem 2. The proof of Theorem 2 is analogous to that of Theorem 1 and follows directly from Proposition 7 and Lemma 27. We omit the details. $\square$

D Restructuring probabilities

In order to compute the restructuring probabilities associated with the rational expectations equilibria in the three models let

$$\tau(y) \equiv \inf\{t \geq 0 : X_t = y\}$$

denote the first time that the cash flow process reaches $y \geq 0$ and define a nonnegative bounded function by setting

$$F(x, T; y, z) \equiv P\left[\tau(z) \leq T \wedge \tau(y) | X_0 = x\right]$$

The probability of restructuring before time T is therefore given by $F(x, T; X_{d0}(1), X_{u0}(1))$ for the frictionless model, and by $F(x, T; X_{d\mathbf{b}}(1), X_{u\mathbf{b}}(1))$ for the model in which the firm bargains with current creditors. The following lemma provides an expression for the function F which can be easily approximated numerically.

Lemma 28 *For* $0 < y < z$ *and* $x \in (y, z)$ *we have that*

$$\begin{aligned} F(x, T; z, y) = &\sum_{n=0}^{\infty} \left[\Phi\left(\frac{b_n - \nu T}{\sqrt{T}}\right) - e^{2\nu b_n} \Phi\left(\frac{-b_n - \nu T}{\sqrt{T}}\right)\right] \\ &- \sum_{n=0}^{\infty} \left[\Phi\left(\frac{a_n - \nu T}{\sqrt{T}}\right) - e^{2\nu a_n} \Phi\left(\frac{-a_n - \nu T}{\sqrt{T}}\right)\right] \end{aligned}$$

where the sequence $(a_n, b_n)_{n\geq 1}$ *is defined by*

$$a_n = a_n(x,y,z) \equiv \log(z/x)^{\frac{1}{\sigma}} + \log(z/y)^{\frac{2n}{\sigma}},$$
$$b_n = b_n(x,y,z) \equiv a_n + \log(x/y)^{\frac{2}{\sigma}} = \log(zx/y^2)^{\frac{1}{\sigma}} + \log(z/y)^{\frac{2n}{\sigma}},$$

the function $\Phi : \mathbb{R} \to (0,1)$ *is the cumulative distribution function of a standard Gaussian random variable, and we have set* $\nu \equiv m/\sigma - \sigma/2$.

Proof. See Borodin and Salminen (2002). □

In the search model, restructuring occurs the first time that the firm meets creditors while the cash flow shock is above the search boundary $X_u^*(1)$. Therefore, it follows from standard results on Poisson point processes (see e.g. Brémaud (1981)) that the associated probability of restructuring can be computed as

$$1 - G(x, T; X_d^*(1), X_u^*(1))$$

where

$$G(x,T;y,z) \equiv E\left[e^{-\lambda \int_0^{T\wedge\tau(y)} 1_{\{X_s \geq z\}} ds} \middle| X_0 = x \right].$$

In order to derive a numerical approximation for this function we start by computing its Laplace transform with respect to the time parameter. To facilitate the presentation let $\Theta = \Theta(q) < 0$, and $\Psi = \Psi(q) \geq 0$ denote the roots of $Q(x;q) = 0$ where the function Q is defined as in the main text.

Lemma 29 *For* $0 < y < z$ *and* $x \geq y$ *we have that the Laplace transform*

$$\hat{G}(x,\phi;y,z) \equiv \int_0^\infty e^{-\phi t} G(x,t;y,z) dt.$$

is explicitly given by

$$\hat{G}(x,\phi;y,z) = 1_{\{y\leq x\leq z\}} \hat{G}_b(x,\phi;y,z) + 1_{\{x\geq z\}} \hat{G}_a(x,\phi;y,z)$$

where the functions $\hat{G}_a$ *and* $\hat{G}_b$ *are defined by*

$$\hat{G}_a(x,\phi;y,z) \equiv \frac{1}{\phi+\lambda}\left[1 + (x/z)^{\Theta(\phi+\lambda)} \frac{\lambda}{\phi} \frac{A(\phi;y,z)}{B(\phi;y,z)}\right],$$
$$\hat{G}_b(x,\phi;y,z) \equiv \frac{1}{\phi}\left[1 + \frac{\lambda}{\phi+\lambda} \frac{\Theta(\phi+\lambda)}{B(\phi;y,z)} \left(x^{\Psi(\phi)} y^{\Theta(\phi)} - x^{\Theta(\phi)} y^{\Psi(\phi)}\right)\right],$$

with

$$A(\phi; y, z) prop: volume \equiv z^{\Psi(\phi)} y^{\Theta(\phi)} \Psi(\phi) - z^{\Theta(\phi)} y^{\Psi(\phi)} \Theta(\phi),$$
$$B(\phi; y, z) \equiv z^{\Theta(\phi)} y^{\Psi(\phi)} \left(\Theta(\phi+\lambda) - \Theta(\phi)\right) + z^{\Psi(\phi)} y^{\Theta(\phi)} \left(\Psi(\phi) - \Theta(\phi+\lambda)\right).$$

Proof. Using the boundedness of the function $G(x,t;y,z)$ together with an application of Fubini's theorem we deduce that

$$\hat{G}(x,\phi;y,z) = E\left[\int_0^\infty e^{-\int_0^{t\wedge\tau(y)} (\phi+\lambda(X_s;z))ds} dt \,\middle|\, X_0 = x\right].$$

where we have set

$$\lambda(x;z) = \lambda 1_{\{x \geq z\}}.$$

Therefore, it follows from Theorem 4.9 in Karatzas and Shreve (1991) that $\hat{G}(x) \equiv \hat{G}(x,\phi;y,z)$ is the unique bounded and piecewise C^2 solution to

$$mx\hat{G}'(x) + \frac{1}{2}\sigma^2 x^2 \hat{G}''(x) + 1 = (\phi + \lambda(x;z))\hat{G}(x), \qquad x > y,$$

subject to the boundary condition

$$\lim_{x \downarrow y} \hat{G}(x) = 1/\phi.$$

The general solution to this second order ODE is given by

$$\hat{G}(x) = 1_{\{y \leq x \leq z\}} \hat{G}_b(x) + 1_{\{x \geq z\}} \hat{G}_a(x)$$

where

$$\hat{G}_b(x) \equiv 1/\phi + C_1 x^{\Theta(\phi)} + C_2 x^{\Psi(\phi)},$$
$$\hat{G}_a(x) \equiv 1/(\phi+\lambda) + C_3 x^{\Theta(\phi+\lambda)} + C_4 x^{\Psi(\phi+\lambda)}$$

for some constants $(C_i)_{i=1}^4$ to be determined. Since the solution has to remain bounded as the state increases, it must be that $C_4 = 0$. In addition, the boundary condition at $x = y$

and the smoothness of the solution require that

$$\lim_{x \downarrow y} \hat{G}_b(x) = 1/\phi,$$
$$\lim_{x \downarrow z} \hat{G}_b(x) = \lim_{x \uparrow z} \hat{G}_a(z),$$
$$\lim_{x \downarrow z} \hat{G}'_b(z) = \lim_{x \uparrow z} \hat{G}'_a(z).$$

Solving this system of three equations for the remaining constants, plugging the solution into the definition of the functions $(\hat{G}_b, \hat{G}_a)$ and simplifying the result gives the desired result. □

To obtain the probability of restructuring before a fixed date we need to invert the Laplace transform. Unfortunately, due to the complex dependence of the transformed function on the transform parameter, this cannot be carried out in closed form. To circumvent this difficulty, we follows Abate and Whitt (1995) and approximate the original function as

$$G(x,T) \approx \sum_{k=0}^{m} \binom{m}{k} \frac{e^{A/2}}{2^{1+m}T} \left[\hat{G}\left(x, \frac{A}{2T}\right) + 2 \sum_{\ell=1}^{n+k} (-1)^{\ell} \Re \hat{G}\left(x, \frac{A}{2T} + \ell \frac{i\pi}{T}\right) \right]$$

where (m, n, A) are constants that control the accuracy of the approximation and we have suppressed the dependence on the thresholds to simplify the notation. In our numerical calculations we use the values

$$m = 11, \quad n = 15, \quad A = 8 \log 10,$$

suggested by Abate and Whitt (1995) to obtain an accuracy of the order of 10^{-8} and verify that the results we obtain are insensitive to that choice.

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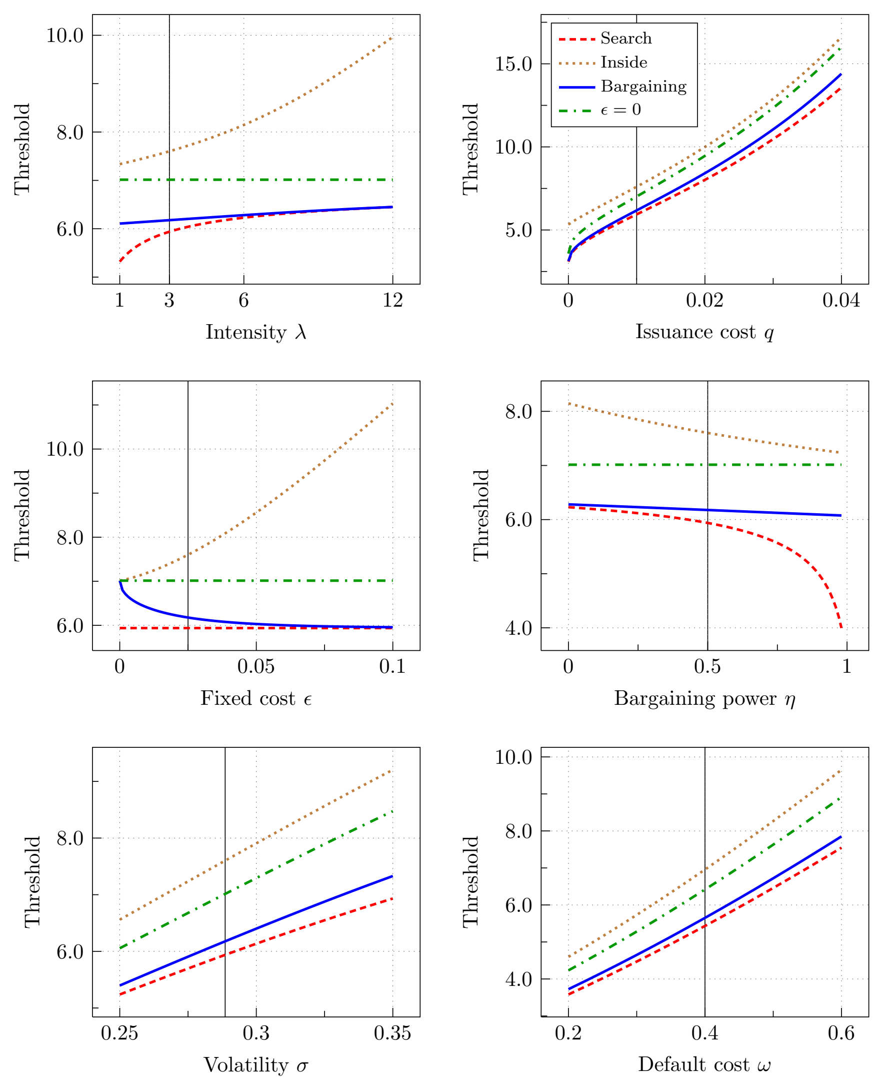
Table 1: Benchmark parameter values

Symbol	Interpretation	Value
A. Firm specific parameters:		
r	Interest rate	0.042
μ	Risk-neutral cash flow rate	0.0067
m	Real cash flow rate	0.0824
σ	Cash flow volatility	0.2886
τ	Corporate tax rate	0.15
ω	Liquidation cost	0.45
B. Credit market parameters:		
λ	Arrival rate of creditors	3.00
q	Proportional issuance cost	0.01
ϵ	Fixed issuance cost	0.025
η	Bargaining power of outside creditors	0.50
θ	Bargaining power of inside creditors	0.50
C. Implied parameters:		
τ^*	Maximal tax rate in the search model	0.9934
$\tau^*(\infty)$	Maximal tax rate in the model with outside creditors	0.8116

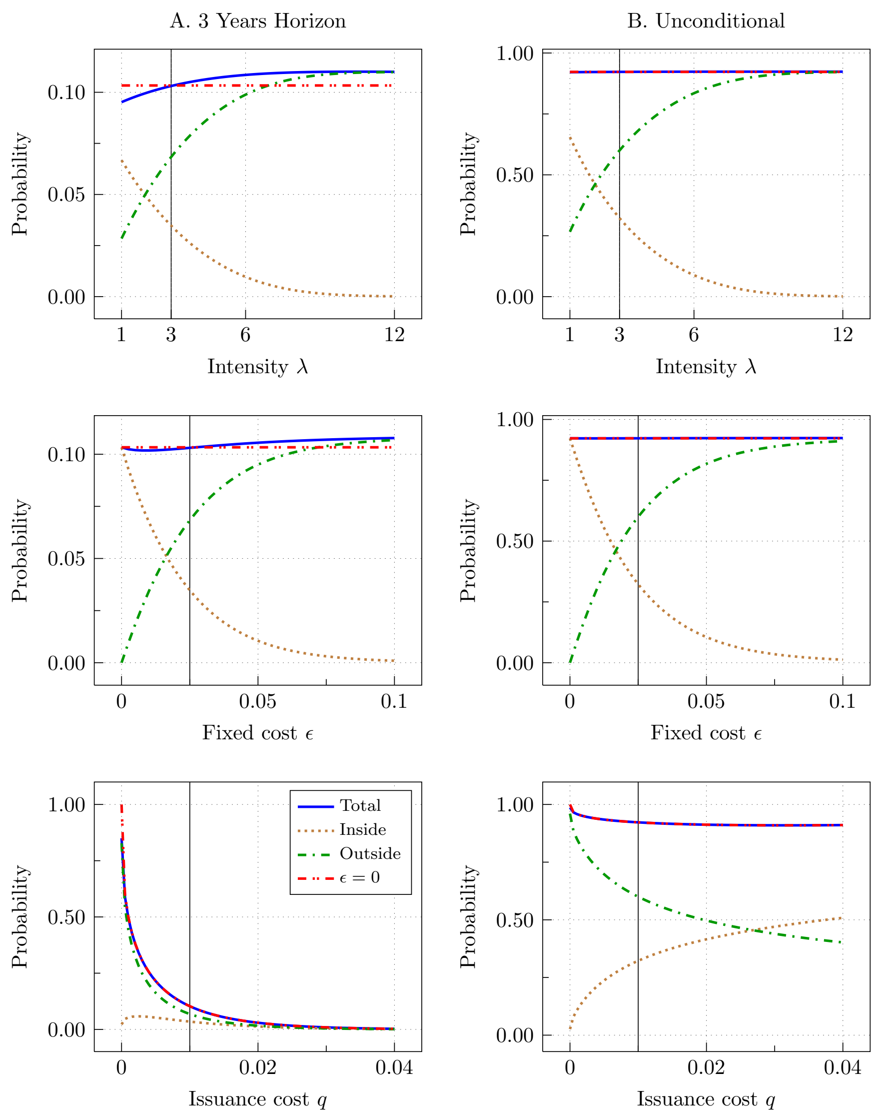
This table gives the benchmark parameter values that we use in our numerical illustrations of the model. The bargaining power of inside creditors is set equal to that of outside creditors. This is without loss of generality since the surplus from issuing debt to inside creditors is always equal to zero in equilibrium.

Figure 1: Restructuring thresholds



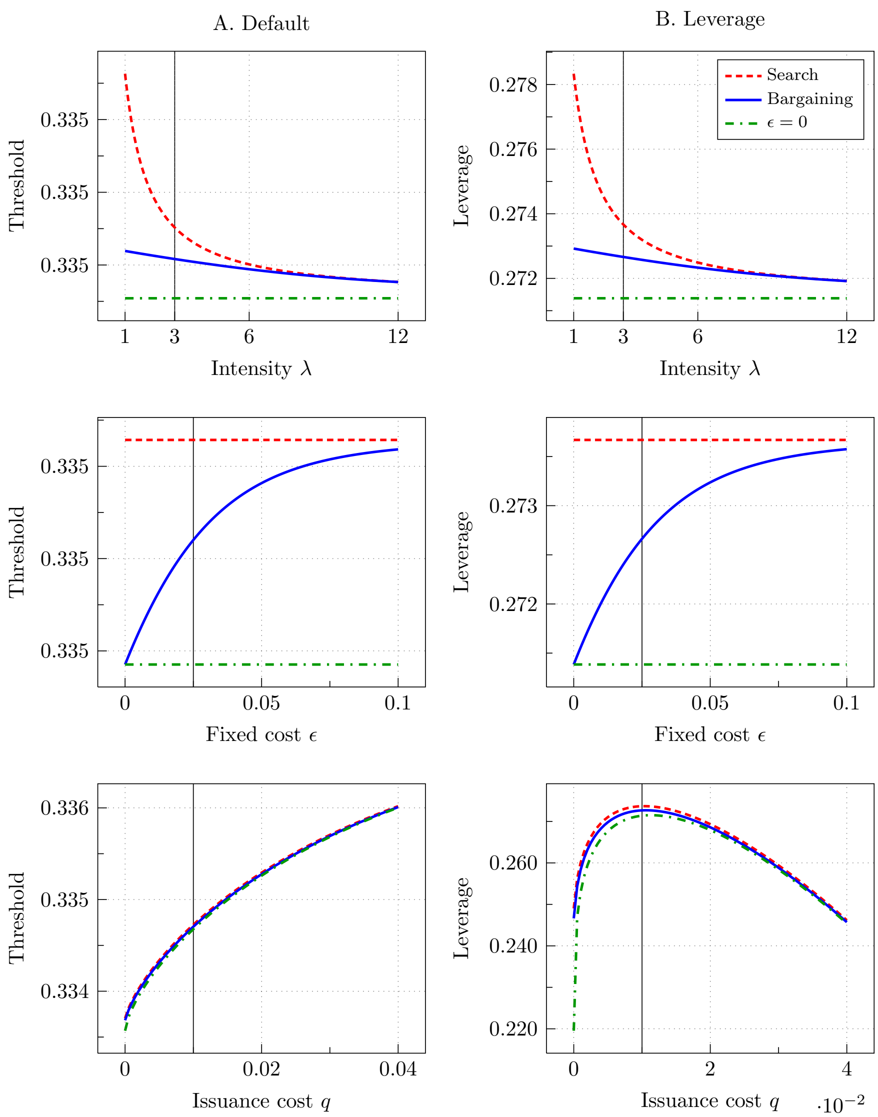
This figure plots the restructuring threshold for the search model $X_u^*(1)$ (dashed), the outside restructuring threshold $X_{\mathbf{b}u}(1)$ (solid), the inside restructuring threshold $X_{\mathbf{bc}}(1)$, and the restructuring threshold for the frictionless model with $\epsilon = 0$ (dash-dotted) as functions of the arrival rate of outside creditors λ, the proportional issuance cost q, the fixed issuance cost ϵ, the bargaining power of outside creditors η, the volatility of the firm's cash flows σ and the cost of default ω. In each panel the vertical line indicates the base case value of the parameter that is being varied.

Figure 2: Probability of restructuring before default



This figure plots the probability of restructuring at a three years horizon (panel A.) and the unconditional probability of restructuring (panel B.) as functions of the arrival rate of outside creditors λ, the fixed issuance cost ϵ and the proportional issuance cost q. The dotted (dashed) line gives the probability of restructuring with inside (outside) creditors, the solid line gives the probability of restructuring with either inside or outside creditors and the dot-dashed line gives probability of restructuring in the frictionless model with $\varepsilon = 0$. In each panel the vertical line indicates the base case value of the parameter that is being varied.

Figure 3: Default and Leverage



This figure plots the default threshold (panel A.) and the target leverage ratio (panel B.) for the search model (dashed), the model with inside creditors (solid) and the frictionless model with $\epsilon = 0$ (dot-dashed) as functions of the arrival rate of outside creditors λ, the fixed issuance cost ϵ and the proportional issuance cost q. In each panel the vertical line indicates the base case value of the parameter that is being varied.