ESANN'2002 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 24-26 April 2002, d-side publi., ISBN 2-930307-02-1, pp. 307-312

Separation of a Mixture of Signals Using Linear Filtering and Second Order Statistics*

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Abstract. Some recent works address the problem of blind source separation with a matrix pencil. In this paper we show that the covariance matrices of the pencil can be computed at the output of a simple linear filter instead of using time-delayed covariance matrices. It is also shown, using block matrix manipulation, that the method might applied when the number of source signals is not equal to the number of mixed signals. An experimental study, comparing different strategies of computing the matrix pencil, is also presented.

1 Introduction

The Blind Source Signal Separation is a problem that arises in many application areas such as communications, speech and biomedical signal processing. The objective is to extract the source signals from some sensor measurements $\mathbf{x}(t)$. Generally, it is assumed that each measured signal is an instantaneous mixture of the source signals. The mathematical model for this problem is $\mathbf{x}(t) = A\mathbf{s}(t)$, where A is the mixing matrix, $\mathbf{s}(t)$ is a vector of source signals at time t. The extraction must be carried on without knowing the structure of the linear combination (the mixing matrix) and the source signals. Most of the solutions comprise two steps[1][2]. In the first step, called the whitening (sphering) phase, the measured data is linearly transformed such that the correlation matrix of the output vector equals the identity matrix. During this phase the dimensionality of the measured vector is also reduced to the dimension of the source vector. After that the separation matrix, between the whitening data and the output, is an orthogonal matrix computed using the fourth-order cumulant or some related method. Then the separation matrix, or an estimate of the inverse of A, is the product of the two matrices computed on the two phases of the method. Another approach comprises the simultaneous diagonalization of a matrix pencil (R_1, R_2),i.e., a generalized eigenvalue

*This work was supported by the "FCT- Fundação para a Ciência e Tecnologia" funding the project POSI/34246/SRI/2000

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decomposition (GSVD). The matrices are calculated with different strategies: Souloumiac [3] consider two segments of signals with distinct energy; Tomé[4] and Stone[5] compute one of matrices in filtered versions of the mixed signals; and Molgedey [6]and Chang [7],[8] computes time-delayed correlation matrices.

This work presents a complete linear algebra formulation for the GSVD approach to the blind source separation problem. The formulation reveals that the matrix pencil can be computed on the filtered signals and, that the quality of the separation is dependent on the eigenvector matrix of an equivalent GSVD statement on the source signals. Some simulation are also presented comparing the results achieved with the GSVD approach and FastICA algorithm [9] which is an algorithm that relies on a prewhitening phase. The simulations also compare the results achieved by GSVD when the matrix pencil is computed using filtered signals with the time-delayed correlation matrices as proposed by Chang [7] and Molgedey[6].

2 The Generalized Eigendecomposition Method

The generalized eigendecomposition comprises the simultaneous diagonalization of two matrices, the so called matrix pencil. Let X be a $m \times N$ matrix containing a segment with N samples of each of m measured signals. The correlation matrix for X is a $m \times m$ matrix and is calculated as

$$R_1 = \frac{1}{N} X X^T \tag{1}$$

Let Y be a matrix $m \times N$ having in each raw a filtered version of each raw in X. Considering that a FIR (finite impulse response) of length M ($M <<$ N) was used, the convolution operation of the linear filtering is expressed as $Y = XH^T$, where H is a $N \times N$ Toeplitz matrix with $h(n)-$the nth sample of impulse response-on the nth diagonal[10]. The correlation matrix of Y is $m \times m$ defined by

$$R_2 = \frac{1}{N} X H^T H X^T \tag{2}$$

Using the matrix pencil (R_1, R_2) the generalized eigenvalue (GSVD) statement is

$$R_1 E = R_2 E D \tag{3}$$

where E is a $m \times m$ eigenvector matrix and D is a diagonal $m \times m$ matrix with the eigenvalues of the matrix pencil (R_1, R_2). Considering that X is an instantaneous mixture of the sources signals, i.e., $X = AS$, the equation can be written as

$$A R_s A^T E = A R_{sf} A^T E D \tag{4}$$

The previous equation shows that the statement (3) is also related with the source pencil (R_s, R_{sf}). In [11] two matrix pencils are called congruent pencils

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if A is an invertible matrix. It is suggested that the eigenvalues are the same for both pencils and that the eigenvectors are related. Those assumptions can be proved [4]:

- The eigenvalues are the roots of the characteristic polynomial which is $\chi(\lambda) = \det(AR_sA^T - \lambda AR_{sf}A^T)$=$\det(R_s - \lambda R_{sf})$, if A is an invertible matrix.
- Considering the matrix pencil of the source signals, the GSVD statement should be $R_sE_s = R_{sf}E_sD$, having D unique values on the diagonal, each eigenvector for a particular eigenvalue are related by

$$E_s = A^TE$$

In what concerns the blind source separation problem the eigenvector matrix E will be an approximation to inverse of mixing matrix, if the E_s is a diagonal matrix (or a permutation). This is a fact when the matrix pencil of the source signals are both diagonal.

2.1 Rectangular Mixing Matrix

When the mixing matrix is a $m \times n$ $(m > n)$ the equation (4) might written using block matrix notation. Considering A and E divided into two blocks: A into A_H, $n \times n$, and A_L, $(m-n) \times n$; E into E_H, $n \times m$and E_L, $(m-n) \times m$,

$$\begin{bmatrix} A_H \\ A_L \end{bmatrix} R_s \begin{bmatrix} A_H^T & A_L^T \end{bmatrix} \begin{bmatrix} E_H \\ E_L \end{bmatrix} = \begin{bmatrix} A_H \\ A_L \end{bmatrix} R_{sf} \begin{bmatrix} A_H^T & A_L^T \end{bmatrix} \begin{bmatrix} E_H \\ E_L \end{bmatrix} D \tag{5}$$

Working the previous expression, two equations are obtained

$$\begin{aligned} A_HR_s\Phi = A_HR_{sf}\Phi D \\ A_LR_s\Phi = A_LR_{sf}\Phi D \end{aligned} \tag{6}$$

where $\Phi = A_H^TE_H + A_L^TE_L = A^TE$ is $n \times m$ matrix. The first equation shows that this case also resumes the relation among equivalent pencils. Φ is a matrix that also represents the eigenvector matrix of the source matrix pencil having $(m-n)$ columns of zeroes paired with the eigenvalues in D that does not belong to eigenvalue decomposition of (R_s, R_{sf}). In the blind source separation, it is possible to find out the number of sources because after the separation $(m-n)$ zero amplitude signals are obtained.

3 Simulation Results

Some simulations are carried on using the signals of the demo created in [9]and colored noise as described in[7]. The first group of signals are used to illustrate the results of the method described here using a two coefficients FIR filter

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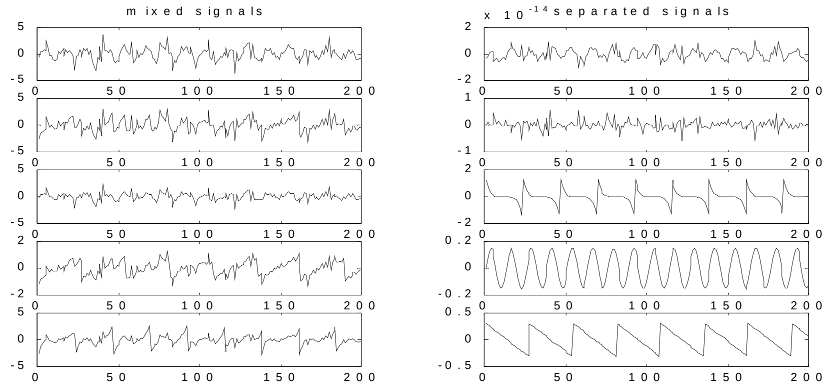

Figure 1: Separation of the 4 source signals having 5 mixed signals

Method	source 1	source 2	source 3	source 4
GSVD/FIR	0.98	0.94	0.96	0.90
GSVD/TD	0.67	0.67	0.70	0.82
FastICA	0.95	0.95	0.94	0.87

Table 1: The performance index (FastIca toolbox signals and 4x4 mixing matrix)

(h = [0.5 0.5]). The figure(1) shows the rectangular mixing cases where the first separated signal has very low amplitude as expected.

The second experiment pretends to evaluate the quality of the separation using GSVD and different strategies to estimate the correlation matrices: on filtered mixed signals (GSVD/FIR); and time-delayed correlation matrices (GSVD/TD) with ($k_1 = 1, k_2 = 2$) as described in [7] but using only one iteration, i.e., computing only one pair of matrices in the complete set of mixed signals. The results of the separation are also compared with FastICA algorithm[9]. The performance of the methods is evaluated using the performance index parameter. The parameter computes the degree of diagonalization of the product (C) of the separation matrix by the mixing matrix. This parameter is computed for each raw i of C and is defined by

$$p_i = \frac{\max(|C_i|)}{\sum |C_{ij}|}.$$

If the matrix C is a permutation of a diagonal matrix, the absolute maximum of each raw must belong to distinct columns. So, in a trial a particular performance index, p_k is considered valid if the maximum belongs to column j with no maximum of other raw($i \neq k$), otherwise the source signal j is not extracted.

Using the FastICA toolbox signals, the mean value of the performance index for each source signal, when a separation is achieved, is computed for the three

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Method	source 1	source 2	source 3	source 4
GSVD/FIR	0.88	0.74	0.70	0.78
GSVD/TD	0.77	0.64	0.59	0.68
FastICA	0.26	0.30	0.23	0.23

Table 2: -The performance index (colored noise signals and 4x4 mixing matrix)

Method	source 1	source 2	source 3	source 4
GSVD/FIR	0.99	0.94	0.96	0.90
GSVD/TD	0.67	0.71	0.70	0.84
FastICA	0.96	0.95	0.95	0.87

Table 3: The performance index (signals of FastIca toolbox and 6x4 mixing matrix)

methods (tables 1 and 3). With this signals both FastICA and GSVD/FIR separate all the sources in every trial (of a total 100) while GSVD/TD does not separate all the signals in 50% of the trials. We can see that the GSVD/TD method has the lowest performance index for all the source signals while GSVD/FIR and FastICA have very similar values. With the colored noise signals (tables 2 and 4): the FastICA algorithm does not converge in 50% of the trials and the mean value of the performance index is very low. The GSVD/TD does not separate all signals in 20% of the trials but the performance index is slightly higher when compared with the other group of signals. The GSVD/ FIR does not separate all the signals in 2% of the trials and the values of the performance indexes, for all the sources, are the highest ones.

4 Conclusions

In this paper a second order statistical method for source separation that is based on a eigendecomposition of a matrix pencil. An alternative formulation based on the definition of congruent pencil and block matrix manipulation was also presented. This method was also used in other works but the matrix pencil is computed with different strategies. The experimental study proves that the strategies used to compute the matrix pencil have influence on the performance of the method. Using correlation matrices computed on the mixed signals and on the filtered mixed signals, the GSVD method performs better than with time-delayed correlation matrices. This experimental study should be further developed in order to understand completely the possible influence of the eigenvalues on the performance as suggested in other works [3] and [8]. Chang proposes an iterative procedure based on the assumption of multiple eigenvalues and Souloumiac considers that the best solution are achieved when there is an eigenvalue spread.

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Method	source 1	source 2	source 3	source 4
GSVD/FIR	0.88	0.76	0.73	0.77
GSVD/TD	0.75	0.66	0.62	0.69
FastICA	0.26	0.26	0.24	0.23

Table 4: The performance index (colored noise signals and 6x4 mixing matrix)

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