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The Nonlinear Dynamic State Neuron

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Abstract. This research is concerned with using nonlinear dynamics to greatly enhance the range of possible behaviours of artificial neurons. A novel neuron model is presented which has a dynamic internal state defined by a set of nonlinear equations, together with a threshold driven spike output mechanism. With the aid of spike feedback control the model is able to stabilise one of a large number of Unstable Periodic Orbits in its internal dynamics. These orbits correspond to dynamic states of the neuron each of which generates a unique periodic spike train as output. The properties of this model are explored through experiments with single neurons and networks of neurons.

1 Introduction

This paper is concerned with using the rich dynamics of nonlinear systems to enhance the range of possible *behaviours* of artificial neurons. The term *behaviour* broadly refers to the information processing activities that neurons engage in, such as arriving at states of activation, generating firing patterns and so on. A long term aim of this research is to use these enhancements in neuron behaviour to overcome some of the limitations found in many artificial neural network models (e.g. low memory capacity in terms of the ratio of patterns that can be stored to the number of neurons deployed). A novel Nonlinear Discrete State (NDS) neuron model is presented which can select from a large range of possible behaviours. Experimental evidence for these behaviours is presented and discussed.

Nonlinear dynamical systems are strong candidates in the search for methods to greatly enhance the range of possible behaviours of artificial neurons. Chaotic systems continuously generate new patterns of behaviour which could be used as a basis for neural computation [1]. Furthermore, several methods of chaos control have been developed which enable the selection and stabilisation of periodic patterns of behaviour called Unstable Periodic Orbits (UPOs) [2, 3, 4]. Chaotic attractors are densely packed with a theoretically infinite number of distinct UPOs. Our research is concerned with the possibility that these UPOs be designated as the set of possible internal dynamic states of a neuron. Although practical limits on computer models of these attractors (e.g. accuracy of floating point representations of numbers) may mean that this set of states is not infinite, nevertheless it would be extremely large. We present experimental evidence that suggests that the NDS neuron can select from an immense number of distinct UPOs each corresponding to a unique internal dynamic state (see Section 3). Each UPO that is stabilised on the internal dynamics of the NDS neuron produces a temporal periodic output pattern of spikes. Experimental evidence suggests that there is a one-to-one correspondence between distinct UPOs and unique periodic spike output patterns (Section 3). If this is the case, then the NDS neuron has at its disposal a very rich range of internal states together with a *vocabulary* for communicating these states to other neurons.

2 The NDS neuron

The Nonlinear Dynamic State (NDS) neuron is a novel spiking neuron model inspired by the Rössler attractor [5]. The NDS neuron has a dynamic internal state that is modelled by three variables ($u(t)$, $x(t)$ and $y(t)$) whose behaviours are determined by the following equations:

$$u_i(t+1) = \begin{cases} \eta_0 & : u_i(t) > \theta \\ \Big[u_i(t) + d(v + u_i(t)(-x_i(t)) + ku_i(t)) \cdots \\ \qquad + w\gamma_i(t-\tau) + w_i I(t) \Big] & : u_i(t) \leq \theta \end{cases} \tag{1}$$

$$x(t+1) = x(t) + b(-y(t) - u(t)) \tag{2}$$

$$y(t+1) = y(t) + c(x(t) + ay(t)) \tag{3}$$

$$\gamma(t) = \begin{cases} 1 & : & u(t) > \theta \\ 0 & : & u(t) \leq \theta \end{cases} \tag{4}$$

where $u(t)$ represents the internal voltage of the neuron, $x(t)$ and $y(t)$ are internal state variables necessary to produce the attractor which governs the chaotic dynamics of the neuron, η_0 is the after-spike reset value for $u(t)$, γ is the spike output, w is the weight of the spike feedback control (see below), w_i is the weight of the external input $I(t)$, and θ is the firing threshold of the neuron. a, b, c, d, v, k are the parameters of the system.

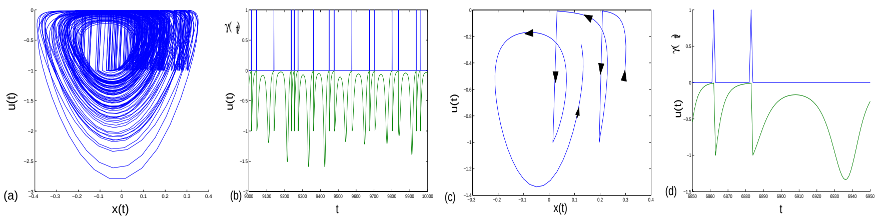

Fig. 1: Phase space plots ((a),(c)) and Time series data ((b),(d))

When $w = 0$ and $w_i = 0$ (i.e. the feedback control is inactive and there is no external input - note that unless explicitly stated otherwise $w_i = 0$ for all experiments) in equation (1), the internal state of the NDS neuron is governed by the attractor illustrated in Figure 1(a). This figure plots the phase space of the neuron in the $u(t)$ vs $x(t)$ plane. Figure 1(b) shows part of the corresponding time series for $u(t)$ and $\gamma(t)$. To illustrate how the dynamics of equations (1)

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to (4) evolve in time, Figure 1(c) shows a short trajectory in $u(t)$ vs $x(t)$ phase space representing a brief evolution of the system and Figure 1(d) shows the corresponding time series for variables $u(t)$ and $\gamma(t)$. As these figures illustrate, the NDS neuron fires whenever $u(t)$ crosses the threshold $\theta(< 0)$ from below. The firing event at time t is signified by assigning the value 1 to variable $\gamma(t)$. After firing, $u(t)$ is reset to η_0.

In the absence of feedback control ($w = 0$ in equation (1)) the internal dynamics and the output ($\gamma(t)$) of the NDS neuron are chaotic (i.e. they display bounded, aperiodic behaviour with sensitive dependence on initial conditions). A simple control mechanism is applied to the model whenever $w > 0$ in equation (1). This spike feedback control mechanism adds the output of the neuron ($\gamma(t)$) delayed by a discrete number of time steps τ to state variable $u(t)$. When spike feedback control is active the internal dynamics of the NDS neuron are stabilised into a UPO, resulting in a periodic spike train for output. An example of such an orbit is shown in Figure 2(a) and 2(b). In this example the weight of the feedback $w = 0.3$ and the delay of the SFC $\tau = 150$. In this case the UPO stabilised in the internal dynamics of the neuron (Figure 2(a)) produce a periodic output consisting of three spikes (Figure 2(b)).

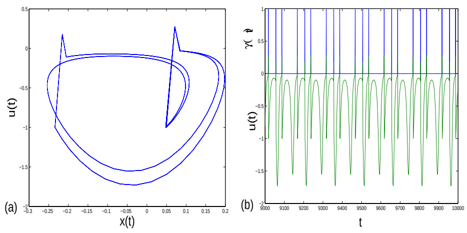

Fig. 2: A UPO stabilised by spike feedback control.

3 Experimental Results

Preliminary experiments with this model have suggested that the NDS neuron can stabilize a large number of unique periodic orbits of the kind shown in Figure 2. Small variations in the length of the feedback delay in the range 150 to 300 resulted in the stabilisation of 151 distinct UPOs on the internal dynamics of the neuron. Each of these UPOs produced a unique periodic spike train as output. Furthermore, due to the sensitive dependence on initial conditions of the NDS neuron, a small change in the initial values of the variables x, y and u will also result in different orbits being stabilized with the feedback spike control mechanism. The results from a second set of experiments have demonstrated this. In these experiments the length of the feedback delay τ is kept constant at 150 and the initial values of state variable $u(t)$ are varied from -0.4 to -0.69 in steps of 0.01. Small changes in the initial condition for the variable u resulted in very different UPOs being stabilized. Furthermore, as with the previous experiments, the periodic spike trains that are output are unique for each set of initial conditions.

From these experiments alone it has been shown that the NDS neuron has access to 151 unique UPOs by varying its feedback delay and 30 unique UPOs by varying its initial conditions within these narrow ranges. Each of these UPOs generates a unique periodic spike pattern as output. In other words, the NDS neuron has available to it a large number of internal dynamic states together with a 'vocabulary' (i.e. a *time structured* neural code) of unique spike trains with which to communicate these states.

Having demonstrated that the NDS neuron can be in one of a vary large number of discrete dynamic states with corresponding periodic spike train output, the next step is to consider how such neurons can be connected to form a functioning network. An essential property of many ANNs is the ability to reconstruct patterns of activation which have previously been stored through training. In the case of the NDS neuron this will necessarily involve reconstructing the UPO on the internal dynamics of the neuron which formed part of the previously associated pattern. As with most ANNs this reconstruction will need to be done through the properties of the various connections in the network. Crucially, it must be possible to reconstruct the UPO via external spike input to the neuron. Preliminary experiments have suggested this is possible with the NDS neuron. One such experiment involved a single NDS neuron that was controlled with a spike feedback delay of length $\tau = 100$. The model was allowed to run for 1000 time steps without feedback control ($w = 0$). Control was activated ($w = 0.5$) at $t = 1001$ and the internal dynamics of the neuron stabilized to period 3 UPO (Figure 3(a)) and the periodic spike output of the neuron was as shown in Figure 3(c). From $t = 3000$ the output of the neuron (γ) was then recorded as a binary sequence with $\gamma(t_1) = 1$ signifying the occurrence of a spike at time t_1, and $\gamma(t_2) = 0$ signifying the absence of a spike at time t_2.

At $t = 4000$ the feedback control was removed from the neuron ($w = 0$). Then the recorded binary sequence was presented as external input (I in equations (1)) to the neuron with $w_i = 0.5$. Solely on the basis of the external binary input the neuron stabilises to the identical UPO which was stabilised when spike feedback control was being applied (3(b) and Figure 3(d)). This experiment shows that it is possible to reconstruct a UPO via external input in the absence of feedback control.

There are two important points to note about this reconstruction property of the NDS neuron. The first is that, unlike spike feedback control, the reconstruction of the UPO is *not* sensitive to initial conditions. The same binary sequence will stabilise the same UPO regardless of the initial conditions of the internal state of the neuron. The second important point is that this one-to-one correspondence between internal state and the binary output sequence means that the sequence is a *compressed* binary representation of the three dimensional dynamic internal state of the neuron. The NDS neuron therefore has a large number of UPOs each with a unique compressed binary representation which can be used

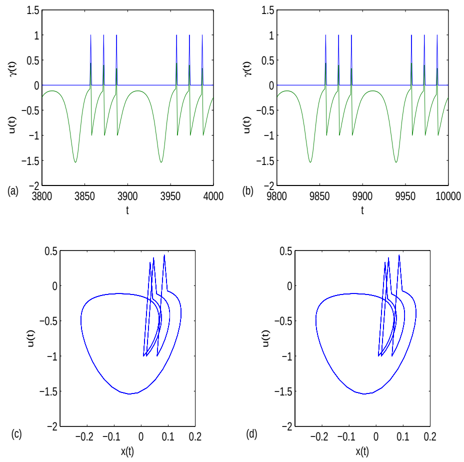

Fig. 3: The original ((a), (c)) and reconstructed ((b), (d)) UPOs

in the reconstruction of that UPO. It is envisaged that the compressed binary representations will be stored through the connection weights and delays of the network in which the NDS neuron operates. We are currently developing network architectures and associated learning mechanisms which will accomplish this.

An initial approach to constructing networks of NDS neurons was to simply connect multiple neurons with mutual weighted time-delay connections. As with the external input I, the weighted inputs from other neurons were summed and added to internal state variable $u(t)$. Each neuron in the network had spike feedback control with the weight $w = 0.3$ and delay $\tau = 100$. Several experiments involving random weights between neurons but with uniform delays on all connections were conducted. One such experiment involved three neurons with the weight matrix shown in Table 1. From the weight matrix it can been seen that neurons A and B have mutual excitatory connections, but both are connected with inhibitory connections to neuron C. After transitions the network stabilises to the UPOs shown in Figure 4. These results show that all neurons in the network stabilise, with neurons A and B stabilising to the same UPO, but different to that stabilised on C. In other words the weights of the network strongly influence the dynamics of the neurons and a globally consistent dynamic state emerges.

4 Conclusion

The results of the experiments presented here demonstrate that the NDS neuron is able to select from a large number of distinct internal dynamic states (UPOs) and can communicate these states to other neurons using a vocabulary of unique

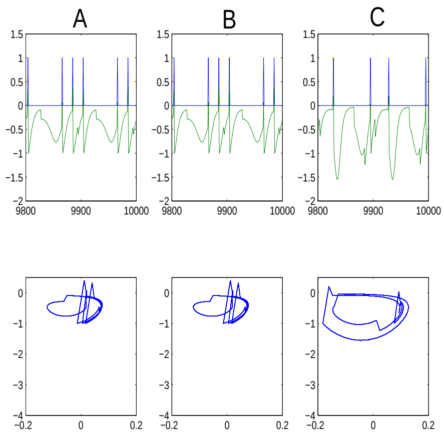

Fig. 4: The UPOs and spike output of a network of three neurons

	A	B	C
A	0.3	0.2	-0.2
B	0.2	0.3	-0.2
C	-0.2	-0.2	0.3

Table 1: The weights of a three neuron network.

periodic spike patterns. The availability of this large number of neuron states enhances the range of possible behaviours of artificial neurons. Future research in this area aims to overcome some of the limitations of artificial neural networks (e.g. memory capacity) using these enhanced behaviours. Preliminary results have been presented showing that UPOs can be reconstructed through external inputs and that small networks of NDS neurons will stabilise to orbits which have some correlation to the weights connecting them. However, possible network architectures and associated learning mechanisms for NDS neurons are the subject of continued research.

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