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Nonlinear Transient Computation and Variable Noise Tolerance

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Abstract. A novel nonlinear transient computation device is presented which is designed to perform computations on multiple spike-train input signals. The input signals perturb the internal dynamic state of the device in a way that is characteristic of the input signal presented in each case. These characteristics are reflected in the output spike train of the device. Experimental evidence is presented in this paper which shows that this output spike train is both a noise tolerant and a noise sensitive response to the input signal presented.

1 Introduction

Devices which process information through perturbations or transients in their internal dynamics can be described as *transient computation machines* [1]. Recent examples of such devices are liquid state machines (LSM) [2] and echo state machines [3]. Inputs to these devices are distributed across a large pool of neurons (referred to as the *liquid layer* in LSMs). These inputs perturb the dynamics of this pool of neurons in a manner that is characteristic of the input presented in each case. A layer of output units (*readout neurons* in LSMs) can be trained to respond to these characteristic transients in the dynamics of the pool of neurons. In this way, the device can map input patterns to target output units.

The Nonlinear Transient Computation Machine (NTCM) is a novel device which performs transient computation without the large pool of neurons required in other devices. Instead, inputs presented to the device perturb the internal dynamics of a single weakly chaotic neuron. The characteristics of this perturbation are reflected in the output spike train generated by the neuron. A detailed discussion of the properties of the NTCM have been presented elsewhere [1]. This paper is concerned with the NTCM's response to noisy input patterns. Specifically, evidence is presented of the NTCM's ability to produce both a noise tolerant and a noise sensitive response to input within the same output spike train.

2 The Nonlinear Transient Computation Machine

The Nonlinear Transient Computation Machine (NTCM) is a novel device for computing time-varying input signals. It consists of two coupled neurons, one of which acts as a pacemaker (denoted N_P) whilst the other provides the locus for the transients (denoted N_T). The internal dynamic states of both neurons are modeled by three variables ($u_i(t)$, $x_i(t)$ and $y_i(t)$ with $i = P$ for the pacemaker, and $i = T$ for the transient neuron) whose behaviors are determined by the following equations:

$$
\begin{aligned}
u_i(t+1) &\overset{(a)}{=} \begin{cases} \eta_0 & : u_i(t) > \theta \\ u_i(t) + d(v - u_i(t)x_i(t) + ku_i(t)) + \Omega_i(t) & : u_i(t) \leq \theta \end{cases} \\
x_i(t+1) &\overset{(b)}{=} x_i(t) + b(-y_i(t) - u_i(t)) + I_i(t) \\
y_i(t+1) &= y_i(t) + c(x_i(t) + ay_i(t)) \\
\gamma_i(t) &= \begin{cases} 1 & : \quad u_i(t) > \theta \\ 0 & : \quad u_i(t) \leq \theta \end{cases}
\end{aligned} \tag{1}
$$

where $u_i(t)$ represents the internal voltage of neuron i at time t, $x_i(t)$ and $y_i(t)$ are internal state variables necessary to produce the attractor which governs the chaotic dynamics of neuron i. η_0 is the after-spike reset value for $u_i(t)$ and a, b, c, d, v, and k are the parameters of the system. θ is the firing threshold and γ_i is the spike output of neuron i. $I_i(t)$ is the weighted sum of the spikes $s_j(t)(j = 1..n)$ occurring on the n external input lines at time t. External input is presented only to the transient neuron (hence $I_P(t) = 0$ for all t):

$$
I_T(t) = f(\sum_{j=1}^{n} w_j s_j(t)) \tag{2}
$$

where $f(x)$ is the sigmoid $1/(1 + exp(-\lambda x))$. For the pacemaker neuron, $\Omega_i(t)$ in Equation 1(a) denotes a time delay self-feedback control (see below) and is defined by $\Omega_P(t) = w_P\gamma_P(t-\tau)$. Whereas for the transient neuron, $\Omega_i(t)$ denotes instantaneous input from the pacemaker neuron: $\Omega_T(t) = w_T\gamma_P(t)$

In the absence of delayed self-feedback ($w_P = 0$) and external input the internal dynamics ($u_i(t)$, $x_i(t)$ and $y_i(t)$) and the output ($\gamma_i(t)$) of both neurons are weakly chaotic (with average Lyapunov exponent of approximately 0.01)[4, 1]. The coupling between the neurons defined by the equation for Ω_T results in the synchronization of the transient neuron with the pacemaker (i.e. $u_P(t) = u_T(t)$, $x_P(t) = x_T(t)$ and $y_P(t) = y_T(t)$ for $t > \chi$, where χ is sufficiently large to allow synchronization to take place.)

When self-feedback is subsequently activated at time t_c in the pacemaker (i.e. $w_P(t) > 0$ for all $t > t_c$) the output of the neuron ($\gamma_P(t)$) delayed by a discrete number of time steps τ is added to state variable $u_P(t)$. The effect of this self-feedback is to stabilize the internal dynamics of the pacemaker into a periodic orbit, resulting in a periodic spike train as output [4].

The purpose of the pacemaker (N_P) is to lead the transient neuron (N_T) into a periodic firing pattern through synchronization. While external input is being presented to N_T, the coupling from N_P is temporarily removed. The external input ($I_T(t)$ in equation 1(b)) perturbs the internal state of N_P which

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will subsequently evolve along a transient away from the periodic firing pattern induced by N_P. This transient is reflected in the output spike train of N_T. After the input has been presented the coupling with N_P is gradually restored and as N_T begins to converge back to the original synchronized periodic firing pattern, the effects of the external input on its internal dynamics fade and eventually disappear.

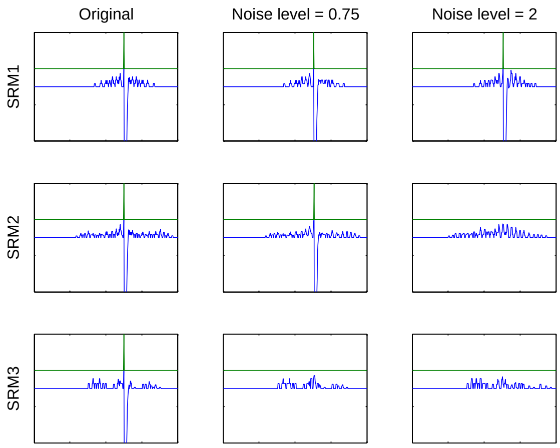

Fig. 1: The activations of the three SRM readout neurons (rows) in response to increasing noise levels in a specific input pattern (columns)

Several important properties of the NTCM have been discussed in detail elsewhere [1], including the so called *separation property* SP and the *approximation property* AP which are said to be prerequisite for computation using transients [2]. This paper is concerned with another property of the NTCM which is that it consistently produces both a noise tolerant and noise sensitive response to input patterns [1]. This occurs because small differences in inputs do not have a significant immediate effect on the internal dynamics of N_T. Hence, two similar input patterns will initially result in similar spike trains from N_T. However, because the internal dynamics of N_T are weakly chaotic, the difference in these spike trains will after some time increase at a low exponential rate. Consequently, small differences in inputs initially produce an almost identical responses from N_T, enabling noise tolerance; but these responses will eventually become quite distinct, enabling a sensitive response to noise within the same spike train.

The following experiment demonstrates how this variable response to noise can be detected by the *readout mechanism* of the NTCM. The readout mechanism is composed of three Spike Response Model (SRM) neurons [5], each of which is sensitive to a specific temporal sub-range (or *time zone*) of the spike train emitted by N_T. The first SRM is sensitive to spikes in the first 100 time steps of N_T's spike train (note that the input spike train is presented to N_T at

Jitter:	0	1	2	3	4	5	6	7	8	9
0.75 WGN:	43.1	23.7	4.1	0.1	0	0	0	0	0	0
2.0 WGN:	16.7	16.4	11.9	5.8	4.2	1.4	0.7	0.1	0.4	0

Jitter:	-9	-8	-7	-6	-5	-4	-3	-2	-1
0.75 WGN:	0	0	0	0	0	0	0	3.8	25.2
2.0 WGN:	0.1	0	0.4	0.4	1.1	3.9	8.6	11.7	16.2

Table 1: The percentage of spikes jittered by from -9 to +9 time steps for two different levels of noise.

$t = 0$). The second is responsive to spikes within the [50..150] time step window. The third is responsive to spikes in the [100..200] window. All three SRM readout neurons should fire if the input closely matches the recognized pattern (see below). As noise is introduced in the input the third readout neuron will cease to fire since it is activated by the most noise sensitive part of the N_T's spike train, whilst the other two will continue to fire. As the noise is increased further the second readout neuron will also cease to fire. The first readout neuron will continue to fire even in the presence of strong noise because it is activated by the most noise tolerant part of the N_T's spike train.

In this experiment the NTCM is first presented with a prototype pattern consisting of five independent spike trains, each containing up to 4 randomly timed spikes within the period [1..100]. The corresponding spike output of N_T is then used to construct multiple time-delayed connections from N_T to each of the readout neurons. The delays in these connections are configured so that an above threshold effect is caused on each readout neuron whenever the pattern of spikes produced by N_T in the readout's time zone matches that produced by the prototype input pattern.

Noisy versions of the prototype were created by adding *jitter* to the timing of each spike. The jitter was determined using white Gaussian noise with a mean of 0. Table 1 shows the percentage of spikes jittered by from -9 to +9 time steps in each pattern for a noise level of 0.75 and a noise level of 2 (see below). The graphs is Figure 1 show the output of the three readout neurons (rows) for three different levels of noise (columns). The first column shows that all three readout neurons fire when the NTCM is presented with the original prototype pattern. In the second column only readout neurons SRM1 and SRM2 fire when presented with a jittered version of the prototype pattern (noise level 0.75 in Table 1). The third column shows only readout neuron SRM1 firing in response to the prototype with significantly increased jitter (noise level of 2.0 in Table 1). In this way the NTCM is able to produce both a noise tolerant (indicated by readout neuron SRM1) and a noise sensitive (indicated by readout neurons SRM2 and SRM3) response to inputs through the same spike train output.

3 Results

This paper explores the variable noise tolerance of the NTCM as outlined in the previous section. Specifically, the aim is to show that the spike output of the three readout neurons is determined by the level of noise present in the input, and that readout neurons receiving early spikes from N_T (SRM1 in the above) will be more noise tolerant than those receiving late spikes from N_T (SRM2 and SRM3 in the above). To this end, the experiment described in the previous section is repeated 400,000 times with a gradual increase in the noise level after every 100 experimental runs. The noise level is simply the weight which multiplies the white Gaussian noise function (mean of 0). This weighted function is used to determine the amount of jitter added to the prototype. As shown in Table 1, larger weights increase the range of time steps by which each spike can be jittered. In the experiment presented here, the weight is increased from 0 to 4.0 in steps of 0.01. 100 experimental runs were made for each value of the weight.

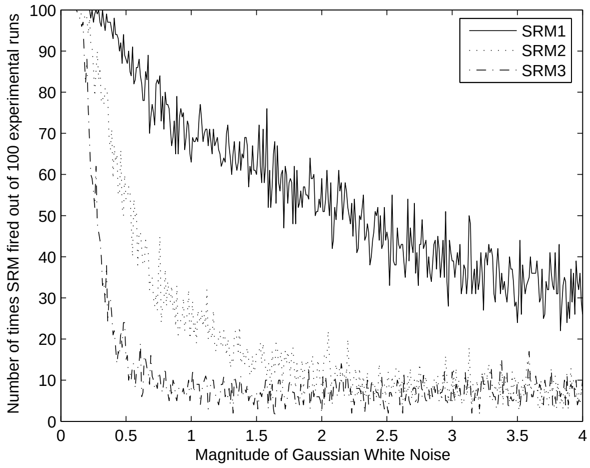

Fig. 2: The responses of the three SRM readout neurons to increasing levels of white Gaussian noise in an input pattern.

The graph in Figure 2 plots the total number of times each readout neuron fires out of the 100 experimental runs for each weight value. The graphs clearly shows that all three neurons fire in all 100 runs when the weight is in the range 0..0.2. As the weight is increased from 0.2 the percentage of cases in which SRM3 fires drops quite rapidly towards zero. The fall in the percentage of cases in which SRM2 fires also drops, but less steeply. SRM1 continues to fire in a significant number of cases even up to a value of 4 for the weight. The variance in the plots as the weight is increased is due to the variance in the amount of jitter applied to each of the 100 patterns for each weight value. The plots would smooth out as the number of runs for each weight is increased beyond 100. The plots for SRM2 and SRM3 level off but don't remain at zero as the weight is increased. This again is the result of the white Gaussian noise where a percentage of spikes in each pattern will not be jittered. Hence a small proportion of jittered patterns out of the 100 in each time step will quite closely resemble the prototype causing SRM2 and SRM3 to fire.

These results show that the spike output of the three readout neurons is determined by the level of noise in the input. They also show that readout neurons that receive early spikes from N_T are more noise tolerant than those that receive late spikes.

4 Conclusion

Experimental evidence has been provided which demonstrates that the NTCM produces both a noise tolerant and a noise sensitive response to an input pattern within the same output spike train. This property is the result of the weakly chaotic internal dynamics of the N_T neuron. Nearby points in the state space of a weakly chaotic attractor will diverge at a relatively low exponential rate. Consequently, transients in the dynamics of N_T caused by similar input patterns will initially evolve in a similar way. As a result, the early spikes of the N_T's output are tolerant of noise. Only later in the evolution will these transients begin to diverge significantly, producing the noise sensitive characteristic of the later spikes of N_T. The potential utility of this property of the NTCM for specific applications is beginning to be explored. It could be useful in the context of speech recognition, for example, where variations in the vocal characteristics of a spoken word might be of interest (for speaker identification, for instance) as well as the correct recognition of the word independent of specific vocal characteristics that communicate it.

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