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Three-dimensional self-organizing dynamical systems for discrete structures memorizing and retrieval

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Abstract. The synthesis concept for dynamical system with the memory of multiple states defined with the quaternion algebra usage is considered. The system memorizes numerous configurations consisting of separate nodes and retrieves any of them from the non-stationary distorted state. Each stored configuration corresponds to the particular attractor of the dynamical system, defined by the set of nonlinear ordinary differential equation in the hypercomplex domain. The model demonstrates the intelligence in sample structure assembling based on the initial desire, that is shown numerically in the paper. Such models can be used in robotics, complex information systems and in pattern recognition tasks.

1 Introduction

One of the key application areas for neural networks and relates techniques is the complex associative system design. Known neural network properties of pattern recognition, approximation and generalization are quite promising for making basics of intelligent technical systems. The neural networks are implemented in various areas including data mining, automatic control, image processing as the efficient mathematical implementation of working concept. At the same time, it would be very interesting to use neural networks or similar approach as the physical core of the system behavior, as it occurs in natural organisms. It is likely to say that artificial neural networks now works as the mental processing engine rather than the interconnected structure of the system components itself. One can mention some interesting works on complex associative dynamical structures analysis and synthesis in technical areas [[1], [2]], where the dynamical networks serve as the mechanism of system activity. This enables the important properties like dynamic structure assembling according to some particular goal or as the result of the existing situation recognition. One of the biggest problems is to develop the generalized approach for system design based on the associative dynamics of the set of interconnected nodes. The neural networks resemble this concept, but require adaptation to the geometrical interpretation of natural structures and engines. The work [[3]] contains the concepts of that approach including very important results for self-organization laws

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implementation for technical systems design. The background of that work has been used further to propose the advanced models of self-organizing systems with memory of multiple states represented by geometrical structures [[4],[5]]. The mentioned papers contain approaches for introducing geometrical invariants for models of pattern recognition by dynamical networks. The concept of the form or shape has been used in order to generate the common dynamical system for structure memorizing and retrieval in two-dimensional Euclidean space. It has been proved analytically that those systems are globally steady and have steady stationary solutions corresponding to memorized configurations. Therefore next step should be the three-dimensional model implementation, as it fully reflects the real natural processes and systems. Numerous papers propose various approaches of using hypercomplex numbers in order to extend the working domain and address the natural properties of artificial systems, as it is shown in [[6]]. Works [[7],[8]] contain important results on using quaternions to generate multilayer perceptrons and extend the backpropagation algorithm onto the hypercomplex domain. The work [[9]] closely relates to the presented paper and demonstrates the quaternion usage in Hopfield-like neural networks as well as outlines the dynamical and topological properties of such models, while [[9]] proposes the approach of using quaternions for self-organized control system in saccadic motion simulation.

The presented paper contains the quaternion-based approach to the synthesis of the dynamical systems which is capable to store and successfully retrieve three-dimensional structures from any non-stationary initial condition. The system consists of finite number of nonlinearly interconnected nodes, while its memory is represented by the matrix of the interconnection coefficients and computed by the Hebbian-like rule. The singular linear mapping allows achieving spatial invariance in structures retrieval. The physical interpretation of the proposed model is illustrated by the series of numerical experiments with two simplified forms memorizing and assembling. The proposed model is analytically composed and very convenient for further investigations and improvements.

2 Model of three-dimensional dynamical system with complex memory

Let define the considered system by the set of units with three coordinates. Then, current configuration in the 3-D Euclidean space can be formalized by the vector **p** with pure quaternions p_j as its components $p_j = i_1 x_j + i_2 y_j + i_3 z_j$, where *x*, *y* and *z* are real numbers representing coordinates and $i_k\,(k = 1,2,3)$ are generalized imaginary units from quaternion algebra. Therefore one can use known operations from quaternion algebra to multiply and divide quaternions. The importance of being joined to the single quaternion variable instead of using coordinates separately is shown in [[5]] for two-dimensional space where complex numbers were used. That algebra is probably the best one among other hypercomplex systems because it has the operation of division in 3D-case exclusively. At the same time, the quaternion algebra reflects linear algebra properties for three-dimensional Euclidean space, and, in particular, corresponds to non-commutative rotations. Last property can limits the usage of the proposed approach, but we can further use the additional method for synthesis of the system invariant to the rotation round the particual axis.

As it has been proposed in [[5]], let us define the system which is invariant towards the spatial transitions, where each system component is the point in the Euclidean space with the origin placed in the centroid of **p**. To enable this automatically, define the spatially invariant configuration through the real mapping $\mathbf{q} = \mathbf{Hp}$, where the matrix **H** obtains the form:

$$\mathbf{H} = \frac{1}{N}\begin{bmatrix} N-1 & -1 & ... & -1 \\ -1 & N-1 & ... & -1 \\ ... & ... & ... & ... \\ -1 & -1 & ... & N-1 \end{bmatrix}$$

Evidently, the mapping **H** converts the vector **p** into **q**, defined relatively the centroid if all system nodes contains unit masses. As in the scheme of making self-organizing systems capable to retrieve multiple configurations from [[3]-[5]], let us take the potential function as the global energy function for the considered system:

$$\Psi(\mathbf{p}) = -\frac{1}{2}\bar{\mathbf{p}}\mathbf{Jp} + \frac{1}{4}(\bar{\mathbf{p}}\mathbf{Hp})^2 + \frac{1}{4}\sum_{k=1}^{M}\sum_{l\neq k}\bar{\mathbf{p}}\mathbf{u}^{(k)}\bar{\mathbf{u}}^{(k)}\mathbf{p}\bar{\mathbf{p}}\mathbf{u}^{(l)}\bar{\mathbf{u}}^{(l)}\mathbf{p} \qquad (1)$$

with $\bar{\mathbf{p}}$ as the quaternion conjugate to **p** according to the quaternion algebra rules $\bar{\mathbf{p}} = -\mathbf{p}'$, where the prime denotes transposition. The matrix **J** plays a role of the system memory and can be computed in a manner similar to Hebbian rule $\mathbf{J} = \sum_{k=1}^{M}\mathbf{v}^{(k)}\bar{\mathbf{u}}^{(k)}$. Here vectors $\mathbf{v}^{(k)}$ (k=1,2,..,M) set up the memorized system configurations in terms of quaternions, while $\mathbf{u}^{(k)}$ are the adjont vector set to ensure the orthonormality between $\mathbf{v}^{(k)}$ and $\mathbf{u}^{(k)}$. Then relations $\bar{\mathbf{u}}^{(k)}\mathbf{v}^{(j)} = \delta_{kj}$ and $\mathbf{U} = \mathbf{V}(\bar{\mathbf{V}}\mathbf{V})^{-1}$ take place, where δ_{kj} is the Kroneker symbol, and matrices V and U consist of vectors $v^{(k)}$ and $u^{(k)}$ correspondingly. The system dynamics will be determined by the following differential equation minimizing the potential (1):

$$\left\{\mathbf{p} = \mathbf{Jp} - \sum_{k=1}^{M}\sum_{l\neq k}\left(\mathbf{u}^{(k)}\bar{\mathbf{u}}^{(k)}\mathbf{p}\right)\left(\bar{\mathbf{p}}\mathbf{u}^{(l)}\bar{\mathbf{u}}^{(l)}\mathbf{p}\right) - \mathbf{Hp}(\bar{\mathbf{p}}\mathbf{Hp})\right. \qquad (2),$$

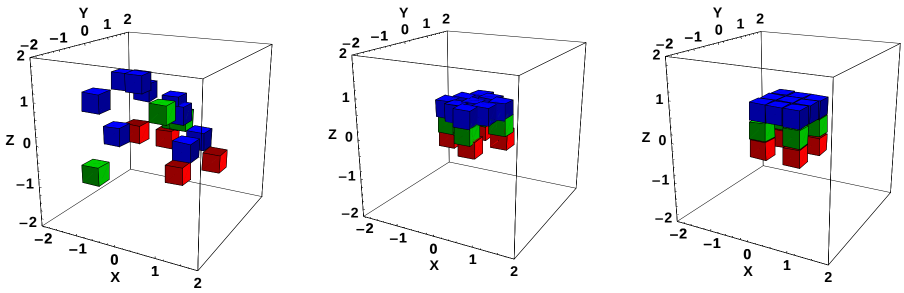
where all vectors and matrices except **H** consist of quaternions. The system (2) is spatially invariant, has M steady solutions $\mathbf{v}^{(k)}$ (k=1,2,..,M) and drifts to them from any non-stationary initial state **p**(0) by the analogy with systems from [[5]].Therefore the system restores the particular memorized configuration $\mathbf{v}^{(l)}$ from the initial state **Hp**(0) without external control, i.e. according to the self-organization taking place in nonlinear system (2). The main difference between two-dimensional systems defined by complex numbers in [[5]] and the presented one is that the last is non-invariant to the arbitrary rotations in 3D. This fact denotes that without necessary adaptation, the final configuration will be oriented in the same way as the memorized $\mathbf{v}^{(l)}$ in three-dimensional space. So if one wishes to restore something like a 'chair', that chair appears in the same orientation as it has been stored in the system memory.

The system with the potential function (1) and dynamics simulated by the equation set (2) moves to some particular memorized $\mathbf{v}^{(l)}$ from the initial state $\mathbf{Hp}(0)$ reduced to the origin, which has been placed to the centroid of the original system, and thus obtains the required form by self-organization. Each stored configuration $\mathbf{v}^{(k)}$ has its own attractor in the phase space of (2) or, in other words, locates in the certain minimum of (1). It has been proved in past works [[4],[5]], that system similar to the proposed here but defined in complex numbers has all $\mathbf{v}^{(k)}$ as steady solutions and that the system is globally steady. Therefore one can estimate that the model implemented in quaternions for three-dimensional space would have same required properties. If so, the proposed system can retrieve any memorized configuration from the closest initial state determined by the 3D structure.

2 Numerical simulations

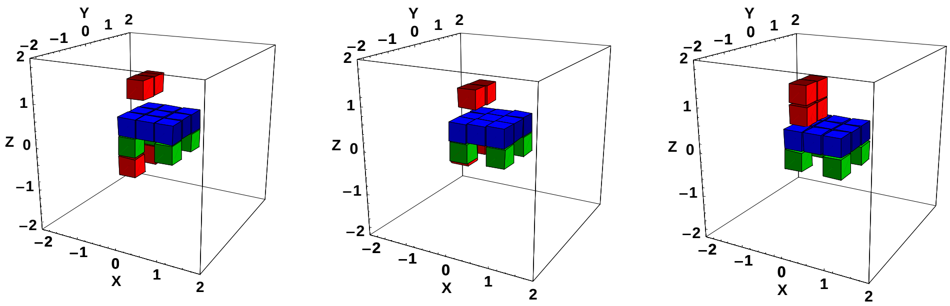
The proposed model has been investigated by series of numerical experiments with several 3D-configurations. Let us illustrate the case of the system (2) which stores two different forms: a simplified chair and a table determined by the number of units in three-dimensional Euclidean space. Each structure consists of 17 units, so M=2 and N=17, while each unit is the pure quaternion with components representing coordinates along the X-,Y- and Z-axes. To define the spatially invariant construction, the mapping $\mathbf{H}$ is used for vectors $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ composition with the origin located in the centroid. Let us then change any of standard structures in order to distort initial configuration and differ it from the chair or table, because the system holds the equilibrium in those points. The numerical experiments with the dynamical system defined through the quaternion algebra have been performed in *Mathematica* 5. The typical picture occurring when the system moves to the form of the table from the certain distorted initial state is shown in Figure 1. One can notice that three-dimensional dynamical structure successfully retrieves the memorized configuration from some state similar to that one, but takes the form exactly as it has been stored (see Figure 1c), so it is not rotation invariant unlike systems in the Euclidean plane [[5]]. In particular, the structure presented in Figure 1c is up to identical to the memorized one, but it is rotating round the Z-axis to take the exact orientation. That feature can be eliminated by using partially limited rotation fixing the rotation axis, which allows using two differential equations instead of (2).

At the same time, the intelligence of the system is demonstrated in Figure 2. The initial configuration differs from the from of the table by the fact that parts of front ножки (two lower units) look like if they are moved up by some definite reason (see Figure 2a). The virtual operator says that he wants to construct the chair and let the system perform the rest of the process. Subsequently, the dynamical system recognizes the operator's wishes and finally obtains the chair form changing own state from the initial one to the steady memorized construction (see Figure 2c) through the series of non-stationary states, with the example shown in Figure 2b.



a) b) c)

Fig.1: The structure ‘table’ assembling



a) b) c)

Fig.2: The structure ‘chair’ assembling as the recognition of the operator’s desire

The considered system demonstrates intelligent capabilities of desired structure reconstruction and assembling from the state similar to the final configuration. The process has quite clear physical interpretation in terms of mechanics and reflects the real processes of natural systems with self-assembling properties realized by the memory of form.

3 Conclusions

The novel dynamical system which is capable to memorize and restore multiple configurations in the three-dimensional Euclidean space is proposed here. The system consists of the number of units located specially to form a desired structure and mathematically introduced as the vector with pure quaternion components. Each component contains the coordinates along three axes. The process of particular structure assembling is defined by the nonlinear ordinary differential equations where each system unit is characterized by the separate dynamic equation connected to others. The initial state of the system can be similar or close to one of the memorized structures, so it is restored during the process of moving from the initial state to the steady one. The system has the desired memory with pre-defined stored configurations memorized according to the rule of Hebbian-like type. It is evidently, that the system defined by quaternions has similar stability properties as the system defined by complex numbers. The singular mapping moves the system origin to the centroid and ensures spatial invariance. The system has been numerically investigated, and the illustrations demonstrates the couple of simplified configurations which have been memorized and retrieved, while the system demonstrated the intelligence in the sense of finishing the operator's command of setting up initial configuration in order to achieve the required final state. The proposed system has some properties very similar to the known abilities of neural networks, and, at the same time, it has obvious physical interpretation in terms of natural patterns formation in three-dimensional space. The results of this work can be successfully used in robotics, pattern recognition theory and applications as well as some branches of computer-aided industrial design and information networks management.

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