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# Synchronization and Acceleration: Complementary Mechanisms of Temporal Coding

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**Abstract**. Temporal coding is studied with an oscillatory network model that is a complex-valued generalization of the Cohen-Grossberg-Hopfield system. The model is considered with synchronization and acceleration, where acceleration refers to a mechanism that causes the units of the network to oscillate with higher-phase velocity in case of stronger and/or more coherent input. Applying Hebbian memory, we demonstrate that acceleration introduces the desynchronization that is needed to segment two overlapping patterns without using inhibitory couplings.

## 1 Introduction

Temporal coding requires synchronizing as well as desynchronizing mechanisms [1]. (A review of temporal coding may be found in [2] and for a recent list of references to implementations with oscillatory networks, see [3].) Desynchronization may result from inhibitory couplings. Here, we consider an alternative approach where desynchronization is due to a mechanism that arises in the context of complex-valued neural networks. This mechanism, described and denoted as acceleration in [4], has a natural interpretation in terms of neural features. It implies that units of an oscillatory network take a higher-phase velocity for stronger and/or more coherent inputs from the other units. In the following, we describe the model and demonstrate the profound and favourable effect of acceleration on the segmentation of two overlapping patterns. More details and additional examples may be found in [4, 5, 6].

In section 2 we give the model, the examples are described in section 3, and section 4 contains the summary.

## 2 Complex-Valued Neural Network Model

The oscillatory networks model that we use is a complex-valued generalization of the classical Cohen-Grossberg-Hopfield (CGH) model [7, 8]. Given a network

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with $N$ units, where the state of each unit $k$ is described in terms of the complex coordinate $z_k$, $k = 1, \ldots, N$, the model is given by

$$\frac{\tau}{z_k}\frac{dz_k}{dt} = -\frac{1}{2}\ln\frac{z_k\bar{z}_k}{1 - z_k\bar{z}_k} + J_{1,k} + J_{2,k}z_k\bar{z}_k + \frac{1}{N}\sum_{l=1}^{N} h_{kl}\left\{a + b\,\frac{z_l\bar{z}_k}{\bar{z}_l z_k}\right\} z_l\bar{z}_l. \tag{1}$$

Here, $\bar{z}_k$ are the complex-conjugate variables, $t$ is the time, $\tau$ is a time-scale, the (Hebbian) weights $h_{kl}$ will be specified below, and $a > 0$ is a real-valued parameter. The remaining parameters may have imaginary parts:

$$J_{1,k} = I_k + \frac{i}{2}\tau\omega_{1,k}\ , \quad J_{2,k} = \frac{i}{2}\tau\omega_{2,k}\ ,$$

and

$$b = \frac{1}{2}\left(\sigma + i\tau\omega_3\right),$$

where $\sigma > 0$, $\omega_3 \geq 0$ are real. The $I_k$ describe inputs, the $\omega_{1,k}$ are eigenfrequencies and the $\omega_{2,k}$ parameterize shear terms. In the following, we find that acceleration arises from non-vanishing $\omega_3$. Notice, equation 1 may be written as a complex-valued gradient system [4].

The relation of equation 1 to the classical CGH model gets obvious by going to real coordinates with amplitudes $V_k$ and phases $\theta_k$ given by

$$z_k^2 = V_k \exp(i\theta_k), \qquad \bar{z}_k^2 = V_k \exp(-i\theta_k).$$

The dynamics is restricted to the punctured unit disk,

$$0 < z_k\bar{z}_k = V_k = g(u_k) < 1,$$

where the signal function $g$ is defined by

$$V_k = g(u_k) = \frac{1}{2}(1 + \tanh(u_k)).$$

Using $dg(u)/du = 2g(u)(1 - g(u))$ and

$$u_k = g^{-1}(V_k) = \frac{1}{2}\ln\frac{V_k}{1 - V_k},$$

we obtain as real-valued formulation of equation 1:

$$\tau\frac{du_k}{dt} = \Lambda(u_k)\left(I_k - u_k + \frac{1}{N}\sum_{l=1}^{N} w_{kl}(\theta_l - \theta_k)V_l\right) \tag{2a}$$

$$\tau\frac{d\theta_k}{dt} = \tau\omega_k\left(u, \theta\right) + \underbrace{\frac{1}{N}\sum_{l=1}^{N} s_{kl}(\theta_l - \theta_k)V_l,}_{\text{synchronization terms}} \tag{2b}$$

with

$$\omega_k(u,\theta) = \omega_{1,k} + \omega_{2,k} V_k + \underbrace{\frac{1}{N}\sum_{l=1}^{N} \Delta\omega_{kl}(\theta_l - \theta_k) V_l}_{\text{acceleration terms}} .$$

The scaling factor is

$$\Lambda(u_k) = \frac{1}{1 - V_k}$$

and the phase-dependent couplings are given by

$$w_{kl}(\theta) = h_{kl}\left(a + \frac{\sigma}{2}\cos(\theta) - \frac{\tau\omega_3}{2}\sin(\theta)\right), \tag{3}$$

$$s_{kl}(\theta) = h_{kl}\,\sigma \sin(\theta), \tag{4}$$

$$\Delta\omega_{kl}(\theta) = h_{kl}\,\omega_3 \cos(\theta). \tag{5}$$

Equation 2a is recognized as being of the CGH type, now with phase dependent couplings and phase dynamics given by equation 2b. The general system with all-order mode couplings is given in [4].

The storage of $P$ patterns $\xi_k^p$, with $p = 1, \ldots, P$, enters equation 2 through the couplings $h_{kl}$. Here, it is sufficient to assume that $\xi_k^p \in \{0, 1\}$, where 1 (0) corresponds to an on-state (off-state). In equations 3 to 5, Hebbian memory may be used that is defined by

$$h_{kl} = \sum_{p=1}^{P} \lambda_p\, \xi_k^p \xi_l^p, \tag{6}$$

with $\lambda_p > 0$. The $\lambda_p$ describe the weights for patterns $p$.

The collective dynamics of the network may be described in terms of coherences $C_p$ and phases $\Psi_p$ of the patterns, given by

$$Z_p = C_p \exp(i\Psi_p) = \frac{1}{N_p}\sum_{k=1}^{N} \xi_k^p V_k \exp(i\theta_k),$$

where $C_p$, $\Psi_p$ are real, and $0 \leq C_p < 1$ [3]. These measures generalize coherence measures that were used by Kuramoto, see [9] for a review.

## 3 Examples: Two Overlapping Patterns

In this section, we demonstrate the effect that acceleration has on the segmentation of $P = 2$ overlapping patterns with equal weights $\lambda_1 = \lambda_2 = 1/2$. We consider networks of $N = 34$ units and consider two cases of stored patterns. Parameters are $\tau = 1$, $a/N = 2$, $\sigma/N = a$, and the inputs $I_k$ are chosen so that all units of the network get active. See [4] for the numerical approach. The initial values for the $u_k$ are close to zero and the initial phases $\theta_k$ are uniformly random distributed. In order to study the desynchronizing effect of acceleration without it with the effect of different eigenfrequencies and shear forces, we set $\omega_{1,k} = \omega_{2,k} = 0$ for every $k$.

### 3.1 Case of one dominating pattern

**Examples 1 and 2.** First, we consider a situation where one pattern is dominating the other, i.e., one pattern has clearly more active units than the other. We set $\xi_k^1 = 1$ for $k = 1, ..., 26$, and $\xi_k^2 = 1$ for $k = 21, ..., N$. All other components are zero. Thus, there is an overlap of 6 units. We compare the case without acceleration, i.e. $\omega_3 = 0$ (example 1), with a case of non-vanishing acceleration, $\omega_3/N = \pi/\tau > 0$ (example 2), see figure 1.

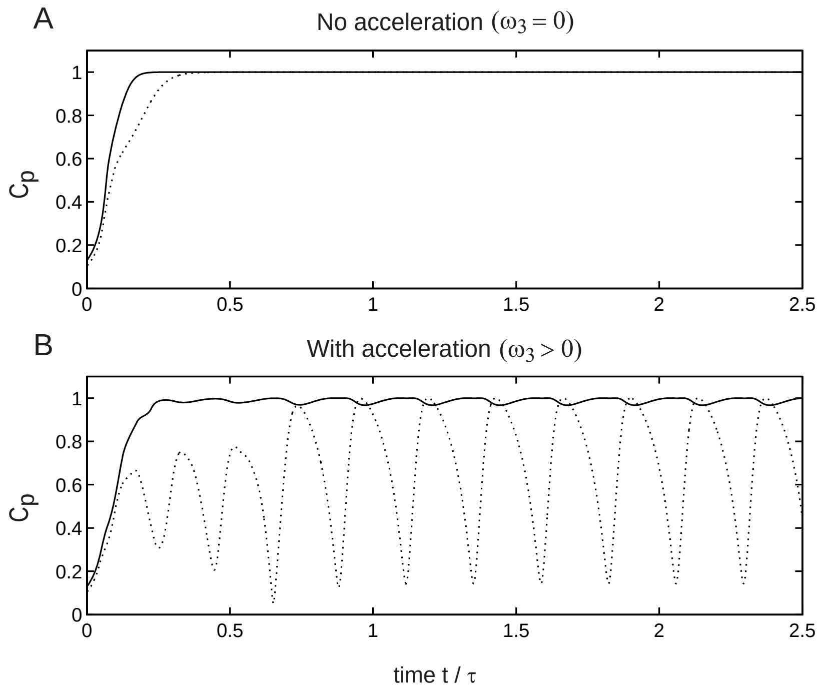


Fig. 1: Pattern coherences $C_p$ in case that pattern $p = 1$ is dominating. (A) Example 1, (B) example 2. The solid lines give $C_1$ and the dotted lines $C_2$.

Without acceleration, both patterns take the same phase and the superposition problem is present. In contrast, with acceleration, the dominating pattern $p = 1$ takes a state of enduring coherence, thereby segmenting itself from the other pattern that shows a different phase dynamics. The superposition problem is resolved with respect to pattern coherences.

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### 3.2 Without dominating pattern: pattern switching

**Examples 3 and 4.** Second, we consider a situation where none of the patterns is dominating the other, i.e., both pattern have the same number of active units. We set $\xi_k^1 = 1$ for $k = 1, ..., 22$, and $\xi_k^2 = 1$ for $k = 13, ..., N$. All other components are zero. Thus, there is an overlap of 10 units. Again, we compare the case without acceleration, i.e. $\omega_3 = 0$ (example 3), with a case of non-vanishing acceleration, $\omega_3/N = \pi/\tau > 0$ (example 4), see figure 2.

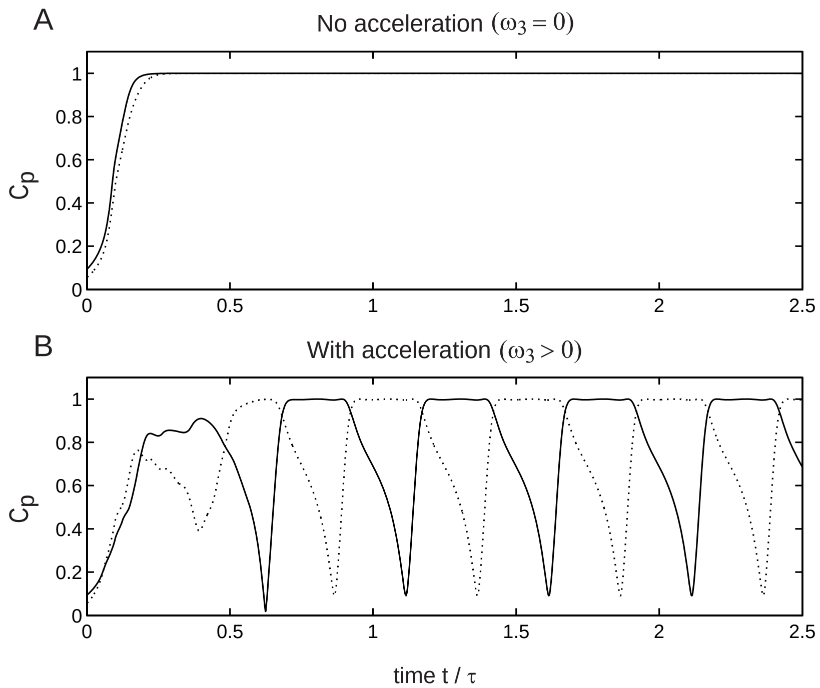


Fig. 2: Pattern coherences $C_p$ without dominating pattern (pattern switching). (A) Example 3, (B) example 4. The solid lines give $C_1$ and the dotted lines $C_2$.

Without acceleration, both patterns take the same phase and the superposition problem is present. In contrast, with acceleration, the two patterns are segmented in time by taking coherent states in alternating order. We refer to this behavior as pattern switching. Again, the superposition problem is not present in the sense that the two patterns do not take coherent states during the same time periods.

## 4 Summary

Temporal coding with synchronizing and desynchronizing mechanisms was studied based on a complex-valued neural network model. Without using inhibitory couplings, the desynchronizing mechanism was due to acceleration [4], causing the phase velocity of the oscillating units to increase with stronger and/or more coherent input from the other units.

The profound and favorable effect of acceleration on segmenting two overlapping patterns was demonstrated with examples. In case that one pattern was dominating the other in terms of a larger number of active units (assuming equal weights for the patterns in equation 6), this pattern was segmented by taking a coherent state in contrast to the other pattern. In case that no pattern was dominating, pattern switching occurred where both patterns took coherent states during time periods of alternating order.

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