

## Informational Cost In Correlation-Based Neuronal Networks

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**Abstract.** The cost of maintaining a given level of activity in a neuronal network depends on its size and degree of connectivity. Should a neural function require large-size fully-connected networks, the cost can easily exceed metabolic resources, especially for high level neural functions. We show that, even in this case, the cost can still match the energetic resources provided the function is broken down into a set of subfunctions, each assigned to a highly-connected, small-size module, all together forming a correlation-based type network. Cell assemblies are the best examples of such type of networks.

### Introduction

The broad range of connectivity found in the brain cortex [1] suggests a variety of wiring schemes, possibly associated with the various brain functions. The way this association is actually done is largely unknown. Highly connected neuronal structures ("cell-assemblies") were first hypothesized by Hebb [2] as being the neurocorrelates of "thought", "expectancy", "interest", "attention", and similar brain states. Hebb believed that a high connectivity was mandatory for these structures to be able to accomplish mass-sustained neural functions as those mentioned above, thus apparently pairing complexity of *architecture* with complexity of *function*. According to modern views, a cell-assembly is nothing but a set of cells that fire synchronously [3], [4]. Reverberations, quasi-periodic oscillations, and chaos are typical outcomes of such assemblies [5]. Thus, we propose a better definition of a cell-assembly as a *functionally coherent* set of cells. Cell memberships in the

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assembly is recognized through “correlation” of its activity with the activities of the other members, rather than upon “connectivity”. The dynamics of such neuronal networks cannot be investigated by looking at the responses of a few cells, however correlated they might be. A brain state is likely to be the result of an almost uncountable interactions among neurons. A *mass*-dynamics is more appropriate to describe distributed mass-functions. The corresponding neurocorrelates should also conform with a distributed character. The related metabolic cost should then be assessed for a distributed architecture.

Hebb’s hypothesis of full connectivity as a necessary condition to achieve specific responses cannot be disregarded solely on the basis of energy and volume constraints [6], as the extension of wiring, and the associated cost, *can* actually be acceptable provided a neural function could be broken down into pieces, each allotted to a different Hebbian module. An architecture of distributed Hebbian modules would be an acceptable support for a neural function. The corresponding cost contains an *informational* contribution that does not appear in the case of a single non-Hebbian neuronal network in-bulk. In what follows, the cost difference is evaluated as a function of the number of Hebbian modules.

## 2. Cost and Connectivity

The axonal cost is proportional to the total *length* of the links that carry an action potential (AP). With  $M$  the connectivity, i.e., the number of links per neuron (which depends on the type of cell), and  $N_1/N=\lambda$  the fraction of active neurons at a time  $t$ , the number of “hot” links is  $\lambda NM$ . On assuming the cells arranged in a simple cubic structure of lattice constant  $a$ , a rough estimate of the average length of a link is  $\frac{1}{2}$  the diagonal of a cube of side  $na$ , with  $n^3=N$  [7], namely:  $\langle \ell \rangle \approx aN^{1/3} \sqrt{3}/2$ . With  $\rho$  the neuronal density (about  $10^{11}$ /liter), the estimate of the lattice constant is  $a=\rho^{-1/3} \approx 0.02$  mm. Thus, the average axonal cost is proportional to  $\lambda NM \langle \ell \rangle$ .

At the end of the axon, the AP triggers a series of events that eventually culminate in the initiation of an AP in the postsynaptic neuron. Assuming a single synapse per link, the pre-synaptic cost is simply proportional to the number of hot links. There is also a post-synaptic cost, which is related to the action of all inputs that activate a cell. With  $M_1=\mu M$  the links between active cells, the post-synaptic cost is proportional to  $N_1 M_1 = \lambda N \mu M$ . For  $N, M$  sufficiently large  $\mu \approx \lambda$ , and  $N_1 M_1 \approx \lambda^2 MN$ .

The total cost of a network of size  $N$ , connectivity  $M$ , and activity  $\lambda$ , is the sum of the three above components. Arranged in a polynomial form, it takes the form:

$$C(\lambda; N) = \alpha(N)\lambda^2 + \beta(N)\lambda \quad (1)$$

where:

$$\begin{aligned} \alpha(N) &= c_{\text{post}} MN = c_{\text{post}} hN^2 \\ \beta(N) &= \left[ c_{\text{ax}} aN^{1/3} + c_{\text{pre}} \right] hN^2 \end{aligned} \quad (2)$$

with  $h=M/N$ , and  $c_{ax}$  ( $10^6$ ATP/mm),  $c_{pre}$  ( $10^5$  ATP/synapse),  $c_{post}$  ( $10^5$  ATP/synapse) specific cost figures [8], [9].

In general,  $M < N$  ( $h < 1$ ), except in the case of full-connectivity (*Hebbian* module), in which  $M=N$  ( $h=1$ ). It must be noticed that full-connectivity can be claimed only for networks whose size is *up to*  $M$ . In this case, in fact, the links of a neuron ( $M$ ) are equal to or exceed the number of neurons in the network. Thus, given the type of cell, there exists a critical size  $N_c=M$  beyond which  $h$  must *decrease* with the size as  $N_c/N$ .

For a Hebbian module Eq.s (2) become:

$$\begin{aligned}\alpha(N) &= c_{post} N^2 = \alpha_H \\ \beta(N) &= \left[ c_{ax} a N^{1/3} + c_{pre} \right] N^2 = \beta_H\end{aligned}\quad (3)$$

The cost increases with the size as  $N^{7/3}$ , reaching a maximum at  $N=N_c$ . For a non-Hebbian module of size  $N=nN_c$  ( $n>1$ ), made of the same type of neurons,  $h=M/N=1/n$ , and eq.s (2) become:

$$\begin{aligned}\alpha(nN_c) &= c_{post} nN_c^2 = \\ \beta(nN_c) &= \left[ c_{ax} a n^{1/3} N_c^{1/3} + c_{pre} \right] nN_c^2\end{aligned}\quad (4)$$

In this case the cost increases with the size as  $N^{4/3}$ , i.e., considerably less than in the case of a Hebbian module.

### 3. Informational Cost

The cost of driving a network made of  $n$  identical, fully-connected ( $h=1$ ) modules each of size  $N_c$ , and with a *distributed* activity, is:

$$C_H = \sum_{i=1}^n \alpha_H \lambda_i^2 + \sum_{i=1}^n \beta_H \lambda_i = n \left[ \alpha_H \langle \lambda^2 \rangle + \beta_H \langle \lambda \rangle \right] \quad (5)$$

Interactions among modules, necessary for correlating their activities, is supposed to be *weak*, e.g., via a few physical links, or through diffusion of chemicals in the extracellular space. Thus the corresponding cost is neglected.

By replacing  $\langle \lambda^2 \rangle = \langle \lambda \rangle^2 + \sigma_\lambda^2$ , eq.(5) becomes:

$$C_H = n \left[ \alpha_H \langle \lambda \rangle^2 + \beta_H \langle \lambda \rangle \right] + n \alpha_H \sigma_\lambda^2 \quad (6)$$

The term  $n\alpha_H\sigma_\lambda^2$  accounts for the correlation of the activities across modules, which results from information processing. We refer to this term as to the *informational cost*.  $C_H$  is minimum when  $\sigma_\lambda=0$ , i.e., when *synchronization* of all activities is reached. In this condition, in fact, there is no information at all to be processed. For  $n=10$ ,  $N_c=10^3$ , the informational cost is of the order of  $10^{14}$  ATPs (per AP), a value that is within the range of the metabolic costs in the brain [9].

For a non-Hebbian module of the same size the cost is:

$$C = \alpha(nN_c)\lambda^2 + \beta(nN_c)\lambda \quad (7)$$

We recall that, for such a module,  $nh=1$ , thus the cost difference  $D(n)=C_H-C$  takes the form:

$$D(n) = N_c^2 n \left[ c_{ax} a N_c^{1/3} \left(1 - n^{1/3}\right) \langle \lambda \rangle + c_{post} \sigma_\lambda^2 \right] \quad (8)$$

For  $n$  sufficiently large,  $D(n)$  is *negative*, which means that a *set* of correlated Hebbian modules is more cost-efficient than a neuronal network in-bulk. Fig. 1 shows  $D(n)$  vs.  $n$  for different values of  $\langle \lambda \rangle$ , with  $\sigma_\lambda=0.02\langle \lambda \rangle$ , and  $N_c=1000$ .

We may conclude that, provided a neural function could actually be *distributed* in  $n$  correlated pieces - each having a Hebbian module as a support - then the cost of carrying out the function is the least possible. Function decomposition might be the preferred way brain deals with high-level neural functions while keeping the cost within the energetic budget.

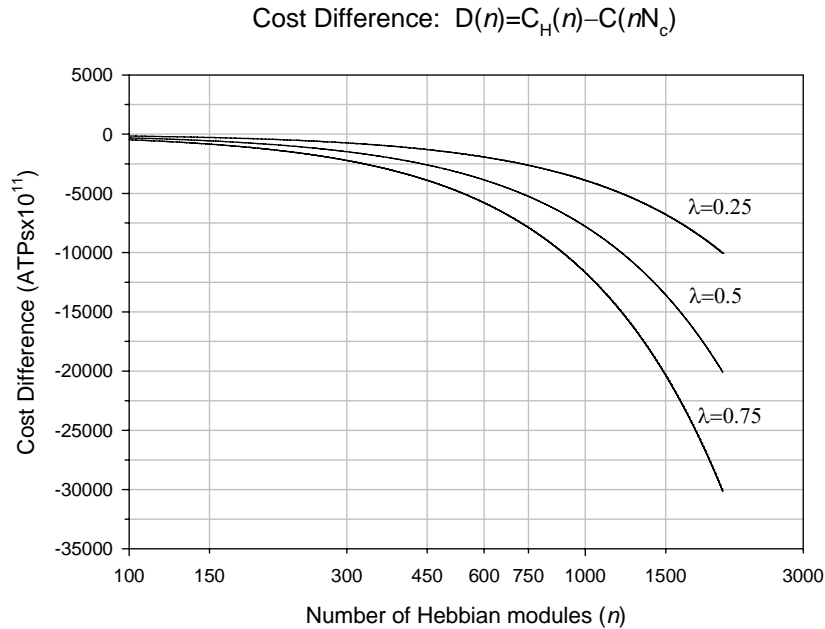


Fig. 1 – Plots of eq.(8) for  $N_c=1000$ ,  $\sigma_n/\langle\lambda\rangle=0.02$ , for three values of  $\langle\lambda\rangle$ .

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