OFDM MIMO Radar Design for Low-Angle Tracking Using Mutual Information †

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***Abstract*—We develop an information theoretic waveform design algorithm for target tracking in the presence of multipath. We employ a co-located multiple-input-multiple-output (MIMO) radar configuration using wideband orthogonal frequency division multiplexing (OFDM) signalling scheme. Apart from the frequency diversity provided by OFDM, we also exploit polarization to resolve the multipath signals by using polarization-sensitive transceivers. Thus, we can track the scattering coefficients of the target at different frequencies along with its position and velocity. We apply a sequential Monte Carlo method (particle filter) to track the target, while integrating an optimal waveform design technique based on mutual information criterion. Our numerical examples demonstrate the achieved performance improvement due to the adaptive waveform design.**

I. INTRODUCTION

Tracking targets in the presence of multipath is one of the challenging problems to the radar community. The areas of application in which multipath effects are significant are in low-angle tracking (sea-skimmers) [1]-[4], height finding [5], [6], command-guidance surface-to-air missiles, and radar-aided navigation and landing systems [7]. Similar problems have also been addressed in sonar literature due to bottom bounce in shallow waters [8], [9].

To resolve the multipath components it is generally known to use short pulse, multi-carrier wideband radar signals [1], [10], [11]. In this work, we employ the orthogonal frequency division multiplexing (OFDM) scheme [12], which is one of the ways to accomplish simultaneous use of several subcarriers. The frequency diversity of OFDM also provides richer information about the target as different scattering centers resonate at different frequencies. Additionally, to gain waveform diversity we employ a co-located multiple-input-multiple-output (MIMO) radar configuration [13]. Furthermore, in [14], [15] it is shown that polarization allows identification of correlated source signals (e.g., multipath) with small separation angles. Hence, we consider polarization-sensitive transceivers in our work.

We use a sequential Monte Carlo method (particle filter) to track the target. However, in contrast to the conventional open-loop tracker, we integrate the tracking procedure with an information theoretic waveform design algorithm achieve better performance. We propose a criterion based on mutual information (MI) [16] between the state and measurement vectors for selecting the optimal waveform. Previous work in the application of information theoretic criteria for radar waveform design includes [17]-[22] and the references therein. In our work, the choice of MI criterion provides a computationally efficient approach than the posterior Cramér-Rao bound based waveform design technique [23].

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In Section II, we develop a target dynamic state model and a parametric OFDM MIMO radar measurement model. Based on these models, in Section III we present a sequential Monte Carlo approach for target tracking. In Section IV, we propose an adaptive waveform design algorithm to maximize the mutual information criterion. Numerical examples and conclusions are presented in Section V and VI, respectively.

II. STATE AND MEASUREMENT MODEL

In this section, we first present a dynamic state model for target tracking. Then, we develop an OFDM MIMO radar signal model that accounts for polarimetric measurements over multiple frequencies.

A. Dynamic State Model

We consider the position, velocity, and scattering parameters at different frequencies of the target into our state model. The scattering matrix of the target at a particular frequency can be represented as

$$\boldsymbol{S}_l^{\mathrm{t}} = \begin{bmatrix} s_l^{\mathrm{hh}} & s_l^{\mathrm{hv}} \\ s_l^{\mathrm{vh}} & s_l^{\mathrm{vv}} \end{bmatrix}, \tag{1}$$

where s_l^{hv} is the complex scattering coefficient of the target in the horizontally polarized component of the received signal due to the vertically polarized component of the transmitted signal at the l-th frequency; similarly for the other quantities. From Huynen's work [24], we also know that $\boldsymbol{S}_l^{\mathrm{t}} \triangleq \boldsymbol{U}_l^* \boldsymbol{D}_l \boldsymbol{U}_l^H$ where

- $\boldsymbol{U}_l$ is a unitary matrix
$$\boldsymbol{U}_l = \begin{bmatrix} \cos\vartheta_l & -\sin\vartheta_l \\ \sin\vartheta_l & \cos\vartheta_l \end{bmatrix} \begin{bmatrix} \cos\varepsilon_l & j\sin\varepsilon_l \\ j\sin\varepsilon_l & \cos\varepsilon_l \end{bmatrix}, \tag{2}$$
where ϑ_l is the orientation angle of the target ellipse with respect to line of sight and relative to the radar $(-90^\circ \leq \vartheta_l \leq 90^\circ)$; ε_l is the ellipticity of the target $(-45^\circ \leq \varepsilon_l \leq 45^\circ)$ [25].
- $\boldsymbol{D}_l$ is a diagonal matrix
$$\boldsymbol{D}_l = m_l\, e^{j\varrho_l} \begin{bmatrix} e^{j\varsigma_l} & 0 \\ 0 & e^{-j\varsigma_l}\tan^2\varpi_l \end{bmatrix}, \tag{3}$$
where m_l is the maximum target amplitude; ϱ_l is the absolute phase of the scattering matrix $(-180^\circ \leq \varrho_l \leq 180^\circ)$; ς_l is called the target skip angle $(-45^\circ \leq \varsigma_l \leq 45^\circ)$; ϖ_l is called the target characteristic angle $(0^\circ \leq \varpi_l \leq 45^\circ)$ [25].

In general, all of these six target variables ϑ_l, ε_l, m_l, ϱ_l, ς_l, and ϖ_l are functions of frequency and aspect direction [24].

Considering a target at position (x, y, z) and moving with velocity $\boldsymbol{v}(= \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k})$, we construct the state vector as follows:

$$\boldsymbol{x} = \left[x, y, z, \dot{x}, \dot{y}, \dot{z}, \boldsymbol{\vartheta}^T, \boldsymbol{\varepsilon}^T, \boldsymbol{m}^T, \boldsymbol{\varrho}^T, \boldsymbol{\varsigma}^T, \boldsymbol{\varpi}^T\right]^T, \tag{4}$$

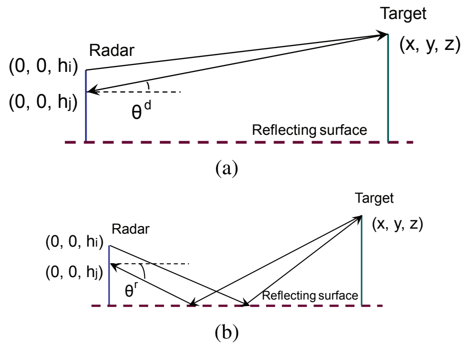

Fig. 1. (a) Direct and (b) specularly reflected signal paths.

where $\boldsymbol{\vartheta} = [\vartheta_1, \ldots, \vartheta_L]^T$, $\boldsymbol{\varepsilon} = [\varepsilon_1, \ldots, \varepsilon_L]^T$, $\boldsymbol{m} = [m_1, \ldots, m_L]^T$, $\boldsymbol{\varrho} = [\varrho_1, \ldots, \varrho_L]^T$, $\boldsymbol{\varsigma} = [\varsigma_1, \ldots, \varsigma_L]^T$, and $\boldsymbol{\varpi} = [\varpi_1, \ldots, \varpi_L]^T$.

Assuming constant velocity movement, we obtain a linear dynamic state equation at k-th time step as

$$\boldsymbol{x}_k = \begin{bmatrix} \begin{bmatrix} \boldsymbol{I}_3 & T_{\mathrm{PRI}}\boldsymbol{I}_3 \\ \boldsymbol{0} & \boldsymbol{I}_3 \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{6L} \end{bmatrix} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k, \tag{5}$$

where T_{PRI} denotes the pulse repetition interval (PRI) and $\boldsymbol{w}$ represents the state noise. In this model, we consider that the scattering coefficients of the target are almost constant. This is true, for example, when the target is far away from the radar. We assume $\boldsymbol{w}$ to be zero-mean Gaussian distributed random vector with covariance matrix

$$\boldsymbol{\Sigma}_{\boldsymbol{w}} = \begin{bmatrix} q_{\mathrm{pv}} \begin{bmatrix} \frac{T_{\mathrm{PRI}}^4}{3}\boldsymbol{I}_3 & \frac{T_{\mathrm{PRI}}^3}{2}\boldsymbol{I}_3 \\ \frac{T_{\mathrm{PRI}}^3}{2}\boldsymbol{I}_3 & T_{\mathrm{PRI}}^2\boldsymbol{I}_3 \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & q_{\mathrm{scat}}\boldsymbol{I}_{6L} \end{bmatrix},$$

where q_{pv} and q_{scat} are constants. Hence, the position and velocity of the target are statistically independent of the scattering coefficients.

B. Measurement Model

We consider an array of L transceivers forming an $L \times L$ colocated MIMO configuration. Each of the transceivers is positioned at $(0, 0, h_l)$, $l = 0, 1, \ldots, L-1$, and transmits with a carrier frequency f_l forming an OFDM signalling system with $\Delta f = B/(L+1) = 1/T$, where B is the total bandwidth and T denotes pulse duration. Each of the transceivers is capable of transmitting and receiving polarized signals. We formulate the complex envelope of the signal at j-th receiver due to i-th transmitter as a summation of three terms

$$\begin{aligned} \boldsymbol{y}_{ij}(t) &= \boldsymbol{y}_{ij}^{\mathrm{sig}}(t) + \boldsymbol{c}_{ij}(t) + \boldsymbol{e}_{ij}(t), \\ &\quad \text{for } i, j = 0, 1, \ldots, L-1,\ t = 0, 1, \ldots, N-1, \end{aligned} \tag{6}$$

where $\boldsymbol{y}_{ij}^{\mathrm{sig}}(t)$ represents the signal part of the measurement, composed of the coherent sum of direct and specularly reflected signals (as shown in Fig. 1); $\boldsymbol{c}_{ij}(t)$ represents the clutter, comprising of any randomly reflected returns within the time-span of interest; $\boldsymbol{e}_{ij}(t)$ is the thermal noise.

The complex envelope of $\boldsymbol{y}_{ij}^{\mathrm{sig}}(t)$ can be represented as

$$\boldsymbol{y}_{ij}^{\mathrm{sig}}(t) = \boldsymbol{V}(\theta_j^{\mathrm{d}}, \phi)\boldsymbol{S}_i^{\mathrm{t}}\boldsymbol{\xi}_{ij}^{\mathrm{d}}(t) + \boldsymbol{V}(\theta_j^{\mathrm{r}}, \phi)\boldsymbol{\Gamma}_j\boldsymbol{S}_i^{\mathrm{t}}\boldsymbol{\Gamma}_i\boldsymbol{\xi}_{ij}^{\mathrm{r}}(t), \tag{7}$$

where

$$\boldsymbol{\xi}_{ij}^{\mathrm{d}}(t) = \boldsymbol{p}(\alpha_i, \beta_i)\, a_i\, e^{-j2\pi(f_i + f_{\mathrm{D}ij}^{\mathrm{d}})\tau_{ij}^{\mathrm{d}}}\, e^{j2\pi f_{\mathrm{D}ij}^{\mathrm{d}} t},$$
$$\boldsymbol{\xi}_{ij}^{\mathrm{r}}(t) = \boldsymbol{p}(\alpha_i, \beta_i)\, a_i\, e^{-j2\pi(f_i + f_{\mathrm{D}ij}^{\mathrm{r}})\tau_{ij}^{\mathrm{r}}}\, e^{j2\pi f_{\mathrm{D}ij}^{\mathrm{r}} t},$$

and

- $\phi = \arctan(y/x)$, $\theta_j^{\mathrm{d}} = \arctan\left((z - h_j)/\sqrt{x^2 + y^2}\right)$, $\theta_j^{\mathrm{r}} = \arctan\left((z + h_j)/\sqrt{x^2 + y^2}\right)$.
- $\boldsymbol{V}(\theta, \phi)$ denotes the array factor for a two-dimensional polarimetric sensor, defined as

$$\boldsymbol{V}(\theta, \phi) = \begin{bmatrix} -\sin\phi & -\cos\phi\sin\theta \\ 0 & \cos\theta \end{bmatrix}.$$

- $\boldsymbol{S}_i^{\mathrm{t}}$ is the scattering matrix of the target. Here we consider same $\boldsymbol{S}_i^{\mathrm{t}}$ for both the direct and reflected path assuming that their angular separation is not large.
- $\boldsymbol{\Gamma}_j = \mathrm{diag}\{\gamma_j^{\mathrm{h}}, \gamma_j^{\mathrm{v}}\}$ denotes the reflection matrix, where

$$\gamma_j^{\mathrm{h}} \triangleq \left[\sin\theta_j^{\mathrm{r}} - \sqrt{\epsilon_0 - \cos^2\theta_j^{\mathrm{r}}}\right] / \left[\sin\theta_j^{\mathrm{r}} + \sqrt{\epsilon_0 - \cos^2\theta_j^{\mathrm{r}}}\right],$$
$$\gamma_j^{\mathrm{v}} \triangleq \left[\epsilon_0\sin\theta_j^{\mathrm{r}} - \sqrt{\epsilon_0 - \cos^2\theta_j^{\mathrm{r}}}\right] / \left[\epsilon_0\sin\theta_j^{\mathrm{r}} + \sqrt{\epsilon_0 - \cos^2\theta_j^{\mathrm{r}}}\right],$$

and ϵ_0 is the relative permittivity at the reflecting surface.
- The transmitting polarization vector is given as

$$\boldsymbol{p}(\alpha_i, \beta_i) \triangleq \begin{bmatrix} \cos\alpha_i & \sin\alpha_i \\ -\sin\alpha_i & \cos\alpha_i \end{bmatrix} \begin{bmatrix} \cos\beta_i \\ j\sin\beta_i \end{bmatrix},$$

with α and β are the orientation and ellipticity of the polarization ellipse.
- $\boldsymbol{a} = [a_0, a_1, \ldots, a_{L-1}]^T$ represents the complex weights transmitted over L transmitters (and also subcarriers).
- The delays and Doppler frequencies are expressed as

$$\begin{aligned} \tau_{ij}^{\mathrm{d}} &= \frac{1}{c}\sum_{\imath}\sqrt{x^2 + y^2 + (z - h_\imath)^2},\ \imath = i, j, \\ \tau_{ij}^{\mathrm{r}} &= \frac{1}{c}\sum_{\imath}\sqrt{x^2 + y^2 + (z + h_\imath)^2}, \\ f_{\mathrm{D}ij}^{\mathrm{d}} &= \frac{f_i}{c}\sum_{\imath}\frac{x\dot{x} + y\dot{y} + (z - h_\imath)\dot{z}}{\sqrt{x^2 + y^2 + (z - h_\imath)^2}}, \\ f_{\mathrm{D}ij}^{\mathrm{r}} &= \frac{f_i}{c}\sum_{\imath}\frac{x\dot{x} + y\dot{y} + (z + h_\imath)\dot{z}}{\sqrt{x^2 + y^2 + (z + h_\imath)^2}}, \end{aligned}$$

with $f_i = f_{\mathrm{c}} + i\,\Delta f$ denoting the $\imath$th transmitting frequency.

Stacking the measurements of all L^2 transmitter-receiver pairs and N temporal instants into a $2L^2N \times 1$ column vector we get

$$\boldsymbol{y} = \boldsymbol{y}^{\mathrm{sig}} + \boldsymbol{c} + \boldsymbol{e}, \tag{8}$$

where $\boldsymbol{y} = [\boldsymbol{y}(t_0), \boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_{N-1})]^T$ and $\boldsymbol{y}(t_n) = [\boldsymbol{y}_{00}(t_n), \ldots, \boldsymbol{y}_{0L-1}(t_n), \boldsymbol{y}_{10}(t_n), \ldots, \boldsymbol{y}_{1L-1}(t_n), \ldots, \boldsymbol{y}_{L-1L-1}(t_n)]^T$; similarly for $\boldsymbol{y}^{\mathrm{sig}}$, $\boldsymbol{c}$, and $\boldsymbol{e}$.

1) Clutter Model: We model the clutter component, which also depends on the transmitted signal [23], as follows:

$$\boldsymbol{c}_{ij}(t) = \boldsymbol{V}(\theta_j^{\mathrm{c}}, \phi)\boldsymbol{S}_i^{\mathrm{c}}\,\boldsymbol{p}(\alpha_i, \beta_i)\, a_i\, e^{-j2\pi f_i \tau_{ij}^{\mathrm{c}}}, \tag{9}$$

where θ^{c} indicates the direction of the radar beam and τ^{c} is the corresponding average clutter delay. Defining [26]

$$\begin{aligned} \boldsymbol{s}_i^{\mathrm{c}} &\triangleq \left[s_i^{\mathrm{hh,c}}, s_i^{\mathrm{vv,c}}, s_i^{\mathrm{hv,c}}, s_i^{\mathrm{vh,c}}\right]^T, \\ \widetilde{\boldsymbol{P}}_{ij}^{\mathrm{c}} &\triangleq \begin{bmatrix} \widetilde{p}_1^{\mathrm{c}} & 0 & \widetilde{p}_2^{\mathrm{c}} & 0 \\ 0 & \widetilde{p}_2^{\mathrm{c}} & 0 & \widetilde{p}_1^{\mathrm{c}} \end{bmatrix}, \\ \left[\widetilde{p}_1^{\mathrm{c}} \quad \widetilde{p}_2^{\mathrm{c}}\right]^T &\triangleq \boldsymbol{p}(\alpha_i, \beta_i)\, a_i\, e^{-j2\pi f_i \tau_{ij}^{\mathrm{c}}}, \end{aligned} \tag{10}$$

we can write (9) as

$$\boldsymbol{c}_{ij}(t) = \boldsymbol{V}(\theta_j^{\mathrm{c}}, \phi)\widetilde{\boldsymbol{P}}_{ij}^{\mathrm{c}}\boldsymbol{s}_i^{\mathrm{c}}. \tag{11}$$

2) Statistical Assumptions: We assume that the clutter response, $\boldsymbol{s}_i^{\mathrm{c}}$, and thermal noise component, $\boldsymbol{e}_{ij}$, are statistically independent complex Gaussian vectors with zero mean. They are completely characterized by their covariance matrices, $\boldsymbol{\Sigma}_{\mathrm{c}}$ and $\sigma_{\mathrm{e}}^2\boldsymbol{I}_2$, respectively. We further assume that the clutter and noise responses are uncorrelated among different frequency channels and spatially and temporally white. Under these assumptions, the measurement vector (8) is distributed as

$$\boldsymbol{y} \quad \sim \quad \mathbb{C}\mathcal{N}\left(\boldsymbol{y}^{\mathrm{sig}}, \boldsymbol{\Sigma}\right), \tag{12}$$

where

$$\boldsymbol{\Sigma} \quad = \quad \boldsymbol{I}_N \otimes \left(\boldsymbol{Q}\left(\boldsymbol{I}_{L^2} \otimes \boldsymbol{\Sigma}_{\mathrm{c}}\right)\boldsymbol{Q}^H + \sigma_{\mathrm{e}}^2\boldsymbol{I}_{2L^2}\right), \tag{13}$$

$$\boldsymbol{Q} \quad \triangleq \quad \text{blkdiag}\left(\boldsymbol{V}(\theta_0^{\mathrm{c}},\phi)\widetilde{\boldsymbol{P}}_{00}^{\mathrm{c}}, \ldots, \boldsymbol{V}(\theta_{L-1}^{\mathrm{c}},\phi)\widetilde{\boldsymbol{P}}_{L-1L-1}^{\mathrm{c}}\right) \tag{14}$$

III. Tracking Filter

We employ a sequential Monte Carlo method (particle filter), which is known to be powerful for solving nonlinear and non-Gaussian Bayesian inference problems. In this approach the key idea is to represent the posterior density function by a set of random sample points with associated weights and to compute the required estimates based on these samples and weights.

Let $\boldsymbol{x}_k^{(i)}, i = 1, 2, \ldots, N_{\mathrm{x}}$, denote the sample points with associated weights $w_k^{(i)}, i = 1, 2, \ldots, N_{\mathrm{x}}$, that characterize the posterior density function at the k-th time instant. Mathematically

$$p(\boldsymbol{x}_k|\boldsymbol{y}_k) \approx \sum_{i=1}^{N_{\mathrm{x}}} w_k^{(i)}\delta\left(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}\right). \tag{15}$$

However, in practice the samples $\boldsymbol{x}_k^{(i)}, i = 1, 2, \ldots, N_{\mathrm{x}}$, are generated from a proposal (or importance) density function $q\left(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k\right)$, which is easier to sample from. Then, the corresponding weights are updated as

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p\left(\boldsymbol{y}_k|\boldsymbol{x}_k^{(i)}\right)p\left(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}\right)}{q\left(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k\right)}. \tag{16}$$

In this work, we use the transitional prior, $p\left(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}\right)$, as the importance density function and the importance weights are realized as $w_k^{(i)} \propto w_{k-1}^{(i)}p\left(\boldsymbol{y}_k|\boldsymbol{x}_k^{(i)}\right)$.

IV. Adaptive Waveform Design

In this section, we propose a new adaptive waveform design technique for improved tracking performance. Our approach is to mathematically formulate a utility function based on mutual information of target state and measurement vectors and then to determine the parameters of the next pulse by maximizing this utility function.

A. Mutual Information

In probability theory and information theory, the mutual information of two random variables is a quantity that measures the mutual dependence of the two variables. Mathematically, the mutual information of two random variables X and Y is defined as [16]:

$$I(X;Y) = \mathrm{E}_{X,Y}\left[\log\frac{p(X,Y)}{p(X)p(Y)}\right] = H(X) - H(X|Y), \tag{17}$$

where $p(X,Y)$ is the joint probability distribution function and $p(X)$, $p(Y)$ are the marginal probability distribution functions of X and Y; marginal entropy $H(X)$ quantifies the amount of uncertainty in X and conditional entropy $H(X|Y)$ gives the measure of the uncertainty remaining in X after Y is known. Thus, the difference of these two quantities corroborates the intuitive meaning of mutual information as the reduction of uncertainty (i.e., amount of information) in X after knowing Y.

B. Waveform Design Criterion

We develop a criterion that selects the optimal waveform at the k-th time instant such that the mutual information between the state and measurement vectors at the $(k+1)$-th time instant is maximized. However, because of their availability we must also exploit the measurement history $\boldsymbol{y}_{1:k} = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_k\}$ to improve our optimization procedure. Hence, we formulate the utility function in terms of conditional mutual information as follows:

$$\widetilde{I}(\boldsymbol{x}_{k+1};\boldsymbol{y}_{k+1}|\boldsymbol{y}_{1:k}) = \mathrm{E}_{\boldsymbol{x}_{k+1},\boldsymbol{y}_{k+1}|\boldsymbol{y}_{1:k}}\left[\log\frac{p(\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1},\boldsymbol{y}_{1:k})}{p(\boldsymbol{y}_{k+1}|\boldsymbol{y}_{1:k})}\right]. \tag{18}$$

Defining

$$\Lambda(\boldsymbol{x}_{k+1},\boldsymbol{y}_{k+1}) = \log\frac{p(\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1})}{\int_{\boldsymbol{x}_{k+1}} p(\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1})p(\boldsymbol{x}_{k+1}|\boldsymbol{y}_{1:k})d\boldsymbol{x}_{k+1}},$$

we can explicitly write (18) as

$$\widetilde{I} = \int_{\mathcal{X}}\left[\int_{\mathcal{Y}}\Lambda(\boldsymbol{x}_{k+1},\boldsymbol{y}_{k+1})\,p(\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1})d\boldsymbol{y}_{k+1}\right]p(\boldsymbol{x}_{k+1}|\boldsymbol{y}_{1:k})d\boldsymbol{x}_{k+1}.$$

We use Monte Carlo integration to compute this integral. To calculate the outer integral we need samples of the state $\boldsymbol{x}_{k+1}$. From the posterior density function at the k-th time step, $p(\boldsymbol{x}_k|\boldsymbol{y}_{1:k})$, we obtain N_{x} samples $\boldsymbol{x}_k^{(i)}$ and associated weights $w_k^{(i)}$. Then the corresponding samples and weights at the $(k+1)$-th time instant are given as $\boldsymbol{x}_{k+1}^{(i)}$ and w_k^i, where $\boldsymbol{x}_{k+1}^{(i)} \sim p(\boldsymbol{x}_{k+1}|\boldsymbol{x}_k^{(i)})$ [23]. To calculate the inner integral we need samples of the measurement $\boldsymbol{y}_{k+1}$. We draw N_{y} i.i.d. samples for each $\boldsymbol{x}_{k+1}^{(i)}$ from the likelihood function $p(\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1}^{(i)})$. Then we approximate (18) as

$$\widetilde{I}(\boldsymbol{x}_{k+1};\boldsymbol{y}_{k+1}) \approx \frac{1}{N_{\mathrm{y}}}\sum_{i=1}^{N_{\mathrm{x}}}\sum_{j=1}^{N_{\mathrm{y}}} w_k^i\,\Lambda(\boldsymbol{x}_{k+1}^{(i)},\boldsymbol{y}_{k+1}^{(j)}), \tag{19}$$

where

$$\Lambda(\boldsymbol{x}_{k+1}^{(i)},\boldsymbol{y}_{k+1}^{(j)}) = \log\frac{p(\boldsymbol{y}_{k+1}^{(j)}|\boldsymbol{x}_{k+1}^{(i)})}{\sum_{i=1}^{N_{\mathrm{x}}} w_k^i p(\boldsymbol{y}_{k+1}^{(j)}|\boldsymbol{x}_{k+1}^{(i)})},$$

and obtain the optimal waveform as

$$\boldsymbol{a}_{\mathrm{opt}} = \underset{\boldsymbol{a}}{\operatorname{argmax}}\ \widetilde{I}(\boldsymbol{x}_{k+1};\boldsymbol{y}_{k+1}), \quad \text{subject to } a^H a = 1. \tag{20}$$

V. Numerical Results

We present below numerical examples to demonstrate the performance improvement due to the proposed adaptive waveform design technique for target tracking.

We consider a target that starts at a position $(x,y,z) = (20\text{ km}, 0, 50\text{ m})$ and is moving with a velocity of $3350\,(= -3350\hat{i} + 0\hat{j} + 0\hat{k})$ m/s. We assume that the target scattering parameters are partially known except for $\boldsymbol{m}$. To track this target we employ an OFDM MIMO radar with the following specifications: carrier frequency $f_{\mathrm{c}} = 1$ GHz; total available bandwidth $B = 100$ MHz; number of subcarriers (and transceivers) $L = 3$ (forming a 3×3 co-located MIMO configuration); subcarrier spacing $\Delta f = B/(L+1) = 25$ MHz; positions of the transceivers are $(0,0,100)$, $(0,0,100.15)$, and $(0,0,100.3)$ m; pulse width $T = 1/\Delta f = 40$ ns; pulse repetition interval $T_{\mathrm{PRI}} = 2$ ms; number of temporal samples $N = 5$.

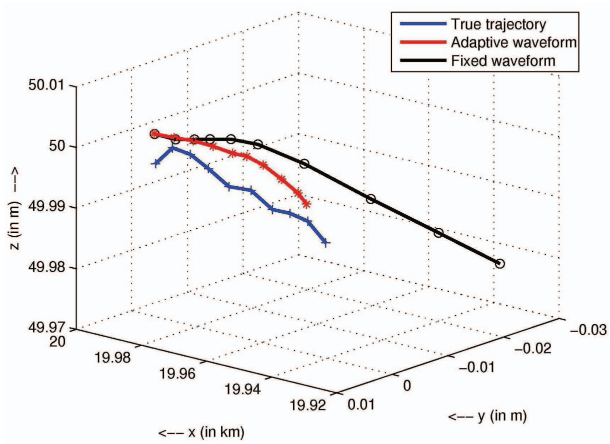

Fig. 2. Comparison of the tracking performances due to fixed and adaptive waveforms.

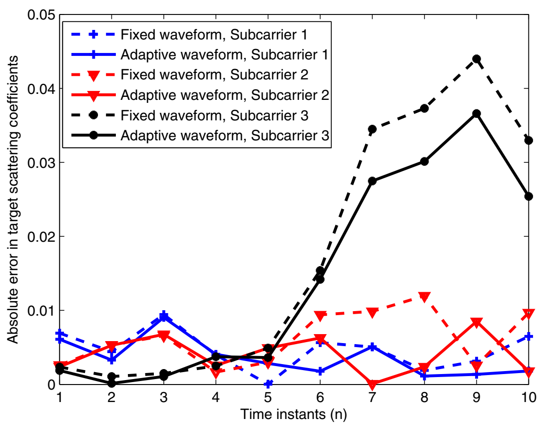

Fig. 3. Absolute error in scattering coefficients at three different subcarriers due to fixed (dotted lines) and adaptive (solid lines) waveforms.

Fig. 2 depicts the tracking performance with respect to the true trajectory for both fixed and adaptive waveforms. It is quite evident that the adaptive waveform performs much better than the fixed waveform. In Fig. 3 we plot the absolute error associated with the target scattering coefficients ($\boldsymbol{m}$) at different subcarriers. Overall the adaptive waveform results in smaller error than the fixed waveform.

VI. Conclusions

We developed an adaptive waveform design algorithm based on mutual information criterion for target tracking in the presence of multipath. We first presented a dynamic state model by considering the target scattering coefficients at different frequencies along with its position and velocity into the state vector. Then, we developed the parametric measurement model of an OFDM MIMO radar system employing polarization-sensitive transceivers. Based on the state and measurement model, we used a sequential Monte Carlo tracker (particle filter) and integrated an adaptive waveform design procedure with it. We selected the optimal waveform by maximizing the mutual information between the state and measurement vectors for one time-step ahead. We presented numerical examples to show the performance improvement achieved due to the waveform design. In our future work, we will consider more realistic modeling of the multipath scenario and address the problem of multi-target tracking.

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