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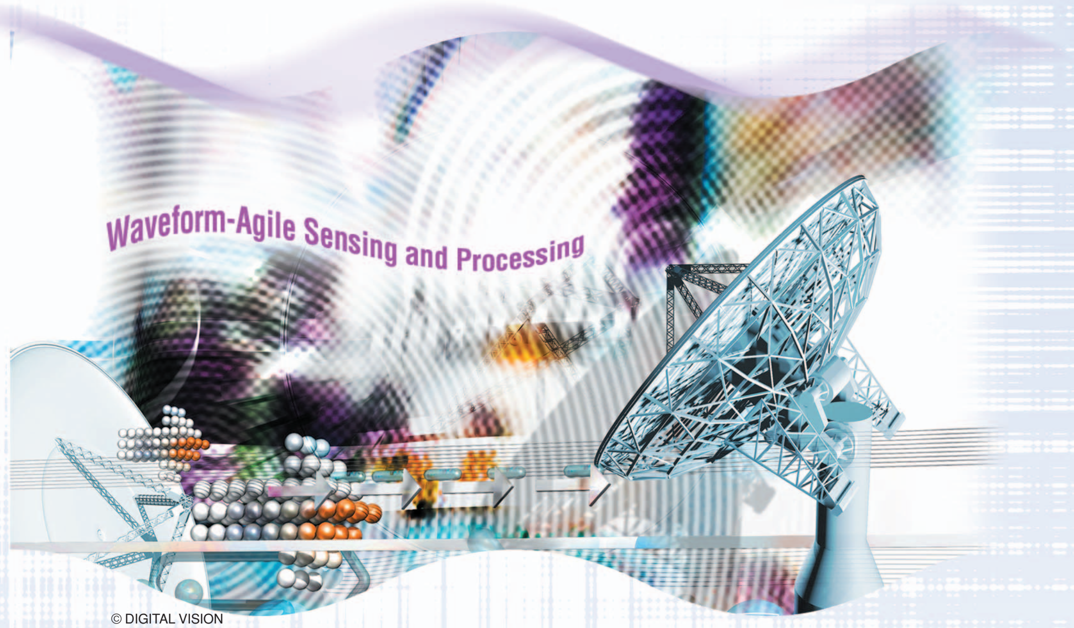

Waveform-Agile Sensing for Tracking

A review perspective

Waveform-agile sensing is motivated by the improvements in performance that can result when the transmitted waveform is dynamically tailored to match the sensing objective and the environment. Waveform agility in active radar systems can provide performance improvements such as reduced target-tracking error, improved target detection, higher target identification accuracy, and increased efficiency of sensor usage. While advances in sensor technology and flexible digital waveform generators that enable dynamic waveform design and adaptation have only been recently developed [2], [11], [36], bats and dolphins have exploited these features in their echolocation for millions of years [1], [16]. In target tracking, where target information is sequentially incorporated, changes in target-sensor geometry and the sensing environment imply that the tracker's requirements for information are constantly changing. Dynamic waveform adaptation provides a sensing methodology by designing the next transmitted waveform to optimally meet the tracker's requirements as depicted in Figure 1. This methodology can be compared to a mathematical game of twenty questions between the sensor and the environment in which the selected waveforms play the role of the questions [9].

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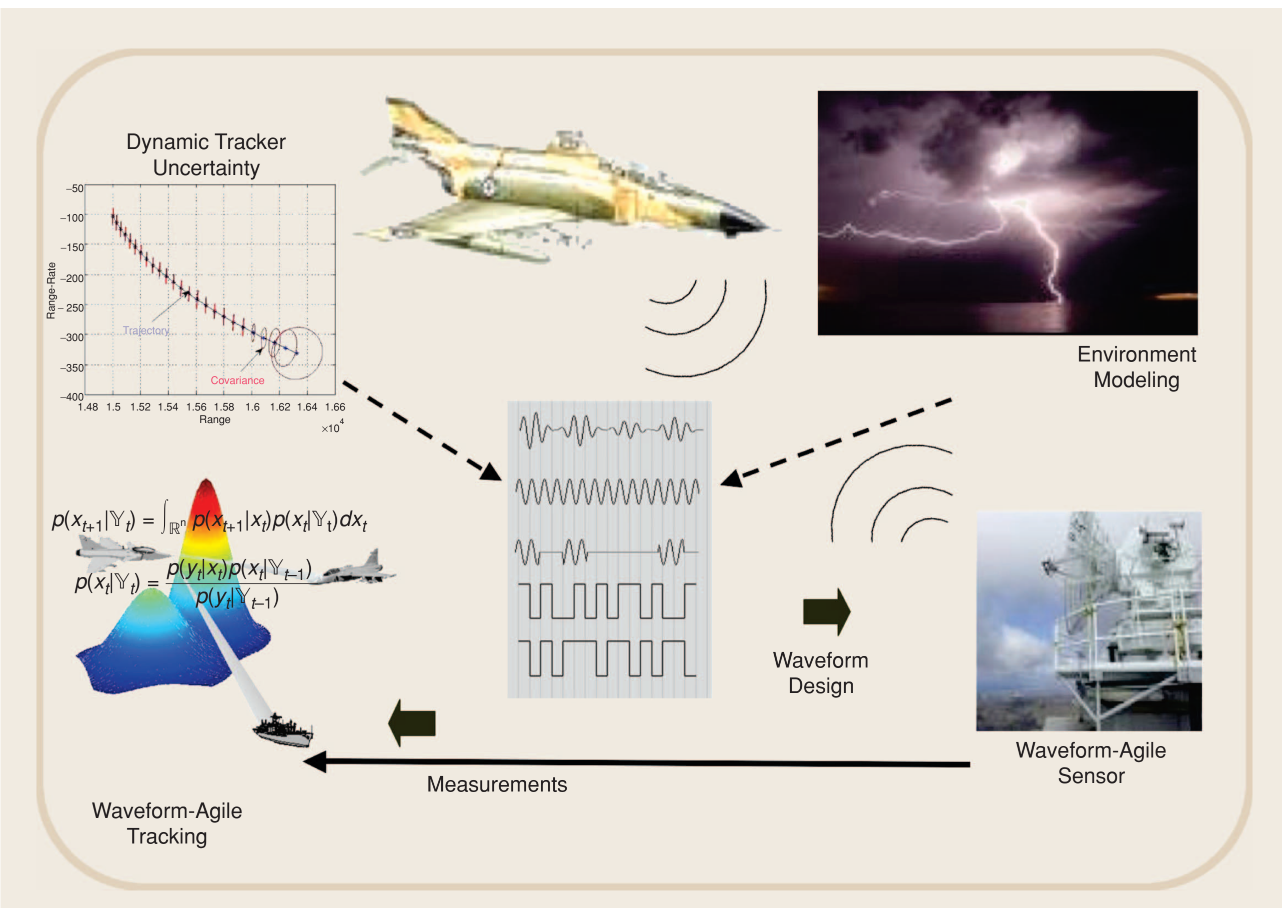
Computational constraints also provide motivation for the use of waveform adaptation. As sensor capabilities have grown, so has the amount of data they gather, with the result that processing bottlenecks severely inhibit system capabilities. Thus, it becomes imperative to only collect data that is matched to the sensing objective. For example, using waveforms that can only increase Doppler resolution does not provide any information in identifying an object that is known to be stationary. Therefore, the ability to intelligently direct sensing resources to gather the most pertinent data takes on a new relevance.

Early attempts at optimizing active tracking systems treated the sensor and tracker as independent subsystems [5]. Accordingly, previous works [10], [33], and [34] sought primarily to improve the received matched filter response in order to maximize the resolution, minimize the effects of mismatched filter design, and optimally design the waveform for reverberation-limited environments or for clutter rejection. With the advent of state-of-the-art waveform-agile techniques, it is now possible to integrate the sensor and tracker subsystems to increase target-tracking performance [36], [58].

In radar, two main approaches to waveform-agile sensing have been considered: the control theoretic approach [4], [17], [22], [23], [31], [41], [45], [47], and the information theoretic approach [6], [18], [26], [59]. From a control theoretic perspective, initial work focused on the selection of waveforms to satisfy constraints on the desired peak or average power of the transmitted waveform [4], [41]. More recent approaches have exploited the idea of optimizing a waveform-dependent cost function, such as the mean-squared tracking error, to update the transmitted waveform parameters for the next time step [17], [22], [23], [45]. This optimization results in a feedback loop, wherein the waveform selected affects the next observation and hence the tracker update, which then directs the next waveform choice. This approach can be treated as a special application of the control problem where the control input is the vector consisting of the parameters of the designed waveform.

Information theory was introduced in radar by Woodward in [57], and then extended to include waveform design by Bell in [6]. The information theoretic approach to waveform-agile sensing is based on the maximization of the mutual information between targets and waveform-dependent observations. This method for dynamic waveform design was incorporated into tracking applications in [18] and [26]. It was also used in multiple-input, multiple-output (MIMO) radar systems that transmit independent waveforms via multiple transmitters [59]. In [24], sensor scheduling actions were taken based on the information expected to be gained by taking the action, while in [7] and [53], a wavelet decomposition was used to design



[FIG1] **Illustration of waveform-agile sensing for target tracking.**

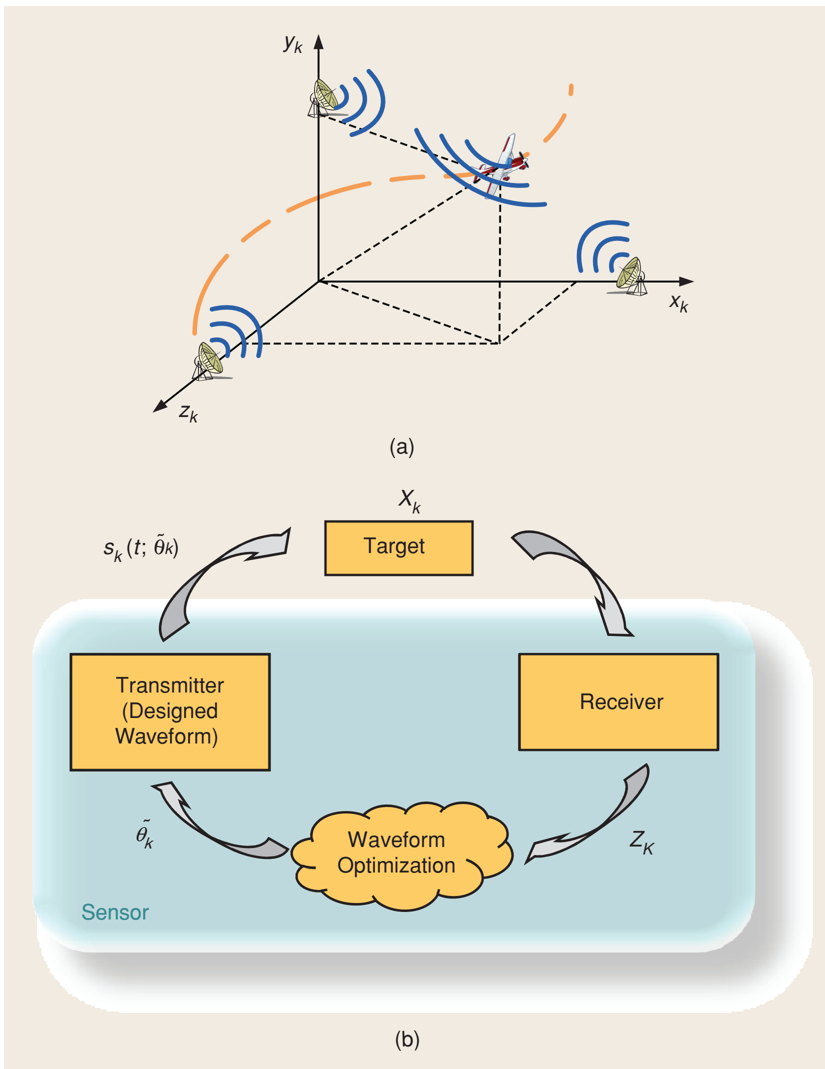
waveforms to increase the extraction of target information in nonstationary environments.

In all waveform-agile tracking applications, a critical component is a mechanism that predicts the expected observation errors that would result from a particular choice of waveform. The narrowband ambiguity function (AF) of the received waveform is a natural starting point for constructing such a mechanism since the AF provides a measure of the estimation accuracy of the delay and Doppler of the target [52]. This follows from the fact that the signal obtained after matched filtering at the receiver is directly related to the AF [35], [48]. For example, in minimizing the mean-squared tracking error under high signal-to-noise ratio (SNR) conditions in waveform design, the Cramér-Rao lower bound (CRLB) characterization is widely used because the CRLB can be obtained directly from the curvature of the peak of the AF at the origin in the delay-Doppler plane [22], [45]. This method was applied to wideband scenarios and environments with clutter and multiple targets [17], [43], [45]. The CRLB was also used in MIMO radar applications as a cost function to optimize the covariance matrix of the waveforms according to a total power constraint [29], [60]. Since the CRLB only captures the local properties of the peak of the AF, it loses relevance in low-SNR situations. An important alternative approach based on resolution cells was presented in [12], [37], and [39] to compare the steady state estimation error performance of a number of waveform combinations. A resolution cell is an area in the delay-Doppler plane enclosed by a contour of the AF (including AF sidelobes) of the transmitted waveform. Within this area, a specified probability of detection is guaranteed for given probability of false alarm and SNR values. In yet another application based on the AF in high-clutter scenarios, an adaptive pulse-diverse waveform design was used that incorporated a constraint on the allowable height of the AF sidelobes [8]. Other approaches to waveform design for tracking include the use of polarization diversity to improve the tracking accuracy in the presence of clutter [19].

This article provides an overview on waveform-agile sensing for target tracking. Although we cite many relevant approaches, this is by no means a comprehensive list due to space limitations. We first formulate the target-tracking problem, then provide overviews of the control theoretic and information theoretic approaches to waveform-agile tracking, and finally consider some tracking examples in different scenarios. Although not discussed here, waveform-agile sensing has also found applications in detection [25], [44], [56], classification [15], [32], [49], and MIMO radar [13], [14], [29], [54], [60].

TARGET-TRACKING FORMULATION

To provide context for our review of dynamic waveform adaptation for tracking, we introduce here a specific target-tracking scenario. In Figure 2(a), the motion of a single target moving in a three-dimensional (3-D) space is tracked by three waveform-agile radar sensors. The target state vector at time step k is denoted by X_k. This vector consists of dynamic parameters of the target, such as position, velocity and direction of arrival, that are to be estimated. For the example in Figure 2, the state $\mathrm{X}_k = [x_k\ y_k\ z_k\ \dot{x}_k\ \dot{y}_k\ \dot{z}_k]^T$ represents the Cartesian position and velocity coordinates, (x_k, y_k, z_k) and $(\dot{x}_k, \dot{y}_k, \dot{z}_k)$, respectively. T denotes vector transpose. The observations from the sensors at time step k are concatenated and denoted by Z_k. Following this



[FIG2] (a) Demonstration of waveform-agile sensing for target tracking in 3-D using three active sensors. (b) The transmitted waveform for each sensor is chosen at each time step k to optimize a specific performance metric.

notation, the target dynamic state and observation equations can be written as

$$X_k = f_k(X_{k-1}, W_{k-1}) \quad \text{and} \quad Z_k = h_k(X_k, V_k), \qquad (1)$$

where $f_k(\cdot)$ and $h_k(\cdot)$ are time-varying (and possibly nonlinear) functions, W_k represents stochastic state modeling errors, and V_k is the observation noise. The tracking objective is to estimate $X_{1:k} = [X_1\ X_2\ \ldots\ X_k]$, based on the available observations $Z_{1:k} = [Z_1\ Z_2\ \ldots\ Z_k]$. The tracker sequentially estimates the probability density function $p(X_k|Z_{1:k})$, and the related mean provides an estimate $\hat{X}_k$ of the target state.

We denote by $s_k(t; \boldsymbol{\theta}_k)$ the transmitted waveform at time step k, which is parameterized by the vector set $\boldsymbol{\theta}_k$. Then, the objective of waveform-agile tracking is to estimate the probability density function $p(X_k|Z_{1:k}, \boldsymbol{\theta}_{1:k})$, whose related mean provides the estimate of the target state at time k. As shown in Figure 2(b), each radar sensor adapts its transmitted waveform from pulse to pulse in order to optimize the target-tracking performance. The waveform selection is implemented by a search through a waveform library or by dynamic waveform design so as to minimize (or maximize) a specified cost function, such as the predicted mean-squared error (MSE) or the mutual information between the target and the observation. If the transmitted waveform is a frequency-modulated (FM) chirp, for example, then the elements of $\boldsymbol{\theta}_k$ can include the phase function, FM rate, duration, and amplitude of the chirp.

[DYNAMIC WAVEFORM ADAPTATION PROVIDES A SENSING METHODOLOGY BY DESIGNING THE NEXT TRANSMITTED WAVEFORM TO OPTIMALLY MEET THE TRACKER'S REQUIREMENTS.]

The target-tracking algorithm depends on the characteristics of the functions in (1), and it is often based on techniques like Kalman filtering, particle filtering, or other sequential Monte Carlo methods [3]. The choice of tracking algorithm has a significant impact on the waveform selection because the chosen algorithm determines the accuracy with which the predicted cost of using a candidate waveform can be computed as well as the computational complexity involved.

WAVEFORM-AGILE SENSING APPROACHES

CONTROL THEORETIC APPROACH

As different waveforms have different resolution properties, they result in different measurement errors. Thus, dynamic waveform adaptation can be applied in tracking applications by choosing waveforms that yield small errors in those dimensions of the target state where the tracker's uncertainty is large. This approach was first used in [22], where the optimal waveform parameters for tracking one-dimensional (1-D) target motion in a clutter-free environment were derived. In this scenario, the target state consists of the range and range-rate from a single sensor so that $X_k = [r_k\ \dot{r}_k]^T$. The sensor provides observations of delay and Doppler that lead to a linear relationship between the target state and the observations. Specifically, (1) can be expressed as

$$X_k = F X_{k-1} + W_{k-1} \quad \text{and} \quad Z_k = H X_k + V_k, \qquad (2)$$

where F and H are known model matrices, and W_k and V_k are additive, zero-mean white Gaussian processes with covariance matrices Q_k and $N(\boldsymbol{\theta}_k)$, respectively. Under an assumption of high SNR, the sensor can be assumed to achieve the CRLB on the measurement error covariance. So $N(\boldsymbol{\theta}_k)$ is set to the CRLB for the waveform specified by the vector $\boldsymbol{\theta}_k$. With this characterization, the state-space model in (2) permits the use of a Kalman filter as the tracker. With a waveform library consisting of linear FM (LFM) chirps with amplitude modulation and an assumption of perfect detection, the authors in [22] derived closed-form solutions for waveforms $s_k(t; \tilde{\boldsymbol{\theta}}_k)$ that resulted in minimizing the tracking MSE. The corresponding configured parameter vector at time step k is given by

$$\tilde{\boldsymbol{\theta}}_k = \arg\min_{\boldsymbol{\theta}_k} \mathrm{Tr}\{P_{k|k}(\boldsymbol{\theta}_k)\}, \qquad (3)$$

where $\mathrm{Tr}\{\cdot\}$ denotes the matrix trace and $P_{k|k}(\boldsymbol{\theta}_k)$ is the state covariance matrix at time k, which can be computed in closed-form using the Kalman filter equations for the given $\boldsymbol{\theta}_k$. The authors also investigated optimal waveform selection when the performance metric is the minimization of the validation gate volume. This is a region in the observation space within which observations are validated as possible target reflections rather than clutter reflections [5]. In this case also, the linear dynamics and observation models permit the waveform design to be obtained in closed form. This work was extended to include clutter and imperfect detection in [23].

In many tracking scenarios, such as the 3-D target motion in the section "Target-Tracking Formulation," the observation models are nonlinear. As a result, the Kalman filter cannot be used as the tracker, and closed-form solutions to the waveform selection problem cannot be found. Kalman filter approximations or sequential Monte Carlo methods must then be used.

The waveform-agile tracking application in [40] and [50] involves maneuvering targets and thus does not use a constant velocity dynamic state model. As the tracker uses an interacting multiple model, the waveform selection is performed by minimizing the track error covariance following a number of possible cost functions. The optimal waveform is chosen from a waveform library formed using the fractional Fourier transform to rotate the AF of a base waveform.

INFORMATION THEORETIC APPROACH

The mutual information between the target state and observation vectors, denoted by $I(X_k; Z_k)$, is a measure of the information that the measurement $\mathbf{Z}_k$ provides about the target state $\mathbf{X}_k$ in (1). The greater the mutual information, the more accurately we expect to be able to estimate the target state information. The mutual information depends on the transmitted waveform $s_k(t;\boldsymbol{\theta}_k)$ with parameter vector $\boldsymbol{\theta}_k$ at time step k, since the observation $\mathbf{Z}_k$ is waveform dependent. Accordingly, maximization of the mutual information between the target and the received signal provides a method for designing the transmitted waveform parameters $\boldsymbol{\theta}_k$.

The use of information theory to design waveforms for extracting extended target information was first introduced in [6]. Specifically, transmitted waveforms are designed that maximize the mutual information $\mathrm{I}(g_k; z_k)$ between the received observation $z_k(t)$ and an ensemble of extended targets (with impulse response $g_k(t)$) that are assumed to be Gaussian with spectral variance $\sigma^2_{g_k}(f)$. From [6], the optimal transmitted waveform that maximizes the mutual information $I(g_k; z_k)$ at time step k is the waveform whose magnitude-squared spectrum is given by $|S_k(f)|^2 = \max\{0, A_k - [0.5\, T_o R_k(f)/\sigma^2_{g_k}(f)]\}$. Here, $R_k(f)$ is the spectral density of additive Gaussian noise, T_o is the total duration of observation, and A_k is obtained by constraining the total energy of the transmitted waveform [6]. Following this approach, the waveform is designed directly without being selected from a waveform library.

DYNAMIC SELECTION OF PARAMETERS FOR FM CHIRP WAVEFORMS CAN REDUCE TRACKING ERRORS.

Although waveforms can be designed by optimally selecting their parameters, they can also be selected from waveform libraries. Such libraries were designed for target tracking following an information theoretic approach [18], [51]. Specifically, the waveform was chosen from a fixed library by maximizing the expected information using the dynamic target model and the sensor observations. The expected mutual information as a function of the waveform parameter vector $\boldsymbol{\theta}_k$ is given by

$$\mathrm{I}(\mathbf{X}_k; \mathbf{Z}_k) = \log(\det[\mathcal{I} + B(\boldsymbol{\theta}_k)^{-1} P_{k|k}]),$$

where $\mathcal{I}$ is the identity matrix, $P_{k|k}$ is the state covariance matrix at time step $k, B(\boldsymbol{\theta}_k)^{-1}$ is the inverse of $B(\boldsymbol{\theta}_k) = \mathcal{A}N(\boldsymbol{\theta}_k)\mathcal{A}^T, N(\boldsymbol{\theta}_k)$ is the measurement error covariance matrix, $\mathcal{A}$ is the transformation matrix between the measurement and the target state, and $\det[\cdot]$ denotes the matrix determinant. With $N(\boldsymbol{\theta}_k)$ computed via the CRLB, the mutual information is used to compute the utility functions of different waveform libraries. Based on this information, it was demonstrated in [18] that the maximum expected information about the target state could be obtained by an LFM chirp waveform whose FM rate is either its minimum or maximum allowable value. This approach was extended to interacting multiple model trackers to allow for different dynamic models [51]. Here, the transmitted waveform was designed to decrease dynamic model uncertainty for the target of interest by maximizing the expected information obtained from the next measurement.

Recently, the mutual information waveform design algorithm in [6] was generalized to multiple target tracking [26]. Following the information theoretic approach and assuming a power constraint, beamforming is used to design waveforms that optimize the mutual information between each beam and each target. Specifically, assuming that the number of targets is known to be L and that $\mathbf{Z}_{k,l}$ is the received observation from the lth target at time step k, then the corresponding transmitted waveform $s_{k,l}(t;\boldsymbol{\theta}_{k,l})$ is designed to maximize the combined mutual information $\sum_{l=1}^{L}\alpha_{k,l}\,\mathrm{I}(\mathrm{H}_{k,l};\mathbf{Z}_{k,l})$ where $\alpha_{k,l}$ is the beamforming coefficient and $\mathrm{H}_{k,l}$ is the frequency response of the lth target.

ADAPTIVE WAVEFORM DESIGN FOR TARGET TRACKING

Following the control theoretic approach to waveform agile sensing discussed earlier, transmitted waveforms need to be designed to minimize the MSE of target state estimation. The performance of the waveform design algorithm can be affected by many factors, including environment characterization (narrowband or wideband), presence of multiple targets, and imperfect detection due to low signal-to-clutter (SCR) ratio or low SNR. In addition, the characteristics of the tracking algorithm have a significant impact on the waveform selection process. This can be seen by examining the cost function

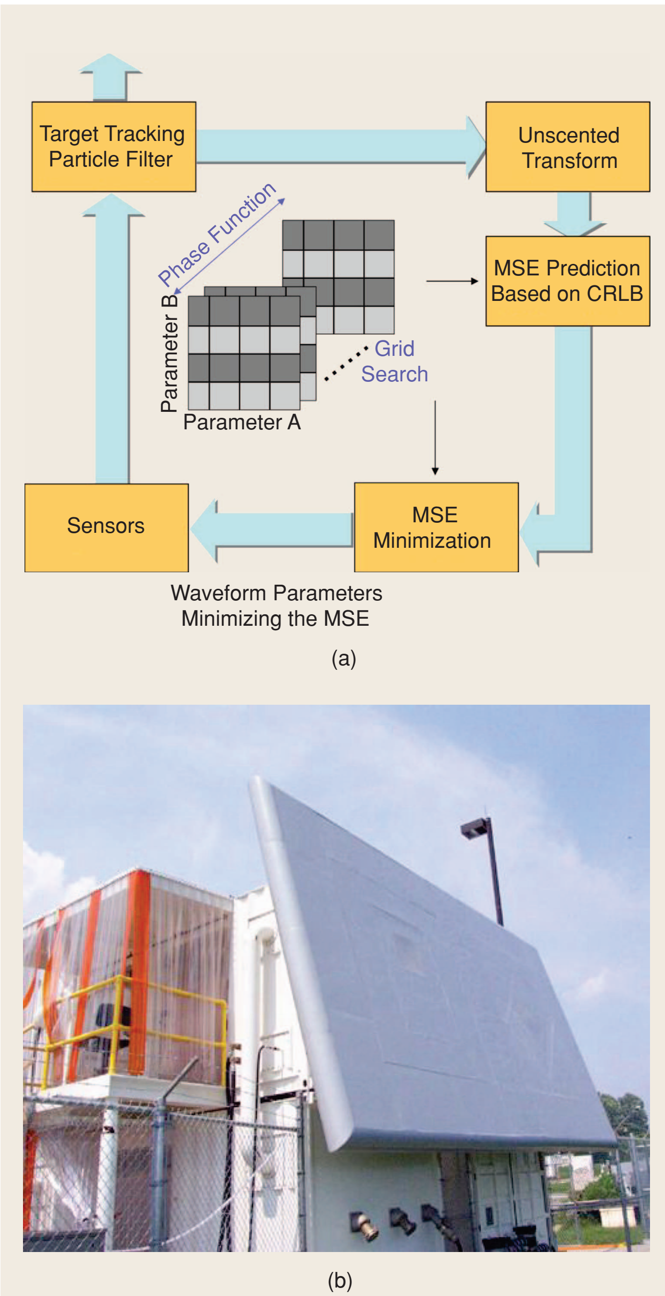
$$J(\boldsymbol{\theta}_k) = E_{\mathbf{X}_k,\mathbf{Z}_k|\mathbf{Z}_{1:k-1}}((\mathbf{X}_k - \hat{\mathbf{X}}_k)^{\mathrm{T}}\Lambda(\mathbf{X}_k - \hat{\mathbf{X}}_k)), \tag{4}$$

where $E(\cdot)$ is an expectation over predicted states and observations, Λ is a weighting matrix that ensures that the units of the cost function are consistent, and $\hat{\mathbf{X}}_k$ is the estimate of $\mathbf{X}_k$ given the sequence of observations $\mathbf{Z}_{1:k}$. The cost function in (4) is the MSE at time step k, and our objective is to select $\boldsymbol{\theta}_k$ as the set of parameters of the transmitted waveform that yields the lowest cost. Selecting $\boldsymbol{\theta}_k$ by executing a search over a possible range of parameter values hinges upon the ability to evaluate the cost in (4) for every candidate waveform. Thus, the first requirement for this is to quantify the relationship between the waveform and the predicted MSE based on the specific tracking scenario considered.

Figure 3(a) demonstrates the control theoretic waveform-agile tracking approach that we are considering under the assumption of high SNR. It can be seen that the MSE cost function is minimized in order to select the waveform parameters A, B, and the phase function. Next, we consider waveform-agile tracking algorithms under different environmental conditions for which we develop relationships between the designed waveform and the predicted MSE. Shown in Figure 3(b) is a waveform-agile test radar with which we tested some of these algorithms [55].

NARROWBAND ENVIRONMENT

In matched filter receivers, the delay and Doppler at each sensor can be estimated from the peak of the correlation between the received waveform with time-frequency shifted versions of the transmitted signal $s(t)$. In narrowband scenarios, the Doppler corresponds to a frequency shift, and the time-frequency correlation function of the waveform is given by the narrowband ambiguity function $\mathrm{AF}_s(\tau, \nu) = \int_{-\infty}^{\infty} s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2})e^{-j2\pi\nu t}dt$ [52]. Assuming that the received waveform is embedded in additive white Gaussian noise, the peak of the AF is the maximum likelihood estimator (MLE) of the delay and Doppler of the received waveform. The CRLB of the matched filter estimator can be obtained by inverting the Fisher information matrix that is computed as the Hessian of the AF, evaluated at the true target delay and Doppler. Note that as the CRLB depends only on the AF evaluated at the origin, the effect of the AF sidelobes (that could affect the location of the peak) are not considered. As a result, the use of the CRLB to characterize the measurement errors is restricted to situations where the SNR is high and the AF sidelobes can be neglected. A number of recent works make this assumption and set the measurement error covariance matrix $N(\boldsymbol{\theta}_k)$ to the CRLB.



[FIG3] **(a) Representation of the waveform-agile tracking algorithm that selects waveform parameters A, B, and the phase function by minimizing the predicted MSE. (b) Naval Research Laboratory (NRL) radar on Chesapeake Bay for testing waveform-agile sensing applications [55].**

When the state and measurement models in (1) are linear and the additive random process in each model is Gaussian, as in (2) for 1-D target motion, the Kalman filter is the optimal state estimator. In this case, the cost function in (4) can be calculated exactly, and the waveform design problem can be solved optimally as in [22]. For nonlinear models, however, sequential Monte Carlo techniques such as particle filtering [3] and unscented Kalman filtering [21] need to be used, and (4) cannot generally be evaluated in closed form. For example, when tracking a target moving in two dimensions, a particle filter [3] can be used as the tracker because the state-space model in (1) has a linear state equation $f_k(\mathrm{X}_{k-1}, \mathrm{W}_{k-1}) = F\mathrm{X}_{k-1} + \mathrm{W}_{k-1}$ and a nonlinear measurement equation $\mathrm{Z}_k = h_k(\mathrm{X}_k) + \mathrm{V}_k$ of range and Doppler. Here, the posterior density function is approximated as $p(\mathrm{X}_k|\mathrm{Z}_{1:k}, \boldsymbol{\theta}_{1:k}) \approx \sum_{j=1}^{N} w_k^j \delta(\mathrm{X}_k - \mathrm{X}_k^j)$, using N random samples X_k^j and associated weights w_k^j. Using this estimated distribution, an estimate of the target state can be obtained as $\hat{\mathrm{X}}_k = \sum_{j=1}^{N} w_k^j \mathrm{X}_k^j$. The particle filter provides a method to estimate the target track that is robust to observation nonlinearities and sensor positioning.

One approach to computing (4) is to evaluate it via Monte Carlo methods. This method was integrated into a stochastic steepest descent algorithm for parameter selection for LFM waveforms in [42]. However, the method is computationally intensive and motivates the need for alternative approximations. The Kalman filter covariance update equation provides a convenient mechanism to approximate $J(\boldsymbol{\theta}_k)$ in (4). Specifically, the predicted error covariance is computed using

$$P_{k|k}(\boldsymbol{\theta}_k) \approx P_{k|k-1} - P_{xz}(P_{zz} + (N(\boldsymbol{\theta}_k))^{-1}P_{xz}{}^T, \qquad (5)$$

where $P_{k|k-1} = FP_{k-1|k-1}F^T + Q$ is obtained from the dynamic model, and $P_{k-1|k-1}$ is the covariance of the state estimate from the previous time step $(k-1)$. The approximation in (5) arises from the fact that the matrices P_{xz} and P_{zz} cannot generally be computed exactly when the observation model is nonlinear. They can be approximated using methods such as the unscented transform [21]. Here, we assume that the density of the state given the observations is Gaussian and use the unscented transform to compute its statistics under a nonlinear transformation. The method consists of selecting a set of sigma points based on the predicted mean $\hat{\mathrm{X}}_{k|k-1} = F\hat{\mathrm{X}}_{k-1|k-1}$ and covariance matrix

$P_{k|k-1}$ at time k. These sigma points are propagated using the nonlinear measurement function $h(\cdot)$ in (1), and the transformed points are used to obtain P_{xz} and P_{zz} in (5). Using the fact that $J(\boldsymbol{\theta}_k)$ is the trace of $\Lambda P_{k|k}(\boldsymbol{\theta}_k)$, the waveform selected is the one that minimizes $\mathrm{Tr}\{\Lambda P_{k|k}(\boldsymbol{\theta}_k)\}$.

This mechanism for waveform selection for tracking a single moving target using two agile sensors and generalized FM chirps was discussed in [45]. The waveform library consists of linear, hyperbolic, exponential, and power FM chirps defined by $s(t) = a(t)\exp(j2\pi b\,\xi(t/t_r))$ where $a(t)$ is the amplitude modulation, b is the FM rate, $\xi(t/t_r)$ is a real-valued, differentiable phase function, and $t_r > 0$ is a reference time [38]. The frequency of these waveforms varies nonlinearly with time according to the waveform's instantaneous frequency $d_{xi}(t/t_r)dt$. Some examples of the time-frequency characteristics of these waveforms are shown in Figure 4(a) with $\lambda = 100$ μs duration and 14 MHz frequency sweep. The waveform design algorithm selects the phase function $\xi_k(t/t_r)$, duration λ_k, and FM rate b_k of the signal for each sensor at each time step k. Using a grid search, these parameters are selected to minimize the predicted MSE.

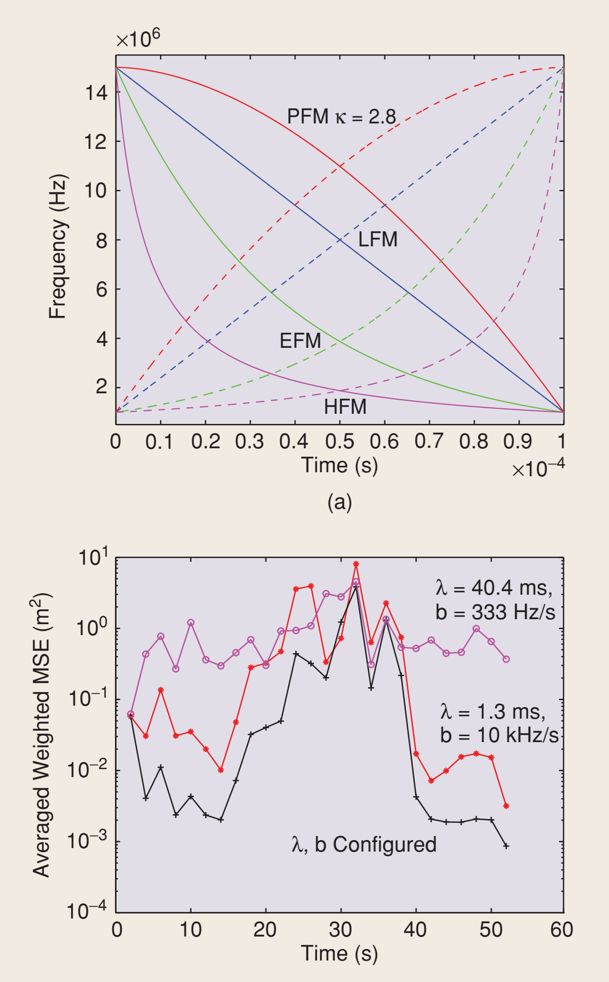
The MSE performance improvement using waveform adaptation was first demonstrated by selecting parameters for the LFM chirp in [42] and the generalized FM chirp in [45]. We demonstrate next the first case, where the simulation consists of two fixed, waveform-agile sensors tracking a single underwater target (with unknown range and range-rate) as it moves in two dimensions. The SNR at the ith sensor is modeled as $\eta_k^i = (r_0/r_k^i)^4$ where r_0 is the range at which a 0 dB SNR is obtained and r_k^i is the range from the ith sensor at time step k. The duration and FM rate are restricted to 1.3 ms $\leq \lambda_k \leq$ 40.4 ms and $0 \leq b \leq 10$ kHz/s, respectively. This is a narrowband application as the maximum frequency sweep is fixed to 100 Hz. The weighting matrix in (4) is set to $\Lambda = \mathrm{diag}[1, 1, 4\,\mathrm{s}^2, 4\,\mathrm{s}^2]$ so that the cost is in units of m^2. All results are averaged over 500 simulation runs. The averaged MSE in Figure 4(b) compares the performance of LFM chirps with typically used fixed parameters and with dynamic parameter settings.

WIDEBAND ENVIRONMENT

The effect of the relative motion between the target and the sensor results in a Doppler scaling (time dilation or compression) on the transmitted waveform. However, received waveforms are treated as narrowband if the Doppler can be approximated by a simple frequency shift. The approximation is valid only when the time-bandwidth (TB) product of the waveform satisfies $\mathrm{TB} \ll c/(2\dot{r})$, where c is the velocity of propagation and $\dot{r}$ is the range-rate or radial velocity of the target with respect to the observation platform. When the approximation is not valid, the Doppler scaling effect must be incorporated into the matched filter output. This is achieved by using the wideband AF (WAF) that is defined as $\mathrm{WAF}_s(\tau, a) = |a| \int s(t)s^*(at - \tau)dt$. As in the narrowband case, the Fisher information matrix is obtained by taking the negative of the second derivatives of the WAF, evaluated at the true target delay and scale [20]. The techniques described previously can now be extended to the wideband scenario with the waveform selected to minimize the predicted MSE [45]. The waveform-agile sensing algorithm is the same as in Figure 3(a) with the CRLB derived from the WAF.

CLUTTERED ENVIRONMENT

When a target is embedded in clutter and perfect detection cannot be assumed, the origin of the measurements becomes uncertain and the tracker must explicitly account for this fact. As a result, the calculation of the predicted covariance update in (5) is modified by the probability of detection. The approach considered in [45] is based on the use of a probabilistic data association filter [5]. Specifically, the observation model includes multiple sensor measurements due to clutter reflections. A validation gate and a clutter density provide a

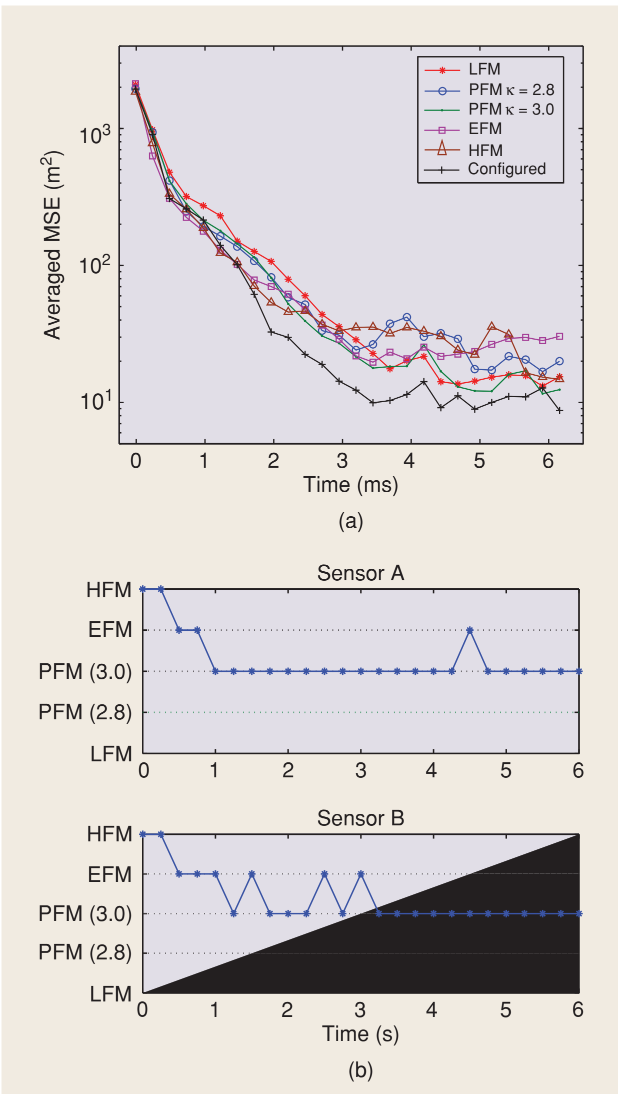


[FIG4] (a) Examples of generalized FM chirps in the time-frequency plane: LFM with phase function $\xi(t/t_r) = (t/t_r)^2$, hyperbolic FM (HFM) with $\xi(t/t_r) = \ln(t/t_r)$, exponential FM (EFM) with $\xi(t/t_r) = \exp(t/t_r)$, and power FM (PFM with parameter κ) with $\xi(t/t_r) = (t/t_r)^\kappa$ [45]. (b) Averaged MSE using fixed and configured waveform parameters (duration λ and FM rate b) for the LFM chirp.

NONLINEAR FM CHIRP WAVEFORMS OFFER SIGNIFICANT ADVANTAGES OVER LINEAR FM CHIRP WAVEFORMS IN WAVEFORM-AGILE TRACKING.

model for the number of false alarms due to clutter. The presence of clutter affects the computation of the likelihood function $p(Z_k|X_k, \boldsymbol{\theta}_k)$ since the probability of obtaining the measurement from the target, given that it is detected, includes the probability that each individual measurement is generated by the target, weighted by the measurement's association probability.

Figure 5 shows the improvement in waveform-agile tracking performance when a single target, moving in two dimensions, is tracked using two radar sensors in a narrowband, cluttered environment. Generalized FM chirps (some of which are depicted in Figure 4(a) in the time-frequency plane) are used for agility configuration such that their phase function, duration, and FM rate can be adaptively selected. The duration is restricted to the range 10 μs $\leq \lambda_k \leq$ 100 μs, the carrier frequency is 10.4 GHz, and the frequency sweep is restricted to 15 MHz. The probability of false alarm is 0.01, the validation gate is taken to be the five-sigma region around the predicted observation [5]. All results are averaged over 500 Monte Carlo simulations, and a track is considered lost when more than four contiguous measurements fall outside the validation gate for either sensor. The clutter density is set so that there are 0.0001 false alarms per unit validation gate volume. As shown in Figure 5(a), when all the parameters of the transmitted waveform (including its time-varying phase function and thus its time-frequency signature) are dynamically selected, the best tracking performance is obtained. The phase function selection at each time step and for each one of the two sensors is shown in Figure 5(b).



[FIG5] (a) Averaged MSE based on waveform agility when all waveform parameters, including the phase function, are configured compared to agility when only the duration and FM rate values are configured. (b) Typical waveform selection for each radar sensor when the phase function is configured [45].

ENVIRONMENT WITH MULTIPLE TARGETS

When multiple targets are present together with clutter, the tracking algorithm must estimate the association between the measurements and the targets as well as the clutter. The waveform adaptation in this scenario is even more challenging than when a single target is present in clutter. The problem is approached by extending the dynamics and observation models to include multiple targets. The covariance matrix $P_{k|k}(\boldsymbol{\theta}_k)$ now contains block matrices corresponding to each target and the calculation of the predicted cost according to (5) proceeds by updating the covariance matrix corresponding to each target in turn. This method, as well as the simulation results for the waveform-agile tracking of two targets, are described in [43].

LOW-SCR ENVIRONMENT

In highly cluttered environments, such as in heavy sea clutter, the problem of tracking a small target is very challenging. This problem is approached by first using waveform design to improve target detection that would then improve tracking performance [28], [44], [46]. Improving detection also improves tracking since, for a given probability of detection, a lower probability of false alarm implies less uncertainty in the origin of the measurement that leads to lower tracking MSE [5]. For the control theoretic approach thus far, the CRLB used is derived from the curvature of the waveform AF at the origin in the delay-Doppler plane. This characterization, however, ignores the AF sidelobes so it is not appropriate in situations involving low-SNR or low-SCR values. Under low SCR, we transmit two waveforms in two subsequent subdwells. The first transmitted waveform in the first subwell is fixed and used to estimate clutter. The waveform in the second subdwell is dynamically designed so that its autocorrelation function is small where the clutter is strong, thus minimizing the effect of the out-of-bin clutter in the predicted target location. This is achieved using a unimodular phase-modulated waveform whose phase is dynamically selected at different time steps. A particle filter based tracker then uses the measurements obtained by the adapted waveform to improve tracking performance [28], [44], [46]. We also use probabilistic data association to counter the uncertainty in the origin of the measurements due to clutter.

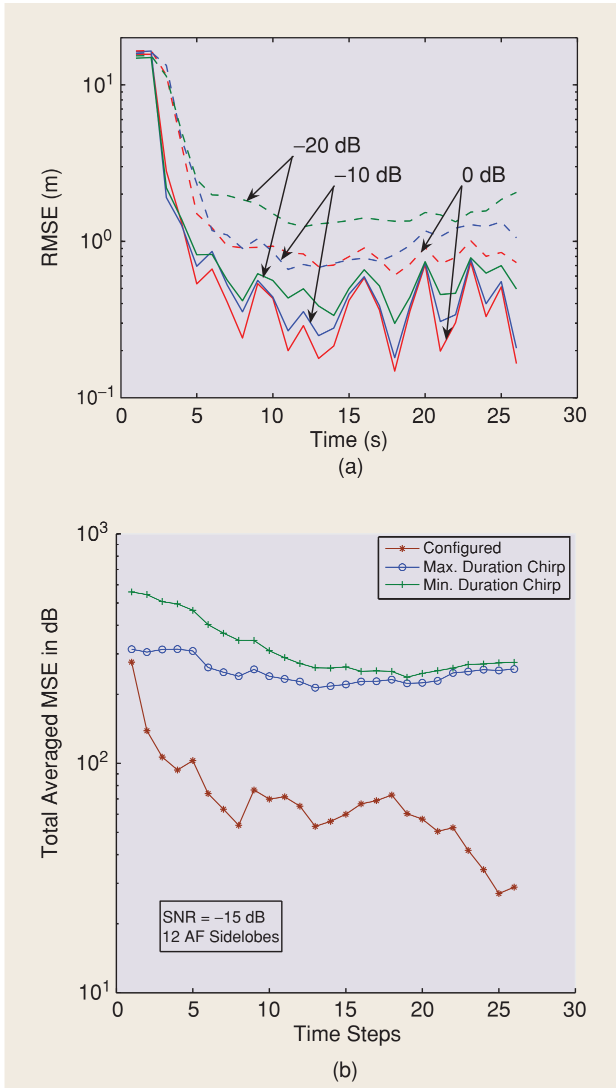
WAVEFORM-AGILE SENSING IS FAST BECOMING AN IMPORTANT TECHNIQUE FOR IMPROVING SENSOR PERFORMANCE IN APPLICATIONS SUCH AS RADAR, SONAR, BIOMEDICINE, AND COMMUNICATIONS.

A simulation study based on parameters derived from real sea clutter measurements is presented in [44] for a target moving in a low-SCR environment. It is assumed that the target is located 10 km away from the sensor and moving away from it at 5 m/s. The clutter scatterers are assumed distributed uniformly in range and uniformly in Doppler over [−1,000, 1,000] Hz. In the first subdwell, an LMF is used with duration 1.5 μs and frequency sweep 100 MHz. Shown in Figure 6(a) is a comparison of the averaged root MSE (RMSE) of the target position by first transmitting a fixed LFM chirp waveform in the first subdwell and then dynamically designing the waveform in the second subdwell. The comparison demonstrates that the tracking performance improves by more than 20 dB SCR (for the same RMSE value) when the waveform is dynamically designed in the second subdwell. These results have recently been extended to rapidly-varying radar scenes by dynamically estimating the space-time covariance matrix [27], [30].

LOW-SNR ENVIRONMENT

Another approach to the CRLB in waveform-agile sensing that directly involves the AF of the transmitted waveform is described in [12], [17], [37], and [39]. The method computes the covariance matrix of the measurement error based on AF resolution cells. A resolution cell is a region in the delay-Doppler plane that tightly encloses the AF contour of the transmitted waveform at a specified probability of detection. The measurement error covariance is computed by assuming a uniform distribution for the true target location within the cell. As different values of probability of detection result in the inclusion of different number of AF sidelobes in the cell, this approach is suitable for waveform-agile tracking in low-SNR environments. Within each resolution cell, each point in the delay-Doppler plane is associated with a specified minimum probability of detection. Ideally, the resolution cell (or resolution cell primitive) should be uniform in geometry, forming a tessellating grid and matching the AF shape [39]. For most waveforms, such a sampling is difficult to achieve. For example, LFM chirps have constant probability of detection contours that are elliptical in shape and cannot form a mutually exclusive and exhaustive sampling grid. The parallelogram enclosing each elliptical area forms a practical resolution cell as it has the desired tessellating shape, and it tightly encloses the resolution cell primitive.

The measurement noise covariance matrix computed using the resolution cell approach was used to compare the steady state tracking MSE of various waveforms in [37], [39]. This method was incorporated into a waveform-agile tracking system in [12] where a sensor is tracking the range and range-rate of a single target moving in 1-D. The sensor is transmitting LFM chirps whose parameter vector $\boldsymbol{\theta}_k$ (duration and FM rate) is allowed to vary at each time step k and is chosen to minimize the tracking MSE. Specifically, based on the desired detection threshold (and thus SNR value),



[FIG6] (a) Comparison of the averaged. (a) Position RMSE at the end of the first sub-dwell (dotted lines) using a fixed LFM and second sub-dwell (solid lines) using the designed waveform. (b) Comparison of the MSE when using fixed and configured LFM chirps at –15 dB SNR when twelve AF sidelobes are incorporated in the resolution cell approach.

the coordinates of a tessellating parallelogram resolution cell are found such that the parallelogram encloses the AF at a level equal or greater than the detection threshold. The value of the measurement noise covariance matrix $N(\boldsymbol{\theta}_k)$ can be derived from the size and shape of this resolution cell. For 1-D target motion, the Kalman filter can be used for waveform design as in (3). In this case, the waveform-dependent error covariance $P_{k|k}(\boldsymbol{\theta}_k)$ in (3) is computed using $N(\boldsymbol{\theta}_k)$ that is obtained from the resolution cell based on the desired probability of detection and SNR.

Simulations demonstrate the increased tracking performance when the LFM chirp parameters are selected in comparison to using an LFM chirp with fixed parameters, as shown in Figure 6(b). The SNR is −15 dB, perfect detection is assumed with 0.2 probability of false alarm, and 12 AF sidelobes are used in the resolution cell. For the same low SNR, the tracking performance deteriorates when only the mainlobe is used [12].

[DYNAMIC WAVEFORM ADAPTATION IS MORE EFFECTIVE IN THE PRESENCE OF CLUTTER THAN IN CLUTTER-FREE ENVIRONMENTS.]

CONCLUSIONS

Waveform-agile sensing is fast becoming an important technique for improving sensor performance in applications such as radar, sonar, biomedicine, and communications. We provided an overview of research work on waveform-agile target tracking. From both control theoretic and information theoretic perspectives, waveforms can be selected to optimize a tracking performance criterion such as minimizing the tracking MSE or maximizing target information retrieval. The waveforms can be designed directly based on their estimation resolution properties, selected from a class of waveforms with varying parameter values over a feasible sampling grid in the time-frequency plane, or obtained from different waveform libraries.

Based on our current research on waveform-agile sensing, we provided detailed information on waveform configuration to minimize the tracking MSE for single or multiple targets moving in different types of environments including narrowband, wideband, highly cluttered, and very noisy environments. We employ a class of FM waveforms with unique signatures in the time-frequency plane. This class includes waveforms with linear time-frequency characteristics that are used extensively in current radar and sonar systems. Waveforms with nonlinear time-frequency signatures are also included as they can have important properties better matched to different environments. For example, waveforms with hyperbolic dispersive signatures have been shown to be invariant to scaling transformations, and are similar to signals used by bats and dolphins for echolocation. It has been shown that the minimal range-Doppler coupling of some of these chirps has an important impact on tracking performance.

Years of target-tracking research have resulted in various factualities on desirable waveform properties for good performance as common practice. With the new element of adaptive waveform configuration, however, the following new conclusions have been reached:

- *Waveforms that maximize the time-bandwidth product do not necessarily provide the best tracking performance.*

It is common practice in radar to use waveforms with the largest allowable duration and maximum possible bandwidth. However, this waveform configuration results in minimizing the conditional variance of range given range-rate estimates, and thus the range and range-rate estimation errors become correlated. This is not useful when both range and range-rate parameters are relatively inaccurately known, which occurs at initialization or under unfavorable sensor-target geometries. Waveform adaptation, on the other hand, matches the waveform to the tracker's needs and alleviates this problem. Echolocating bats also vary their transmitted waveforms through various phases of their search, approach, and capture prey, and they do not necessarily use waveforms that maximize the waveform time-bandwidth product.

- *Nonlinear FM chirp waveforms offer significant advantages over linear FM chirp waveforms.*

Nonlinear FM chirp waveforms have not been used much, if at all, for target tracking, unlike the more popular linear FM chirp waveforms. However, when waveforms are dynamically adapted, nonlinear time-frequency signatures, such as hyperbolic, exponential, or power, provide better tracking performance than the linear signatures. This is common practice in bats that use nonlinear FM chirps to exploit their properties such as Doppler tolerance.

- *Dynamic selection of parameters for FM chirp waveforms can reduce tracking errors.*

It has been recently shown that for an LFM chirp waveform with a given duration, allowable range of frequency sweep rates, and other constraints such as average power, the maximum information about the target state can be obtained when the frequency sweep rate is chosen to be either the maximum or the minimum allowed [18]. This finding was based on a linear observation model and an information theoretic criterion. When the performance metric is the tracking MSE however, and when the tracking formulations involve nonlinear models as well as constantly changing sensor and target positions, we find that dynamic selection of linear and nonlinear FM chirp waveform parameters, including duration and sweep rate, provides appreciable improvement in tracking performance.

- *Dynamic waveform adaptation is more effective in the presence of clutter than in clutter-free environments.*

In the presence of clutter, uncertainty is introduced in the measurements. This needs to be compensated for by the use of the validation gate so that only those measurements that fall within the gate are considered as possibly having originated from the target. The validation gate volume depends upon the choice of waveform, and this provides an added dimension in which waveform adaptation can be exploited.

- *In low-SNR environments, waveform design methods that consider the impact of AF sidelobes provide improved tracking performance.*

In low-SNR tracking scenarios, the resolution cell approach with configured LFM waveforms results in improved tracking performance when the approach incorporates AF sidelobes together with the AF mainlobe. When AF sidelobes are included in the algorithm, the averaged MSE is lower than when only the AF mainlobe is used. However, if too many sidelobes are included, then more errors are introduced in the algorithm and the performance does not improve.

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