Optimal Taxation of Multinational Enterprises: A Ramsey Approach*

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Abstract

What is the optimal design of the international corporate tax system? We revisit this classic question in a multi-country general equilibrium model that incorporates three key features of the modern globalized economy: multinational production; intangible capital; and international profit shifting. Our model's competitive equilibrium is inefficient due to an externality that arises from international spillovers in intangible investment. In the absence of profit shifting, there is little, if anything, a Ramsey planner can do with corporate income taxes to improve the allocation of intangible investment across countries. However, profit shifting allows the planner to use corporate income taxes to internalize the externality and achieve an efficient allocation of intangible investment. To quantitatively investigate the properties of the Ramsey planner's optimal policy in a more realistic setting, we extend our model to an environment with firm heterogeneity and selection into multinational production. Without spillovers, it would be optimal to shut down profit shifting as much as possible. With spillovers, it would be optimal to allow MNEs to continue to shift profits, and if the planner is restricted to Pareto-improving policies, it would be optimal to allow even more profit shifting than under the status quo.

Keywords: Multinational enterprise; intangible capital; profit shifting; corporate taxation; optimal policy; Ramsey problem.

JEL Codes: E6, F23, H25, H27

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1 Introduction

How should the international corporate tax system be organized? What economic trade-offs do policymakers face when designing this system? The public finance literature has long studied these questions, but globalization and the rising importance of multinational enterprises (MNE) have the potential to alter the answers dramatically. The modern global economy is increasingly dominated by large MNEs that invest heavily in intangible capital, which makes them more productive at home and abroad and allows them to shift profits to subsidiaries in low-tax countries. In this paper, we study optimal corporate tax policy using a model that puts these phenomenona front and center.

We study the problem of a cooperative global Ramsey planner in a multi-country neoclassical growth model with three key ingredients: multinational production; nonrival intangible capital; and international profit shifting. In modeling the first two ingredients, we emphasize international spillovers that rise to an externality—intangible investment in one country leads to more output and also more intangible investment abroad—which implies that the allocation of intangible investment across countries in a competitive equilibrium is inefficient. In the theoretical part of the paper, we study the extent to which the Ramsey planner can use corporate income taxes to improve the allocation of intangible investment in the face of this spillover. In the absence of profit shifting, corporate income taxes have little, if any, utility; at best, they can improve one country's intangible investment efficiency while worsening others'. In the presence of profit shifting, however, the planner can use corporate income taxes to not only improve the efficiency of intangible investment in all countries, but in fact to fully correct the externality and achieve a Pareto-optimal allocation of intangible investment across countries.

In the quantitative part of the paper, we analyze the Ramsey planner's problem in a richer environment with heterogeneous firms and selection into multinational production. In this environment, the planner can achieve welfare gains by cutting corporate income taxes, which boosts MNEs' incentives to invest in intangible capital and makes worldwide allocation of resources more efficient. In the absence of a spillover externality, it would be optimal to shut down profit shifting as much as possible. However, if such an externality is present, MNEs should be allowed to shift profits, and the optimal level of profit shifting is increasing in the size of the externality. If the planner is constrained to Pareto-improving tax reforms that at least weakly benefit all regions, it would actually be optimal to increase the amount of profit shifting by MNEs relative to the status quo.

Our theoretical analysis builds on Chari, Nicolini, and Teles (2023) (henceforth CNT) who study the cooperative Ramsey problem in a multi-country Backus, Kehoe, and Kydland

(1994) environment. We extend their analysis in two ways to capture the three ingredients described above. First, we incorporate multinational production and nonrival intangible capital as in McGrattan and Prescott (2009, 2010) (henceforth MP). Each country has a representative MNE that produces a distinct intermediate good, which can be sold abroad by exporting domestic output as in Backus et al. (1994), but also by producing locally via foreign affiliates. MNEs produce their products using local rival factors and nonrival intangible capital, which is in turn produced by employing workers in MNEs' home countries. We depart from MP by assuming that there are spillovers in the production of intangible capital: an increase in one country's usage of foreign intangible capital makes its own investments more effective (Javorcik, 2004; Bitzer and Kerekes, 2008). Second, we incorporate the theory of transfer pricing and profit shifting that we developed in Dyrda, Hong, and Steinberg (2022) (henceforth DHS). MNEs' foreign affiliates pay licensing fees to use intangible capital according to arms-length transfer pricing rules. Normally, these fees are paid to MNEs' domestic parent companies, but MNEs can shift the rights to this licensing income to a tax haven by paying a convex cost.

We begin our theoretical analysis by studying the efficiency properties of our environment. Several of the conditions that characterize an efficient allocation are familiar from CNT. For example, the marginal rate of substitution between labor and consumption should equal the marginal product of labor, and the social return to tangible capital should be equated across countries. In our environment, though, an additional condition characterizes the optimal allocation of intangible investment. This condition states that allocating an additional unit of labor to producing intermediate goods should yield the same worldwide utility gains as allocating it to producing intangible investment, highlighting the externality created by spillovers in this investment. When MNEs from one country invest in additional intangible capital, that increases the production of their foreign affiliates in other countries, but it also makes these other countries' MNEs better at producing intangible capital. The first country's MNEs internalize the first effect, but not the second one.

We proceed to characterize competitive equilibria with distortive taxes under three scenarios. First, a scenario without transfer pricing or profit shifting in which MNEs' foreign affiliates use intangible capital free of charge as in McGrattan and Waddle (2020). Second, a scenario in which foreign affiliates pay licensing fees to use intangible capital but profit shifting is not allowed. Third, our main scenario is with both transfer pricing and profit shifting. Our baseline tax system includes a set of standard instruments that are used to finance government expenditures: taxes on consumption, labor income, exports and imports, and corporate income. In the free transfer and transfer pricing scenarios, the allocation of intangible investment is always inefficient regardless of the tax structure. In the former, corporate income taxes affect intangible investment, but do so in a way that improves efficiency in one country while worsening it in others. In the latter, corporate income taxes are even less effective at improving the efficiency of intangible investment—in fact, they have no effect at all. In the profit shifting scenario, however, the situation is markedly different. Here, raising corporate income taxes increases the gains from profit shifting, which in turn increases the returns on intangible investment, leading MNEs to do more of it. Raising corporate income taxes worldwide—or reducing taxes in the tax haven, if that is possible—improves the efficiency of intangible investment in all countries, unambiguously moving the economy closer to the Pareto frontier. In fact, we show corporate income taxes can be used to fully internalize the spillover externality and achieve an efficient allocation of intangible investment.

We then formalize the Ramsey problem in our economy and equip the planner with a rich set of instruments including consumption, labor income, trade and corporate taxes. The Ramsey planner has two goals in our framework. The first goal is standard: minimizing distortions associated with financing exogenous government expenditures. The second goal is new and unique to our environment: to internalize the spillover externality and efficiently allocate intangible capital across countries. We first argue that with a benchmark set of instruments, the planner can not implement a Pareto-efficient allocation in any of the scenarios. Notably, this includes the profit shifting scenario, where it is possible to achieve an efficient allocation of intangible investment. This is due to the tension between achieving a statically-efficient allocation of intangible investment and a dynamically-efficient allocation of tangible investment. We then demonstrate that if we equip the planner with an additional instrument, a tax (or subsidy) on tangible capital income, the planner can achieve both goals in the profit shifting scenario and implement a Pareto-optimal allocation. In the free transfer and transfer pricing scenarios, however, even a planner equipped with this additional instrument can not restore efficiency. In the profit shifting scenario, where Pareto-optimality can be attained, implementing the Ramsey allocation requires generically non-zero corporate income taxes that differ across countries and non-zero capital income taxes to correct intertemporal distortions. Thus, corporate income tax harmonization is not optimal, and the classic result of Chamley (1986) and Judd (1985) does not hold.

In our quantitative analysis, we dig deeper into the tension between correcting the spillover externality in intangible investment and attaining dynamic efficiency in tangible investment in a richer model with firm heterogeneity and selection into multinational activity. This allows us to account for the fact that while MNEs represent a small fraction of the total number of firms in the economy (and those that shift profits an even smaller fraction), they account for a large fraction of production and an even larger fraction of intangible investment. Firms in this version of the model differ in productivity and must pay fixed costs to export, larger fixed costs to establish foreign affiliates in other productive countries, and even larger fixed costs to establish affiliates in the tax haven in order to shift profits. Following DHS, we divide the world into five regions: two high-tax rich regions, North America and Europe; a low-tax rich region that includes Ireland, Switzerland, and other profit-shifting destinations that nevertheless have productive, diversified economies; a tax haven that includes small countries like Bermuda and the Bahamas where much of the economy is centered around profit shifting; and the rest of the world, which has relatively high taxes and a much lower standard of living than the other four regions. We then calibrate the model to reproduce salient facts about production, trade, multinational activity, and international profit shifting under the current international corporate tax regime. We then solve numerically for the Ramsey planner's optimal tax regime in each of the three scenarios described above. We also consider the problem of a constrained Ramsey planner who is restricted to Pareto-improving policies that all countries prefer, at least weakly, to the current tax regime.

The unconstrained planner's solution highlights four takeaways. First, it would be optimal to shut down profit shifting completely in the absence of a spillover externality. When spillovers are present it would be optimal to allow MNEs to continue to shift profits, although the optimal level of profit shifting would be lower relative to the status quo. Second, dynamic efficiency plays an important role in allocating intangible investment efficiently. Corporate income taxes boost intangible investment through the profit shifting channel but also reduce tangible investment, which in turn drags down intangible investment as well. Our results show that the second channel is stronger than the first: intangible investment moves in the opposite direction as corporate income taxes in equilibrium. Third, spillovers allow the planner to achieve larger welfare gains by enacting larger corporate income tax cuts. Fourth, the unconstrained planner would benefit high-tax regions at the expense of the low-tax region. This is due to the small Pareto weight the planner puts on the low-tax region, but also because reducing profit shifting requires raising this region's corporate tax rate, which discourages its MNEs from investing in intangible capital.

The constrained planner's solution differs in two key ways. First, it would never be optimal for the constrained planner to shut down profit shifting, regardless of whether there is a spillover externality. This is because reducing profit shifting hurts the low-tax region, which the constrained planner is not allowed to do. In fact, it would be optimal for the constrained planner to actually increase the level of profit shifting that occurs in equilibrium relative to the status quo, especially when there is a spillover externality. Second, the constrained planner is limited to smaller tax reforms that generate commensurately smaller welfare gains for North America and the rest of the world, the two largest regions on which the planner's objective puts the most weight.

2 Related Literature

This paper is closely related to two main strands of literature. First, we contribute to the literature on the optimal design of the international tax system, which has a long tradition and a variety of approaches. Notable papers are Feldstein and Hartman (1979), Gordon (1986), Keen and Wildasin (2004) and more recently Lyon and Waugh (2018), Costinot, Rodríguez-Clare, and Werning (2020), Costinot and Werning (2022), Hosseini and Shourideh (2022), and Chari, Nicolini, and Teles (2023). In an influential paper Keen and Wildasin (2004) argue that in a static trade model where countries have distinct government budget constraints, the production efficiency result established by Diamond and Mirrlees (1971) fails to hold. Recent work by Chari et al. (2023) challenges this result, however. They develop a dynamic trade model based on Backus, Kehoe, and Kydland (1994) and use it to examine how countries should coordinate their fiscal and trade policies when government spending is financed by distortionary taxes. They show, among other things, that a Ramsey planner can achieve production efficiency, and that free flow of goods, services, and capital across borders is optimal.

Our paper builds upon CNT, extending their model in two dimensions. First, we incorporate multinational production and nonrival intangible capital as in McGrattan and Prescott (2009, 2010). Second, we incorporate the theory of transfer pricing and profit shifting that we developed in Dyrda et al. (2022). Third, we model international spillovers of nonrival intangible capital investment (Javorcik, 2004). The intangible spillovers create an externality, which implies that a competitive equilibrium in our framework will generally not be efficient event absent distortionary taxes. Thus, a Ramsey planner in our framework will aim to minimize distortions associated with financing government expenditure and to correct the externality. It leads to an interesting trade-off between ensuring production efficiency and allocating intangible capital efficiently, which is absent in CNT. One potential resolution of this trade-off is to include a capital income taxation on top of the other standard taxes we consider, which breaks the Chamley-Judd (Judd, 1985; Chamley, 1986) result in our framework. Again, this is in contrast to CNT.

Second, we contribute to the line of research studying the macroeconomic and policy implications of intangible capital and technology transfer. McGrattan and Prescott (2009,

2010) incorporate multinational production and technology capital into a standard, neoclassical growth framework and show that this channel substantially increases the gains to openness to FDI. Holmes, McGrattan, and Prescott (2015) use a version of this model to quantify the impact of China's quid pro quo policy and show that it has had a significant effect on global innovation and welfare. In Dyrda et al. (2022), we develop a framework to study the aggregate implications of taxing multinational enterprises that shift profits to tax havens by transferring ownership of nonrival intangible capital. Santacreu (2023) develops an Armington trade model of innovation and international technology licensing and uses it to rationalize the bilateral royalty payments across countries observed in the data. Other studies in this strand of the literature are Yang and Maskus (2001), Glass and Saggi (2002), Benhabib et al. (2017), and Mandelman and Waddle (2020). The vast majority of papers in this line of research concentrate on a positive analysis. In contrast, in this paper, we embed the theory of intangibles and profit shifting we developed in DHS into a multi-country neoclassical growth model and use this framework to conduct normative analysis. Our study is the first to analyze an optimal policy in a framework where global production, nonrival intangible capital, and profit shifting are explicitly modeled.

Perhaps the most similar paper to our is Quadrini and Ríos Rull (2023), also published in this volume, who study international tax competition in a model with multinational production, intangible capital, and profit shifting. There are numerous differences in our approaches to modeling and quantifying these forces, but the primary distinction between between their paper and ours is that we ask a normative question—what should the optimal international corporate tax system look like if governments could agree to coordinate—whereas they ask a positive question—what system emerges in equilibrium if such coordination is impossible. We view their paper and ours as highly complementary.

3 Theoretical analysis

We begin with a theoretical treatment of the implications of multinational production, intangible capital, and profit shifting for optimal corporate income taxation. Our starting point is a Backus et al. (1994) multi-country growth model with distortionary taxation as studied by CNT. To account for these key features of the modern global economy, we first incorporate a variation of MP's theory of multinational production with nonrival intangible capital with an international spillover externality, add then add profit shifting as in DHS. We first describe the environment and characterize its Pareto frontier. We then describe the market structure and characterize a competitive equilibrium with distortionary taxes. We conclude with an analysis of the problem of a cooperative global Ramsey planner who has access to corporate income taxes in addition to the standard set of distortionary taxes.

3.1 Preferences and technology

There are I countries indexed by i and j. Each country has a representative household with preferences over sequences of consumption, $\{c_{it}\}_{t=0}^{\infty}$, and labor, $\{h_{it}\}_{t=0}^{\infty}$, given by

$$U^i = \sum_{t=0}^{\infty} \beta^t u^i \left(c_{it}, h_{it}\right). \tag{1}$$

The utility function, u^i, satisfies the usual properties and the endowment of time is normalized to be one. We use $u^i_{c,t}$ and $u^i_{h,t}$ to denote the marginal utility of country i's household with respect to consumption and labor.

Each country produces a distinct intermediate good. Different from Backus et al. (1994), intermediate goods can be produced both at home as well as in each foreign country. The technology according to which country i's good is produced in country j is given by

$$y_{ijt} = F^{ij} \left(z_{it}, k_{ijt}, l_{ijt}\right), \tag{2}$$

where k_{ijt} and ℓ_{ijt} are tangible capital and labor, which are rival and specific to the country in which production takes place, and z_{it} is country i's intangible capital, which is nonrival and can be used in all production locations simultaneously. We assume that domestically-produced intermediates (i.e., country i's intermediate produced in i, y_{iit}) can be exported, while intermediates produced abroad (i.e., y_{ijt} for $j \neq i$) must be used domestically.[1] We assume F^{ij} has constant return to scale, following Chari et al. (2023). The marginal products of intangible capital, tangible capital, and labor in producing country i's good in country j are denoted by $F^{ij}_{z,t}$, $F^{ij}_{k,t}$, and $F^{ij}_{l,t}$, respectively.

Intermediate goods are used to produce nontradable final goods according to a constant-returns-to-scale technology

$$q_{it} = G^i \left(q_{1it}, ..., q_{Iit}, \hat{q}_{1it}, ..., \hat{q}_{Iit}\right). \tag{3}$$

The first I elements are domestically-produced intermediates (which are imported when $j \neq i$), while the last $I - 1$ elements, accented with hats, are foreign intermediates produced

[1] We do not impose any assumption about the substitutability between y_{iit} and y_{ijt} at this stage, but we assume they are imperfect substitutes in our quantiative analysis. Our theoretical results are robust to allowing for export platforms.

locally in country i. The marginal product of a domestically-produced intermediate from country j in producing country i's final good is denoted by $G^i_{j,t} := \partial G^i/\partial q_{jit}$. Similarly, the marginal product of a locally-produced foreign intermediate from country j is $G^i_{\hat{j},t} := \partial G^i/\partial \hat{q}_{jit}$. Tangible capital follows the standard law of motion

$$x_{it} = k_{it+1} - (1-\delta)\, k_{it}, \tag{4}$$

where δ is the depreciation rate.

We assume that intangible capital, z_{it}, is produced according to the technology

$$z_{it} = H^i\left(\ell^z_{1t}, \ldots, \ell^z_{It}\right). \tag{5}$$

This technology depends on the vector of labor inputs used to produce intangible capital by all countries, not just the home country. We refer to l^z_{it} as research labor. This technology captures, in a parsimonious way, the technology spillover effects. As the intangible capital can be used by all foreign subsidiaries to product the intermediate goods locally, this can be viewed as a vertical spillover from intermediate good production by FDI to intangible capital production by the host country, which is in line with the empirical literature on productivity spillovers of FDI (Javorcik, 2004). We denote the marginal product of an additional unit of research labor in country j in producing intangible capital in country i as $H^i_j := \partial H^i/\partial l^z_{jt}$. If $H^i_j > 0$ for $j \neq i$, then the spillover effect is positive.[2]

For the sake of notation, define an allocation for country i as

$$\mathcal{A}_i \equiv \left\{c_{it}, h_{it}, l^z_{it}, z_{it}, \{l_{jit}, k_{jit}, q_{jit}\}_{\forall j}, \{\hat{q}_{jit}\}_{\forall j \neq i}\right\}_{t=0}^{\infty}$$

and let the vector of global allocations be denoted $\mathcal{A} = \{\mathcal{A}_i\}_{i=1}^{I}$. A feasible allocation vector satisfies several resource constraints. The resource constraints for intermediate goods are

$$y_{iit} = q_{iit} + \sum_{j \neq i} q_{ijt} \tag{6}$$

$$y_{ijt} = \hat{q}_{ijt} \quad \forall_{j \neq i}. \tag{7}$$

Final goods are used for private consumption, c_{it}, public consumption, g_{it}, and tangible

[2] In a quantitative model, we relax the assumption that every country has subsidiaries in all other locations. There, the location choice is endogenous, and the spillover effect in country i depends only on the intangible investment of countries with subsidiaries in i.

investment, x_{it}. Thus, the resource constraint for final goods is

$$c_{it} + g_i + x_{it} = q_{it} \tag{8}$$

and the market clearing condition for tangible capital is

$$k_{it} = \sum_{j=1}^{I} k_{jit}, \tag{9}$$

which says that the stock of tangible capital in country i must be split between the production of domestic intermediates and locally-produced foreign intermediates. The market clearing condition for labor is

$$h_{it} = \sum_{j=1}^{I} l_{jit} + l_{it}^{z}. \tag{10}$$

3.2 Pareto frontier

Let $\omega = (\omega_1, \ldots, \omega_I)$ denote a vector of Pareto weights and define $\mathcal{A}^P(\omega)$ as the allocation vector that maximizes global welfare weighted by ω:

$$\mathcal{A}^P(\omega) = \arg\max_{\mathcal{A}} \left\{ \sum_{i=1}^{I} \omega_i \sum_{t=0}^{\infty} \beta^t u^i\left(c_{it}, h_{it}\right) \right\} \quad \text{subject to (2)–(10)} \tag{11}$$

The Pareto frontier is then defined as $\boldsymbol{\mathcal{A}^P} = \left\{\mathcal{A}^P(\omega) : \omega \in \mathbb{R}_+^I\right\}$. Allocations on the Pareto frontier satisfy the following five efficiency conditions.[3]

First is the *no intratemporal wedges* condition:

$$-\frac{u_{c,t}^i}{u_{h,t}^i} = \frac{1}{G_{i,t}^i F_{l,t}^{ii}} = \frac{1}{G_{\hat{j},t}^i F_{l,t}^{ji}} \quad \forall_i, \forall_{j \neq i}. \tag{12}$$

This condition states that the marginal rate of substitution between consumption and leisure should equal the marginal rate of transformation of labor into final goods. Note that this condition holds for all goods produced in location i, both domestic and foreign.

Second is the *no intertemporal wedges* condition that links the marginal rate of substitution between consumption across periods with the marginal product of capital:

$$\frac{u_{c,t}^i}{\beta u_{c,t+1}^i} = (1-\delta) + G_{i,t+1}^i F_{k,t+1}^{ii} = (1-\delta) + G_{\hat{j},t+1}^i F_{k,t+1}^{ji} \qquad \forall_i, \forall_{j \neq i}. \tag{13}$$

[3] We relegate all the detailed derivations to the Appendix A.

Note that this condition, too, holds for all goods produced in location i, both domestic and foreign.

Third is *static production efficiency*, which states that the marginal rate of technical substitution between any pair of imported goods (m, n) should be equated across countries:

$$\frac{G^i_{n,t}}{G^i_{m,t}} = \frac{G^n_{n,t} u^n_{c,t}}{G^m_{m,t} u^m_{c,t}} \qquad \forall_i, \forall_{m,n \neq i}. \tag{14}$$

Fourth, we have *dynamic production efficiency*, which states that the social return to tangible capital should be equated across countries:

$$\frac{G^i_{j,t}}{G^i_{j,t+1}} \left((1-\delta) + G^i_{i,t+1} F^{ii}_{k,t+1}\right) = \left(\frac{G^j_{j,t} u^j_{c,t}}{G^j_{j,t+1} \beta u^j_{c,t+1}}\right) \qquad \forall_i, \forall_{j \neq i}. \tag{15}$$

If an allocation satisfies both static and dynamic production efficiency we say it is simply *production efficient*.

Before moving on to the fifth and most important efficiency condition, it is useful to compare the first four to the ones in CNT. Their Pareto frontier is characterized by similar conditions, but the addition of multinational production and nonrival intangible capital alters several of them. Specifically, the intra- and intertemporal wedge conditions (12) and (13) hold across for all goods produced in each country i, both foreign and domestic (hence the second equalities in both conditions). This implies that the marginal rates of technological transformation between labor and tangible capital are equated across all goods produced in country i, both foreign and domestic: $\frac{F^{mi}_{k,t}}{F^{mi}_{l,t}} = \frac{F^{ni}_{k,t}}{F^{ni}_{l,t}}$ for all m, n.

The last Pareto-efficiency condition, *efficiency of intangible investment*, describes how intangible investment should be allocated across countries:

$$\frac{F^{ii}_{l,t}}{F^{ii}_{z,t} H^i_{i,t}} = 1 + \overbrace{\sum_{j \neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}}}^{\text{Nonrivalry effect}} \tag{16}$$
$$+ \underbrace{\sum_{j \neq i} \left[\frac{H^j_{i,t}}{H^i_{i,t}} \left(\frac{G^i_{\hat{j},t} F^{ji}_{z,t}}{G^i_{i,t} F^{ii}_{z,t}} + \frac{u^j_{c,t} G^j_{j,t} F^{jj}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \right) + \sum_{k \neq i,j} \frac{H^k_{i,t}}{H^i_{i,t}} \frac{u^j_{c,t} G^j_{\hat{k},t} F^{kj}_{z,t}}{u^c_{i,t} G^i_{i,t} F^{ii}_{z,t}} \right]}_{\text{Spillover effect: } \Sigma_{it}}.$$

This condition states that allocating labor to producing the domestic intermediate good, q_{iit}, should yield the same worldwide utility gains as allocating it to producing intangible capital, z_{it}. The left-hand side is the marginal rate of technical substitution between research labor and productive labor in producing the domestic good. It consists of two parts: the marginal rate of technical substitution between productive labor and intangible capital, $F^{ii}_{l,t}/F^{ii}_{z,t}$, and the marginal rate of transformation between research labor and intangible capital, $H^i_{i,t}$. Holding fixed the latter, the greater the marginal rate of technical substitution between productive labor and intangible capital, the lower the ratio of labor to intangible capital. Thus, loosely, speaking, the greater the right-hand side of (16), the more intangible-intensive country ought to be.

To unpack what the right-hand side of this condition represents, it's helpful to first shut down the spillover effect, i.e. set $H^j_{i,t} = 0 \ \forall j \neq i$, so that $\Sigma_{it} = 0$. The term $\frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}}$ represents the increase in country j's utility that comes from the additional output of country i's good in country j generated by an increase z_{it}, measured relative to the increase in country i's utility that comes from increased domestic production. This effect is due solely to nonrivalry, and as we will see, it will be internalized by the market in a competitive equilibrium.

Now add the spillover effect, which we denote by Σ_{it}. Each part of the spillover effect represents a different channel through which increased intangible investment in country i affects welfare around the world by increasing other countries' intangible capital. Consider first the part in parentheses multiplied by $\frac{H^j_{i,t}}{H^i_{i,t}}$, which itself consists of two terms. The term $\frac{G^i_{j,t} F^{ji}_{z,t}}{G^i_{i,t} F^{ii}_{z,t}}$ reflects the increase in the home country's utility caused by the increase in locally-produced foreign goods. The term $\frac{u^j_{c,t} G^j_{j,t} F^{jj}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}}$ reflects the increase in foreign countries' utility caused by the increase in their own domestic production. The second part multiplied by $\frac{H^k_{i,t}}{H^i_{i,t}}$ represents the marginal impact of additional intangible capital on utility in each foreign country j due to increased output of locally-produced foreign goods from each third country $k \neq i, j$.

Having laid out and described all of the conditions that characterize the Pareto frontier, we are now in a position to formally define an efficient allocation:

Definition 1 (Efficient allocation) *An allocation $\mathcal{A}$ is efficient, i.e., $\mathcal{A} \in \boldsymbol{\mathcal{A}^P}$, if it satisfies the following conditions:*

1. *feasibility constraints for intangible capital, tangible capital and labor:* (5), (9), (10).
2. *resource constraints for intermediate and final goods,* (6),(7), (8);
3. *no intratemporal and intertemporal wedges,* (12), (13);
4. *static and dynamic production efficiency,* (14), (15); *and*
5. *efficient allocation of intangible investment,* (16).

3.3 Market arrangements and competitive equilibrium

We now specify the market arrangements in our economy and define a competitive equilibrium. As in a standard Backus et al. (1994) environment, households in each country choose consumption, labor supply, tangible investment, and holdings of international bonds to maximize lifetime utility subject to budget constraints, and each country has a competetive final-good producer. As in McGrattan and Waddle (2020), each country also has a representative multinational intermediate-good producer (MNE) that chooses intangible investment in the home country and rival factor inputs in each production location around the world to maximize global profits. As in DHS, multinationals use transfer pricing to account for the income generated by intangible capital, and can shift the rights to this income to a tax haven by paying a cost. Finally, as in CNT, each country has a government that finances public consumption using proportional taxes on consumption, labor income, and international trade, as well as a tax on corporate income, which is the focus of our analysis.

3.3.1 Fiscal policy

Public consumption, g_{it}, is exogenous. The taxes on consumption and labor income are denoted by τ_{it}^c and τ_{it}^h, respectively. Trade taxes consist of a bilateral import tax, τ_{ijt}^m, on goods shipped from country j to country i, and an analogous export tax, τ_{ijt}^x. The corporate income tax is denoted by τ_{it}^p. It applies to all taxable income earned by MNEs operating in country i regardless of their home country. We denote the sequence of government spending for a given country i as $\mathcal{G}_i = \{g_{it}\}_{t=0}^{\infty}$ and a vector of such sequences for all countries as $\mathcal{G} = \{\mathcal{G}_i\}_{i=1}^{I}$. We denote a collection of taxes for a given country i by $\mathcal{T}_i = \left\{\tau_{it}^p, \tau_{it}^h, \tau_{it}^c, \left\{\tau_{ijt}^x, \tau_{jit}^m\right\}_{\forall j \neq i}\right\}_{t=0}^{\infty}$ and the vector of worldwide tax policies by $\mathcal{T} = \{\mathcal{T}_i\}_{i=1}^{I}$.

3.3.2 Final goods producers

Final goods producers choose inputs of intermediates to maximize profits period by period:

$$\max_{\{q_{jit}\}_{\forall j}, \{\hat{q}_{jit}\}_{\forall j \neq i}} p_{it} q_{it} - \sum_j \left(1 + \tau_{jit}^m\right) p_{jit} q_{jit} - \sum_{j \neq i} \hat{p}_{jit} \hat{q}_{jit} \tag{17}$$

subject to (3), where p_{it} denotes the price of the final good in country i, p_{jit} denotes the price of domestically-produced intermediate goods imported from country j, and $\hat{p}_{jit}$ denotes the price of locally-produced foreign intermediates.

3.3.3 Intermediate goods producers

Each country has a representative MNE with a subsidiary in each of its production locations around the world. The MNE's objective is to maximize the total profits generated by all of its subsidiaries. We describe this problem in three steps: first, in a standard environment without transfer pricing or profit shifting; second, with transfer pricing but without profit shifting; and third, with profit shifting as modeled in DHS.

Before describing these scenarios and the MNE's problem in full, it is helpful to first define the simpler problem of maximizing the profits in a single production location taking as given the MNE's level of intangible capital, z_{it}, as this problem and the associated equilibrium objects are the same in all three scenarios. For the domestic parent division (i.e., the subsidiary that produces y_{iit}), this problem is given by

$$\pi_{iit}\left(z_{it}\right) = \max_{\{l_{iit},k_{iit},q_{ijt}\}_{j=1}^{I}} (1-\tau_{it}^{p})\left[p_{iit}q_{iit} + \sum_{j\neq i}(1-\tau_{ijt}^{x})p_{ijt}q_{ijt} - w_{it}l_{iit} - \delta p_{it}k_{iit}\right] - r_{it}k_{iit} \tag{18}$$

subject to (2) and (6). Note that depreciation of tangible capital, $\delta p_{it}k_{it}$, is tax-deductable, while the remainder of the cost of renting tangible capital, $r_{it}k_{it}$, is not. This is consistent with standard accounting practices and implies that demand for tangible capital is decreasing in the corporate tax rate. The analogous problem for foreign divisions (i.e., that produce y_{ijt}, $j \neq i$) is

$$\pi_{ijt}\left(z_{it}\right) = \max_{\hat{q}_{ijt},l_{ijt},k_{ijt}} \left(1-\tau_{jt}^{p}\right)\left[\hat{p}_{ijt}\hat{q}_{ijt} - w_{jt}l_{ijt} - \delta p_{jt}k_{ijt}\right] - r_{jt}k_{ijt} \tag{19}$$

subject to (2) and (7). Having defined these objects, we can now describe the MNE's problem in full.

Free transfer scenario (FT). In this scenario, all of an MNE's foreign subsidiaries use intangible capital at no cost, just as in McGrattan and Waddle (2020). The MNE's problem is to choose intangible capital to maximize its worldwide dividends after taxes:

$$d_{it} = \max_{z_{it},l_{it}^{z}} \left\{\pi_{iit}\left(z_{it}\right) - \left(1-\tau_{it}^{p}\right)w_{it}l_{it}^{z} + \sum_{j\neq i}\pi_{ijt}\left(z_{it}\right)\right\} \tag{20}$$

subject to (5). Note that the cost of intangible capital, $w_{it}\ell_{it}^{z}$, is tax-deductable.

Transfer pricing scenario (TP). In this scenario, a foreign subsidiary in country j of an

MNE from country i pays a licensing fee of ϑ_{ijt} to the parent division in the MNE's home country for the rights to use each unit of the MNE's intangible capital. We assume that licensing fee income and costs are tax deductable. The MNE's worldwide profits in this scenario can be written as

$$d_{it} = \max_{z_{it}, l_{it}^z} \left\{ \pi_{iit}(z_{it}) - (1 - \tau_{it}^p) w_{it} l_{it}^z + \sum_{j \neq i} \left[\pi_{ijt}(z_{it}) + (\tau_{jt}^p - \tau_{it}^p) \vartheta_{ijt} z_{it} \right] \right\} \tag{21}$$

subject to (5). Note that the terms ϑ_{ijt} enter both as income for the parent division, which is taxed at a rate of τ_{it}^p, and as costs for foreign divisions, which are deducted from these divisions at rates of τ_{ijt}^p. For now, we leave the parameterization of these licensing fees unspecified, but it is worth stating that the appropriate arms-length licensing fee that would prevail if an MNE's foreign affiliates rented intangible capital in a competitive market would be equal to the affiliates' marginal revenue products of intangible capital, i.e., $\vartheta_{ijt} = \hat{p}_{ijt} F_{z,t}^{ij}$. We will adopt this specification later on.

Profit Shifting scenario (PS). In this scenario, firms can choose to sell a fraction λ of their intangible capital to a tax haven that taxes corporate income at a rate of τ_{TH}^p. This sale occurs at a markdown φ below the intangible capital's market price, which is given by $\nu_{it} := \sum_{j=1}^{I} \vartheta_{ijt}$. Note that this object includes licensing fees that the parent "pays" to itself. The sale also incurs a cost $\mathcal{C}(\lambda)$ in proportion to ν_{it}. We follow DHS and assume that this cost function is given by

$$\mathcal{C}(\lambda) \equiv \lambda - (1 - \lambda) \log(1 - \lambda), \tag{22}$$

which implies that $\mathcal{C}(0) = 0$, $\lim_{\lambda \to 1} \mathcal{C}(\lambda) = 1$, and that $\mathcal{C}$ is increasing and convex over $[0, 1)$. The tax haven then collects a fraction λ of the worldwide licensing fees generated by the MNE's intangible capital, while the parent division collects the remainder. The MNE's profits in this scenario are

$$\begin{aligned} d_{it} = \max_{z_{it}, l_{it}^z, \lambda_{it}} \Bigg\{ & \pi_{iit}(z_{it}) + (1 - \tau_{it}^p) \left[-w_{it} l_{it}^z + (1 - \lambda_{it}) \sum_{j \neq i} \vartheta_{ijt} z_{it} + \lambda_{it} \Big((\varphi - \mathcal{C}(\lambda_{it})) \nu_{it} z_{it} - \vartheta_{iit} z_{it} \Big) \right] \\ & + \sum_{j \neq i} \left[\pi_{ijt}(z_{it}) - (1 - \tau_{jt}^p) \vartheta_{ijt} z_{it} \right] + (1 - \tau_{THt}^p) \lambda_{it} (1 - \varphi) \nu_{it} z_{it} \end{aligned} \tag{23}$$

subject to (5). The first line contains the parent division's after-tax profits, which now include: licensing fee income from the portion of intangible capital that is retained, $(1 - \lambda_{it}) \sum_{j \neq i} \vartheta_{ijt}(z_{it})$; a licensing fee payment to the tax haven on the portion of the intangible capital that is sold, $\lambda_{it} \vartheta_{iit}(z_{it})$; income from selling intangible capital to the tax haven, $\lambda_{it}\varphi\nu_{it}(z_{it})$; and the cost of profit shifting, $\lambda_{it}\mathcal{C}(\lambda_{it})\nu_{it}(z_{it})$. The second line contains productive foreign affiliates' after-tax profits, which are the same as in the transfer pricing scenario; and the after-tax income of the affiliate in the tax haven, which consists solely of the licensing fees this affiliate collects minus the cost of purchasing the intangible capital from the parent division. The dividends of the the MNE's affiliates in productive countries are the same as in the transfer pricing scenario.

3.3.4 Households

The household's problem is formulated in the same way as in CNT. The household in country i chooses sequences of consumption, labor, and tangible capital, $\{c_{it}, h_{it}, k_{it}\}_{t=0}^{\infty}$, to maximize lifetime utility (1) subject to the law of motion for capital (4) and a lifetime budget constraint,

$$\sum_{t=0}^{\infty} Q_t \left[(1+\tau_{it}^c)\, p_{it}c_{it} + p_{it}\left(k_{it+1} - (1-\delta+r_{it})k_{it}\right) - \left(1-\tau_{it}^h\right) w_{it}h_{it}\right] = a_{i0}, \qquad (24)$$

where Q_t is the intertemporal price in units of the numeraire in period 0. The right-hand side represents the present value of the household's lifetime wealth in period 0,

$$a_{i0} = R_{i0} + Q_{-1}b_{i0} + \left(1+r^f\right) f_{i0} + V_{i0}, \qquad (25)$$

where $R_{i0} = (1-\delta+r_{i0})k_{i0}$, initial holdings of domestic public debt are denoted by $Q_{-1}b_{i0}$, and $\left(1+r^f\right) f_{i0}$ denotes initial net holdings of international bonds. V_{i0} denotes the initial stock value of the domestic multinational, which is given by

$$V_{i0} = \sum_{t=0}^{\infty} Q_t\, d_{it}. \qquad (26)$$

We assume that interest parity holds and world capital markets are fully integrated, so that the return on one-period bonds, $r_{t+1}^f := Q_t/Q_{t+1}$, is the same in all countries.

3.3.5 Equilibrium conditions

The market-clearing conditions for labor, tangible capital, and final goods have already been defined in (10), (9), and (8). In addition to these conditions, the government's budget constraint must hold. The government in each country i collects the following stream of revenues:

$$REV_i = \sum_{t=0}^{\infty} Q_t \left[\tau^c_{it} p_{it} c_{it} + \tau^h_{it} w_{it} h_{it} + \tau^p_{it} \left(\hat{\pi}_{iit} + \sum_{j \neq i} \hat{\pi}_{jit} \right) + \tau^m_{jit} \sum_{j \neq i} p_{jit} q_{jit} + \tau^x_{ijt} \sum_{j \neq i} p_{ijt} q_{ijt} \right]$$

where $\hat{\pi}_{jit}$ denotes the corporate tax base of the MNE from country i's affiliate in country j. Then the government budget constraint can be written as

$$REV_i = \sum_{t=0}^{\infty} Q_t p_{it} g_{it} + Q_{-1} b_{i0} \tag{27}$$

The last condition we need to complete a description of a competitive equilibrium is the balance of payments. This condition is different in each of the three scenarios defined above.

Free transfer scenario (FT). In the absence of transfer pricing and profit shifting, the balance of payments for country i is

$$\sum_{t=0}^{\infty} Q_t \sum_{j \neq i} (p_{ijt} q_{ijt} - p_{jit} q_{jit}) + \sum_{t=0}^{\infty} Q_t \sum_{j \neq i} (d_{ijt} - d_{jit}) = -\left(1 + r^f\right) f_{i0} \tag{28}$$

where d_{ijt} are dividends paid to country i by its MNE's affiliate in country j, which in this scenario are simply $d_{ijt} = \pi_{ijt}$. The first term on the left is net exports of intermediate goods and the second is net factor payments. The right is the (negative of the) financial account.

Transfer pricing scenario (TP). Licensing fees enter the balance of payments in two ways: indirectly, through the dividend payments of foreign MNEs, and directly, as net exports of intangible capital services. In the transfer pricing scenario the balance of payments is

$$\sum_{t=0}^{\infty} Q_t \sum_{j \neq i} [p_{ijt} q_{ijt} - p_{jit} q_{jit} + \vartheta_{ijt} z_{it} - \vartheta_{jit} z_{jt} + d_{ijt} - d_{jit}] = -(1 + r^f) f_{i0} \tag{29}$$

Note that in this case, foreign affiliate dividends are $d_{ijt} = \pi_{ijt} - (1 - \tau^p_{jt}) \vartheta_{ijt}(z_{it})$. Note also that licensing fees affect net exports and net foreign payments differently. The full value of the licensing fees shows up on the left, while the only the after-tax portion shows up on the right. Thus, transfer pricing affects net exports of intermediate goods, either through quantities or prices, in equilibrium. Specifically, countries that export relatively large amounts of intangible capital services see their net exports of intermediate goods fall.[4]

[4] The United States, which consistently runs a trade surplus in services and a deficit in goods, exemplifies

Profit shifting scenario (PS). In the profit shifting scenario we need to adjust net exports of intangible capital services to account for the fact that the parent division takes in fewer licensing fees and now pays some out. We also need to include the sale of intangible capital to the tax haven in the financial account. The balance of payments now becomes

$$\sum_{t=0}^{\infty} Q_t \left[\sum_{j \neq i} \left(p_{ijt} q_{ijt} - p_{jit} q_{jit} + d_{ijt} - d_{jit} + (1 - \lambda_{it}) \vartheta_{ijt} z_{it} - \vartheta_{jit} z_{jt} \right) - \lambda_{it} \vartheta_{iit} z_{it} \right] \\ = - \sum_{t=0}^{\infty} Q_t \varphi \lambda_{it} \nu_{it} z_{it} - \left(1 + r^f\right) f_{i0} \tag{30}$$

We also need to include the tax haven's balance of payments. The tax haven's exports of intangible capital services are $\sum_{i=1}^{I} \lambda_{it} \nu_{it} z_{it}$. It makes net foreign payments in the form of dividends repatriated to MNEs' home countries, which are equal to $\sum_{i=1}^{I} \lambda_{it} \nu_{it} z_{it} (1 - \tau_{TH}^{p})$. Its financial account includes purchases of intangible capital in the amount of $\sum_{i=1}^{I} \lambda_{it} \varphi \nu_{it} z_{it}$. And it imports intermediate goods from other countries for consumption in the amount of $\sum_{j \neq TH}^{I} p_{jTHt} q_{jTHt}$. Hence its balance of payments can be written as

$$\sum_{t=0}^{\infty} Q_t \left(\sum_{i=1}^{I} \lambda_{it} \tau_{TH}^{p} \nu_{it} z_{it} - \sum_{j \neq TH}^{I} p_{jTHt} q_{jTHt} \right) = \sum_{t=0}^{\infty} Q_t \sum_{i=1}^{I} \lambda_{it} \varphi \nu_{it} z_{it} \tag{31}$$

We are now ready to define a competitive equilibrium. First, for the sake of notation denote a collection of prices for country i as $\mathcal{P}_i := \left\{ p_{it}, w_{it}, \{p_{ijt}\}_{\forall j}, \{\hat{p}_{ijt}\}_{\forall j \neq i} \right\}_{t=0}^{\infty}$ and denote a vector of worldwide prices as $\mathcal{P}$. Then we can define a competitive equilibrium as follows.

Definition 2 *Given government expenditures and tax policies, $(\mathcal{G}, \mathcal{T})$, and initial conditions, $\{f_{i0}, b_{i0}, k_{i0}, Q_{-1}\}_{i=1}^{I}$, a competitive equilibrium is an allocation and prices, $(\mathcal{A}, \mathcal{P})$, such that in each country, households maximize utility subject to their budget constraints; final goods producers and MNEs maximize profits; goods and factor market clearing conditions hold; government budget constraints hold; and the balance of payments holds.*

3.4 Competitive equilibrium characterization

Now that we have defined a competitive equilibrium in our environment, we turn now to its characterization and a comparison to the Pareto frontier. All derivations and proofs are relegated to Appendix B.2 for brevity.

this effect.

Manipulating the first-order conditions of the household's problem and the firms' problems, we obtain counterparts to the *no intratemporal wedges* condition (12) and *no intertemportal wedges* condition (13):

$$-\frac{u^i_{c,t}}{u^i_{h,t}} = \frac{(1+\tau^c_{it})}{\left(1-\tau^h_{it}\right)}\frac{1}{G^i_{i,t}F^{ii}_{l,t}} = \frac{(1+\tau^c_{it})}{\left(1-\tau^h_{it}\right)}\frac{1}{G^i_{\hat{j},t}F^{ji}_{l,t}} \quad \forall_i, \forall_{j\neq i}, \tag{32}$$

and

$$\begin{aligned}\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} &= \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\left[1+\left(1-\tau^p_{it+1}\right)\left(G^i_{i,t}F^{ii}_{k,t+1}-\delta\right)\right] \\ &= \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\left[1+\left(1-\tau^p_{it+1}\right)\left(G^i_{\hat{j},t}F^{ji}_{k,t+1}-\delta\right)\right] \quad \forall_i, \forall_{j\neq i}.\end{aligned} \tag{33}$$

The first-order conditions of the final good producers can be combined to yield counterparts to the *static efficiency* condition (14) and *dynamic efficiency* condition (15):

$$\frac{(1-\tau^x_{nit})\left(1+\tau^m_{mit}\right)}{(1+\tau^m_{nit})\left(1-\tau^x_{mit}\right)}\frac{G^i_{n,t}}{G^i_{m,t}} = \frac{p_{nt}}{p_{mt}}\frac{G^n_{n,t}}{G^m_{m,t}} \quad \forall_i, \forall_{m,n\neq i}, \tag{34}$$

and

$$\begin{aligned}&\frac{\left(1+\tau^m_{jit+1}\right)}{\left(1-\tau^x_{jit+1}\right)}\frac{\left(1-\tau^x_{jit}\right)}{\left(1+\tau^m_{jit}\right)}\frac{G^i_{j,t}}{G^i_{j,t+1}}\left[1+\left(1-\tau^p_{it+1}\right)\left(G^i_{i,t+1}F^{ii}_{k,t+1}-\delta\right)\right] = \\ &\qquad \frac{G^j_{j,t}}{G^j_{j,t+1}}\left[1+\left(1-\tau^p_{jt+1}\right)\left(G^j_{j,t+1}F^{jj}_{k,t+1}-\delta\right)\right] \quad \forall_i, \forall_{j\neq i}.\end{aligned} \tag{35}$$

As in our characterization of the Pareto frontier in section 3.2, these conditions are similar to those found in CNT. Most notably, perhaps, (34) says that, just as in CNT, static efficiency can be achieved if export subsidies are used to offset import tariffs, i.e., $\tau^x_{ijt} = -\tau^m_{ijt}$ for all i and j. There are two primary differences, however. First, as in the characterization of the Pareto frontier, (32) and (33) hold for locally-produced foreign intermediate goods as well as domestic intermediates. Second, and more important, corporate tax rates τ^p_{it+1} enter the dynamic equilibrium conditions (33) and (35). In CNT, the planner can achieve dynamic efficiency and no intertemporal wedges without corporate income taxes by setting consumption taxes to be constant over time. Here, if corporate income taxes are non-zero, this is not the case even if consumption taxes are constant. As we will see, this creates a tension between achieving dynamic efficiency and an efficient allocation of intangible investment. It is useful to establish the following lemma before moving on to the last equilibrium condition.

Lemma 1 *In any competitive equilibrium in which static efficiency (14) holds, we have*

$$\frac{u_{ct}^i}{u_{ct}^j}\frac{G_{j,t}^i}{G_{j,t}^j} = 1, \ \forall_{i,j}.$$

The last condition we need to characterize the equilibrium governs the allocation of labor to production of domestic intermediates, l_{iit}, and intangible capital, l_{it}^z. This condition, which we obtain by manipulating the MNE's first-order conditions, is different in each of the three scenarios described above.

Free transfer scenario (FT). In the absence of transfer pricing and profit shifting, the allocation of labor to these two activities is characterized by

$$\frac{F_{l,t}^{ii}}{H_{i,t}^i F_{z,t}^{ii}} = 1 + \sum_{j \neq i} \frac{(1-\tau_{jt}^p)\hat{p}_{ijt} F_{z,t}^{ij}}{(1-\tau_{it}^p) p_{iit} F_{z,t}^{ii}} \tag{36}$$

The right-hand side represents the additional after-tax profits that MNEs earn in foreign affiliates relative to the domestic parent from additional intangible capital. The higher (lower) the domestic (foreign) corporate tax rate, the higher the ratio of intangible capital to productive labor. This is because foreign affiliates' returns to intangible investment are taxed at different rates than rate at which the cost of intangible investment is deducted in the home country. It is important to note that this expression differs from (16), which describes an efficient allocation of intangible investment across countries, due to the externality created by the nonrivalry of intangible capital in an intuitive way. The social planner cares about how much additional utility each country gains from additional intangible investment in country i, whereas MNEs care about the additional profits they earn.

In order to shed further light on how an equilibrium allocation of intangible investment differs from an efficient one (and to define the Ramsey planner's problem later on), it is helpful to write (36) solely in terms of policies and allocations:

$$\frac{F_{l,t}^{ii}}{H_{i,t}^i F_{z,t}^{ii}} = 1 + \sum_{j \neq i} \left(\frac{u_{c,t}^j G_{\hat{i},t}^j F_{z,t}^{ij}}{u_{c,t}^i G_{i,t}^i F_{z,t}^{ii}}\right)\left(\frac{\left(1-\tau_{jt}^p\right)}{\left(1-\tau_{it}^p\right)}\frac{\left(1-\tau_{jit}^x\right)}{\left(1+\tau_{jit}^m\right)}\frac{G_{jt}^i}{G_{jt}^j}\frac{u_{c,t}^i}{u_{c,t}^j}\right) \tag{37}$$

Comparing this with (16), we can now see clearly that firms in the free transfer scenario do not internalize the spillover externality. While the first term in (16) that captures the utility gains from increased global production of country i's good are present, all of the terms that involve H_j^i for $j \neq i$ that capture the gains from greater production of foreign goods are missing.

To more sharply characterize the role of corporate income taxes in the equilibrium allocation of intangible investment, suppose that static efficiency holds, so that $(1-\tau^x_{jit})G^i_{jt}u^i_{c,t}$ is equal to $(1+\tau^m_{jit})G^j_{jt}u^j_{c,t}$, and that taxes on corporate income are zero in all countries. Then the right-hand side of (37) is lower than the right-hand side of (16) in all countries. This implies that the marginal rate of transformation between productive labor and intangible capital is lower than in a Pareto-optimal allocation for all countries; all countries do too little intangible investment. More generally, for any vector of corporate tax rates $\{\tau^p_{it}\}_{i=1}^I$, it must be that for at least one country, the second term in parentheses in (37) is less than one. For this country, the marginal rate of transformation between productive labor and intangible is unambiguously lower than in a Pareto-optimal allocation; this country does too little intangible investment. This implies that it is impossible to achieve an efficient allocation of intangible investment in the free transfer scenario using corporate income taxes as long as static efficiency holds.

Transfer pricing scenario (TP). In the transfer pricing scenario, without putting further structure on the licensing fees, $\vartheta_{ijt}(z_{it})$, this condition becomes

$$\frac{F^{ii}_{l,t}}{H^i_{i,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i}\left[\frac{(1-\tau^p_{jt})\hat{p}_{ijt}F^{ij}_{z,t}}{(1-\tau^p_{it})p_{iit}F^{ii}_{z,t}} + \frac{\left(\tau^p_{jt}-\tau^p_{it}\right)\vartheta_{ijt}}{(1-\tau^p_{it})p_{iit}F^{ii}_{z,t}}\right]. \tag{38}$$

The second term in parentheses is decreasing in the domestic corporate income tax rate and increasing in the foreign tax rate. Thus, the effects of corporate income taxes on the marginal rate of transformation between productive labor and intangible investment are now ambiguous. This is because some of the intangible income generated by an MNE's foreign affiliates are now taxed at the home instead of abroad.

To provide a sharper characterization, from this point forward we assume that licensing fees are set according to the arms-length principle, which states that transfer prices should be set to the prices that would prevail in a competitive secondary market. In such a market, foreign affiliates would choose to rent intangible capital so that the cost equalled the marginal benefit, i.e., the marginal revenue product of intangible capital. Thus, we set $\vartheta_{ijt} = \hat{p}_{ijt}F^{ij}_{z,t}$. Under this assumption, the equilibrium condition (38) above simplifies to

$$\frac{F^{ii}_{l,t}}{H^i_{i,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i}\frac{\hat{p}_{ijt}F^{ij}_{z,t}}{p_{iit}F^{ii}_{z,t}} = 1 + \sum_{j\neq i}\left(\frac{u^j_{c,t}G^j_{\hat{i},t}F^{ij}_{z,t}}{u^i_{c,t}G^i_{i,t}F^{ii}_{z,t}}\right)\left(\frac{(1-\tau^x_{jit})}{(1+\tau^m_{jit})}\frac{G^i_{jt}}{G^j_{jt}}\frac{u^i_{c,t}}{u^j_{c,t}}\right) \tag{39}$$

Now, none of the corporate income taxes enters at all. This is because all of the worldwide income generated by an MNE's intangible capital is ultimately taxed in the MNE's home country at the same rate at which the cost of this capital is deducted. Here, if static efficiency holds, all countries do too little intangible investment regardless of how corporate income taxes are set across countries, and the planner has no ability at all to improve the allocation of intangible investment using these taxes.

Profit shifting scenario (PS). Maintaining our assumption that lincensing fees are set according to the arms-length principle, in the scenario with both transfer pricing and profit shifting, this condition changes to

$$\frac{F_{l,t}^{ii}}{H_{i,t}^{i}F_{z,t}^{ii}} = \left(1+\sum_{j\neq i}\frac{\hat{p}_{ijt}F_{z,t}^{ij}}{p_{iit}F_{z,t}^{ii}}\right)\left(1-\mathcal{C}\left(\lambda_{it}\right)+\frac{\lambda_{it}\left(1-\varphi\right)\left(\tau_{it}^{p}-\tau_{THt}^{p}\right)}{\left(1-\tau_{it}^{p}\right)}\right) \tag{40}$$
$$= \left[1+\sum_{j\neq i}\left(\frac{u_{c,t}^{j}G_{\hat{i},t}^{j}F_{z,t}^{ij}}{u_{c,t}^{i}G_{i,t}^{i}F_{z,t}^{ii}}\right)\left(\frac{\left(1-\tau_{jit}^{x}\right)}{\left(1+\tau_{jit}^{m}\right)}\frac{G_{jt}^{i}}{G_{jt}^{j}}\frac{u_{c,t}^{i}}{u_{c,t}^{j}}\right)\right]\underbrace{\left(1-\mathcal{C}\left(\lambda_{it}\right)+\frac{\lambda_{it}\left(1-\varphi\right)\left(\tau_{it}^{p}-\tau_{THt}^{p}\right)}{\left(1-\tau_{it}^{p}\right)}\right)}_{\Omega(\tau_{it}^{p})}.$$

The first term in square brackets is the same as (39) for the transfer pricing scenario. The second term, $\Omega(\tau_{it}^{p})$, is the gain from profit shifting per unit of intangible capital, where λ_{it} endogenous and given by the formula (B.29) derived in Appendix B.1. Note that only the corporate tax rate of an MNE's home country and the tax haven's tax rate matter; the rates at which foreign affiliates profits are taxed still do not show up. The marginal rate of technical substitution between productive labor and intangible capital now unambiguously increases with the home country's tax rate as in the free transfer scenario, but it also decreases with the tax haven's tax rate. Additionally, $\Omega(\tau_{it}^{p})$, which represents the per-unit net gain from profit shifting, is weakly greater than one, with strict inequality when $\tau_{it}^{p} > \tau_{TH}^{p}$. This indicates that MNEs do more intangible investment in the profit shifting scenario as we showed in DHS.

Since the spillover term Σ_{it} in (16) is strictly positive, this result implies that each country's intangible investment level can be improved to the Pareto-optimal level by increasing its corporate income tax rate, which increases the returns to profit shifting and thereby increases the incentive to invest in intangible capital. In other words, profit shifting helps competitive firms internalize the effects of the spillover externality. If it were possible, lowering the tax haven's tax rate to further increase profit shifting would accomplish the same thing, and we consider this possibility in our quantitative analysis. More importantly, $\Omega(\tau_{it}^{p})$ is increasing and approaches infinity as τ_{it}^{p} approaches one. Intuitively, this increases the gain from profit shifting relative to the cost, and the latter goes to zero as τ_{it}^{p} approaches one due to tax-deductability. We formalize this result in the following lemma.

Lemma 2 *We can establish the following properties of the return on profit shifting, $\Omega(\tau_{it}^p)$:*

1. $\Omega(\tau_{it}^p) = 1$ *when* $\tau_{it}^p = \tau_{THt}^p$*;*
2. $\Omega(\tau_{it}^p) > 1$ *when* $\tau_{it}^p > \tau_{THt}^p$*; and*
3. $\Omega(\tau_{it}^p)$ *is increasing in* τ_{it}^p*, with* $\Omega(\tau_{it}^p) \to \infty$ *as* $\tau_{it}^p \to 1$.

This lemma implies that in the profit shifting scenario, corporate taxes can always be set high enough to achieve an efficient level of intangible investment in each country. This is in stark contrast to the free transfer and transfer pricing scenarios, where it is impossible to achieve an efficient allocation of intangible investment using corporate income taxes. This is our first main result, which we formalize in the proposition below.

Proposition 1 (Intangible investment efficiency in competitive equilibrium) *Consider competitive equilibria that satisfy the static efficiency condition (14). In both the free transfer and the transfer pricing scenarios, the competitive equilibrium allocation associated with any tax vector $\mathcal{T}$ always has an inefficient allocation of intangible investment. In the profit shifting scenario, the competitive equilibrium allocation features an efficient level of intangible investment in each country if $\Omega(\tau_{it}^p)$ is such that (40) is equivalent to (16); otherwise it is inefficient.*

Note that it is not necessarily the case that there exists a Pareto efficient competitive equilibrium in the profit shifting scenario, i.e., a competitive equilibrium that satisfies all of the other efficiency conditions (12)–(15) as well as (16). The purpose of this proposition is to highlight the fact that the only scenario in which it is even possible to attain Pareto efficiency is the profit shifting scenario; in the free-transfer and transfer-pricing scenarios, where (16) cannot be satisfied, attaining Pareto efficiency in a competitive equilibrium is impossible.

It is useful to contrast these results with the case in which there are no spillovers in intangible capital production, i.e., $z_{it} = H^i(\ell_{it}^z)$ so that $H_{j,t}^i = 0$ for all $j \neq i$. In this case, the intangible allocation condition in the free transfer scenario (37) is the same as the Pareto-optimal condition (16) with zero corporate income taxes if trade taxes are set to ensure static efficiency. In the free transfer scenario, where corporate income taxes do not enter (39) at all, intangible investment is efficient under static efficiency for any corporate income taxes at all. The difference is perhaps the most striking in the profit shifting scenario. In the presence of spillovers, improving the efficiency of intangible investment dictates raising corporate income taxes to increase the gains from profit shifting. In the absence of spillovers, on the other hand, efficiency can be achieved under static efficiency by eliminating profit shifting by setting corporate income taxes zero (or, alternatively, to any value below the tax haven's tax rate), in which case (40) collapses to the transfer pricing scenario's version (39). The following remark formalizes this point.

Remark 1 *If there are no spillovers in intangible capital production, then a competitive equilibrium has an efficient allocation of intangible investment in the following circumstances:*

- *In the free transfer scenario: under static efficiency and zero corporate income taxes, or more generally when corporate income taxes are set so that* $(1-\tau^p_{jt})(1-\tau^x_{jit})G^i_{jt}u^i_{c,t}$ *is equal to* $(1-\tau^p_{it})(1+\tau^m_{jit})G^j_{jt}u^j_{c,t}$.
- *In the transfer pricing scenario: under static efficiency for any corporate income taxes.*
- *In the profit shifting scenario: under static efficiency with either zero corporate income taxes or when each country's corporate income tax is equal to the tax haven's tax rate.*

The final piece of our characterization is a description of the conditions required for an allocation to be implementable as a competitive equilibrium with an appropriately-chosen tax vector. The following proposition summarizes these conditions. They are essentially the same as in CNT and thus merit no further discussion.

Proposition 2 (Necessary conditions for implementation) *Given a government spending vector* $\mathcal{G}$ *and initial conditions* $\{k_{i0}, b_{i0}, f_{i0}\}_{i=1}^I$, *an allocation* $\mathcal{A}$ *and a collection of period-0 prices* $\{p_{i0}\}_{i=1}^I$ *the resource constraints (2)–(10) and the following implementability condition:*

$$\sum_{t=0}^{\infty}\beta^t\left[u^i_{c,t}c_{it}-u^i_{h,t}h_{it}\right]=\frac{u^i_{c0}}{(1+\tau^c_{i0})}\left(R_{i0}+Q_{-1}\frac{b_{i0}}{p_{i0}}+\left(1+r^f\right)\frac{f_{i0}}{p_{i0}}\right),\ \forall i=1,\dots,I. \tag{41}$$

where $R_{i0}=(1-\delta+r_{i0})\,k_{i0}$.

3.5 Corporate taxes and intangible investment in an example

Before moving on to the Ramsey planner's problem, we use an example economy with parameterized production technologies to provide a sharper characterization of how corporate income taxes affect a given country's intangible capital investment in partial equilibrium, taking all other prices and quantities as given. In order to do so we depart from the constant-returns-to-scale assumption that we imposed in Section 3.1. Suppose now that the production technology for intermediate goods is

$$F^{ij}\left(z_{it},k_{ijt},l_{ijt}\right)=A_j\left(N_jz_{it}\right)^{\phi}k^{\alpha}_{ijt}l^{\gamma}_{ijt},\quad \phi+\alpha+\gamma<1 \tag{42}$$

and that the production technology for intangible capital is

$$z_{it} = H^i\left(l_{it}^z\right) = B_i l_{it}^z \tag{43}$$

as in DHS. Note here that since we are taking other countries' intangible investments as given, we can ignore spillovers for the moment; implicitly, we are summarizing the effects of all of these spillovers by the constant B_i. Under these assumptions, we can solve for each country's equilibrium level of intangible capital in closed form in each of the three scenarios described above.

Free transfer scenario (FT). The equilibrium intangible investment in this scenario is

$$z_{it} = \left(\underbrace{\hat{r}_{it}(\tau_{it})}_{\searrow \text{ in } \tau_{it}^p} \Lambda_{iit} + \sum_{j \neq i} \underbrace{\frac{(1-\tau_{jt}^p)}{(1-\tau_{it}^p)} \hat{r}_{jt}(\tau_{jt})}_{\nearrow \text{ in } \tau_{it}^p,\ \searrow \text{ in } \tau_{jt}^p} \Lambda_{ijt} \right)^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}} \tag{44}$$

where $\Lambda_{ijt} := \phi \frac{B_i}{w_{it}} \left[\hat{p}_{ijt} A_j N_j^\phi \left(\frac{\gamma}{w_{jt}}\right)^\gamma \left(\frac{\alpha}{r_{jt}+p_{jt}\delta}\right)^\alpha \right]^{\frac{1}{1-\gamma-\alpha}}$ and $\hat{r}_{jt}(\tau_{jt}) := \left[\frac{(1-\tau_{jt}^p)(r_{jt}+p_{jt}\delta)}{r_{jt}+(1-\tau_{jt}^p)p_{jt}\delta}\right]^{\frac{\alpha}{1-\gamma-\alpha}}$. The impact of domestic corporate income taxes is ambiguous in this scenario because there are two opposing effects. The first effect, which is negative, operates through domestic tangible capital. Since tangible capital costs are not fully tax-deductable, an increase in τ_{it}^p reduces domestic tangible investment, which in turn lowers the return to intangible capital deployed domestically. The second effect, which is positive, operates through multinational production. The costs of intangible investment are deducted from the taxes of an MNE's domestic parent division, so when τ_{it}^p rises the after-tax return to intangible capital deployed abroad increases relative to the after-tax costs. Note that if there were no tangible capital—or, alternatively, tangible capital costs were fully tax deductable—the first effect would vanish because $\hat{r}_{jt}(\tau_{jt}^p) = 1$, and intangible investment would unambiguously increase with the domestic tax rate. On the other hand, foreign corporate income taxes unambiguously reduce domestic intangible investment, as both effects above operate in the same direction. Higher foreign taxes reduce foreign subsidiaries' tangible investment, and also reduce the after-tax return to intangible capital deployed abroad relative to the after-tax cost.

Transfer pricing scenario (TP). In this scenario, the solution for z_i is

$$z_{it} = \left(\underbrace{\hat{r}_{it}\left(\tau_{it}\right)}_{\searrow \text{ in } \tau^p_{it}} \Lambda_{iit} + \sum_{j \neq i} \underbrace{\hat{r}_{jt}\left(\tau_{jt}\right)}_{\searrow \text{ in } \tau^p_{jt}} \Lambda_{ijt} \right)^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}} \tag{45}$$

Here, where all of an MNE's worldwide intangible income is booked in the home country and taxed at the home country's tax rate, the only effect of corporate taxes on intangible investment operates through the tangible investment channel. An increase in corporate income taxes, either at home or abroad, reduces an MNE's tangible capital in that location and thereby its return to intangible investment. Note that in this scenario, if there were no tangible capital or its costs were fully deductable, corporate taxes would have no effect on intangible investment at all.

Profit shifting scenario (PS). The solution for z_i in this scenario is

$$z_{it} = \left[\left(\underbrace{\hat{r}_{it}\left(\tau_{it}\right)}_{\searrow \text{ in } \tau^p_{it}} \Lambda_{iit} + \sum_{j \neq i} \underbrace{\hat{r}_{jt}\left(\tau_{jt}\right)}_{\searrow \text{ in } \tau^p_{jt}} \Lambda_{ijt} \right) \left(\underbrace{1 - \mathcal{C}\left(\lambda_{it}\right) + \frac{\lambda_{it}\left(1-\varphi\right)\left(\tau^p_{it} - \tau^p_{THt}\right)}{\left(1-\tau^p_{it}\right)}}_{\Omega(\tau^p_{it}):\nearrow \text{ in } \tau_{it},\ \geq 1} \right) \right]^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}} \tag{46}$$

The first part in square brackets is the same as in the transfer pricing scenario. The second part, $\Omega(\tau^p_{it})$, is familiar from the characterization above. As mentioned, it is increasing in the domestic tax rate, and so it operates in a qualitatively similar fashion to the second effect in the free transfer scenario. When the domestic tax rate rises, the after-tax income of the MNE's subsidiary in the tax haven rises relative to the after-tax cost of intangible investment at home. Thus, the effect of domestic tax rates is now ambiguous again. Also as mentioned, $\Omega(\tau^p_{it})$ is greater than one as long as $\tau^p_{it} > \tau^p_{TH}$, which means that intangible investment is higher in the profit shifting scenario than in the transfer pricing scenario as in DHS.

This example highlights the fact that corporate income taxes affect MNEs' intangible investment through their negative effects on tangible investment as well as their positive effects on the returns to deploying intangible capital in foreign affiliates. Although increasing corporate income taxes can help improve the allocation of intangible investment in the presence of profit shifting, it may not be optimal to do so if the first effect dominates. This caveat is particularly relevant if some firms do not engage in multinational production; for these firms, corporate taxes affect intangible investment only through the tangible investment channel.

Our quantitative model, in which the vast majority of firms are not MNEs, takes this into account.

3.6 Cooperative Ramsey planner's problem

Given that competitive equilibria are generally inefficient in our environment, how should governments coordinate to set taxes? What trade-offs do they face in doing so? To formalize these issues, we now study the problem of a cooperative global Ramsey planner who chooses the best allocation that can be supported with prices and policies as a competitive equilibrium. We first characterize the Ramsey problem in the environment described above, where competitive equilibria are not Pareto optimal in all three scenarios. We then equip the planner with an additional fiscal instrument, a tax on tangible capital income, and show that the Ramsey allocation can be implemented in the profit shifting scenario, but not in the free transfer or transfer pricing scenarios.

The Ramsey problem is to choose an allocation, $\mathcal{A}$, a vector of prices, $\mathcal{P}$, and a vector of tax policies, $\mathcal{T}$ to maximize a weighted sum of utilities across countries,

$$\sum_{i=1}^{I} \omega^i U^i \tag{47}$$

subject to the restriction that $(\mathcal{A}, \mathcal{P})$ is the competitive equilibrium associated with $\mathcal{T}$. Ultimately, though, what the planner cares about is the allocation, so we follow CNT and analyze the relaxed Ramsey problem, which consists of choosing an allocation $\mathcal{A}$ and period-zero prices and policies to maximize (47) subject to the implementability condition (41) and the resource constraints (2)-(10).[5] As in CNT, we also impose a restriction that each country must be allowed to keep an exogenous value of initial wealth $\mathcal{W}_i$ measured in units of utility.[6]

Definition 3 (Relaxed Ramsey problem) *Given a vector of Pareto weights ω, the relaxed Ramsey problem is defined as*

$$\max_{\mathcal{A}} \left\{ \sum_{i=1}^{I} \omega_i \sum_{t=0}^{\infty} \beta^t v^i \left(c_{it}, h_{it}; \varphi^i\right) - \varphi^i \mathcal{W}_{i0} \left(\tau_{i0}^c\right) \right\} \quad \textit{subject to } (2)\text{–}(10)$$

[5] Of course, in order to demonstrate that the solution to the relaxed Ramsey problem can in fact be implemented as a competitive equilibrium, one has to construct policies that, together with the allocation, constitute a competitive equilibrium. In particular, one has to show that the balance of payments condition (28) (or (29), or (30) depending on the scenario) is satisfied. See CNT for the details of this argument.

[6] This is a technical condition that restricts the planner's ability to confiscate initial wealth.

where

$$v^{i}\left(c_{it},h_{it};\varphi^{i}\right):=u^{i}(c_{it},h_{it})+\varphi^{i}\left(u^{i}_{c,t}c_{it}-u^{i}_{h,t}h_{it}\right) \tag{48}$$

$$W_{i0}\left(\tau^{c}_{i0}\right):=\frac{u^{i}_{c0}}{(1+\tau^{c}_{i0})}\left(R_{i0}+Q_{-1}\frac{b_{i0}}{p_{i0}}+\left(1+r^{f}\right)\frac{f_{i0}}{p_{i0}}\right) \tag{49}$$

and φ^{i} is the multiplier on country i's implementability condition (41). The solution is denoted by $\mathcal{A}^{R}(\omega)$. The set of Ramsey allocations is defined as $\boldsymbol{\mathcal{A}^{R}}=\left\{\mathcal{A}^{R}(\omega):\omega\in\mathbb{R}^{I}_{+}\right\}$.

Ramsey allocations are characterized by the following conditions, with details shown in Appendix B.4. First, we have the planner's versions of the intra- and intertemporal wedge conditions:

$$-\frac{v^{i}_{c,t}}{v^{i}_{h,t}}=\frac{1}{G^{i}_{i,t}F^{ii}_{l,t}}=\frac{1}{G^{i}_{\hat{j},t}F^{ji}_{l,t}}\qquad\forall_{i}\forall_{j\neq i} \tag{50}$$

$$\frac{v^{i}_{c,t}}{\beta v^{i}_{c,t+1}}=\left((1-\delta)+G^{i}_{i,t+1}F^{ii}_{k,t+1}\right)=\left((1-\delta)+G^{i}_{\hat{j},t+1}F^{ji}_{k,t+1}\right)\qquad\forall_{i}\forall_{j\neq i}. \tag{51}$$

Second, we have the planner's versions of static and dynamic efficiency conditions, i.e. counterparts of (B.30) and (B.34). The static efficiency states that for any imported goods (m,n) in location i we have

$$\frac{G^{i}_{n,t}}{G^{i}_{m,t}}=\frac{G^{n}_{n,t}v^{n}_{c,t}}{G^{m}_{m,t}v^{m}_{c,t}}\qquad\forall_{i}\forall_{m,n\neq i}$$

The dynamic efficiency of the Ramsey planner reads as:

$$\frac{G^{i}_{j,t}}{G^{i}_{j,t+1}}\left((1-\delta)+G^{i}_{i,t+1}F^{ii}_{k,t+1}\right)=\left(\frac{G^{j}_{j,t}v^{j}_{c,t}}{G^{j}_{j,t+1}\beta v^{j}_{c,t+1}}\right)\qquad\forall_{j}$$

Finally, we have a condition that describes how the planner allocates intangible investment across countries:

$$\frac{F^{ii}_{l,t}}{F^{ii}_{z,t}H^{i}_{i,t}}=\left(1+\sum_{j\neq i}\frac{v^{j}_{c,t}G^{j}_{\hat{i},t}F^{ij}_{z,t}}{v^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}}\right)+\sum_{j\neq i}\left(\frac{H^{j}_{i,t}}{H^{i}_{i,t}}\left(\frac{G^{i}_{\hat{j},t}F^{ji}_{z,t}}{G^{i}_{i,t}F^{ii}_{z,t}}+\frac{v^{j}_{c,t}G^{j}_{j,t}F^{jj}_{z,t}}{v^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}}\right)+\sum_{k\neq i,j}\frac{H^{k}_{i,t}}{H^{i}_{i,t}}\frac{v^{j}_{c,t}G^{j}_{\hat{k},t}F^{kj}_{z,t}}{v^{c}_{i,t}G^{i}_{i,t}F^{ii}_{z,t}}\right). \tag{52}$$

Notice that:

$$v^i_{c,t} = u^i_{c,t}\left[1+\varphi^i\left(1+\sigma^{icc}_t-\sigma^{ich}_t\right)\right]$$
$$v^i_{h,t} = u^i_{h,t}\left[1+\varphi^i\left(1+\sigma^{ihh}_t-\sigma^{ihc}_t\right)\right]$$

where σ^{icc}_t and σ^{ihh}_t are the own elasticities of the marginal utilities of consumption and labor, and σ^{ich}_t and σ^{ihc}_t are the cross elasticities. In general, these elasticities depend on allocations and vary across countries and over time, which means that the ratios of the derivatives of v^i do not necessarily coincide with marginal rates of substitution. Importantly, if countries are asymmetric, it is generally the case that

$$\frac{-v^i_{c,t}/v^i_{h,t}}{-u^i_{c,t}/u^i_{h,t}} \neq \frac{-v^j_{c,t}/v^j_{h,t}}{-u^j_{c,t}/u^j_{h,t}} \quad \text{and} \quad \frac{-v^i_{c,t}/v^i_{c,t+1}}{-u^i_{c,t}/u^i_{c,t+1}} \neq \frac{-v^j_{c,t}/v^j_{c,t+1}}{-u^j_{c,t}/u^j_{c,t+1}} \quad \text{and} \quad \frac{v^j_{c,t}}{v^i_{c,t}} \neq \frac{u^j_{c,t}}{u^i_{c,t}}. \tag{53}$$

Beyond that, there is little we can say without putting further structure on the problem. Implementations, given the tax instruments $\boldsymbol{\mathcal{T}}$, may have intra- and/or intertemporal wedges, i.e., they may not satisfy one or more of (50) and (51) or may violate dynamic efficiency (15). As we have discussed above, in the profit shifting scenario, the planner can achieve an efficient allocation of intangible investment using corporate income taxes. Unless countries are symmetric, however, this requires non-zero corporate income taxes that differ across countries, which leads to intertemporal wedges and violates dynamic efficiency. The tension between these two margins implies that in general, the planner must make sacrifices on both. In the free transfer and transfer pricing scenarios, where it is altogether impossible to achieve an efficient allocation of intangible investment, it is not clear that eliminating intra- and intertemporal wedges is the second-best option.

3.6.1 Adding taxes on tangible capital income

One way to make further progress is to equip the planner with an additional policy instrument: a tax on tangible capital income, τ^k_{it}. In this case, the planner can implement Ramsey allocations in the profit shifting scenario, but not in the free transfer or transfer pricing. Only two equilibrium conditions are affected by capital income taxes. The intertemporal wedge condition becomes

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\left[1+(1-\tau^k_{it+1})(1-\tau^p_{it+1})(G^i_{i,t}F^{ii}_{k,t+1}-\delta)\right], \tag{54}$$

and the dynamic efficiency condition now says that

$$\frac{\left(1+\tau^m_{jit+1}\right)}{\left(1-\tau^x_{jit+1}\right)}\frac{\left(1-\tau^x_{jit}\right)}{\left(1+\tau^m_{jit}\right)}\frac{G^i_{j,t}}{G^i_{j,t+1}}\left[1+\left(1-\tau^k_{it+1}\right)\left(1-\tau^p_{it+1}\right)\left(G^i_{i,t+1}F^{ii}_{k,t+1}-\delta\right)\right]. \tag{55}$$

is the same across countries.

To support a Ramsey allocation in the profit shifting scenario, first choose the trade taxes to be offsetting, i.e. $\tau^m_{ijt}=-\tau^x_{ijt}$. This ensures static efficiency, i.e. that (34) coincides with (14). It also implies that the term $\frac{\left(1+\tau^m_{jit+1}\right)}{\left(1-\tau^x_{jit+1}\right)}\frac{\left(1-\tau^x_{jit}\right)}{\left(1+\tau^m_{jit}\right)}$ in (55) is equal to one. Then, as we show in the Appendix following arguments by CNT, that one can set τ^x_{ijt} so that the balance of payments holds.

Now, compare (32) with (50), (54) with (51), and (55) with (15). It is clear that given a set of corporate income taxes, the taxes on consumption, labor, and capital income that eliminate intratemporal wedges, eliminate intertemporal wedges, and ensure dynamic production efficiency in Ramsey equilibrium are given by

$$\frac{-u^i_{c,t}/u^i_{h,t}}{-v^i_{c,t}/v^i_{h,t}}=\frac{\left(1+\tau^c_{it}\right)}{\left(1-\tau^h_{it}\right)};\qquad \frac{-u^i_{c,t}/\beta u^i_{c,t+1}}{-v^i_{c,t}/\beta v^i_{c,t+1}}=\frac{\left(1+\tau^c_{it}\right)}{\left(1+\tau^c_{it+1}\right)};\qquad \tau^k_{it}=-\tau^p_{it}. \tag{56}$$

We are left to determine how to set corporate income taxes to achieve a Ramsey allocation of intangible investment. For the free transfer case (37) using result in Lemma 1 under static efficiency we have:

$$\frac{F^{ii}_{l,t}}{H^i_{i,t}F^{ii}_{z,t}}=1+\sum_{j\neq i}\left(\frac{u^j_{c,t}G^j_{\hat{i},t}F^{ij}_{z,t}}{u^i_{c,t}G^i_{i,t}F^{ii}_{z,t}}\right)\left(\frac{1-\tau^p_{jt}}{1-\tau^p_{it}}\right) \tag{57}$$

Now comparing (57) with (52) and following the argument laid out in Proposition 1 we conclude the Ramsey planner can not implement an efficient allocation in general. For the transfer pricing using Lemma 1 in (39) we get:

$$\frac{F^{ii}_{l,t}}{H^i_{i,t}F^{ii}_{z,t}}=1+\sum_{j\neq i}\left(\frac{u^j_{c,t}G^j_{\hat{i},t}F^{ij}_{z,t}}{u^i_{c,t}G^i_{i,t}F^{ii}_{z,t}}\right) \tag{58}$$

Now comparing (58) with (52) it's immediate that the Ramsey planner does not have any instruments to correct the externality from the spillover. Thus an efficient can not be implemented in this case. Finally, for the profit shifting case we have, using Lemma 1, that (40) becomes:

$$\frac{F^{ii}_{l,t}}{H^{i}_{i,t}F^{ii}_{z,t}} = \left[1 + \sum_{j\neq i}\left(\frac{u^{j}_{c,t}G^{j}_{\hat{i},t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}}\right)\right]\Omega(\tau^{p}_{it}). \tag{59}$$

where $\Omega(\tau^p_{it})$ is the gain from profit shifting per unit of intangible capital as defined in (40). Now, compare (59) with (52). Using the properties of $\Omega(\tau^p_{it})$ summarized in Lemma 2 and invoking mean value theorem it's immediate that there exists $\hat{\tau}^p_{it}$ that implements Ramsey allocation of intangible capital. This together with other taxes as specified in (56) implements Ramsey allocation as a competitive equilibrium. We summarize this discussion in the following proposition.[7] The remark that follows provides some additional, and we think quite interesting, context about how the Ramsey problem in our environment departs from the classic Chamley-Judd result that holds in CNT.

Proposition 3 (Ramsey allocations with tangible capital income taxes) *A Ramsey planner equipped with tangible capital income taxes in addition to the instruments $\boldsymbol{\mathcal{T}}$ specified in section 3.3.1 can implement an efficient allocation in profit shifting case, but not in free transfer and transfer pricing cases.*

Remark 2 *It is important to note that the tax system described above that supports a Ramsey allocation in the profit shifting scenario has non-zero taxes on capital income. As stated in* (56)*, tangible capital income taxes should be chosen to offset corporate income taxes, and* (59) *says that as long as countries marginal utilities differ, corporate income taxes should be non-zero and differ across countries. Hence, tangible capital income taxes are non-zero as well. This is in sharp contrast to CNT, where the classic Chamley-Judd result holds where zero taxes on capital income are optimal.*

Remark 3 *It is also important to note that, as is widely known in the literature on Ramsey taxation, there are other sets of instruments that the planner could use to implement an efficient allocation. In our context, the most obvious is an intangible investment subsidy, which the planner could use to correct the spillover externality regardless of the presence of transfer pricing or profit shifting. In fact, many countries do subsidize research and development and other forms of intangible investment. In this paper, we focus on corporate taxation because this is where the current efforts at cooperation between international policymakers are; there is widespread discussion among policymakers about corporate tax harmonization and shutting down profit shifting, but there is little, if any, discussion about coordinating R&D subsidies.*

[7]The proof containing additional details is relegated to the Appendix C.

Remark 4 *Note that we are not restricting the planner's ability to set date-zero taxes, see the definition of $\mathcal{T}$. Thus, by setting the initial taxes τ^c_{i0} or τ^k_{i0} the Ramsey planner can effectively make the implementability condition non-binding. This implies $v^i_{j,t} = u^i_{j,t}$ for $j = c, h$ and the planner can implement an efficient allocation as defined in Definition 1. Importantly, if we restrict planner's ability to set initial taxes and therefore make the implementability constraint binding our Proposition 3 still holds. In that case, we generically have $v^i_{j,t} \neq u^i_{j,t}$ for $j = c, h$ and the Ramsey allocation is constrained efficient.*

4 Quantitative analysis

We now turn to our quantitative analysis. First, we describe the main differences between our quantitative model and the theoretical model described in the previous section. Second, we discuss our calibration strategy. Third, we study the optimal policies of cooperative global Ramsey planners who choose corporate income taxes in all countries to maximize worldwide welfare.

4.1 Overview of the quantitative model

Our quantitative model builds closely on DHS. Each region i (the term "region" better describes the way we take the model to the data than "country") has a continuum of firms that differ in productivity, produce differentiated varieties of intermediate goods, and compete monopolistically as in Chaney (2008). Exporting, multinational production, and profit shifting now require firms to pay fixed costs, and so only a subset of firms choose to engage in these activities in equilibrium. Final-goods producers in each region aggregate the set of available varieties into a single nontradable final good. Households and the government are modeled in the same way as in the theory section. Below, we briefly describe the main differences between the quantitative model and the theory, suppressing time subscripts where possible for the sake of brevity. See Appendix D for a full description of the quantitative model.

There are two significant differences between the quantitative model in this paper and DHS. The first is the international spillover in intangible investment, which creates an externality that is not present in DHS and significantly complicates the computation of the model's equiliibrium. The second is the addition of tangible capital, which changes the effects of corporate income taxes on intangible investment. In DHS, an increase in domestic corporate income taxes has no effect on non-MNEs' intangible investment because this investment is tax deductable, and actually increases MNEs' intangible investment by reducing foreign subsidiaries' relative tax rates. The addition of tangible capital makes these effects more complicated. Tangible capital costs are not fully tax deductable, which means that tangible investment is decreasing in the domestic corporate income tax rate. All else equal, a lower tangible capital stock reduces the desired level of intangible investment. Consequently, the overall effect of corporate income taxes on MNEs' intangible investment ambiguous as shown in the theory section above, and corporate income taxes now unambiguously reduce non-MNEs' intangible investment.

Demand for intermediates. The final good in each region is a constant-elasticity-of-substitution aggregate of all of the varieties available in that region:

$$q_{it} = \left[\sum_{j=1}^{I} \int_{\Omega_{jit}} q_{jit}(\omega)^{\frac{\rho-1}{\rho}} d\omega \right]^{\frac{\rho}{\rho-1}}, \tag{60}$$

where ω indexes varieties, Ω_{jit} is the set of varieties from j available in i, and ρ is the elasticity of substitution between varieties. This implies a standard CES demand function,

$$p_{jit}(\omega) = p_{it} q_{it}^{\frac{1}{\rho}} q_{jit}(\omega)^{-\frac{1}{\rho}}, \tag{61}$$

where the aggregate price index p_{it} is given by the usual ideal price index formula.

Production technology. A firm from region i with productivity $a(\omega)$ produces output in region j according to the constant-returns-to-scale technology

$$y_{ijt}(\omega) = \sigma_{ij} A_j a(\omega) \left(n_j z_{it}(\omega)\right)^{\phi} k_{ijt}(\omega)^{\alpha} \ell_{ijt}(\omega)^{1-\phi-\alpha} \tag{62}$$

where A_j is region j's aggregate total factor productivity, ϕ is the share of intangible capital, α is the share of tangible capital, and σ_{ij} captures barriers to FDI as in McGrattan and Waddle (2020). We assume that $\sigma_{ij} \in [0, 1]$ and that $\sigma_{ii} = 1$. We assume constant returns to scale here, rather than decreasing returns as in the theory section, because we have moved to a monopolistic-competition framework.

Research & development. As in the theory section, firms hire workers in their home countries to produce intangible capital. We model spillovers from other countries' R&D efforts in a slightly different fashion. We assume now that the productivity of domestic R&D labor is augmented by the total amount of intangible capital deployed by foreign MNEs' local affiliates. This differs from the setup in the theory section in two ways. First, only foreign MNEs with local affiliates create spillovers; the R&D done by foreign firms without operations in region i has no effect on that region's R&D productivity. Second, spillovers are created by foreign intangible capital, not the workers employed abroad to produce it. This is without loss of generality, but makes programming the model slightly simpler. The intangible capital production technology is specified as follow:

$$z_{it}(\omega) = H^i\left(\ell^z_{it}(\omega), \sum_{j\neq i}\int_{\Omega_{jit}} z_{jt}(\omega')\ \mathrm{d}\omega'\right) := A_i \ell^z_{it}(\omega)\left[\frac{\sum_{j\neq i}\int_{\Omega_{jit}} z_{jt}(\omega')\ \mathrm{d}\omega'}{\sum_{j\neq i}\int_{\bar{\Omega}_{ji}} \bar{z}_j(\omega')\ \mathrm{d}\omega'}\right]^{\upsilon}, \tag{63}$$

where ω indexes the domestic firm, $\ell^z_{it}(\omega)$ is the firm's R&D labor, Ω_{jit} is the mass of firms from region $j \neq i$ with affiliates in region i, and ω' indexes these foreign firms. The parameter υ controls the degree of spillovers. When $\upsilon = 0$ there are no spillovers as in DHS. The higher υ, the greater the spillovers. We normalize the spillover term in square brackets to one in the benchmark equilibrium calibrated under the current tax code. This facilitates calibration by ensuring that the benchmark equilibrium is the same with and without spillovers. Bars denote equilibrium objects in the benchmark. It is important to point out that the spillover effect greatly increases the computation burden relative to the model in DHS. The R&D technology (63) depicts a fixed-point problem that needs to be solved each time one tries a new candidate price vector in trying to satisfy the equilibrium conditions. Holding prices fixed, one needs to iterate on firms' R&D decisions until the term in brackets above converges.

Trade and foreign direct investment. Firms can sell their products to foreign countries through exporting and/or multinational production. Firms based in region i must pay a fixed cost κ^X_i for each country to which they export, and a fixed cost κ^F_i for each country in which they operate a foreign affiliate. Fixed costs are denominated in units of the home country's labor. Each unit of goods shipped abroad incurs an iceberg transportation cost ξ_{ij}. As in McGrattan and Waddle (2020) and Garetto et al. (2019), exports and locally-produced products are treated as distinct varieties; firms can simultaneously export to, and produce locally for, the same foreign country. We use $J^X_{it}(\omega) \subseteq I \setminus \{i\}$ to denote a firm's export destinations and $J^F_{it}(\omega) \subseteq I \setminus \{i\}$ to denote its portfolio of foreign affiliates. The firm's resource constraints can then be written as follows:

$$y_{iit}(\omega) = q_{iit}(\omega) + \sum_{j\in J^X_{it}(\omega)} \xi_{ij} q_{ijt}(\omega) \tag{64}$$

$$y_{ijt} = \hat{q}_{ijt}(\omega),\ j \in J^F_{it}(\omega) \tag{65}$$

where $q_{ijt}(\omega)$ and $\hat{q}_{ijt}(\omega)$ represent exports and locally-produced products, respectively.

Transfer pricing. Transfer pricing works in the same way as in the theory section. An affiliate in region j of a firm based in region i with intangible capital z pays a licensing fee $\phi p_{ijt}(\omega)y_{ijt}(\omega)/z_{it}(\omega) \equiv \vartheta_{ijt}(\omega)z_{it}(\omega)$, where $\vartheta_{ijt}(\omega)$ is the the firm's marginal revenue product of intangible capital in j. The total amount of licensing fees across the firm's portfolio of foreign affiliates is $\nu_{it}(\omega)z_{it}(\omega) \equiv \sum_{j\in J^F_{it}(\omega)\cup\{i\}} \vartheta_{ijt}(\omega)z_{it}(\omega)$. Note that this includes the licensing fee the parent corporation "pays" itself for the use of its own intangible capital.

Profit shifting. Profit shifting works in basically the same way as in the theory section, except that we allow firms to shift profits to the productive region with the lowest tax rate, denoted by LT, as well as an unproductive, zero-population tax haven, denoted by TH. Suppose a firm from region i sells a fraction $\lambda_{i,LT,t}(\omega)$ of its intangible capital to the low-tax region and a fraction $\lambda_{i,TH,t}(\omega)$ to the tax haven. Its affiliate in the low-tax region receives licensing fees of $\lambda_{i,LT,t}(\omega)\sum_{j\in J^F_{it}(\omega)\cup\{i\}\setminus\{LT\}} \vartheta_{ijt}(\omega)z_{it}(\omega)$ and its affiliate in the tax haven receives $\lambda_{i,TH,t}(\omega)\sum_{j\in J^F_{it}(\omega)\cup\{i\}} \vartheta_{ijt}(\omega)z_{it}(\omega)$. The firm's domestic parent corporation receives the remaining fees, $[1-\lambda_{i,LT,t}(\omega)-\lambda_{i,TH}(\omega)]\sum_{j\in J^F_{it}(\omega)} \vartheta_{ijt}z_{it}(\omega)$. The total variable profit shifting cost is $\mathcal{C}_{i,LT}\left(\lambda_{i,LT,t}(\omega)\right)+\mathcal{C}_{i,TH}(\lambda_{i,TH,i}(\omega))$, where $\mathcal{C}_{ij}(\lambda) = \left[\lambda + (1-\lambda)\log(1-\lambda)\right]\psi_{ij}$. Setting up an affiliate in the tax haven also requires a fixed cost κ^{TH}_i. See DHS for a detailed treatment of our quantitative theory of profit shifting.

Tax treatment of capital expenditures. Like McGrattan and Waddle (2020), we follow standard accounting practices in modeling the tax treatment of firms' expenses associated with intangible and tangible capital. As in the theory section, R&D expenditures and depreciation of tangible capital are tax-deductible, but other tangible capital costs are not deductable. As before, this makes corporate income taxes have an ambiguous effect on MNEs' intangible investment. But most firms in the quantitative model are not MNEs, and these accounting rules imply that corporate income taxes unambiguously reduce non-MNEs' intangible investment.

4.2 Calibration

We follow the same calibration procedure as in DHS, so we describe this procedure only briefly here. We first partition to world into five regions: North America (NA), Europe (EU), the low-tax region (LT), the rest of the world (RW), and the tax haven (TH). The low-tax region includes includes Belgium, Ireland, Hong Kong, the Netherlands, Singapore, and Switzerland. The tax-haven region includes Luxembourg, small European countries and territories like Cyprus, Malta, and the Isle of Man, and a number of Caribbean countries. Data for each region are obtained by aggregating or averaging country-level data as appropriate. We then choose parameter values for each region so that, under the current international tax regime, the model's equilibrium matches data on production, international trade, foreign direct investment, and profit shifting. The key parameters and the data moments that discipline them are listed below; Table 1 reports the values of these parameters and moments.

- population (n_i), which is taken directly from the data;
- aggregate TFP (A_i), which is identified by real GDP per capita;
- corporate tax rates (τ_i^p), which are taken from Tørsløv et al. (2022);
- labor income tax rates (τ_i^l) are set to 22.4% as in McGrattan and Waddle (2020);
- the intangible share (γ), which is identified by the fraction of income generated by foreign MNEs' local affiliates that accrues to intangible capital;
- trade costs (κ_i^F and ξ_{ij}), which are identified by export participation and bilateral trade flows;
- FDI costs (κ_i^F and σ_i), which are identified by the fraction of firms that engage in multinational production and the fraction of each region's gross value added that is produced by local affiliates of foreign MNEs;
- and profit-shifting costs ($\kappa_{i,TH}$, $\psi_{i,LT}$, and $\psi_{i,TH}$), which are identified by the number of firms in each high-tax region with affiliates in the tax haven region, the aggregate amount of profits shifted from each high-tax region to the low-tax region, and the aggregate amount of profits shifted from high-tax regions to the tax-haven region.

We do not attempt to calibrate the spillover parameter, υ, by matching moments, as identifying it from macro data is challenging. Our specification of the R&D technology (63) normalizes the spillover effect to one in the calibrated benchmark equilibrium, which ensures that this equilibrium matches the data regardless of the strength of the spillover effect. Our focus is not on how spillovers affect the world economy under the current system, but rather on how they shape the optimal corporate income tax system, In our counterfactual policy experiments, we consider three values of υ: zero (no spillovers, 0.1 (weak spillovers), and 0.2 (strong spillovers). The weak spillover calibration implies a similar effect of FDI on domestic firms' productivity as estimated by Javorcik (2004).

4.3 Experiments

Having laid out the model and its calibration, we turn now to our quantitative optimal policy experiments. We restrict attention here to long-run steady states; we do not study the transition dynamics that would follow policy changes. We allow the planner to choose corporate income taxes τ_i^p in each country, letting the taxes on labor income adjust to ensure that the government's budget in each region clears. We first consider a cooperative global Ramsey planner that seeks to maximize a weighted sum of the welfare of consumers in all regions. We then study a constrained global planner that seeks to maximize the same objective, but is restricted to Pareto improvements relative to the benchmark equilibrium. We solve each planner's problem six times: (a) without spillovers, holding fixed the tax haven't tax rate τ_{TH}^p; (b) with weak spillovers, holding τ_{TH}^p fixed; (c) with strong spillovers, holding τ_{TH}^p fixed; (d) without spillovers, allowing the planner to choose τ_{TH}^p as well as the other regions' corporate tax rates; (e) with weak spillovers, allowing the planner to choose τ_{TH}^p; and (f) with strong spillovers, allowing the planner to choose τ_{TH}^p.

It is important to state explicitly that we focus our experiments on optimal policies in long-run stationary equilibria; we do not model the transition dynamics from the status quo to the new stationary equilibria associated with these policies. As we will show, these policies would entail cutting corporate income taxes, which would stimulate investment in both tangible and intangible capital, and the dynamics of these investments could have significant implications for welfare. However, modeling the dynamics of multinational firms' decisions about intangible investment and profit shifting would be an extremely challenging computational task, especially in the presents of spillovers. Accomplishing this task would represent a major contribution to the literature, and we leave it for future research.

4.3.1 Unconstrained planners

The unconstrained global planner maximizes worldwide welfare by choosing a vector of corporate income tax rates $\boldsymbol{\tau} = (\tau_1^p, \ldots, \tau_I^p)$. This problem is given formally by

$$\max_{\boldsymbol{\tau}} \left\{ \sum_{i \in I} \omega_i u^i \left(c_i^*(\boldsymbol{\tau}), h_i^*(\boldsymbol{\tau}) \right) \right\}, \tag{66}$$

where $c_i^*(\boldsymbol{\tau})$ and $h_i^*(\boldsymbol{\tau})$ denote region i's consumption and labor allocations in the steady-state equilibrium associated with the policy vector $\boldsymbol{\tau}$, and ω_i is the Pareto weight on region i. We assume that these weights are proportional to each region's real GDP in the benchmark calibrated equilibrium.[8] Note that in versions (c) and (d) of this experiment, the choice vector $\boldsymbol{\tau}$ also includes the tax haven's tax rate, τ^p_{TH}. The optimal corporate tax rates and welfare implications are shown in Table 2. Table 3 shows additional results about the effects of these policy changes on public finances and macro variables in the case where the tax haven's tax rate is fixed, and Table 4 shows additional results in the case where the planner chooses the tax haven's tax rate.

There are four main takeaways. First, and most important, it is optimal to shut down profit shifting in the absence of spillovers, but profit shifting should remain active when spillovers are present, just as the theory shows. In the no-spillover calibration, the unconstrained planner sets high enough tax rates in the low tax region and, if possible, the tax haven to completely eliminate profit shifting. In the calibrations with spillovers, on the other hand, the planner reduces profit shifting but still allows it to operate to both the low tax region and the tax haven, even then she can choose the latter's tax rate. The optimal amount of profit shifting is higher in the strong-spillover calibration than in the weak-spillover calibration, just as one would expect. In both cases, the optimal amount of profit shifting is highest in the rest of the world and lowest in Europe.

Second, lowering corporate tax rates increases intangible investment, which means that the intertemporal distortion channel is stronger than the profit shifting channel. Recall that the effect of corporate taxes on intangible investment is ambiguous from equation (46). On the one hand, higher corporate taxes increase the return to profit shifting, which also increases the return to intangible investment. On the other hand, they reduce tangible investment, which also reduces the returns to intangible investment. The results show that tangible investment reacts strongly to corporate tax changes, and that intangible investment moves in the same direction in equilibrium. Thus, in order to stimulate intangible investment, the planner needs to cut corporate income taxes.

Third, spillovers allow the planner to achieve larger welfare gains by cutting high-tax regions' corporate taxes more aggressively. The planner cuts corporate taxes in all three high-tax regions regardless of the strength of the spillover effect, but the optimal tax cuts are smallest in the no-spillover calibration and largest in the strong-spillover calibration.

Fourth, the unconstrained planner would always benefit the three high tax regions at the expense of the low-tax region. This is because the planner puts more weight on the high tax regions as they have larger economies, but also because it would be optimal to reduce profit

[8] This is similar to the standard approach in which Pareto weights are related to country wealth. Alternatively, one could assume the planner's objective is weighted by population, which would put much more weight on the rest of the world and incentivize the planner to redistribute resources towards this poorer region.

shifting (even in the presence of spillovers). This requires the planner to raise the low-tax region's corporate tax rate, which discourages its firms from investing in intangible capital. In the next set of experiments, we restrict the planner to Pareto-improving tax reforms.

4.4 Constrained global planner

The constrained global planner has the same objective function as the unconstrained global planner, but is constrained to choose a Pareto-improving policy vector that leaves no region worse off relative to the status-quo equilibrium. This constraint can be written as

$$u_i\left(c_i^*(\boldsymbol{\tau}), h_i^*(\boldsymbol{\tau})\right) \geq u_i\left(c_i^*(\bar{\boldsymbol{\tau}}), h_i^*(\bar{\boldsymbol{\tau}})\right),\ i \in I, \tag{67}$$

where $\bar{\boldsymbol{\tau}}$ is the status-quo policy vector, and $c_i^*(\bar{\boldsymbol{\tau}})$ and $h_j^*(\bar{\boldsymbol{\tau}})$ are the associated allocations in the status-quo stationary equilibrium. Note that this problem can be formulated equivalently in the language of the relaxed Ramsey problem (3) with $\mathcal{W}_i > 0$ chosen appropriately. The results of this set of experiments are shown in Tables 5–7.

There are two takeaways from this second set of experiments. First, it is never optimal for the constrained planner to shut down profit shifting, regardless of the presence of spillovers. As discussed above, reducing the amount of profits that are shifted from high-tax regions to the low-tax region reduces the latter's welfare, which the constrained planner cannot do. Therefore, the constrained planner always sets corporate taxes in the low-tax region below the high-tax regions' tax rates. In all six versions of this experiment, the amount of profits shifted to the low-tax region actually rise. Perhaps more interestingly, the constrained planner never shuts down profit shifting to the tax haven, even in the absence of spillovers. In experiments (d)–(f), where the contrained planner can choose the tax haven's tax rate, she sets it well below the high-tax regions' tax rates.

Second, the constrained planner would choose smaller corporate tax cuts, especially in Europe, and would generally achieve more modest welfare gains. North America and the rest of the world would always gain less than under the unconstrained planner's optimal policy. With no spillovers or weak spillovers, Europe would also gain less, but with strong spillovers Europe would actually gain more. The low-tax region would always be indifferent between the status quo and the constrained planner's optimal policy.

5 Conclusion

We study optimal corporate income taxation in an environment that emphasizes three closely related features of the modern globalized economy: multinational enterprises, intangible capital, and profit shifting. In our model, MNEs use nonrival intangible capital to produce output in many locations around the world as in McGrattan and Prescott (2009, 2010), and they can use transfer pricing to shift the income generated by this capital to subsidiaries in foreign tax havens as in Dyrda et al. (2022). We assume that there are international spillovers in intangible investment, which creates an externality that implies competitive equilibria without distortionary taxes are inefficient.

In our theoretical analysis, we show that in the absence of profit shifting it is impossible for a Ramsey planner to use corporate income taxes to correct this externality, but in the absence of profit shifting corporate income taxes can be used to implement an efficient allocation of intangible investment. Under profit shifting, higher corporate income taxes increase the return to intangible investment because the costs are tax-deductable, and the planner can use this to her advantage to internalize the spillover externality. However, there is a tension between achieving a statically-efficient allocation of intangible investment and a dynamically-efficient level of tangible investment. If the planner is equipped with taxes on tangible capital income as well, we show that she can implement a Pareto-optimal allocation.

In our quantitative analysis, we study the Ramsey planner's problem in a richer version of our model with asymmetric countries, firm heterogeneity, selection into multinational activity. We find that the optimal policy entails cutting corporate income taxes to boost intangible investment, especially in the presence of a spillover externality, and raising taxes on labor income to restore fiscal balance. Consistent with the theory, it would be optimal to shut down profit shifting completely in the absence of a spillover externality, but profit shifting should be allowed to operate, albeit at a lower level, if such an externality is present. When the planner is constrained to Pareto improvements that leave all regions weakly better off, the optimal policy would actually cause the level of profit shifting to increase relative to the status quo.

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Table 1: Calibration

Statistic or parameter value	North America	Europe	Low-tax	RoW	Tax haven
(a) Assigned parameters and target moments					
Population (NA = 100)	100	92	11	1,323	–
Real GDP (NA = 100)	100	80.78	14.57	297.10	–
Corporate tax rate (%)	22.5	17.3	11.4	17.4	3.3
Labor tax rate (%)	22.4	22.4	22.4	22.4	–
Foreign MNEs' VA share (%)	11.12	19.82	28.73	9.55	–
Total lost profits ($B)	143	216	–	257	–
Lost profits to TH (%)	66.4	44.5	–	71.1	–
Imports from... (% GDP)					
North America	–	1.28	1.77	1.74	–
Europe	1.70	–	12.39	3.78	–
Low tax	0.35	2.98	–	0.59	–
Row	6.15	7.96	6.78	–	–
(b) Calibrated parameter values					
TFP (A_i)	.00	0.90	1.43	0.28	–
Prod. dispersion (η_i)	4.30	4.32	4.87	4.15	–
Utility weight on leisure (ψ_i)	1.46	1.49	1.51	1.47	–
Fixed export cost (κ_i^X)	2.5e-3	5.2e-3	1.5e-3	2.1e-2	–
Variable FDI cost (σ_i)	0.46	0.55	0.52	0.55	–
Fixed FDI cost (κ_i^F)	2.56	2.27	0.65	12.70	–
Cost of shifting profits to LT (ψ_{iLT})	3.73	0.42	–	2.73	–
Cost of shifting profits to TH (ψ_{iTH})	2.46	1.37	–	2.05	–
Fixed FDI cost to TH (κ_i^{TH})	0.13	0.08	–	0.75	–
Variable trade cost from...					
North America	–	3.25	3.45	2.12	–
Europe	1.87	–	1.69	1.35	–
Low tax	2.00	1.59	–	1.58	–
RoW	2.19	2.56	2.96	–	–

Notes: Population and real GDP from World Bank WDI. Corporate tax rate from Tørsløv et al. (2022). Foreign MNEs' VA share from OECD AMNE database. Fractions of firms with foreign affiliates from Compustat. Lost profits from Tørsløv et al. (2022). Imports/GDP from WIOD. Dashes (–) represent "not applicable."

Table 2: Unconstrained planner's problem: optimal corporate tax rates and welfare implications

Variable	North America	Europe	Low tax	Rest of world	Tax haven
(a) No spillovers, τ_{TH} fixed					
Corp. tax rate (%)	18.71	13.64	17.61	17.87	3.30
Chg. in corp tax rate (p.p.)	-3.79	-3.66	6.21	0.47	0.00
Welfare (% chg.)	0.092	0.116	-1.051	0.063	–
(b) Weak spillovers, τ_{TH} fixed					
Corp. tax rate (%)	15.52	9.96	13.76	16.69	3.30
Chg. in corp tax rate (p.p.)	-6.98	-7.34	2.36	-0.71	0.00
Welfare (% chg.)	0.097	0.051	-0.730	0.118	–
(c) Strong spillovers, τ_{TH} fixed					
Corp. tax rate (%)	14.36	5.06	9.40	15.01	3.30
Chg. in corp tax rate (p.p.)	-8.14	-12.24	-2.00	-2.39	0.00
Welfare (% chg.)	0.224	-0.099	-0.462	0.242	–
(d) No spillovers, planner chooses τ_{TH}					
Corp. tax rate (%)	20.21	15.82	19.78	20.55	20.50
Chg. in corp tax rate (p.p.)	-2.29	-1.48	8.38	3.15	17.20
Welfare (% chg.)	0.142	0.224	-1.179	0.218	–
(e) Weak Spillovers, planner chooses τ_{TH}					
Corp. tax rate (%)	18.60	12.04	17.02	18.88	18.57
Chg. in corp tax rate (p.p.)	-3.90	-5.26	5.62	1.48	15.27
Welfare (% chg.)	0.161	0.123	-0.925	0.234	–
(f) Strong Spillovers, planner chooses τ_{TH}					
Corp. tax rate (%)	15.10	8.54	10.29	16.89	15.20
Chg. in corp tax rate (p.p.)	-7.40	-8.76	-1.11	-0.51	11.90
Welfare (% chg.)	0.190	0.067	-0.468	0.303	–

Notes: Table shows corporate tax rates and welfare under unconstrained planner's optimal policy. Panel (a): $\upsilon = 0$, planner cannot choose τ_{TH}. Panel (b): $\upsilon = 0.1$, planner cannot choose τ_{TH}. Panel (c): $\upsilon = 0.2$, planner cannot choose τ_{TH}. Panel (d): $\upsilon = 0$, planner can choose τ_{TH}. Panel (e): $\upsilon = 0.1$, planner can choose τ_{TH}. Panel (f): $\upsilon = 0.2$, planner can choose τ_{TH}.

Table 3: Unconstrained planner's problem: public-finance and macro implications

Variable	North America	Europe	Low tax	Rest of world
(a) No spillovers, τ_{TH} fixed				
Corporate tax revenue (% chg.)	-17.54	-20.50	27.46	3.51
Lost profits (benchmark = 1)	0.54	0.31	0.00	0.83
Labor income tax rate (p.p. chg.)	1.88	1.61	-1.36	-0.30
Tangible capital (% chg.)	2.07	1.86	-3.36	-0.25
Domestic intangible capital (% chg.)	2.50	2.41	-2.98	-0.23
Foreign MNE intangible capital (% chg.)	0.38	0.52	-0.30	1.12
Employment (% chg.)	-1.11	-0.95	0.97	0.15
Real GDP (% chg.)	-0.40	-0.33	-0.58	-0.11
Consumption (% chg.)	-0.63	-0.51	-0.40	0.16
(b) Weak spillovers, τ_{TH} fixed				
Corporate tax revenue (% chg.)	-32.60	-42.73	0.36	-4.06
Lost profits (benchmark = 1)	0.40	0.16	0.18	0.84
Labor income tax rate (p.p. chg.)	3.49	3.36	0.07	0.33
Tangible capital (% chg.)	3.61	3.46	-1.41	0.34
Domestic intangible capital (% chg.)	5.15	5.37	0.04	1.24
Foreign MNE intangible capital (% chg.)	2.22	2.33	1.43	3.34
Employment (% chg.)	-2.04	-1.93	0.12	-0.20
Real GDP (% chg.)	-0.62	-0.59	-0.61	-0.15
Consumption (% chg.)	-1.22	-1.22	-0.65	-0.01
(c) Strong spillovers, τ_{TH} fixed				
Corporate tax revenue (% chg.)	-38.11	-71.53	-30.77	-14.50
Lost profits (benchmark = 1)	0.41	0.02	0.35	0.81
Labor income tax rate (p.p. chg.)	4.05	5.62	1.70	1.18
Tangible capital (% chg.)	4.20	5.42	0.66	1.20
Domestic intangible capital (% chg.)	7.14	9.55	3.77	3.60
Foreign MNE intangible capital (% chg.)	4.80	4.82	3.71	6.06
Employment (% chg.)	-2.37	-3.22	-0.84	-0.68
Real GDP (% chg.)	-0.50	-0.89	-0.60	-0.08
Consumption (% chg.)	-1.31	-2.20	-1.02	-0.20

Notes: Table shows macroeconomic effects of unconstrained planner's optimal policy. Panel (a): $\upsilon = 0$, planner cannot choose τ_{TH}. Panel (b): $\upsilon = 0.1$, planner cannot choose τ_{TH}. Panel (c): $\upsilon = 0.2$, planner cannot choose τ_{TH}.

Table 4: Unconstrained planner's problem: public-finance and macro implications. continued

Variable	North America	Europe	Low tax	Rest of world
(d) No spillovers, planner chooses τ_{TH}				
Corporate tax revenue (% chg.)	-8.74	-5.42	45.58	22.68
Lost profits (benchmark = 1)	0.00	0.00	0.00	0.00
Labor income tax rate (p.p. chg.)	0.92	0.42	-2.31	-1.93
Tangible capital (% chg.)	1.52	0.96	-4.45	-1.43
Domestic intangible capital (% chg.)	1.26	0.66	-4.73	-2.30
Foreign MNE intangible capital (% chg.)	-1.12	-1.21	-1.84	-0.35
Employment (% chg.)	-0.54	-0.30	1.52	0.99
Real GDP (% chg.)	-0.21	-0.09	-0.50	0.21
Consumption (% chg.)	-0.21	0.02	-0.16	0.87
(e) Small Spillovers, planner chooses τ_{TH}				
Corporate tax revenue (% chg.)	-16.93	-29.49	25.06	11.46
Lost profits (benchmark = 1)	0.03	0.00	0.12	0.08
Labor income tax rate (p.p. chg.)	1.79	2.32	-1.23	-0.98
Tangible capital (% chg.)	2.28	2.64	-3.02	-0.60
Domestic intangible capital (% chg.)	2.73	3.51	-2.56	-0.70
Foreign MNE intangible capital (% chg.)	0.38	0.51	-0.31	1.51
Employment (% chg.)	-1.04	-1.35	0.87	0.48
Real GDP (% chg.)	-0.28	-0.36	-0.51	0.14
Consumption (% chg.)	-0.52	-0.77	-0.34	0.55
(f) Large Spillovers, planner chooses τ_{TH}				
Corporate tax revenue (% chg.)	-34.01	-50.95	-22.86	-1.80
Lost profits (benchmark = 1)	0.10	0.00	0.41	0.30
Labor income tax rate (p.p. chg.)	3.61	3.99	1.28	0.11
Tangible capital (% chg.)	3.93	4.11	0.35	0.39
Domestic intangible capital (% chg.)	6.07	6.81	2.56	1.81
Foreign MNE intangible capital (% chg.)	3.23	3.08	2.48	4.44
Employment (% chg.)	-2.10	-2.29	-0.60	-0.11
Real GDP (% chg.)	-0.49	-0.58	-0.54	0.08
Consumption (% chg.)	-1.17	-1.44	-0.87	0.23

Notes: Table shows macroeconomic effects of unconstrained planner's optimal policy. Panel (d): $\upsilon = 0$, planner can choose τ_{TH}. Panel (e): $\upsilon = 0.1$, planner can choose τ_{TH}. Panel (f): $\upsilon = 0.2$, planner can choose τ_{TH}.

Table 5: Constrained planner's problem: optimal corporate tax rates and welfare implications

Variable	North America	Europe	Low tax	Rest of world	Tax haven
(a) No spillovers, τ_{TH} fixed					
Corp. tax rate (%)	19.32	16.32	10.32	17.46	3.30
Chg. in corp tax rate (p.p.)	-3.18	-0.98	-1.08	0.06	0.00
Welfare (% chg.)	0.051	0.000	0.000	0.007	–
(b) Weak spillovers, τ_{TH} fixed					
Corp. tax rate (%)	18.05	15.48	9.10	17.22	3.30
Chg. in corp tax rate (p.p.)	-4.45	-1.82	-2.30	-0.18	0.00
Welfare (% chg.)	0.075	0.024	0.000	0.038	–
(c) Strong spillovers, τ_{TH} fixed					
Corp. tax rate (%)	14.04	15.64	7.46	14.94	3.30
Chg. in corp tax rate (p.p.)	-8.46	-1.66	-3.94	-2.46	0.00
Welfare (% chg.)	0.095	0.107	0.000	0.100	–
(d) No spillovers, planner chooses τ_{TH}					
Corp. tax rate (%)	18.74	16.43	10.77	18.51	5.77
Chg. in corp tax rate (p.p.)	-3.76	-0.87	-0.63	1.11	2.47
Welfare (% chg.)	0.086	0.024	0.000	0.035	–
(e) Weak Spillovers, planner chooses τ_{TH}					
Corp. tax rate (%)	17.20	15.87	9.62	17.84	5.27
Chg. in corp tax rate (p.p.)	-5.30	-1.43	-1.78	0.44	1.97
Welfare (% chg.)	0.088	0.039	0.000	0.058	–
(f) Strong Spillovers, planner chooses τ_{TH}					
Corp. tax rate (%)	14.88	15.61	7.62	15.57	6.45
Chg. in corp tax rate (p.p.)	-7.62	-1.69	-3.78	-1.83	3.15
Welfare (% chg.)	0.135	0.122	0.000	0.119	–

Notes: Table shows corporate tax rates and welfare under constrained planner's optimal policy. Panel (a): $\upsilon = 0$, planner cannot choose τ_{TH}. Panel (b): $\upsilon = 0.1$, planner cannot choose τ_{TH}. Panel (c): $\upsilon = 0.2$, planner cannot choose τ_{TH}. Panel (d): $\upsilon = 0$, planner can choose τ_{TH}. Panel (e): $\upsilon = 0.1$, planner can choose τ_{TH}. Panel (f): $\upsilon = 0.2$, planner can choose τ_{TH}.

Table 6: Constrained planner's problem: public-finance and macro implications

Variable	North America	Europe	Low tax	Rest of world
(a) No spillovers, τ_{TH} fixed				
Corporate tax revenue (% chg.)	-15.07	-6.25	-12.70	0.48
Lost profits (benchmark = 1)	0.75	1.01	1.11	1.08
Labor income tax rate (p.p. chg.)	1.61	0.49	0.68	-0.04
Tangible capital (% chg.)	1.73	0.48	0.71	-0.07
Domestic intangible capital (% chg.)	2.21	0.85	1.17	0.09
Foreign MNE intangible capital (% chg.)	0.70	0.47	0.45	1.00
Employment (% chg.)	-0.93	-0.27	-0.38	0.02
Real GDP (% chg.)	-0.29	-0.14	-0.16	-0.07
Consumption (% chg.)	-0.56	-0.18	-0.25	0.02
(b) Weak spillovers, τ_{TH} fixed				
Corporate tax revenue (% chg.)	-21.03	-11.41	-22.55	-1.10
Lost profits (benchmark = 1)	0.68	0.99	1.18	1.10
Labor income tax rate (p.p. chg.)	2.25	0.90	1.20	0.09
Tangible capital (% chg.)	2.39	0.89	1.25	0.06
Domestic intangible capital (% chg.)	3.25	1.62	2.16	0.54
Foreign MNE intangible capital (% chg.)	1.26	1.07	1.33	1.82
Employment (% chg.)	-1.30	-0.50	-0.65	-0.05
Real GDP (% chg.)	-0.38	-0.19	-0.24	-0.08
Consumption (% chg.)	-0.77	-0.31	-0.45	0.01
(c) Strong spillovers, τ_{TH} fixed				
Corporate tax revenue (% chg.)	-39.49	-11.10	-36.21	-14.87
Lost profits (benchmark = 1)	0.43	1.16	1.26	0.89
Labor income tax rate (p.p. chg.)	4.23	0.86	1.93	1.25
Tangible capital (% chg.)	4.31	0.79	1.91	1.18
Domestic intangible capital (% chg.)	6.46	2.29	3.85	2.77
Foreign MNE intangible capital (% chg.)	3.30	2.86	2.82	3.72
Employment (% chg.)	-2.48	-0.49	-1.07	-0.70
Real GDP (% chg.)	-0.77	-0.25	-0.41	-0.29
Consumption (% chg.)	-1.50	-0.22	-0.71	-0.36

Notes: Table shows macroeconomic effects of constrained planner's optimal policy. Panel (a): $\upsilon = 0$, planner cannot choose τ_{TH}. Panel (b): $\upsilon = 0.1$, planner cannot choose τ_{TH}. Panel (c): $\upsilon = 0.2$, planner cannot choose τ_{TH}.

Table 7: Constrained planner's problem: public-finance and macro implications, continued

Variable	North America	Europe	Low tax	Rest of world
(d) No spillovers, planner chooses τ_{TH}				
Corporate tax revenue (% chg.)	-17.47	-5.02	-8.81	7.21
Lost profits (benchmark = 1)	0.59	0.87	1.09	1.02
Labor income tax rate (p.p. chg.)	1.88	0.40	0.47	-0.61
Tangible capital (% chg.)	2.06	0.48	0.52	-0.55
Domestic intangible capital (% chg.)	2.47	0.71	0.87	-0.62
Foreign MNE intangible capital (% chg.)	0.40	0.22	0.20	0.89
Employment (% chg.)	-1.11	-0.22	-0.26	0.34
Real GDP (% chg.)	-0.38	-0.16	-0.20	-0.04
Consumption (% chg.)	-0.64	-0.12	-0.17	0.26
(e) Weak Spillovers, planner chooses τ_{TH}				
Corporate tax revenue (% chg.)	-24.74	-8.72	-18.24	2.99
Lost profits (benchmark = 1)	0.52	0.92	1.16	1.02
Labor income tax rate (p.p. chg.)	2.65	0.68	0.98	-0.26
Tangible capital (% chg.)	2.82	0.74	1.01	-0.22
Domestic intangible capital (% chg.)	3.68	1.32	1.79	0.09
Foreign MNE intangible capital (% chg.)	1.16	1.02	0.95	1.61
Employment (% chg.)	-1.56	-0.38	-0.54	0.14
Real GDP (% chg.)	-0.51	-0.21	-0.28	-0.09
Consumption (% chg.)	-0.92	-0.22	-0.36	0.15
(f) Strong Spillovers, planner chooses τ_{TH}				
Corporate tax revenue (% chg.)	-35.46	-10.72	-34.59	-10.75
Lost profits (benchmark = 1)	0.36	1.01	1.28	0.74
Labor income tax rate (p.p. chg.)	3.79	0.83	1.85	0.90
Tangible capital (% chg.)	3.94	0.85	1.85	0.91
Domestic intangible capital (% chg.)	5.78	2.18	3.58	2.20
Foreign MNE intangible capital (% chg.)	2.78	2.62	2.27	3.15
Employment (% chg.)	-2.24	-0.48	-1.02	-0.51
Real GDP (% chg.)	-0.68	-0.23	-0.41	-0.23
Consumption (% chg.)	-1.31	-0.20	-0.68	-0.22

Notes: Table shows macroeconomic effects of constrained planner's optimal policy. Panel (d): $\upsilon = 0$, planner can choose τ_{TH}. Panel (e): $\upsilon = 0.1$, planner can choose τ_{TH}. Panel (f): $\upsilon = 0.2$, planner can choose τ_{TH}.

Appendix

A Pareto Frontier

In this section, we derive the optimality conditions for allocations in our economy for two cases. We first present the benchmark case with no spillovers from non-rival intangible capital. We then add the spillovers and compare the optimality conditions. The Pareto frontier is characterized by a solution to the following problem

$$\max_{\mathcal{A}_t} \sum_{i=1}^{I} \sum_{t=0}^{\infty} N_i \beta^t \left[u^i \left(\frac{c_{it}}{N_i}, \frac{h_{it}}{N_i} \right) \right] \tag{PF-NR}$$

$$\begin{aligned} &\text{subject to} \\ F^{ii}\left(z_{it}, k_{iit}, l_{iit}\right) &= q_{iit} + \sum_{j \neq i} q_{ijt} \quad \forall_i \\ F^{ij}\left(z_{it}, k_{ijt}, l_{ijt}\right) &= \hat{q}_{ijt} \quad \forall_{j \neq i} \forall_i \\ G^i\left(q_{1it}, ..., q_{Iit}, \hat{q}_{1it}, ..., \hat{q}_{Iit}\right) &= c_{it} + g_{it} + x_{it} \quad \forall_i \\ x_{it} &= k_{it+1} - (1-\delta)\, k_{it} \\ H^i\left(l_{it}^z\right) &= z_{it} \quad \forall_i \\ h_{it} &= \sum_j l_{jit} + l_{it}^z \\ k_{it} &= \sum_j k_{jit} \\ k_0 &\quad \text{given} \end{aligned}$$

where the set of constraints can be simplified to

$$c_{it} + g_{it} + \sum_j k_{jit+1} - (1-\delta) \sum_j k_{jit} =$$

$$G^i \left(q_{1it}, ..., F^{ii}\left(H^i\left(l_{it}^z\right), k_{iit}, l_{iit}\right) - \sum_{j \neq i} q_{ijt}, ..., q_{Iit}, F^{1i}\left(H^1\left(l_{1t}^z\right), k_{1it}, l_{1it}\right), ..., F^{Ii}\left(H^I\left(l_{It}^z\right), k_{Iit}, l_{Iit}\right) \right)$$

and then the first-order conditions of the problem (PF-NR) are

$$c_{it} : 0 = \beta^t u^i_{c,t} - \lambda^i_t \tag{A.1}$$

$$l_{jit} : 0 = \beta^t u^i_{h,t} + \lambda^i_t G^i_{\hat{j},t} F^{ji}_{l,t} \tag{A.2}$$

$$l_{iit} : 0 = \beta^t u^i_{h,t} + \lambda^i_t G^i_{i,t} F^{ii}_{l,t} \tag{A.3}$$

$$l^z_{it} : 0 = \beta^t u^i_{h,t} + \lambda^i_t G^i_{i,t} F^{ii}_{z,t} H^i_{l,t} + \sum_{j \neq i} \lambda^j_t G^j_{\hat{i},t} F^{ij}_{z,t} H^i_{l,t} \tag{A.4}$$

$$q_{jit} : 0 = -\lambda^j_t G^j_{j,t} + \lambda^i_t G^i_{j,t} \tag{A.5}$$

$$k_{jit+1} : 0 = (1-\delta)\,\lambda^i_{t+1} - \lambda^i_t + \lambda^i_{t+1} G^i_{\hat{j},t+1} F^{ji}_{k,t+1} \tag{A.6}$$

$$k_{iit+1} : 0 = (1-\delta)\,\lambda^i_{t+1} - \lambda^i_t + \lambda^i_{t+1} G^i_{i,t+1} F^{ii}_{k,t+1} \tag{A.7}$$

and thus we get from (A.1), (A.3) and (A.2) we get the no intratemporal wedge condition

$$-\frac{u^i_{c,t}}{u^i_{h,t}} = \frac{1}{G^i_{i,t} F^{ii}_{l,t}} = \frac{1}{G^i_{\hat{j},t} F^{ji}_{l,t}} \qquad \forall_i \forall_{j \neq i} \tag{A.8}$$

which equalizes the marginal rate of substitution between consumption and labor with the marginal rate of technical substitution between final consumption good and labor input. The second equality implies that

$$\frac{G^i_{i,t}}{G^i_{\hat{j},t}} = \frac{F^{ji}_{l,t}}{F^{ii}_{l,t}} \tag{A.9}$$

Next, from (A.1), (A.7) and (A.6) we get

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \left((1-\delta) + G^i_{i,t+1} F^{ii}_{k,t+1}\right) = \left((1-\delta) + G^i_{\hat{j},t+1} F^{ji}_{k,t+1}\right) \qquad \forall_i \forall_{j \neq i} \tag{A.10}$$

and the second equality implies that

$$\frac{G^i_{i,t+1}}{G^i_{\hat{j},t+1}} = \frac{F^{ji}_{k,t+1}}{F^{ii}_{k,t+1}} \tag{A.11}$$

which put together with (A.9) implies

$$\frac{F^{ji}_{l,t}}{F^{ji}_{k,t+1}} = \frac{F^{ii}_{l,t}}{F^{ii}_{k,t+1}} \tag{A.12}$$

i.e. the marginal rate of technological transformation between tangible capital and labor is equalized across goods produced domestically by all MNEs operating in location i. An implication of that is symmetric tax treatment of these two goods. Now, toward static efficiency, we have that for any imported goods (m, n) in location i we have from (A.5) and (A.1) that

$$\frac{G^i_{n,t}}{G^i_{m,t}} = \frac{G^n_{n,t} u^n_{c,t}}{G^m_{m,t} u^m_{c,t}} \tag{A.13}$$

and note that the right-hand side is independent on i, thus the ratio of marginal rates of technical substitution for any pair of intermediate, imported goods are equated across countries. Now toward dynamic efficiency we have from (A.5) in two consecutive periods and using (A.7) that

$$\frac{G^i_{j,t}}{G^i_{j,t+1}} = \frac{1}{\left((1-\delta) + G^i_{i,t+1} F^{ii}_{k,t+1}\right)} \left(\frac{G^j_{j,t} \lambda^j_t}{G^j_{j,t+1} \lambda^j_{t+1}} \right)$$

implying using (A.1) that

$$\frac{G^i_{j,t}}{G^i_{j,t+1}} \left((1-\delta) + G^i_{i,t+1} F^{ii}_{k,t+1}\right) = \left(\frac{G^j_{j,t} u^j_{c,t}}{G^j_{j,t+1} \beta u^j_{c,t+1}} \right) \tag{A.14}$$

and note that the right-hand side is independent of i. Toward labor-intangible condition, from (A.3), (A.4) and (A.1) we get

$$u^i_{c,t} G^i_{i,t} F^{ii}_{l,t} = H^i_{l,t} \left(u^i_{c,t} G^i_{i,t} F^{ii}_{z,t} + \sum_{j \neq i} u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t} \right)$$

which rearranged yields

$$\frac{F^{ii}_{l,t}}{F^{ii}_{z,t} H^i_{l,t}} = 1 + \sum_{j \neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \tag{A.15}$$

This concludes the characterization of the Pareto frontier for the non-rival case.

A.1 With the externality

Suppose now that

$$z_{it} = H^i(\ell^z_1, \ldots, \ell^z_I) = H^i\left(\ell^z_i, \sum_{j \neq i} \ell^z_j\right)$$

The notation H^i_k is shorthand for $\partial H^i(\cdot)/\partial \ell^z_{kt}$. Then the FOC for intantible labor becomes

$$l^z_{it}: 0 = \beta^t u^i_{h,t} + \lambda^i_t \left[G^i_{i,t} F^{ii}_{z,t} H^i_{i,t} + \sum_{k \neq i} G^i_{\hat{k},t} F^{ki}_{zt} H^k_{i,t} \right] + \sum_{j \neq i} \lambda^j_t \left[G^j_{\hat{i},t} F^{ij}_{z,t} H^i_{i,t} + G^j_{j,t} F^{jj}_{z,t} H^j_{i,t} + \sum_{k \neq j} G^j_{\hat{k},t} F^{kj}_{zt} H^k_{i,t} \right]$$

Note that in the case of two countries, this collapses to

$$l^z_{it}: 0 = \beta^t u^i_{h,t} + \lambda^i_t \left[G^i_{i,t} F^{ii}_{z,t} H^i_{i,t} + G^i_{\hat{j},t} F^{ji}_{zt} H^k_{i,t} \right] + \lambda^j_t \left[G^j_{\hat{i},t} F^{ij}_{z,t} H^i_{i,t} + G^j_{j,t} F^{jj}_{z,t} H^j_{i,t} \right]$$

Combining with the FOCs for consumption and labor as before, we get

$$u^i_{c,t} G^i_{i,t} F^{ii}_{l,t} = H^i_{i,t} \left(u^i_{c,t} G^i_{i,t} F^{ii}_{z,t} + \sum_{j \neq i} u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t} \right) + \sum_{j \neq i} H^j_{i,t} \left(u^i_{c,t} G^i_{\hat{j},t} F^{ji}_{z,t} + u^j_{c,t} G^j_{j,t} F^{jj}_{z,t} \right) + \sum_{j \neq i} \sum_{k \neq j} H^k_{i,t} u^j_{c,t} G^j_{\hat{k},t} F^{kj}_{z,t}$$

Now divide both sides by $u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}$ and then by $H^i_{i,t}$ as before:

$$\frac{F^{ii}_{l,t}}{F^{ii}_{z,t} H^i_{i,t}} = \left(1 + \sum_{j \neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \right) + \sum_{j \neq i} \frac{H^j_{i,t}}{H^i_{i,t}} \left(\frac{G^i_{\hat{j},t} F^{ji}_{z,t}}{G^i_{i,t} F^{ii}_{z,t}} + \frac{u^j_{c,t} G^j_{j,t} F^{jj}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \right) + \sum_{j \neq i} \sum_{k \neq i,j} \frac{H^k_{i,t}}{H^i_{i,t}} \frac{u^j_{c,t} G^j_{\hat{k},t} F^{kj}_{z,t}}{u^c_{i,t} G^i_{i,t} F^i i_{z,t}}$$

B Competitive equilibrium

B.1 Derivation of Balance of Payment Conditions

Start with the budget constraint of the household

$$\sum_{t=0}^{\infty} Q_t \left[(1+\tau_{it}^c)\, p_{it} c_{it} + p_{it} \left(k_{it+1} - (1-\delta + r_{it}) k_{it} \right) - \left(1-\tau_{it}^h\right) w_{it} h_{it} \right] = a_{i0}$$

where $a_{i0} = Q_{-1} b_{i0} + \left(1+r^f\right) f_{i0} + V_{i0}$ and $V_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$. The government budget constraint reads as

$$\sum_{t=0}^{\infty} Q_t p_{it} g_{it} + Q_{-1} b_{i0} = \sum_{t=0}^{\infty} Q_t \left[\tau_{it}^c p_{it} c_{it} + \tau_{it}^h w_{it} h_{it} + \tau_{it}^p \left(\hat{\pi}_{iit} + \sum_{j\neq i} \hat{\pi}_{jit} \right) + \tau_{jit}^m \sum_{j\neq i} p_{jit} q_{jit} + \tau_{ijt}^x \sum_{j\neq i} p_{ijt} q_{ijt} \right].$$

Combining the two budget constraints, we can obtain

$$\begin{aligned} a_{i0} = \sum_{t=0}^{\infty} Q_t p_{it} \Bigg(& c_{it} + g_{it} + [k_{it+1} - k_{it}] - w_{it} h_{it} - R_{it} k_{it} \\ & - \left[\tau_{it}^p \left(\hat{\pi}_{iit} + \sum_{j\neq i} \hat{\pi}_{jit} \right) + \tau_{jit}^m \sum_{j\neq i} p_{jit} q_{jit} + \tau_{ijt}^x \sum_{j\neq i} p_{ijt} q_{ijt} \right] \Bigg) + P_{-1} b_{i0} \end{aligned} \tag{B.1}$$

We now drive the balance of payment conditions for the three scenarios.

Free transfer

In this scenario, we have the headquarter corporate tax base $\hat{\pi}_{iit}$ as:

$$\hat{\pi}_{iit} = p_{iit} q_{iit} + \sum_{j\neq i} \left(1-\tau_{ijt}^x\right) p_{ijt} q_{ijt} - w_{it} l_{iit} - p_{it} \delta k_{iit} - w_{it} l_{iz}^z \tag{B.2}$$

and the final good market clearing condition as

$$G_t^i = c_{it} + \left(k_{it+1} - (1-\delta)\, k_{it}\right) + g_{it}.$$

Plugging these into equation (B.1), we have

$$\sum_{t=0}^{\infty} Q_t p_{it} G_t^i = \sum_{t=0}^{\infty} Q_t \Bigg(\Bigg[p_{iit} q_{iit} + \sum_{j \neq i} p_{ijt} q_{ijt} - w_{it} l_{iit} - p_{it} \delta k_{iit} - w_{it} l_{iz}^z \Bigg] - r_{it} k_{iit}$$

$$+ \sum_{j \neq i} \left(\left(1 - \tau_{jt}^p\right) \hat{\pi}_{ijt} - r_{jt} k_{ijt} \right) \Bigg)$$

$$+ \sum_{t=0}^{\infty} Q_t \Bigg[w_{it} h_{it} + p_{it} \delta k_{it} + R_{it} k_{it} + \tau_{it}^p \sum_{j \neq i} \hat{\pi}_{jit} + \tau_{jit}^m \sum_{j \neq i} p_{jit} q_{jit} \Bigg] + \left(1 + r^f\right) f_{i0}.$$

And the labor market clearing condition

$$h_{it} = \sum_j l_{jit} + l_{it}^z$$

implies

$$\sum_{t=0}^{\infty} Q_t p_{it} G_t^i = \sum_{t=0}^{\infty} Q_t \Bigg(\Bigg[p_{iit} q_{iit} + \sum_{j \neq i} p_{ijt} q_{ijt} - p_{it} \delta k_{iit} \Bigg] - r_{it} k_{iit} + \sum_{j \neq i} \left(\left(1 - \tau_{jt}^p\right) \hat{\pi}_{ijt} - r_{jt} k_{ijt} \right) \Bigg) +$$

$$+ \sum_{t=0}^{\infty} Q_t \Bigg[\sum_{j \neq i} l_{jit} + p_{it} \delta k_{it} + R_{it} k_{it} + \tau_{it}^p \sum_{j \neq i} \hat{\pi}_{jit} + \tau_{jit}^m \sum_{j \neq i} p_{jit} q_{jit} \Bigg] + \left(1 + r^f\right) f_{i0}.$$

Now, using the zero profit condition of the final good producer

$$p_{it} G_t^i (\cdot) = p_{iit} q_{iit} + \sum_{j \neq i} \hat{p}_{jit} \hat{q}_{jit} + \sum_{j \neq i} \left(1 + \tau_{jit}^m\right) p_{jit} q_{jit}$$

and the market clearing condition of tangible capital

$$k_{it} = \sum_j k_{jit}$$

we have

$$\sum_{t=0}^{\infty} Q_t \sum_{j \neq i} \left(p_{jit} q_{jit} - p_{ijt} q_{ijt} \right) = \sum_{t=0}^{\infty} Q_t \Bigg(\Bigg[- \sum_{j \neq i} \hat{p}_{jit} \hat{q}_{jit} - p_{it} \delta k_{iit} \Bigg] - r_{it} k_{iit} + \sum_{j \neq i} \left(\left(1 - \tau_{jt}^p\right) \hat{\pi}_{ijt} - r_{jt} k_{ijt} \right) \Bigg)$$

$$+ \sum_{t=0}^{\infty} Q_t \Bigg[w_{it} \Bigg(\sum_{j \neq i} l_{jit} \Bigg) + p_{it} \delta \sum_j k_{jit} + R_{it} \sum_j k_{jit} + \tau_{it}^p \sum_{j \neq i} \hat{\pi}_{jit} \Bigg]$$

$$+ \left(1 + r^f\right) f_{i0}.$$

Finally, using the definition of foreign affiliate's corporate tax base $\hat{\pi}_{ijt}$

$$\hat{\pi}_{ijt} = \hat{p}_{ijt}\hat{q}_{ijt} - w_{jt}l_{ijt} - p_{jt}\delta k_{ijt}$$

and the definition of affiliate dividend d_{ijt}

$$d_{ijt} = (1 - \tau_{jt})\,\hat{\pi}_{ijt} - r_{jt}k_{ijt}$$

we can derive the BoP condition as

$$\sum_{t=0}^{\infty} Q_t \sum_{j \neq i} (p_{ijt}q_{ijt} - p_{jit}q_{jit} + d_{ijt} - d_{jit}) = -\left(1 + r^f\right) f_{i0} \tag{B.3}$$

with the condition that

$$\sum_{t=0}^{\infty} Q_t \sum_{i} \left[\sum_{j \neq i} (d_{ijt} - d_{jit})\right] = -\sum_{i} \left(1 + r^f\right) f_{i0}$$

Towards expressing the condition in terms of only allocations and policy instruments, first note that with the intermediate good producer's FOCs,

$$\begin{aligned} w_{jt} &= \hat{p}_{ijt}F^{ij}_{l,t} \\ r_{jt} &= \left(1 - \tau^p_{jt}\right)\left(\hat{p}_{ijt}F^{ij}_{k,t} - p_{jt}\delta\right) \end{aligned}$$

we have

$$d_{ijt} = \left(1 - \tau^p_{jt}\right)\left[\hat{p}_{ijt}\hat{q}_{ijt} - w_{jt}l_{ijt} - \delta p_{jt}k_{ijt}\right] - r_{jt}k_{ijt} = \left(1 - \tau^p_{jt}\right)\hat{p}_{ijt}F^{ij}_{z,t}z_{it}.$$

Also, with the final good producer's FOCs

$$\begin{aligned} p_{iit} &= \left(1 - \tau^x_{ijt}\right) p_{ijt} \\ p_{iit} &= p_{it}G^i_{i,t} \\ \left(1 + \tau^m_{ijt}\right) p_{jit} &= p_{it}G^i_{j,t} \\ \hat{p}_{ijt} &= p_{jt}G^j_{\hat{i},t} \end{aligned}$$

we can then restate the BoP condition as

$$\sum_{t=0}^{\infty} Q_t p_{it} \sum_{j\neq i} \left(\frac{G^i_{i,t} q_{ijt}}{\left(1-\tau^x_{ijt}\right)} - \frac{G^i_{j,t} q_{jit}}{\left(1+\tau^m_{ijt}\right)} + \left(1-\tau^p_{jt}\right) \frac{p_{jt}}{p_{it}} G^j_{\hat{i},t} F^{ij}_{z,t} z_{it} - \left(1-\tau^p_{it}\right) G^i_{\hat{j},t} F^{ji}_{z,t} z_{jt} \right) = -\left(1+r^f\right) f_{i0}$$

where we will show later that

$$\frac{p_{jt}}{p_{it}} = \frac{\left(1-\tau^x_{jit}\right)}{\left(1+\tau^m_{jit}\right)} \frac{u^i_{c,t} G^i_{j,t}}{u^j_{c,t} G^j_{j,t}}$$

together with the household's intertemporal condition

$$\frac{Q_t p_{it}}{Q_{t+1} p_{it+1}} = 1 + \frac{R_{it+1}}{p_{it+1}}$$

where $R_{it} = \left(1-\tau^p_{it}\right)\left(p_{iit} F^{ii}_{k,t} - p_{it}\delta\right) = p_{it}\left(1-\tau^p_{it}\right)\left(G^i_{i,t} F^{ii}_{k,t} - p_{it}\delta\right)$. We thus have the BoP condition as

$$\sum_{t=0}^{\infty} \frac{1}{\Pi^t_{s=0}\left[1+\left(1-\tau^p_{it}\right)\left(G^i_{i,t} F^{ii}_{k,t} - \delta\right)\right]} \sum_{j\neq i} \left(\frac{G^i_{j,t} q_{jit}}{\left(1+\tau^m_{ijt}\right)} - \frac{G^i_{i,t} q_{ijt}}{\left(1-\tau^x_{ijt}\right)} - \right. \tag{B.4}$$
$$\left. \left(1-\tau^p_{jt}\right) \frac{\left(1-\tau^x_{jit}\right)}{\left(1+\tau^m_{jit}\right)} \frac{u^i_{c,t} G^i_{j,t}}{u^j_{c,t} G^j_{j,t}} G^j_{\hat{i},t} F^{ij}_{z,t} z_{it} + \left(1-\tau^p_{it}\right) G^i_{\hat{j},t} F^{ji}_{z,t} z_{jt} \right) = \left(1+r^f\right) \frac{f_{i0}}{p_{i0}}.$$

Transfer pricing

The BoP condition in the transfer pricing can be obtained by following the same procedure. The only difference is the definition of corporate tax bases. In this scenario, we have

$$\hat{\pi}_{iit} = p_{iit} q_{iit} + \sum_{j\neq i} \left(1-\tau^x_{ijt}\right) p_{ijt} q_{ijt} - w_{it} l_{iit} - p_{it}\delta k_{iit} - w_{it} l^z_{iz} + \sum_{j\neq i} \vartheta_{ijt} z_{it}$$

and

$$\hat{\pi}_{ijt} = \hat{p}_{ijt} \hat{q}_{ijt} - w_{jt} l_{ijt} - p_{jt}\delta k_{ijt} - \vartheta_{ijt} z_{it}.$$

With these definitions, the BoP condition in this scenario can be written as

$$\sum_{t=0}^{\infty} Q_t \sum_{j\neq i} \left(p_{ijt} q_{ijt} - p_{jit} q_{jit} + d_{ijt} - d_{jit} + \vartheta_{ijt} z_{it} - \vartheta_{jit} z_{jt}\right) = -\left(1+r^f\right) f_{i0} \tag{B.5}$$

with the condition that

$$\sum_{t=0}^{\infty} Q_t \sum_i \left[\sum_{j \neq i} (d_{ijt} - d_{jit}) \right] = - \sum_i \left(1 + r^f\right) f_{i0}.$$

We can easily show that, in the transfer pricing scenario, $d_{ijt} = 0, \forall j \neq i$ given constant return-to-scale production function F. Following the steps in the free transfer scenario, the BoP condition can be restated as

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{s=0}^{t} \left[1 + \left(1 - \tau_{it}^{p}\right)\left(G_{i,t}^{i} F_{k,t}^{ii} - \delta\right)\right]} \sum_{j \neq i} \left(\frac{G_{j,t}^{i} q_{jit}}{\left(1 + \tau_{ijt}^{m}\right)} - \frac{G_{i,t}^{i} q_{ijt}}{\left(1 - \tau_{ijt}^{x}\right)} - \frac{\left(1 - \tau_{jit}^{x}\right)}{\left(1 + \tau_{jit}^{m}\right)} \frac{u_{c,t}^{i} G_{j,t}^{i}}{u_{c,t}^{j} G_{j,t}^{j}} G_{\hat{\imath},t}^{j} F_{z,t}^{ij} z_{it} + G_{\hat{\jmath},t}^{i} F_{z,t}^{ji} z_{jt} \right) = \left(1 + r^f\right) \frac{f_{i0}}{p_{i0}} \tag{B.6}$$

Profit shifting

The BoP condition in the profit shifting can be obtained by following the same procedure, as well. There are two differences: the definition of corporate tax bases and the final good market clearing condition. The corporate tax bases in this scenario are defined as

$$\hat{\pi}_{iit} = p_{iit} q_{iit} + \sum_{j \neq i} \left(1 - \tau_{ijt}^{x}\right) p_{ijt} q_{ijt} - w_{it} l_{iit} - p_{it} \delta k_{iit} - w_{it} l_{iz}^{z} + \sum_{j \neq i} \vartheta_{ijt} z_{it} + \left(\varphi \lambda_{it} - \mathcal{C}\left(\lambda_{it}\right) - \lambda_{it}\right) \sum_j \vartheta_{ijt} z_{it}$$

and

$$\hat{\pi}_{ijt} = \hat{p}_{ijt} \hat{q}_{ijt} - w_{jt} l_{ijt} - p_{jt} \delta k_{ijt} - \vartheta_{ijt} z_{it}.$$

And the final good's market clearing condition, due to costly profit shifting, is modified as

$$G_t^i = c_{it} + \left(k_{it+1} - (1 - \delta) k_{it}\right) + g_{it} + \mathcal{C}\left(\lambda_{it}\right) \sum_j \vartheta_{ijt} z_{it}.$$

With these changes, we can derive the BoP condition in the profit shifting scenario, of non-tax-haven countries, as

$$\sum_{t=0}^{\infty} Q_t \left[\sum_{j \neq i} \left(p_{ijt} q_{ijt} - p_{jit} q_{jit} + d_{ijt} - d_{jit} + \left(1 - \lambda_{it}\right) \vartheta_{ijt} z_{it} - \vartheta_{jit} z_{jt}\right) - \lambda_{it} \vartheta_{iit} z_{it} \right] = - \sum_{t=0}^{\infty} Q_t \left[\varphi \lambda_{it} \sum_j \vartheta_{ijt} z_{it} \right] - \left(1 + r^f\right) f_{i0}. \tag{B.7}$$

For the tax haven, we assume that it imports intermediate goods from other countries for the balance of payments

$$\sum_{t=0}^{\infty} Q_t \left(\sum_{i=1}^{I} \lambda_{it} \tau_{TH}^{p} \nu_{it} z_{it} - \sum_{j \neq TH} p_{jTHt} q_{jTHt} \right) = \sum_{t=0}^{\infty} Q_t \sum_{i=1}^{I} \varphi \lambda_{it} \nu_{it} z_{it}.$$

Summing up the BoP of all countries, we have

$$\sum_{t=0}^{\infty} Q_t \left(\sum_{i} \left[\sum_{j \neq i} (d_{ijt} - d_{jit}) - (1 - \tau_{TH}^{p}) \lambda_{it} \nu_{it} z_{it} \right] \right) = - \sum_{i} \left(1 + r^f\right) f_{i0}.$$

We can again show that $d_{ijt} = 0, \forall j \neq i, TH$. We can restate the BoP condition in the profit shifting scenario as

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{s=0}^{t} \left[1 + (1 - \tau_{it}^{p}) \left(G_{i,t}^{i} F_{k,t}^{ii} - \delta\right)\right]} \left[\sum_{j \neq i} \left(\frac{G_{j,t}^{i} q_{jit}}{(1 + \tau_{ijt}^{m})} - \frac{G_{i,t}^{i} q_{ijt}}{(1 - \tau_{ijt}^{x})} - \right. \right. \tag{B.8}$$

$$\left. \left. \frac{(1 - \tau_{jit}^{x})}{(1 + \tau_{jit}^{m})} \frac{u_{c,t}^{i} G_{j,t}^{i}}{u_{c,t}^{j} G_{j,t}^{j}} G_{\hat{i},t}^{j} F_{z,t}^{ij} z_{it} + G_{\hat{j},t}^{i} F_{z,t}^{ji} z_{jt} \right) + (1 - \varphi) \lambda_{it} \sum_{j} \frac{(1 - \tau_{jit}^{x})}{(1 + \tau_{jit}^{m})} \frac{u_{c,t}^{i} G_{j,t}^{i}}{u_{c,t}^{j} G_{j,t}^{j}} G_{\hat{j},t}^{i} F_{z,t}^{ij} z_{it} \right] = \left(1 + r^f\right) \frac{f_{i0}}{p_{i0}}.$$

B.2 Characterization of the competitive equilibrium

We show how to derive the conditions that characterize the competitive equilibrium here. Start from the FOCs characterizing households' choices. We have the intratemporal consumption-labor condition as

$$-\frac{u_{c,t}^{i}}{u_{h,t}^{i}} = \frac{(1 + \tau_{it}^{c}) p_{it}}{\left(1 - \tau_{it}^{h}\right) w_{it}}$$

and then from FOCs of the intermediate producers that

$$w_{it} = p_{iit} F_{l,t}^{ii} = \hat{p}_{jit} F_{l,t}^{ji}$$

and then from the FOC of the final producer that

$$G_{i,t}^{i} = \frac{p_{iit}}{p_{it}}$$

$$G_{\hat{j},t}^{i} = \frac{\hat{p}_{jit}}{p_{it}}$$

which put together gives

$$-\frac{u^i_{c,t}}{u^i_{h,t}} = \frac{(1+\tau^c_{it})}{\left(1-\tau^h_{it}\right)}\frac{1}{G^i_{i,t}F^{ii}_{l,t}} = \frac{(1+\tau^c_{it})}{\left(1-\tau^h_{it}\right)}\frac{1}{G^i_{\hat{j},t}F^{ji}_{l,t}} \quad \forall_i, \forall_{j\neq i} \tag{B.9}$$

Now consider the intertemporal FOC of the household, which reads as

$$\frac{u^i_{c,t}}{u^i_{c,t+1}} = \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\beta\left[1+\frac{R_{it+1}}{p_{it+1}}\right]$$

and we have that

$$R_{it} = (1-\tau^p_{it})\left(p_{iit}F^{ii}_{k,t} - p_{it}\delta\right) = (1-\tau^p_{it})\left(\hat{p}_{jit}F^{ji}_{k,t} - p_{it}\delta\right)$$

thus we have

$$\begin{aligned}\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} &= \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\left[1+\left(1-\tau^p_{it+1}\right)\left(G^i_{i,t}F^{ii}_{k,t+1}-\delta\right)\right] \\ &= \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\left[1+\left(1-\tau^p_{it+1}\right)\left(G^i_{\hat{j},t}F^{ji}_{k,t+1}-\delta\right)\right] \quad \forall_i, \forall_{j\neq i}\end{aligned} \tag{B.10}$$

For the final goods producers and two imported goods (m,n) we have that

$$\frac{G^i_{n,t}}{G^i_{m,t}} = \frac{(1+\tau^m_{nit})\,p_{nit}}{(1+\tau^m_{mit})\,p_{mit}}$$

then notice that we have from the condition for the intermediate goods producer that

$$\begin{aligned}\frac{1}{(1-\tau^x_{nit})} &= \frac{p_{nit}}{p_{nnt}} \\ \frac{1}{(1-\tau^x_{mit})} &= \frac{p_{mit}}{p_{mmt}}\end{aligned}$$

and thus combining these equations we have

$$\frac{(1-\tau^x_{nit})\,(1+\tau^m_{mit})}{(1+\tau^m_{nit})\,(1-\tau^x_{mit})}\frac{G^i_{n,t}}{G^i_{m,t}} = \frac{p_{nnt}}{p_{mmt}}$$

and notice that the RHS of the equation is the same for all i. This is equivalent result to Chari, Nicolini, Teles (2022) equation (17). Using the final good producer's FOC, we have

$$\frac{\left(1-\tau_{nit}^{x}\right)\left(1+\tau_{mit}^{m}\right)}{\left(1+\tau_{nit}^{m}\right)\left(1-\tau_{mit}^{x}\right)}\frac{G_{n,t}^{i}}{G_{m,t}^{i}}=\frac{p_{nt}}{p_{mt}}\frac{G_{n,t}^{n}}{G_{m,t}^{m}} \quad \forall_{i},\forall_{m,n\neq i} \tag{B.11}$$

Toward the dynamic production efficiency, we have from the final good producer's problem:

$$\begin{aligned} G_{i,t}^{i} &= \frac{p_{iit}}{p_{it}} \\ G_{i,t}^{j} &= \frac{\left(1+\tau_{ijt}^{m}\right)p_{ijt}}{p_{jt}} \end{aligned}$$

and from the intermediate good producer's problem:

$$\begin{aligned} p_{iit} &= \left(1-\tau_{ijt}^{x}\right)p_{ijt} \\ p_{jjt} &= \left(1-\tau_{jit}^{x}\right)p_{jit} \end{aligned}$$

From the final good producer's problem, for the same good intertemporally we have

$$\begin{aligned} G_{i,t}^{i} &= \frac{p_{iit}}{p_{it}} \\ G_{i,t+1}^{i} &= \frac{p_{ii,t+1}}{p_{i,t+1}} \\ G_{i,t}^{j} &= \frac{\left(1+\tau_{ijt}^{m}\right)p_{ijt}}{p_{jt}} \\ G_{i,t+1}^{j} &= \frac{\left(1+\tau_{ijt+1}^{m}\right)p_{ijt+1}}{p_{jt+1}} \end{aligned}$$

We then have

$$\begin{aligned} \frac{G_{i,t}^{i}}{G_{i,t+1}^{i}} &= \frac{p_{i,t+1}}{p_{it}}\frac{p_{iit}}{p_{ii,t+1}} \\ \frac{G_{i,t}^{j}}{G_{i,t+1}^{j}} &= \frac{p_{j,t+1}}{p_{jt}}\frac{\left(1+\tau_{ijt}^{m}\right)p_{ijt}}{\left(1+\tau_{ijt+1}^{m}\right)p_{ijt+1}} \end{aligned}$$

Then using the intermediate good producer's problem

$$p_{iit}=\left(1-\tau_{ijt}^{x}\right)p_{ijt}$$

we have

$$\frac{G_{i,t}^{j}}{G_{i,t+1}^{j}}=\frac{p_{j,t+1}}{p_{jt}}\frac{\left(1+\tau_{ijt}^{m}\right)/\left(1-\tau_{ijt}^{x}\right)\cdot p_{iit}}{\left(1+\tau_{ijt+1}^{m}\right)/\left(1-\tau_{ijt+1}^{x}\right)\cdot p_{iit+1}}$$

Hence

$$\frac{G_{i,t}^{j}}{G_{i,t+1}^{j}}\frac{p_{jt}}{p_{j,t+1}}=\frac{\left(1+\tau_{ijt}^{m}\right)/\left(1-\tau_{ijt}^{x}\right)}{\left(1+\tau_{ijt+1}^{m}\right)/\left(1-\tau_{ijt+1}^{x}\right)}\frac{p_{it}}{p_{i,t+1}}\frac{G_{i,t}^{i}}{G_{i,t+1}^{i}} \tag{B.12}$$

Now we derive the intertemporal price ratio from the household's problem. Note that

$$\begin{aligned} k_{it+1}: &\quad 0=-\lambda_i Q_t p_{it}+Q_{t+1}\lambda_i\left[p_{it+1}+R_{it+1}\right] \\ k_{jt+1}: &\quad 0=-\lambda_j Q_t p_{jt}+Q_{t+1}\lambda_j\left[p_{it+1}+R_{jt+1}\right] \end{aligned}$$

we have

$$\begin{aligned} \frac{p_{it}}{p_{it+1}} &= \frac{Q_{t+1}}{Q_t}\left(1+\frac{R_{it+1}}{p_{it+1}}\right) \\ \frac{p_{jt}}{p_{jt+1}} &= \frac{Q_{t+1}}{Q_t}\left(1+\frac{R_{jt+1}}{p_{jt+1}}\right) \end{aligned}$$

and

$$\begin{aligned} R_{it} &= \left(1-\tau_{it}^{p}\right)\left(p_{iit}F_{k,t}^{ii}-p_{it}\delta\right) \\ R_{jt} &= \left(1-\tau_{jt}^{p}\right)\left(p_{jjt}F_{k,t}^{jj}-p_{jt}\delta\right) \end{aligned}$$

Therefore,

$$\frac{G_{i,t}^{j}}{G_{i,t+1}^{j}}\frac{Q_{t+1}}{Q_t}\left(1+\frac{R_{jt+1}}{p_{jt+1}}\right)=\frac{\left(1+\tau_{ijt}^{m}\right)/\left(1-\tau_{ijt}^{x}\right)}{\left(1+\tau_{ijt+1}^{m}\right)/\left(1-\tau_{ijt+1}^{x}\right)}\frac{Q_{t+1}}{Q_t}\left(1+\frac{R_{it+1}}{p_{it+1}}\right)\frac{G_{i,t}^{i}}{G_{i,t+1}^{i}}$$

and further, we have

$$\begin{aligned} &\frac{G_{i,t}^{j}}{G_{i,t+1}^{j}}\left(1+\left(1-\tau_{jt+1}^{p}\right)\left(G_{j,t+1}^{j}F_{k,t+1}^{jj}-\delta\right)\right)= \\ &\qquad \frac{\left(1+\tau_{ijt}^{m}\right)\left(1-\tau_{ijt+1}^{x}\right)}{\left(1+\tau_{ijt+1}^{m}\right)\left(1-\tau_{ijt}^{x}\right)}\frac{G_{i,t}^{i}}{G_{i,t+1}^{i}}\left(1+\left(1-\tau_{it+1}^{p}\right)\left(G_{i,t+1}^{i}F_{k,t+1}^{ii}-\delta\right)\right) \end{aligned} \tag{B.13}$$

Toward the labor-intangible condition, in the free transfer scenario we have

$$
\begin{aligned}
l_{iit} &: w_{it} = p_{iit} F^{ii}_{l,t} \\
l^z_{it} &: w_{it} = p_{iit} F^{ii}_{z,t} H^i_{l,t} + \sum_{j \neq i} \frac{\left(1-\tau^p_{jt}\right)}{\left(1-\tau^p_{it}\right)} \left(\hat{p}_{ijt} F^{ij}_{z,t} H^i_{l,t}\right)
\end{aligned}
$$

and combining these equations, we get

$$
\frac{F^{ii}_{l,t}}{H^i_{l,t} F^{ii}_{z,t}} = 1 + \sum_{j \neq i} \frac{\left(1-\tau^p_{jt}\right)}{\left(1-\tau^p_{it}\right)} \frac{\hat{p}_{ijt}}{p_{iit}} \frac{F^{ij}_{z,t}}{F^{ii}_{z,t}}
$$

and notice that from the FOCs of the final goods producers we have

$$
\begin{aligned}
p_{iit} &= p_{it} G^i_{i,t} \\
\hat{p}_{ijt} &= p_{jt} G^j_{\hat{i},t}
\end{aligned}
$$

thus we have

$$
\begin{aligned}
\frac{F^{ii}_{l,t}}{H^i_{l,t} F^{ii}_{z,t}} &= 1 + \sum_{j \neq i} \frac{\left(1-\tau^p_{jt}\right) p_{jt}}{\left(1-\tau^p_{it}\right) p_{it}} \frac{G^j_{\hat{i},t}}{G^i_{i,t}} \frac{F^{ij}_{z,t}}{F^{ii}_{z,t}} \\
&= 1 + \sum_{j \neq i} \frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \left[\frac{\left(1-\tau^p_{jt}\right)}{\left(1-\tau^p_{it}\right)} \frac{u^i_{c,t} p_{jt}}{u^j_{c,t} p_{it}} \right] \qquad \text{(B.14)}
\end{aligned}
$$

Further, we have

$$
\begin{aligned}
p_{it} &= \frac{(1+\tau^m_{jit}) p_{jit}}{G^i_{j,t}} = \frac{(1+\tau^m_{jit}) p_{jjt}}{(1-\tau^x_{jit}) G^i_{j,t}} \\
p_{jt} &= \frac{p_{jjt}}{G^j_{j,t}}.
\end{aligned}
$$

We can then rewrite the above condition as

$$
\frac{F^{ii}_{l,t}}{H^i_{l,t} F^{ii}_{z,t}} = 1 + \sum_{j \neq i} \frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \left[\frac{\left(1-\tau^p_{jt}\right)}{\left(1-\tau^p_{it}\right)} \frac{(1-\tau^x_{jit})}{(1+\tau^m_{jit})} \frac{u^i_{c,t} G^i_{j,t}}{u^j_{c,t} G^j_{j,t}} \right] \qquad \text{(B.15)}
$$

The labor-intangible capital condition is different in the transfer pricing and profit shifting scenarios. In the transfer pricing scenario, the FOCs with respect to l_{iit} and l^z_{it} for the intermediate good producer is

$$
\begin{aligned}
l_{iit} &: w_{it} = p_{iit} F^{ii}_{l,t} \\
l^{z}_{it} &: w_{it} = p_{iit} F^{ii}_{z,t} H^{i}_{l,t} + \sum_{j\neq i} \vartheta_{ijt} H^{i}_{l,t} + \sum_{j\neq i} \frac{\left(1-\tau^{p}_{jt}\right)}{\left(1-\tau^{p}_{it}\right)} \left(\hat{p}_{ijt} F^{ij}_{z,t} H^{i}_{l,t} - \vartheta_{ijt} H^{i}_{l,t}\right)
\end{aligned}
$$

Following the same steps as in the free transfer scenario, we can obtain:

$$
\frac{F^{ii}_{l,t}}{H^{i}_{l,t} F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \left[\frac{u^{j}_{c,t} G^{j}_{\hat{i},t} F^{ij}_{z,t}}{u^{i}_{c,t} G^{i}_{i,t} F^{ii}_{z,t}} \frac{\left(1-\tau^{p}_{jt}\right)}{\left(1-\tau^{p}_{it}\right)} \frac{u^{i}_{c,t} p_{jt}}{u^{j}_{c,t} p_{it}} + \frac{\left(\tau^{p}_{jt}-\tau^{p}_{it}\right)}{\left(1-\tau^{p}_{it}\right)} \frac{\vartheta_{ijt}}{p_{iit} F^{ii}_{z,t}} \right] \tag{B.16}
$$

When setting $\vartheta_{ijt} = \hat{p}_{ijt} F^{ij}_{z,t}$, we can rewrite equation (B.16) as

$$
\begin{aligned}
\frac{F^{ii}_{l,t}}{H^{i}_{l,t} F^{ii}_{z,t}} &= 1 + \sum_{j\neq i} \left[\frac{u^{j}_{c,t} G^{j}_{\hat{i},t} F^{ij}_{z,t}}{u^{i}_{c,t} G^{i}_{i,t} F^{ii}_{z,t}} \frac{\left(1-\tau^{p}_{jt}\right)}{\left(1-\tau^{p}_{it}\right)} \frac{u^{i}_{c,t} p_{jt}}{u^{j}_{c,t} p_{it}} + \frac{\left(\tau^{p}_{jt}-\tau^{p}_{it}\right)}{\left(1-\tau^{p}_{it}\right)} \frac{\hat{p}_{ijt} F^{ij}_{z,t}}{p_{iit} F^{ii}_{z,t}} \right] \\
&= 1 + \sum_{j\neq i} \left[\frac{u^{j}_{c,t} G^{j}_{\hat{i},t} F^{ij}_{z,t}}{u^{i}_{c,t} G^{i}_{i,t} F^{ii}_{z,t}} \frac{\left(1-\tau^{p}_{jt}\right)}{\left(1-\tau^{p}_{it}\right)} \frac{u^{i}_{c,t} p_{jt}}{u^{j}_{c,t} p_{it}} + \frac{u^{j}_{c,t} G^{j}_{\hat{i},t} F^{ij}_{z,t}}{u^{i}_{c,t} G^{i}_{i,t} F^{ii}_{z,t}} \frac{\left(\tau^{p}_{jt}-\tau^{p}_{it}\right)}{\left(1-\tau^{p}_{it}\right)} \frac{u^{i}_{c,t} p_{jt}}{u^{j}_{c,t} p_{it}} \right] \\
&= 1 + \sum_{j\neq i} \frac{u^{j}_{c,t} G^{j}_{\hat{i},t} F^{ij}_{z,t}}{u^{i}_{c,t} G^{i}_{i,t} F^{ii}_{z,t}} \left(\frac{u^{i}_{c,t} p_{jt}}{u^{j}_{c,t} p_{it}} \right) \\
&= 1 + \sum_{j\neq i} \frac{u^{j}_{c,t} G^{j}_{\hat{i},t} F^{ij}_{z,t}}{u^{i}_{c,t} G^{i}_{i,t} F^{ii}_{z,t}} \left(\frac{\left(1-\tau^{x}_{jit}\right)}{\left(1+\tau^{m}_{jit}\right)} \frac{u^{i}_{c,t} G^{i}_{j,t}}{u^{j}_{c,t} G^{j}_{j,t}} \right)
\end{aligned} \tag{B.17}
$$

In the profit shifting senario, the FOCs with respect to l_{iit} and l^{z}_{it} for the intermediate good producer is

$$
\begin{aligned}
l_{iit} &: w_{it} = p_{iit} F^{ii}_{l,t} \\
l^{z}_{it} &: w_{it} = \Big[p_{iit} F^{ii}_{z,t} H^{i}_{l,t} + \left(1-\lambda_{it}\right) H^{i}_{l,t} \sum_{j\neq i} \vartheta_{ijt}\left(z_{it}\right) - \lambda_{it} H^{i}_{l,t} \vartheta_{iit}\left(z_{it}\right) \\
&\qquad - \mathcal{C}_{i}\left(\lambda_{it}\right) H^{i}_{l,t} \sum_{j} \vartheta_{ijt}\left(z_{it}\right) + \varphi \lambda_{it} H^{i}_{l,t} \sum_{j} \vartheta_{ijt}\left(z_{it}\right) \Big] \\
&\quad + \sum_{j\neq i,TH} \frac{\left(1-\tau^{p}_{jt}\right)}{\left(1-\tau^{p}_{it}\right)} \left[\hat{p}_{ijt} F^{ii}_{z,t} H^{i}_{l,t} - H^{i}_{l,t} \vartheta_{ijt}(z_{it}) \right] + \frac{\left(1-\tau^{p}_{THt}\right)}{\left(1-\tau^{p}_{it}\right)} \lambda_{it} \left(1-\varphi\right) H^{i}_{l,t} \sum_{j} \vartheta_{ijt}\left(z_{it}\right)
\end{aligned}
$$

Setting $\vartheta_{ijt} = \hat{p}_{ijt} F^{ij}_{z,t}$, we can derive the labor-intangible condition as

$$\frac{F^{ii}_{l,t}}{H^i_{l,t} F^{ii}_{z,t}} = \left[1 + \sum_{j \neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \left(\frac{\left(1 - \tau^x_{jit}\right)}{\left(1 + \tau^m_{jit}\right)} \frac{u^i_{c,t} G^i_{j,t}}{u^j_{c,t} G^j_{j,t}} \right) \right] \left(1 - \mathcal{C}\left(\lambda_{it}\right) + \frac{\lambda_i \left(1 - \varphi\right) \left(\tau^p_{it} - \tau^p_{THt}\right)}{\left(1 - \tau^p_{it}\right)} \right). \tag{B.18}$$

Lastly, when intangible capital is rival, we have the FOCs as:

$$l_{iit} : w_{it} = p_{iit} F^{ii}_{l,t}$$
$$l^z_{iit} : w_{it} = p_{iit} F^{ii}_{z,t} H^i_{l,t}$$

where l^z_{iit} is the amount of worker headquarter i employs to produce the rival intangible capital for itself. Then we have the labor-intangible capital equilibrium condition as

$$\frac{F^{ii}_{l,t}}{F^{ii}_{z,t} H^i_{l,t}} = 1.$$

B.3 The Example

We solve for the optimal intangible capital investment with specific functional forms imposed in Section 3.5.

Free transfer

With the functional forms imposed, we can derive the following FOCs for the intermediate good firm using equation (20):

$$l_{iit} : w_{it} = p_{iit} \gamma A_i \left(N_i z_{it}\right)^{\phi} k^{\alpha}_{iit} l^{\gamma-1}_{iit} \tag{B.19}$$
$$l_{ijt} : w_{jt} = \hat{p}_{ijt} \gamma A_j \left(N_i z_{it}\right)^{\phi} k^{\alpha}_{ijt} l^{\gamma-1}_{ijt} \qquad \forall j \neq i \tag{B.20}$$
$$k_{iit} : r_{it} = \left(1 - \tau^p_{jt}\right) \left(p_{iit} \alpha A_i \left(N_i z_{it}\right)^{\phi} k^{\alpha-1}_{iit} l^{\gamma}_{iit} - p_{it} \delta \right) \tag{B.21}$$
$$k_{ijt} : r_{jt} = \left(1 - \tau^p_{jt}\right) \left(\hat{p}_{ijt} \alpha A_j \left(N_j z_{it}\right)^{\phi} k^{\alpha-1}_{ijt} l^{\gamma}_{ijt} - p_{jt} \delta \right) \tag{B.22}$$
$$l^z_{it} : w_{it} = p_{iit} F^i_{z,t} A_i + \sum_{j \neq i} \frac{\left(1 - \tau^p_{jt}\right)}{\left(1 - \tau^p_{it}\right)} \left(\hat{p}_{ijt} F^{ij}_{z,t} A_i \right) \tag{B.23}$$
$$q_{ijt} : p_{iit} = \left(1 - \tau^x_{ijt}\right) p_{ijt} \qquad \forall j \neq i \tag{B.24}$$

From the FOCs of labor and tangible capital inputs, equations (B.19), (B.20), (B.21), (B.22), we can get

$$l_{iit} = \left(\frac{\gamma}{w_{it}}\right)^{\frac{1-\alpha}{1-\gamma-\alpha}} \left(\frac{\left(1-\tau_{it}^{p}\right)\alpha}{r_{it}+\left(1-\tau_{it}^{p}\right)p_{jt}\delta}\right)^{\frac{\alpha}{1-\gamma-\alpha}} \left(p_{iit}A_i\left(N_i z_{it}\right)^{\phi}\right)^{\frac{1}{1-\gamma-\alpha}} \tag{B.25}$$

$$k_{iit} = \left(\frac{\gamma}{w_{it}}\right)^{\frac{\gamma}{1-\gamma-\alpha}} \left(\frac{\left(1-\tau_{it}^{p}\right)\alpha}{r_{it}+\left(1-\tau_{it}^{p}\right)p_{jt}\delta}\right)^{\frac{1-\gamma}{1-\gamma-\alpha}} \left(p_{iit}A_i\left(N_i z_{it}\right)^{\phi}\right)^{\frac{1}{1-\gamma-\alpha}} \tag{B.26}$$

and

$$l_{ijt} = \left(\frac{\gamma}{w_{jt}}\right)^{\frac{1-\alpha}{1-\gamma-\alpha}} \left(\frac{\left(1-\tau_{jt}^{p}\right)\alpha}{r_{jt}+\left(1-\tau_{jt}^{p}\right)p_{jt}\delta}\right)^{\frac{\alpha}{1-\gamma-\alpha}} \left(\hat{p}_{ijt}A_j\left(N_j z_{it}\right)^{\phi}\right)^{\frac{1}{1-\gamma-\alpha}} \tag{B.27}$$

$$k_{ijt} = \left(\frac{\gamma}{w_{jt}}\right)^{\frac{\gamma}{1-\gamma-\alpha}} \left(\frac{\left(1-\tau_{jt}^{p}\right)\alpha}{r_{jt}+\left(1-\tau_{jt}^{p}\right)p_{jt}\delta}\right)^{\frac{1-\gamma}{1-\gamma-\alpha}} \left(\hat{p}_{ijt}A_j\left(N_j z_{it}\right)^{\phi}\right)^{\frac{1}{1-\gamma-\alpha}} \tag{B.28}$$

Plugging these into the FOC of l_{it}^{z}, equation (B.23), we can obtain the optimal intangible capital as:

$$z_{it}^{FT} = \left(\sum_{j}\frac{\left(1-\tau_{jt}^{p}\right)}{\left(1-\tau_{it}^{p}\right)}\hat{r}_{jt}\left(\tau_{jt}\right)\Lambda_{ijt}\right)^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}}$$

where we define

$$\Lambda_{jt} \equiv \phi\frac{A_i}{w_{it}}\left(\hat{p}_{ijt}A_j N_j^{\phi}\left(\frac{\gamma}{w_{jt}}\right)^{\gamma}\left(\frac{\alpha}{r_{jt}+p_{jt}\delta}\right)^{\alpha}\right)^{\frac{1}{1-\gamma-\alpha}}$$

and

$$\hat{r}_{jt}\left(\tau_{jt}\right) \equiv \left(\frac{\left(1-\tau_{jt}^{p}\right)\left(r_{jt}+p_{jt}\delta\right)}{r_{jt}+\left(1-\tau_{jt}^{p}\right)p_{jt}\delta}\right)^{\frac{\alpha}{1-\gamma-\alpha}}$$

Transfer pricing

The maximization problem of the transfer pricing case is specified in equation (21). Optimal labor and tangible capital inputs can be derived from the same FOCs as in the free transfer case. The FOC for l_{it}^{z} with transfer pricing is different:

$$l_i^z : 0 = \left(1-\tau_{it}^{p}\right)\left[p_{iit}F_{zt}^{ii}A_i - w_i + A_i\sum_{j\neq i}\vartheta_{ijt}\right] + \sum_{j\neq i}\left(1-\tau_{jt}^{p}\right)\left[\hat{p}_{ijt}F_{zt}^{ij}A_i - A_i\vartheta_{ijt}\right]$$

Plugging in optimal labor and tangible inputs using equations (B.25), (B.26), (B.27), (B.28), we can derive the optimal intangible capital z_{it} as

$$z_{it}^{TP} = \left(\sum_j \hat{r}_{jt} \left(\tau_{jt} \right) \Lambda_{ijt} \right)^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}}$$

Profit shifting

The maximization problem of the profit shifting case is specified in equation (23). As before, we obtain the optimal labor and tangible capital inputs can be derived from the same set of FOCs. In the profit shifting case, firms need to decide the share of intangible capital to shift to the tax haven λ_{it}, in addition to intangible capital investment z_{it}. The FOC with respect to λ_{it} is

$$\lambda_{it} : 0 = -\left(1 - \tau_{it}^{p}\right) z_{it} \sum_j \vartheta_{ijt} \cdot \left[-1 - \mathcal{C}_i^{\prime} \left(\lambda_{it} \right) + \varphi \right] + \left(1 - \tau_{THt}^{p}\right) z_{it} \sum_j \vartheta_{ijt} \cdot \left[1 - \varphi \right]$$

Then

$$\mathcal{C}_i^{\prime} \left(\lambda_{it} \right) = \frac{\left(\tau_{it}^{p} - \tau_{THt}^{p} \right)}{\left(1 - \tau_{it} \right)} \left(1 - \varphi \right)$$

and

$$\lambda_{it} = \mathcal{C}_i^{\prime -1} \left(\frac{\left(\tau_{it}^{p} - \tau_{THt}^{p} \right)}{\left(1 - \tau_{it} \right)} \left(1 - \varphi \right) \right) \tag{B.29}$$

The FOC wrt to l_{it}^{z} is

$$\begin{aligned} l_{it}^{z} : 0 = \left(1 - \tau_{it}^{p}\right) \Bigg[& p_{iit} F_{zt}^{ii} A_i - w_{it} + A_i \Bigg(- \lambda_{it} \vartheta_{iit}(z_{it}) + \left(1 - \lambda_{it}\right) \sum_{j \neq i} \vartheta_{ijt} \left(z_{it} \right) - \mathcal{C}_i \left(\lambda_{it} \right) \sum_j \vartheta_{ijt} \left(z_{it} \right) \\ & + \varphi \lambda_{it} \sum_j \vartheta_{ijt} \left(z_{it} \right) \Bigg) \Bigg] \\ & + \sum_{j \neq i, TH} \left(1 - \tau_j^{p} \right) \left[\hat{p}_{ijt} F_{zt}^{ij} A_i - \vartheta_{ijt}(z_{it}) \right] \\ & + \left(1 - \tau_{THt}^{p} \right) \lambda_{it} \left(1 - \varphi \right) A_i \sum_j \vartheta_{ijt} \left(z_{it} \right) \end{aligned}$$

Plugging in optimal labor and tangible inputs using equations (B.25), (B.26), (B.27), (B.28), we can derive the optimal intangible capital z_{it} in this case as

$$z_{it}^{PS} = \left(\left(\sum_j \hat{r}_{jt}\Lambda_{ijt}\right)\left(1-\mathcal{C}_i\left(\lambda_{it}\right)+\frac{\lambda_{it}\left(1-\varphi\right)\left(\tau_{it}^p-\tau_{THt}^p\right)}{\left(1-\tau_{it}^p\right)}\right)\right)^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}}$$
$$= z_{it}^{TP}\cdot\left(\left(1-\mathcal{C}_i\left(\lambda_{it}\right)+\frac{\lambda_{it}\left(1-\varphi\right)\left(\tau_{it}^p-\tau_{THt}^p\right)}{\left(1-\tau_{it}^p\right)}\right)\right)^{\frac{1-\gamma-\alpha}{1-\gamma-\alpha-\phi}}$$

B.4 Derivation of Ramsey Equilibria

The relaxed Ramsey problem is characterized by a solution to the following problem

$$\max_{\mathcal{A}_t}\sum_{i=1}^{I}\omega_i\sum_{t=0}^{\infty}\beta^t v^i\left(c_{it},h_{it};\varphi^i\right)-\varphi^i\mathcal{W}_{i0}\left(\tau_{i0}^c\right)$$

subject to the following constraints

$$\begin{aligned}
F^{ii}\left(z_{it},k_{iit},l_{iit}\right) &= q_{iit}+\sum_{j\neq i}q_{ijt} \quad \forall_i\\
F^{ij}\left(z_{it},k_{ijt},l_{ijt}\right) &= \hat{q}_{ijt} \quad \forall_{j\neq i}\forall_i\\
G^i\left(q_{1it},...,q_{Iit},\hat{q}_{1it},...,\hat{q}_{Iit}\right) &= c_{it}+g_{it}+x_{it} \quad \forall_i\\
x_{it} &= k_{it+1}-\left(1-\delta\right)k_{it}\\
H^i\left(l_{1t}^z,...,l_{It}^z\right) &= z_{it} \quad \forall_i\\
h_{it} &= \sum_j l_{jit}+l_{it}^z\\
k_{it} &= \sum_j k_{jit}\\
k_0 &\quad \text{given}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{W}_{i0}\left(\tau_{i0}^c\right) &= \frac{u_{c0}^i}{\left(1+\tau_{i0}^c\right)}\left(R_{i0}+Q_{-1}\frac{b_{i0}}{p_{i0}}+\left(1+r^f\right)\frac{f_{i0}}{p_{i0}}\right)\\
R_{i0} &= \left(1-\delta+r_{i0}\right)k_{i0}\\
v^i\left(c_{it},h_{it};\varphi^i\right) &= u^i\left(c_{it},h_{it}\right)+\varphi^i\left(u_{c,t}^i c_{it}-u_{h,t}^i h_{it}\right)
\end{aligned}$$

and the set of constraints can be simplified to

$$c_{it} + g_{it} + \sum_j k_{jit+1} - (1-\delta) \sum_j k_{jit} =$$

$$G^i \left(q_{1it}, ..., F^{ii} \left(H^i \left(l_{it}^z \right), k_{iit}, l_{iit} \right) - \sum_{j \neq i} q_{ijt}, ..., q_{Iit}, F^{1i} \left(H^1 \left(l_{1t}^z \right), k_{1it}, l_{1it} \right), ..., F^{Ii} \left(H^I \left(l_{It}^z \right), k_{Iit}, l_{Iit} \right) \right)$$

and φ^i is the multiplier on the implementability condition

$$\sum_{t=0}^{\infty} \beta^t \left[u_{c,t}^i c_{it} - u_{h,t}^i h_{it} \right] = \frac{u_{c0}^i}{(1+\tau_{i0}^c)} \left(R_{i0} + Q_{-1} \frac{b_{i0}}{p_{i0}} + \left(1 + r^f\right) \frac{f_{i0}}{p_{i0}} \right), \; \forall i = 1, \ldots, I.$$

If the implementability condition is held in i, then we have $\varphi^i = 0$ and $v^i(c_{it}, h_{it}; \varphi) = u^i(c_{it}, h_{it})$. More generally, the first-order conditions of this problem for periods $t > 0$ are

$$c_{it} : 0 = \beta^t v_{c,t}^i - \lambda_t^i \tag{B.30}$$

$$l_{jit} : 0 = \beta^t v_{h,t}^i + \lambda_t^i \varphi^i G_{\hat{j},t}^i F_{l,t}^{ji} \tag{B.31}$$

$$l_{iit} : 0 = \beta^t v_{h,t}^i + \lambda_t^i G_{i,t}^i F_{l,t}^{ii} \tag{B.32}$$

$$\begin{aligned} l_{it}^z : 0 = \beta^t v_{h,t}^i + \lambda_t^i & \left[G_{i,t}^i F_{z,t}^{ii} H_{i,t}^i + \sum_{k \neq i} G_{\hat{k},t}^i F_{zt}^{ki} H_{i,t}^k \right] + \\ & \lambda_t^j \left[G_{\hat{i},t}^j F_{z,t}^{ij} H_{i,t}^i + G_{j,t}^j F_{z,t}^{jj} H_{i,t}^j + \sum_{k \neq j} G_{\hat{k},t}^j F_{zt}^{kj} H_{i,t}^k \right] \end{aligned} \tag{B.33}$$

$$q_{jit} : 0 = -\lambda_t^j G_{j,t}^j + \lambda_t^i G_{j,t}^i \tag{B.34}$$

$$k_{jit+1} : 0 = (1-\delta) \lambda_{t+1}^i - \lambda_t^i + \lambda_{t+1}^i G_{\hat{j},t+1}^i F_{k,t+1}^{ji} \tag{B.35}$$

$$k_{iit+1} : 0 = (1-\delta) \lambda_{t+1}^i - \lambda_t^i + \lambda_{t+1}^i G_{i,t+1}^i F_{k,t+1}^{ii} \tag{B.36}$$

For the initial period t=0, the first-order condition for c_{i0} is

$$c_{i0} : 0 = v_{c,t}^i - \lambda_t^i + \varphi^i \frac{\partial \mathcal{W}_{i0}}{\partial c_{i0}} \tag{B.37}$$

Optimal conditions when $t > 0$

We can first get the no intratemporal wedge condition from equations (B.30)-(B.32) as

$$-\frac{v_{c,t}^i}{v_{h,t}^i} = \frac{1}{G_{i,t}^i F_{l,t}^{ii}} = \frac{1}{G_{\hat{j},t}^i F_{l,t}^{ji}} \qquad \forall_i \forall_{j \neq i}$$

Next, from equations (B.30), (B.36) and (B.35), we can get the no intertemporal wedge condition as

$$\frac{v^i_{c,t}}{\beta v^i_{c,t+1}} = \left((1-\delta) + G^i_{i,t+1}F^{ii}_{k,t+1}\right) = \left((1-\delta) + G^i_{\hat{j},t+1}F^{ji}_{k,t+1}\right) \qquad \forall_i \forall_{j\neq i}$$

Now, towards static efficiency, for any imported goods (m,n) in location i we have from equations (B.30) and (B.34) that

$$\frac{G^i_{n,t}}{G^i_{m,t}} = \frac{G^n_{n,t}v^n_{c,t}}{G^m_{m,t}v^m_{c,t}} \qquad \forall_i \forall_{m,n\neq i}$$

Towards dynamic efficiency, from equations (B.34) and (B.36) we can derive

$$\frac{G^i_{j,t}}{G^i_{j,t+1}}\left((1-\delta) + G^i_{i,t+1}F^{ii}_{k,t+1}\right) = \left(\frac{G^j_{j,t}v^j_{c,t}}{G^j_{j,t+1}\beta v^j_{c,t+1}}\right) \qquad \forall_j$$

Finally, for the intangible efficiency, from equations (B.30), (B.32) and (B.33) we have

$$\begin{aligned}\frac{F^{ii}_{l,t}}{F^{ii}_{z,t}H^i_{i,t}} &= 1 + \sum_{j\neq i}\frac{v^j_{c,t}G^j_{\hat{i},t}F^{ij}_{z,t}}{v^i_{c,t}G^i_{i,t}F^{ii}_{z,t}} \\ &+ \sum_{j\neq i}\left[\frac{H^j_{i,t}}{H^i_{i,t}}\left(\frac{G^i_{\hat{j},t}F^{ji}_{z,t}}{G^i_{i,t}F^{ii}_{z,t}} + \frac{v^j_{c,t}G^j_{j,t}F^{jj}_{z,t}}{v^i_{c,t}G^i_{i,t}F^{ii}_{z,t}}\right) + \sum_{k\neq i,j}\frac{H^k_{i,t}}{H^i_{i,t}}\frac{v^j_{c,t}G^j_{\hat{k},t}F^{kj}_{z,t}}{v^c_{i,t}G^i_{i,t}F^{ii}_{z,t}}\right] \qquad \forall_i\end{aligned}$$

C Proofs

Proof of Lemma (1).

From the two FOCs of static efficiency characterizing the Pareto frontier

$$\begin{aligned} c_{it} &: 0 = \beta^t u^i_{c,t} - \lambda^i_t \\ q_{jit} &: 0 = -\lambda^j_t G^j_{j,t} + \lambda^i_t G^i_{j,t}\end{aligned}$$

we can immediately obtain

$$\begin{aligned}\lambda^j_t G^j_{j,t} &= \lambda^i_t G^i_{j,t} \\ u^j_{c,t}G^j_{j,t} &= u^i_{c,t}G^i_{j,t}\end{aligned}$$

hence

$$\frac{u^i_{c,t}}{u^j_{c,t}}\frac{G^i_{j,t}}{G^j_{j,t}} = 1.$$

■

Proof of Lemma (2).

We prove the properties of the return to profit shifting, $\Omega(\tau^p_{it})$, here. We first show that $\Omega(\tau^p_{it}) > 1$ if $\tau^p_{it} > \tau^p_{THt}$, and $\Omega(\tau^p_{it}) = 1$ if $\tau^p_{it} = \tau^p_{THt}$. Start from the FOC with respect to λ_{it} of the firm's problem in the profit shifting scenario:

$$\lambda_{it} : 0 = -\left(1-\tau^p_{it}\right) z_{it} \sum_j \vartheta_{ijt} \cdot \left[-1 - \mathcal{C}^{'}_i\left(\lambda_{it}\right) + \varphi\right] + \left(1-\tau^p_{THt}\right) z_{it} \sum_j \vartheta_{ijt} \cdot \left[1-\varphi\right].$$

Then, we have

$$\mathcal{C}^{'}_i\left(\lambda_{it}\right) = \frac{\left(\tau^p_{it} - \tau^p_{THt}\right)}{\left(1-\tau_{it}\right)}\left(1-\varphi\right).$$

Assuming that $\mathcal{C}\left(\lambda\right) = \lambda - (1-\lambda)\log(1-\lambda)$, we can solve for λ_{it} analytically as

$$\lambda_{it} = 1 - \exp\left(-\frac{(1-\varphi)(\tau^p_{it} - \tau^p_{THt})}{1-\tau^p_{it}}\right).$$

Then, we can show that when $\tau^p_{it} > \tau^p_{THt}$, $\lambda_{it} > 0$ and

$$\begin{aligned}\Omega(\tau^p_{it}) &= 1 - \mathcal{C}_i\left(\lambda_{it}\right) + \frac{\lambda_{it}\left(1-\varphi\right)\left(\tau^p_{it} - \tau^p_{THt}\right)}{\left(1-\tau^p_{it}\right)} \\ &= 1 - \mathcal{C}_i\left(\lambda_{it}\right) + \lambda_{it}\mathcal{C}^{'}_i\left(\lambda_{it}\right) > 1.\end{aligned}$$

The inequality comes from the fact that $\mathcal{C}\left(\lambda\right)$ is an increasing and convex function in λ. It is straightfoward to see that when $\tau^p_{it} = \tau^p_{THt}$, $\lambda_{it} = 0$ and $\Omega(\tau^p_{it}) = 1$. We then show that $\Omega(\tau^p_{it})$ is increasing in τ^p_{it}

$$\begin{aligned}\frac{\partial\Omega\left(\tau^p_{it}\right)}{\partial\tau^p_{it}} &= -\mathcal{C}^{'}\left(\lambda_{it}\right)\frac{\partial\lambda_{it}}{\partial\tau^p_{it}} + \frac{\partial\lambda_{it}}{\partial\tau^p_{it}}\mathcal{C}^{'}\left(\lambda_{it}\right) + \lambda_{it}\mathcal{C}^{''}\left(\lambda_{it}\right)\frac{\partial\lambda_{it}}{\partial\tau^p_{it}} \\ &= \lambda_{it}\mathcal{C}^{''}\left(\lambda_{it}\right)\frac{\partial\lambda_{it}}{\partial\tau^p_{it}} \\ &= \lambda_{it}\left(1-\varphi\right)\frac{1-\tau^p_{THt}}{\left(1-\tau^p_{it}\right)^2} > 0.\end{aligned}$$

■

Proof of Proposition (1). We start by proving in the transfer pricing scenario, all competitive equilibria that satisfy static efficiency will have inefficient intangible capital investment. The competitive equilibrium condition governing intangible capital in this scenario reads

$$\frac{F^{ii}_{l,t}}{H^{i}_{l,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \frac{u^{j}_{c,t}G^{j}_{i,t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}} \left[\frac{\left(1-\tau^{x}_{jit}\right)}{\left(1+\tau^{m}_{jit}\right)} \frac{G^{i}_{jt}u^{i}_{ct}}{G^{j}_{jt}u^{j}_{ct}} \right]$$

Note that *static efficiency* condition requires no trade tax wedges, *i.e.* $\frac{\left(1-\tau^{x}_{jit}\right)}{\left(1+\tau^{m}_{jit}\right)} = 1, \forall_{j\neq i}$, and that $\frac{G^{i}_{jt}}{G^{j}_{jt}} \frac{u^{i}_{c,t}}{u^{j}_{c,t}} = 1, \forall_{j\neq i}$. Hence, when *static efficiency* is satisfied, the condition above becomes:

$$\frac{F^{ii}_{l,t}}{H^{i}_{l,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \frac{u^{j}_{c,t}G^{j}_{\hat{i},t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}}. \tag{C.1}$$

The same condition on the Pareto frontier is

$$\frac{F^{ii}_{l,t}}{H^{i}_{l,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \frac{u^{j}_{c,t}G^{j}_{\hat{i},t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}} + \Sigma_{it} \tag{C.2}$$

with $\Sigma_{it} > 0$ in presence of positive spillovers. We have thus shown that in the transfer pricing scenario when the *static efficiency* condition is met, the marginal rate of technical substitution between labor and intangible capital is always smaller in the competitive equilibrium. Hence, the competitive equilibrium intangible capital allocation is inefficient. We now show that there is a set of corporate income taxes $\{\tau^{p}_{it}\}_{\forall i}$ that achieves efficient intangible capital allocation in the profit shifting scenario. The competitive equilibrium condition of intangible capital, with static efficiency imposed, in this scenario is

$$\frac{F^{ii}_{l,t}}{H^{i}_{l,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \frac{u^{j}_{c,t}G^{j}_{i,t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}} \cdot \Omega(\tau^{p}_{it}) \tag{C.3}$$

where $\Omega(\tau^{p}_{it}) = 1 - \mathcal{C}_i\left(\lambda_{it}\right) + \frac{\lambda_{it}(1-\varphi)\left(\tau^{p}_{it}-\tau^{p}_{THt}\right)}{\left(1-\tau^{p}_{it}\right)}$. Achieving efficient allocation of intangible capital amounts to find τ^{p}_{it} that satisfies

$$\frac{F^{ii}_{l,t}}{H^{i}_{l,t}F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \frac{u^{j}_{c,t}G^{j}_{i,t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}} \cdot \Omega(\tau^{p}_{it}) = 1 + \sum_{j\neq i} \frac{u^{j}_{c,t}G^{j}_{\hat{i},t}F^{ij}_{z,t}}{u^{i}_{c,t}G^{i}_{i,t}F^{ii}_{z,t}} + \Sigma_{it}, \forall_i$$

which can be re-written as

$$\Omega(\tau^p_{it}) = \left(\sum_{j\neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}}\right)^{-1} \left(\sum_{j\neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} + \Sigma_{it}\right) > 1 \tag{C.4}$$

We have shown in Lemma 2 that $\Omega(\tau^p_{it})$ is a continuous and increasing in τ^p_{it}, with $\Omega(\tau^p_{it}) = 0$ when $\tau^p_{it} = \tau^p_{THt}$ and $\Omega(\tau^p_{it}) \to \infty$ as $\tau^p_{it} \to 1$. Therefore, by the mean value theorem there exists a unique τ^p_{it} that satisfies (C.4) for all $i \in \{1, 2, ..., I\}$.

Lastly, we prove that in the free transfer scenario, all competitive equilibria that satisfy static efficiency have inefficiency intangible capital investment. The competitive equilibrium condition of intangible capital in this scenario, with static efficiency, is

$$\frac{F^{ii}_{l,t}}{H^i_{l,t} F^{ii}_{z,t}} = 1 + \sum_{j\neq i} \frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \frac{(1-\tau^p_{jt})}{(1-\tau^p_{it})} \tag{C.5}$$

It amounts to show that there does not exist a set of corporate income taxes $\{\tau^p_{it}\}_{\forall i}$ that satisfies

$$1 + \sum_{j\neq i} \frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} \frac{(1-\tau^p_{jt})}{(1-\tau^p_{it})} = 1 + \sum_{j\neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} + \Sigma_{it}, \forall i$$

which can be written as a system of I linear equations with I unknowns:

$$\underbrace{\begin{bmatrix} -\Psi_1 & \Upsilon_{21} & ... & \Upsilon_{I1} \\ \Upsilon_{12} & -\Psi_2 & ... & \Upsilon_{I2} \\ ... & ... & ... & ... \\ \Upsilon_{1I} & \Upsilon_{2I} & ... & -\Psi_I \end{bmatrix}}_{\Xi} \begin{bmatrix} 1-\tau^p_{1t} \\ 1-\tau^p_{2t} \\ ... \\ 1-\tau^p_{It} \end{bmatrix} = 0$$

where we define $\Psi_I = \sum_{j\neq i} \left[\frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}}\right] + \Sigma_{it}$ and $\Gamma_{ij} = \frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}}$. It is straightforward to show that Ξ is a strictly diagonally dominant matrix as

$$|\Psi_i| = \sum_{j\neq i} \frac{u^j_{c,t} G^j_{\hat{i},t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} + \Sigma_{it} > \sum_{j\neq i} \frac{u^j_{c,t} G^j_{i,t} F^{ij}_{z,t}}{u^i_{c,t} G^i_{i,t} F^{ii}_{z,t}} = \sum_{j\neq i} |\Upsilon_{ji}|$$

By Gershgorin's circle theorem, matrix Ξ is non-singular, then the above linear system of equations has a unique solution:

$$\tau^p_{it} = 1, \forall i.$$

However, corporate income taxes are strictly smaller than 1. Hence, we cannot find corporate income taxes that implements efficient intangible capital allocation in the free transfer scenario. ■

Proof of Proposition (2).

The budget constraint of the household is given by

$$\sum_{t=0}^{\infty} Q_t \left[\left(1+\tau_{it}^c\right) p_{it} c_{it} + p_{it} \left(k_{it+1} - \left(1-\delta+r_{it}\right) k_{it}\right) - \left(1-\tau_{it}^h\right) w_{it} h_{it} \right] = Q_{-1} b_{i0} + \left(1+r^f\right) f_{i0} + V_{i0} \tag{C.6}$$

where $V_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$. Using the household's first order conditions

$$\beta^t u_{c,t}^i - \lambda Q_t \left(1+\tau_{it}^c\right) p_{it} = 0$$
$$\beta^t u_{h,t}^i - \lambda Q_t \left(1-\tau_{it}^h\right) w_{it} = 0$$

and the Euler equation

$$-\lambda Q_t p_{it} + \lambda Q_{t+1} p_{it+1} \left(1-\delta+r_{it}\right) = 0$$

we can obtain

$$Q_t \left(1+\tau_{it}^c\right) p_{it} c_{it} = \beta^t \frac{u_{ct}^i}{u_{c0}^i} \left(1+\tau_{i0}^c\right) p_{i0} c_{it}$$
$$Q_t \left(1-\tau_{it}^h\right) w_{it} h_{it} = \beta^t \frac{u_{ht}^i}{u_{c0}^i} \left(1+\tau_{i0}^c\right) p_{i0} h_{it}$$

and

$$\frac{Q_t p_{it}}{Q_{t+1} p_{it+1}} = 1 - \delta + r_{it}.$$

Plugging these into equation (C.6), we then have

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{u_{ct}^i}{u_{c0}^i} \left(1+\tau_{i0}^c\right) p_{i0} c_{it} - \frac{u_{ht}^i}{u_{c0}^i} \left(1+\tau_{i0}^c\right) p_{i0} h_{it} \right] - \left[p_{i0}(1-\delta+r_{it})k_{i0}\right] = Q_{-1} b_{i0} + \left(1+r^f\right) f_{i0} + V_{i0}$$

with the transversality condition that

$$\lim_{t\to\infty} Q_t k_{it+1} = 0.$$

The constraint can be further simplified into

$$\sum_{t=0}^{\infty} \beta^t \left[u_{ct}^i c_{it} - u_{ht}^i h_{it}\right] = \frac{u_{c0}^i}{(1+\tau_{i0}^c)} \left(R_{i0} + Q_{-1}\frac{b_{i0}}{p_{i0}} + (1+r^f)\frac{f_{i0}}{p_{i0}} + \frac{V_{i0}}{p_{i0}}\right),\ \forall i = 1, ..., I$$

where $R_{i0} = (1-\delta+r_{i0})k_{i0}$.

We now show $V_{i0} = 0$ under all three model scenarios. The dividend in the free transfer scenario is:

$$d_{it} = \pi_{iit}(z_{it}) - (1-\tau_{it}^p)\, w_{it} l_{it}^z + \sum_{j\neq i} \pi_{ijt}(z_{it})$$

where

$$\begin{aligned}
\pi_{iit}(z_{it}) &= (1-\tau_{it}^p)\left[p_{iit}q_{iit} + \sum_{j\neq i}(1-\tau_{ijt}^x)p_{ijt}q_{ijt} - w_{it}l_{iit} - \delta p_{it}k_{iit}\right] - r_{it}k_{iit} \\
&= (1-\tau_{it}^p)\left[p_{iit}q_{iit} + \sum_{j\neq i}(1-\tau_{ijt}^x)p_{ijt}q_{ijt} - w_{it}l_{iit}\right] - \left(r_{it} + \delta\,(1-\tau_{it}^p)\, p_{it}\right) k_{iit}
\end{aligned}$$

and

$$\begin{aligned}
\pi_{ijt}(z_{it}) &= \left(1-\tau_{jt}^p\right)\left[\hat{p}_{ijt}\hat{q}_{ijt} - w_{jt}l_{ijt} - \delta p_{jt}k_{ijt}\right] - r_{jt}k_{ijt} \\
&= \left(1-\tau_{jt}^p\right)\left[\hat{p}_{ijt}\hat{q}_{ijt} - w_{jt}l_{ijt}\right] - \left(r_{jt} + \delta\left(1-\tau_{jt}^p\right)p_{it}\right)k_{ijt}.
\end{aligned}$$

The sum of discounted dividends is

$$\begin{aligned}
\sum_{t=0}^{\infty} Q_t d_{it} &= \sum_{t=0}^{\infty} Q_t \left[\pi_{iit}(z_{it}) - (1-\tau_{it}^p)\, w_{it}l_{it}^z + \sum_{j\neq i}\pi_{ijt}(z_{it})\right] \\
&= \sum_{t=0}^{\infty} Q_t \Bigg[(1-\tau_{it}^p)\left[p_{iit}q_{iit} + \sum_{j\neq i}(1-\tau_{ijt}^x)p_{ijt}q_{ijt} - w_{it}l_{iit}\right] - \left(r_{it} + \delta\,(1-\tau_{it}^p)\,p_{it}\right)k_{iit} \\
&\qquad + \sum_{j\neq i}\left[\left(1-\tau_{jt}^p\right)\left[\hat{p}_{ijt}\hat{q}_{ijt} - w_{jt}l_{ijt}\right] - \left(r_{jt} + \delta\left(1-\tau_{jt}^p\right)p_{jt}\right)k_{ijt}\right] - (1-\tau_{it}^p)\, w_{it}l_{it}^z \Bigg].
\end{aligned}$$

The first-order conditions for the intermediate-good producers are:

$$\begin{aligned}
l_{iit} &: w_{it} = p_{iit} F^{ii}_{l,t} = \hat{p}_{jit} F^{ji}_{l,t} \\
l^z_i &: w_{it} = p_{iit} F^{ii}_{z,t} H^i_{l,t} + \sum_{j \neq i} \frac{\left(1-\tau^p_{jt}\right)}{\left(1-\tau^p_{it}\right)} \left(\hat{p}_{ijt} F^{ij}_{z,t} H^i_{l,t}\right) \\
k_{ijt} &: r_{jt} = \left(1-\tau^p_{jt}\right)\left(\hat{p}_{ijt} F^{ij}_{k,t} - p_{jt}\delta\right) \\
q_{ijt} &: p_{iit} = \left(1-\tau^x_{ijt}\right) p_{ijt}.
\end{aligned}$$

Plugging in these equations, we have

$$\sum_{t=0}^{\infty} Q_t d_{it} = \sum_{t=0}^{\infty} Q_t \Bigg[\left(1-\tau^p_{it}\right) p_{iit} \left[F^{ii}\left(l_{iit}, k_{iit}, z_{it}\right) - F^{ii}_{l,t} l_{iit} - F^{ii}_{k,t} k_{iit} - F^{ii}_{z,t} l^z_{it} H^i_{l,t}\right] \\ + \sum_{j \neq i} \left[\left(1-\tau^p_{jt}\right) \hat{p}_{ijt} \left[F^{ij}\left(l_{ijt}, k_{ijt}, z_{it}\right) - F^{ij}_{l,t} l_{ijt} - F^{ij}_{k,t} k_{ijt} - F^{ij}_{z,t} l^z_{it} H^i_{l,t}\right]\right] \Bigg]$$

With constant return-to-scale production functions F and H, we have

$$F^{ii}\left(l_{iit}, k_{iit}, z_{it}\right) - F^{ii}_{l,t} l_{iit} - F^{ii}_{k,t} k_{iit} - F^{ii}_{z,t} l^z_{it} H^i_{l,t} = 0$$

and

$$F^{ij}\left(l_{ijt}, k_{ijt}, z_{it}\right) - F^{ij}_{l,t} l_{ijt} - F^{ij}_{k,t} k_{ijt} - F^{ij}_{z,t} l^z_{it} H^i_{l,t} = 0, \ \forall j \neq i.$$

Thus, we have $d_{it} = 0$ and

$$V_{i0} = \sum_{t=0}^{\infty} Q_t d_{it} = 0.$$

The dividend in the transfer pricing scenario is:

$$d_{it} = \pi_{iit}(z_{it}) - (1-\tau^p_{it}) w_{it} l^z_{it} + \sum_{j \neq i} \left[\pi_{ijt}(z_{it}) + (\tau^p_{jt} - \tau^p_{it}) \vartheta_{ijt} z_{it}\right].$$

The first-order conditions for the intermediate-good producers are:

$$\begin{aligned}
l_{iit} &: w_{it} = p_{iit} F^{ii}_{l,t} = \hat{p}_{jit} F^{ji}_{l,t} \\
l^z_i &: w_{it} = p_{iit} F^{ii}_{z,t} H^i_{l,t} + \sum_{j \neq i} \hat{p}_{ijt} F^{ij}_{z,t} H^i_{l,t} \\
k_{ijt} &: r_{jt} = \left(1-\tau^p_{jt}\right)\left(\hat{p}_{ijt} F^{ij}_{k,t} - p_{jt}\delta\right) \\
q_{ijt} &: p_{iit} = \left(1-\tau^x_{ijt}\right) p_{ijt}.
\end{aligned}$$

Following the same procedure and assuming that $\vartheta_{ijt} = \hat{p}_{ijt} F^{ij}_{z,t}$, the sum of discounted dividends can be derived as

$$\sum_{t=0}^{\infty} Q_t d_{it} = \sum_{t=0}^{\infty} Q_t \Bigg[(1-\tau^p_{it})\, p_{iit} \left[F^{ii}(l_{iit}, k_{iit}, z_{it}) - F^{ii}_{l,t} l_{iit} - F^{ii}_{k,t} k_{iit} - F^{ii}_{z,t} l^z_{it} H^i_{l,t} \right]$$
$$+ \sum_{j \neq i} \left[\left(1-\tau^p_{jt}\right) \hat{p}_{ijt} \left[F^{ij}(l_{ijt}, k_{ijt}, z_{it}) - F^{ij}_{l,t} l_{ijt} - F^{ij}_{k,t} k_{ijt} - F^{ij}_{z,t} l^z_{it} H^i_{l,t} \right] \right] \Bigg]$$
$$= 0.$$

The dividend in the profit shifting scenario is:

$$d_{it} = \pi_{iit}(z_{it}) + (1-\tau^p_{it}) \left[-w_{it} l^z_{it} + (1-\lambda_{it}) \sum_{j \neq i} \vartheta_{ijt} z_{it} + \lambda_{it} \Big((\varphi - \mathcal{C}(\lambda_{it})) \nu_{it} z_{it} - \vartheta_{iit} z_{it} \Big) \right]$$
$$+ \sum_{j \neq i} \left[\pi_{ijt}(z_{it}) - (1-\tau^p_{jt}) \vartheta_{ijt} z_{it} \right] + (1-\tau^p_{THt}) \lambda_{it} (1-\varphi) \nu_{it} z_{it}.$$

The first-order conditions for the intermediate-good producers are:

$$l_{iit} : w_{it} = p_{iit} F^{ii}_{l,t} = \hat{p}_{jit} F^{ji}_{l,t}$$
$$l^z_i : w_{it} = \Big[p_{iit} F^{ii}_{z,t} H^i_{l,t} + (1-\lambda_{it}) H^i_{l,t} \sum_{j \neq i} \vartheta_{ijt} - \lambda_{it} H^i_{l,t} \vartheta_{iit}$$
$$- \mathcal{C}_i(\lambda_{it}) H^i_{l,t} \sum_j \vartheta_{ijt} + \varphi \lambda_{it} H^i_{l,t} \sum_j \vartheta_{ijt}$$
$$+ \sum_{j \neq i, TH} \frac{\left(1-\tau^p_{jt}\right)}{\left(1-\tau^p_{it}\right)} \left[\hat{p}_{ijt} F^{ii}_{z,t} H^i_{l,t} - H^i_{l,t} \vartheta_{ijt} \right] + \frac{\left(1-\tau^p_{THt}\right)}{\left(1-\tau^p_{it}\right)} \lambda_{it} (1-\varphi) H^i_{l,t} \sum_j \vartheta_{ijt}$$
$$k_{ijt} : r_{jt} = \left(1-\tau^p_{jt}\right) \left(\hat{p}_{ijt} F^{ij}_{k,t} - p_{jt} \delta \right)$$
$$q_{ijt} : p_{iit} = \left(1-\tau^x_{ijt}\right) p_{ijt}.$$

Following the same procedure and assuming that $\vartheta_{ijt} = \hat{p}_{ijt} F^{ij}_{z,t}$, the sum of discounted dividends can be derived as

$$\sum_{t=0}^{\infty} Q_t d_{it} = \sum_{t=0}^{\infty} Q_t \Bigg[(1-\tau_{it}^p)\, p_{iit} \left[F^{ii}(l_{iit}, k_{iit}, z_{it}) - F_{l,t}^{ii} l_{iit} - F_{k,t}^{ii} k_{iit} - F_{z,t}^{ii} l_{it}^z H_{l,t}^i\right] + \sum_{j\neq i} \left[\left(1-\tau_{jt}^p\right) \hat{p}_{ijt} \left[F^{ij}(l_{ijt}, k_{ijt}, z_{it}) - F_{l,t}^{ij} l_{ijt} - F_{k,t}^{ij} k_{ijt} - F_{z,t}^{ij} l_{it}^z H_{l,t}^i\right]\right] \Bigg] = 0.$$

■

Proof of Proposition (3)**.** The proof that balance of payments is satisfied follows directly from arguments in Chari et al. (2023). Toward showing that one can implement efficient allocation of intangible capital, first impose static efficiency to get

$$\frac{F_{l,t}^{ii}}{F_{z,t}^{ii} H_{i,t}^i} = \left(1 + \sum_{j\neq i} \left(\frac{u_{c,t}^j G_{\hat{\imath},t}^j F_{z,t}^{ij}}{u_{c,t}^i G_{i,t}^i F_{z,t}^{ii}}\right)\right) \Omega\left(\tau_{it}^p\right) \tag{C.7}$$

now we want to find taxes that implement the (52) using C.7. Thus equating the right hand sides we get:

$$\left(1 + \sum_{j\neq i} \left(\frac{u_{c,t}^j G_{\hat{\imath},t}^j F_{z,t}^{ij}}{u_{c,t}^i G_{i,t}^i F_{z,t}^{ii}}\right)\right) \Omega\left(\tau_{it}^p\right) = 1 + \sum_{m\neq i} \frac{v_{c,t}^m G_{\hat{\imath},t}^m F_{z,t}^{im}}{v_{c,t}^i G_{i,t}^i F_{z,t}^{ii}} + \sum_{m\neq i} \frac{H_{i,t}^m}{H_{i,t}^i} \left[\frac{G_{\hat{m},t}^i F_{z,t}^{mi}}{G_{i,t}^i F_{z,t}^{ii}} + \frac{v_{c,t}^m}{v_{c,t}^i} \frac{G_{m,t}^m F_{z,t}^{mm}}{G_{i,t}^i F_{z,t}^{ii}}\right] + \sum_{m\neq i} \frac{v_{c,t}^m}{v_{c,t}^i} \sum_{n\neq i,m} \frac{H_{i,t}^n}{H_{i,t}^i} \frac{G_{\hat{n},t}^m F_{z,t}^{nm}}{G_{i,t}^i F_{z,t}^{ii}}$$

thus we have

$$\Omega\left(\tau_{it}^p\right) = \left(1 + \sum_{m\neq i} \frac{v_{c,t}^m G_{\hat{\imath},t}^m F_{z,t}^{im}}{v_{c,t}^i G_{i,t}^i F_{z,t}^{ii}} + \sum_{m\neq i} \frac{H_{i,t}^m}{H_{i,t}^i} \left[\frac{G_{\hat{m},t}^i F_{z,t}^{mi}}{G_{i,t}^i F_{z,t}^{ii}} + \frac{v_{c,t}^m}{v_{c,t}^i} \frac{G_{m,t}^m F_{z,t}^{mm}}{G_{i,t}^i F_{z,t}^{ii}}\right] + \sum_{m\neq i} \frac{v_{c,t}^m}{v_{c,t}^i} \sum_{n\neq i,m} \frac{H_{i,t}^n}{H_{i,t}^i} \frac{G_{\hat{n},t}^m F_{z,t}^{nm}}{G_{i,t}^i F_{z,t}^{ii}}\right) \times \left(1 + \sum_{j\neq i} \left(\frac{u_{c,t}^j G_{\hat{\imath},t}^j F_{z,t}^{ij}}{u_{c,t}^i G_{i,t}^i F_{z,t}^{ii}}\right)\right)^{-1}$$

where we have from Lemma 2 that

$$\lim_{\tau_{it}^p \to 1} \Omega\left(\tau_{it}^p\right) = \infty$$

$$\lim_{\tau_{it}^p \to \tau_{TH}^p} \Omega\left(\tau_{it}^p\right) = 0$$

then by mean value theorem there exists a unique $\widehat{\tau}^{p}_{it} > \tau^{p}_{TH}$ that implements Ramsey allocation. ■

Optimal conditions when $t = 0$

In period 0, we use replace equation (B.30) with (B.37) as the first-order condition with respect to consumption We can first get the no intratemporal wedge condition as

$$-\frac{v^i_{c,0} + \varphi^i \frac{\partial \mathcal{W}_{i0}}{\partial c_{i0}}}{v^i_{h,0}} = \frac{1}{G^i_{i,t} F^{ii}_{l,0}} = \frac{1}{G^i_{\hat{j},t} F^{ji}_{l,0}} \qquad \forall_i \forall_{j \neq i}$$

The no intertemporal wedge condition is

$$\frac{v^i_{c,0} + \varphi^i \frac{\partial \mathcal{W}_{i0}}{\partial c_{i0}}}{\beta v^i_{c,1}} = \left((1-\delta) + G^i_{i,1} F^{ii}_{k,1}\right) = \left((1-\delta) + G^i_{\hat{j},1} F^{ji}_{k,1}\right) \qquad \forall_i \forall_{j \neq i}$$

The static efficiency condition is

$$\frac{G^i_{n,0}}{G^i_{m,0}} = \frac{G^n_{n,0}\left(v^n_{c,0} + \varphi^n \frac{\partial \mathcal{W}_{n0}}{\partial c_{n0}}\right)}{G^m_{m,0}\left(v^m_{c,0} + \varphi^n \frac{\partial \mathcal{W}_{m0}}{\partial c_{m0}}\right)} \qquad \forall_i \forall_{m,n \neq i}$$

And the dynamic efficiency condition is

$$\frac{G^i_{j,0}}{G^i_{j,1}}\left((1-\delta) + G^i_{i,1} F^{ii}_{k,1}\right) = \left(\frac{G^j_{j,0}\left(v^c_{c,0} + \varphi^j \frac{\partial \mathcal{W}_{j0}}{\partial c_{j0}}\right)}{G^j_{j,1} \beta v^j_{c,1}}\right) \qquad \forall_j$$

Finally, the intangible efficiency is

$$\frac{F^{ii}_{l,0}}{F^{ii}_{z,0} H^i_{i,0}} = 1 + \sum_{j \neq i} \frac{v^j_{c,0} G^j_{\hat{i},0} F^{ij}_{z,0}}{v^i_{c,0} G^i_{i,0} F^{ii}_{z,0}}$$
$$+ \sum_{j \neq i} \left[\frac{H^j_{i,0}}{H^i_{i,0}}\left(\frac{G^i_{\hat{j},0} F^{ji}_{z,0}}{G^i_{i,0} F^{ii}_{z,0}} + \frac{v^j_{c,0} G^j_{j,0} F^{jj}_{z,0}}{v^i_{c,0} G^i_{i,0} F^{ii}_{z,0}}\right) + \sum_{k \neq i,j} \frac{H^k_{i,0}}{H^i_{i,0}} \frac{v^j_{c,0} G^j_{\hat{k},0} F^{kj}_{z,0}}{v^c_{i,0} G^i_{i,0} F^{ii}_{z,0}}\right] \qquad \forall_i$$

D Quantitative model details

Here we provide more details about the quantitative model, which builds closely on Dyrda et al. (2022). Time is discrete and indexed by $t = 1, 2, \ldots$. There are I regions indexed by i and j, each populated by a representative household, a measure of heterogeneous firms, and a government. Regions differ in population, total factor productivity, trade costs, FDI costs, labor income taxes, and corporate income taxes. Households choose consumption, labor supply, tangible investment, and bond holdings. Firms in each region decide the following: where to export and where to open foreign subsidiaries; how much labor to hire and tangible capital to rent in the parent division and each subsidiary; and how much intangible capital to produce in the parent division. As in McGrattan and Prescott (2009). intangible capital is nonrival and is used simultaneously in all of a firm's divisions, both foreign and domestic.

Multinational firms (firms with foreign affiliates) use transfer pricing to allocate the costs of producing intangible capital across their foreign affiliates in proportion to the scale at which these affiliates use this capital. Affiliates license the right to use intangible capital from the division that owns this capital, and MNEs can shift profits by selling their intangible capital to affiliates in lower-tax regions. We denote the region with the lowest corporate income tax rate by LT. Additionally, there is an unproductive tax haven that is populated by a representative household and a government, labelled as TH, where no economic activity takes place. MNEs based in high-tax regions can transfer their intangible capital rights to either the low-tax region or the tax haven, provided that they have established affiliates there.

Throughout this appendix, we use capitals to denote aggregate variables and lower-cases to denote microeconomic firm-level variables. We omit time subscripts where appropriate for brevity,

D.1 Households

Each region i has a representative household with preferences over sequences of consumption, $\{C_{it}\}_{t=0}^{\infty}$, and labor supply, $\{L_{it}\}_{t=0}^{\infty}$, given by

$$\sum_{t=0}^{\infty} \beta^t \left[\log\left(\frac{C_{it}}{N_i}\right) + \psi_i \log\left(1 - \frac{L_{it}}{N_i}\right)\right]. \tag{D.1}$$

Households choose consumption, labor supply, tangible investment, $\{X_{it}\}_{t=0}^{\infty}$, and internationally-traded bonds, $\{B_{it+1}\}_{t=0}^{\infty}$ to maximize utility subject to a sequence of budget constraints,

$$P_{it}[(1+\tau_{ict})C_{it} + X_{it}] + P_{bt}B_{it+1} = (1-\tau_{i\ell t})\, W_{it}L_{it} + (1-\tau_{ikt})R_{it}K_{it} + B_{it} + D_{it}, \tag{D.2}$$

a law of motion for tangible capital,

$$K_{it+1} = (1-\delta)K_{it} + X_{it}, \tag{D.3}$$

and initial conditions on capital and bonds, K_{i0} and B_{i0}. Households take the wage, W_{it}, the labor income tax rate, $\tau_{i\ell t}$, the rental rate, R_{it}, the bond price, P_{bt}, and dividends, D_{it}, as given. The first-order conditions of the household's problem are

$$\frac{C_{it}/N_i}{1-L_{it}/N_i} = \frac{(1-\tau_i)W_{it}}{(1+\tau_{cit})P_{it}}, \tag{D.4}$$

$$P_{bt} = \beta \frac{C_{it}}{C_{it+1}} \frac{P_{t+1}}{P_t}, \tag{D.5}$$

$$1 = \beta \frac{C_{it}}{C_{it+1}} \left(1 - \delta + \frac{(1-\tau_{ikt+1})R_{it+1}}{P_{it+1}}\right). \tag{D.6}$$

We define r_{it} as the ex-depreciation rental rate on tangible capital. Note that in a steady state, the Euler equation for capital is

$$1 = \beta \left(1 - \delta + \frac{(1-\tau_{ik})R_i}{P_i}\right) \Rightarrow R_i = \frac{P_i}{(1-\tau_{ik})} \left[\frac{1-\beta}{\beta}\delta\right]. \tag{D.7}$$

D.2 Final goods

Each region has a representative final-good producer that combines domestic and foreign products into a nontradable aggregate that is bought by households and the government for consumption. The final good is a constant-elasticity-of-substitution aggregate of products from different source countries.

$$Q_{it} = \left[\sum_{j=1}^{J} \int_{\Omega_{jit}} q_{jit}(\omega)^{\frac{\rho-1}{\rho}} d\omega\right]^{\frac{\rho}{\rho-1}}, \tag{D.8}$$

where $q_{jit}(\omega)$ is the quantity of variety ω from region j, Ω_{jit} is the set of goods from j available in i (determined by firms' exporting and FDI decisions specified later), and ρ is the elasticity of substitution between varieties. The demand curve for each variety can be written as

$$p_{jit}(\omega) = P_{it} Q_{it}^{\frac{1}{\rho}} q_{jit}(\omega)^{-\frac{1}{\rho}}. \tag{D.9}$$

The aggregate price index is

$$P_{it} = \left[\sum_{j=1}^{J} \int_{\Omega_{jit}} p_{jit}(\omega)^{1-\rho} d\omega\right]^{\frac{1}{1-\rho}}. \tag{D.10}$$

D.3 Intermediate firm's problem

A firm's objective is to maximize its dividend payout,

$$d_{it}(a) = \pi_{iit}(a, z; J_X) + \sum_{j \in J_F} \pi_{ijt}(a, z) + \mathbb{1}_{\{\lambda_{TH}>0\}} \pi_{i,TH,t}(a, z; J_F), \tag{D.11}$$

in each period by choosing the following objects: where to export and open foreign affiliates, J_X and J_F; how much intangible capital to produce, z; how much local labor and tangible capital to hire in each of its divisions, $\boldsymbol{\ell} = (\ell_{ij})_{j \in I}$ and $\boldsymbol{k} = (k_{ij})_{j \in I}$; how much to sell to each of its markets through exporting and/or FDI, $\boldsymbol{q} = (q_{ij}, \hat{q}_{ij})_{j \in I}$; and how much of its intangible capital property rights to shift, $\boldsymbol{\lambda} = (\lambda_{LT}, \lambda_{TH})$.

The domestic parent corporation's after-tax profits, $\pi_{ii}(a, z; J_x)$, are given by

$$\begin{aligned} \pi_{iit}(a, z; J_X) = (1 - \tau_{it}) \Bigg\{ & p_{iit}(q_{iit}) q_{iit} + \sum_{j \in J_{Xt}} p_{ijt}(q_{ijt}) q_{ijt} + \sum_{j \in J_F} (1 - \lambda_{LTt} - \lambda_{THt}) \vartheta_{ijt}(z) z \\ & - W_{it} \left(\ell_{iit} + \frac{z}{A_i} + \sum_{j \in J_X} \kappa_{iX} + \sum_{j \in J_F} \kappa_{iF} + \kappa_{i,TH} \mathbb{1}_{\{\lambda_{TH}>0\}} \right) - \delta P_{it} k_{it} \\ & - W_{it} \left[\mathcal{C}_{i,TH}(\lambda_{TH}) + \mathcal{C}_{i,LT}(\lambda_{LT}) \right] \nu_{it}(z) z - (\lambda_{TH} + \lambda_{LT}) \vartheta_{iit}(z) z \Bigg\} - r_{it} k_{it}. \end{aligned} \tag{D.12}$$

The first line contains revenues from sales and licensing the portion of intangible capital that is not transferred to the low-tax region or the tax haven. The second line contains labor costs of domestic production workers, workers hired to set up export relationships and foreign affiliates, and depreciation expenses. The last line contains labor costs of workers hired to engage in profit shifting, licensing fees paid to the low-tax region and the tax haven, and capital expenditures net of depreciation. After-tax profits in foreign affiliates in high-tax regions are given by

$$\pi_{ijt}(a, z) = (1 - \tau_{jt}) \left[p_{ijt}(\hat{q}_{ijt}) \hat{q}_{ijt} - W_{jt} \ell_j - \delta P_{jt} k_{jt} - \vartheta_{ijt}(z) z \right] - r_{jt} k_{jt}, \; j \in J_F \backslash \{LT\}. \tag{D.13}$$

The low-tax affiliates' after-tax profits are

$$\begin{aligned} \pi_{i,LT,t}(a, z; J_X) = (1 - \tau_{LTt}) \Bigg[& p_{i,LT,t}(\hat{q}_{i,LT,t}) \hat{q}_{i,LT,t} + \sum_{j \in J_F \cup \{i\} \backslash \{LT\}} \lambda_{LT,t} \vartheta_{ijt}(z) z \\ & - W_{LT,t} \ell_{LT,t} - \delta P_{LT,t} k_{LT,t} - (1 - \lambda_{LT,t}) \vartheta_{i,LT,t}(z) z \Bigg] - r_{LT,t} k_{LT,t}. \end{aligned} \tag{D.14}$$

The first line includes revenues from sales and licensing fees generated by the portion of intangible capital that is transferred to this affiliate. The second line includes labor and capital costs, and licensing fees paid on the portion of intangible capital that is retained by the parent. Finally, the after-tax profits of the tax-haven affiliate, which only include licensing fees, are given by

$$(1-\tau_{TH,t})\left[\sum_{j\in J_{Ft}\cup\{i\}} \lambda_{TH}\vartheta_{ijt}(z)z\right]. \tag{D.15}$$

We denote the firm's policy functions by $z_{it}(a)$, $J_{iXt}(a)$, $J_{iFt}(a)$, $\boldsymbol{\ell}_t(a)$, $\boldsymbol{k}_t(a)$, $\boldsymbol{q}_{it}(a)$, $\boldsymbol{p}_{it}(a)$, $\boldsymbol{\lambda}_t(a)$, and $\boldsymbol{\vartheta}_{it}(a)$. It is also useful to define $\boldsymbol{\pi}_t(a)$ and $\tilde{\boldsymbol{\pi}}_t(a)$ as the after-tax and taxable profits, respectively, in each division implied by these policy functions.

D.4 Aggregation and accounting measures

We revert to expressing firms' choices as functions of their varieties (ω) for notational brevity in defining national accounting measures and other macroeconomic aggregates.

Gross domestic product. Nominal GDP in each region i is the total value of goods produced by domestic firms and local affiliates of foreign MNEs:

$$GDP_{it} = \sum_{j=1}^{I} \int_{\omega\in\Omega_j, i\in J_{Ft}(\omega)} p_{jit}(\omega) y_{jit}(\omega)\ d\omega. \tag{D.16}$$

We compute real GDP by deflating by the consumer price index P_{it} defined in (D.10).

Goods trade. Aggregate exports and imports of goods are given by

$$EX_{it}^{G} = \sum_{j\neq i} \int_{\Omega_i} p_{ijt}^{X}(\omega)\left(1+\xi_{ij}\right) q_{ijt}^{X}(\omega)\ d\omega, \tag{D.17}$$

$$IM_{it}^{G} = \sum_{j\neq i} \int_{\Omega_j} p_{jit}^{X}(\omega)\left(1+\xi_{ji}\right) q_{jit}^{X}(\omega)\ d\omega. \tag{D.18}$$

Services trade. Intangible capital licensing fees enter the national accounts as net exports of intellectual property services. High-tax regions' services trade flows are given by

$$EX_{it}^{S} = \sum_{j \neq i} \int_{\Omega_i} \left[1 - \lambda_{LT,t}(\omega) - \lambda_{TH,t}(\omega)\right] \vartheta_{ijt}(\omega) z_{it}(\omega)\ d\omega, \tag{D.19}$$

$$IM_{it}^{S} = \sum_{j \neq i} \int_{\Omega_i} \left[\lambda_{LT,t}(\omega) + \lambda_{TH,t}(\omega)\right] \vartheta_{ijt}(\omega) z_{it}(\omega)\ d\omega + \sum_{j \neq i} \int_{\Omega_j} \vartheta_{jit}(\omega) z_{it}(\omega)\ d\omega. \tag{D.20}$$

The low-tax region's services trade flows are

$$EX_{LT,t}^{S} = \sum_{j \neq i} \int_{\Omega_i} \left[1 - \lambda_{TH,t}(\omega)\right] \vartheta_{ijt}(\omega) z_{it}(\omega)\ d\omega + \sum_{j \neq i} \int_{\Omega_j} \lambda_{LT,t} \vartheta_{jit}(\omega) z_{it}(\omega)\ d\omega, \tag{D.21}$$

$$IM_{LT,t}^{S} = \sum_{j \neq i} \int_{\Omega_i} \lambda_{TH,t}(\omega) \vartheta_{ijt}(\omega) z_{it}(\omega)\ d\omega + \sum_{j \neq i} \int_{\Omega_j} [1 - \lambda_{LT,t}(\omega)] \vartheta_{jit}(\omega) z_{it}(\omega)\ d\omega. \tag{D.22}$$

We can also write the tax haven's services exports as

$$EX_{TH,t}^{S} = \sum_{j=1}^{I} \int_{\Omega_j} \lambda_{TH,t} \vartheta_{jit}(\omega) z_{it}(\omega)\ d\omega. \tag{D.23}$$

Net factor receipts and payments. Net factor receipts from (payments to) are the sum total of the dividends paid by foreign subsidiaries of domestic multinationals (domestic subsidiaries of foreign multinationals):

$$NFR_{it} = \sum_{j \neq i} \int_{\Omega_i} (1 - \tau_j) \pi_{ijt}(\omega)\ d\omega, \tag{D.24}$$

$$NFP_{it} = \sum_{j \neq i} \int_{\Omega_j} (1 - \tau_i) \pi_{jit}(\omega)\ d\omega. \tag{D.25}$$

Profit shifting. Following Dyrda et al. (2022), we define $\hat{\pi}_{ijt}(\omega)$ as the profits a firm would have reported in region j if it did not shift profits, holding fixed all of its other policy functions. Then, we can define the profits shifted out of region j by firm ω as

$$ps_{ijt}(\omega) = \hat{\pi}_{ijt}(\omega) - \pi_{ijt}(\omega). \tag{D.26}$$

When $ps_{ijt}(\omega) > 0$, this indicates that the firm would book more profits in region j in the absence of profit shifting, i.e., the firm has shifted profits away from region j. Aggregating firm-level shifted profits yields the total profits shifted out of region j:

$$PS_{jt} = \sum_{i=1}^{I} \int_{\Omega_i} ps_{ijt}(\omega)\ d\omega. \tag{D.27}$$

D.5 Market clearing and equilibrium

In equilibrium, the government's budget constraint must be satisfied, the markets for labor, capital, and final goods must be satisfied, and the balance of payments must hold in each productive region.

Government budget constraint. Government spending, G_i, must equal revenue from labor income taxes and corporate taxes:

$$P_{it}G_{it} = \tau_{i\ell t}W_{it}L_{it} + \tau_i \sum_{j=1}^{I} \int_{\Omega_j} \tilde{\pi}_{jit}(\omega)\ d\omega. \tag{D.28}$$

Government consumption, G_i, is an exogenous parameter. In our calibration we set it equal to total tax revenues, but in our counterfactual experiments we hold it fixed and adjust the labor income tax rate, τ_{ilt}, to restore fiscal balance.

Labor market. Labor demand comes from four sources: production of intermediate goods; production of intangible capital; fixed costs of exporting and setting up foreign affiliates; and the costs of transferring intangible capital. The labor market clearing condition can be written as

$$L_{it} = \sum_{j=1}^{I} \int_{\Omega_j} \ell_{jit}(\omega)\ d\omega + \int_{\Omega_i} \left[\frac{z_{it}(\omega)}{A_i} + \sum_{j \in J_{Xt}(\omega)} \kappa_{iX} + \sum_{j \in J_{Ft}(\omega)} \kappa_{iF} + \mathbb{1}_{\{\lambda_{TH,t}(\omega)>0\}} \kappa_{i,TH} \right] d\omega$$
$$+ \int_{\Omega_i} \left[\mathcal{C}_{i,TH}(\lambda_{TH,t}(\omega)) + \mathcal{C}_{i,LT}(\lambda_{LT,t}(\omega)) \right] \nu_{it}(\omega) z_{it}(\omega)\ d\omega. \tag{D.29}$$

Capital market. The capital market clearing condition is

$$K_{it} = \sum_{j=1}^{I} \int_{\Omega_j} k_{jit}(\omega)\ d\omega. \tag{D.30}$$

Final goods market. Final goods market clearing requires that production of final goods equals the sum of private consumption, public consumption, and investment in each region:

$$Q_{it} = C_{it} + G_i + X_{it}. \tag{D.31}$$

Balance of payments. Each region's balance of payments must hold:

$$EX_{it}^G + EX_{it}^S - IM_{it}^G - IM_{it}^S + NFR_{it} - NFP_{it} = P_{bt}B_{it+1} - B_{it}. \tag{D.32}$$

Competitive equilibrium. Given a set of parameters, an equilibrium in our model is a sequence of bond prices, $\{Q_t\}_{t=0}^{\infty}$, a sequence aggregate prices and quantities for each region, $\{W_{it}, P_{it}, C_{it}, L_{it}\}_{t=0}^{\infty}$, and a sequence of firm-level policy functions for each region, $\{J_{iXt}(\omega), J_{iFt}(\omega), z_{it}(\omega), \boldsymbol{\ell}_{it}(\omega), \boldsymbol{k}_{it}(\omega)\boldsymbol{q}_{it}(\omega), \boldsymbol{p}_{it}(\omega), \boldsymbol{\pi}_{it}(\omega), \boldsymbol{\lambda}_{it}(\omega)\}_{t=0}^{\infty}$, that satisfy

1. the representative household's utility maximization problem (D.1)–(D.10);
2. the firm's profit-maximization problem (D.11);
3. the labor market clearing condition (D.29);
4. the capital market clearing condition (D.30);
5. the government's budget constraint (D.28); and
6. the balance of payments (D.32).

A stationary equilibrium is a competitive equilibrium in which the objects listed above are constant over time. In this paper, we restrict attention to stationary equilibria in which all regions have balanced current accounts, i.e., $B_{it+1} = 0$ for all i.[9]

[9]In this class of model there are many possible stationary equilibria, each associated with a different vector of bond holdings (Kehoe et al., 2018). Given a set of initial conditions, the stationary equilibrium to which the model will converge in the long run is unique, but solving for transition dynamics is an immense computational undertaking. Our assumption that all countries have balanced current accounts is common in the literatures on both trade and international macro.