Uncertainty Modelling in Multi-agent Information Fusion Systems

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ABSTRACT
In the field of informed decision-making, the usage of a single diagnostic expert system has limitations when dealing with complicated circumstances. The usage of a multi-agent information fusion (MAIF) system can mitigate this situation, as it allows multiple agents collaborating to solve the problems in a complex environment. However, the MAIF system needs to handle the uncertainty problem between different agents objectively at the same time. Target to this goal, this study reconstructs the generation of basic probability assignments (BPAs) based on the framework of evidence theory, and presents the uncertainty relationship between recognition sets, which are beneficial to the applications of the MAIF system. On the basis of evidence distance measurement, our method demonstrates the effectiveness and extendibility in numerical examples, and improves the accuracy and anti-interference ability during the identification process in the MAIF system.

KEYWORDS
Evidence Theory; Uncertainty; Multi-agent Information Fusion; Reconstructed BPA

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1 INTRODUCTION
In modern engineering applications, electronic information systems tend to be highly integrated and have many components and complex functions. Thus, concurrency, suddenness, complexity are the three main problems that might occur when equipment fails [29, 38, 45]. Among many information systems, multi-source information systems occupy a certain degree of proportion, which are often used to represent complex information [50] from multiple sources. However, how to effectively integrate multi-source information [36] and measure its uncertainty to ensure the correctness and anti-interference in the process of information fusion and diagnosis has become the focus of many scholars [8, 21, 31]. The single diagnostic method like neural networks [11] or expert evaluation [22, 43] used in the past has certain limitations and is not able to accurately feedback the status of the current information system in the realistic environment [20]. Therefore, it is necessary to comprehensively weigh the information [25] from many aspects to get the correct conclusion. The multi-agent information fusion (MAIF) system based on the fusion agent provides a solution from the distribution to the unity.

The multi-agent information fusion system mainly studies the interactive communication, coordinative cooperation, and conflict resolution between various agents. It focuses on the fusion analysis of information between multiple agents, rather than the autonomy and development of individual agent. In the process of multi-agent information fusion, considering that the inference models used by each agent are not necessarily the same, the given conclusions may be inconsistent even under the circumstances that all agents use the same original detected data at the starting point. This inconsistency is mainly reflected in two situations. Firstly, different agents conclude the same answer while their credibility degree towards it is different. For instance, suppose a MAIF system needs to identify the type of failure, both agent A and B thought the fault was attributed to "connection failure", but agent A gave the probability of 90% while agent B did 75%, saying that agent A's credibility is higher even if they have the same conclusion. Secondly, different conclusions emerge between different agents, indicating that the information is contradictory. For the first situation, we require to optimize the information fusion process for obtaining a result with higher accuracy. As for the second situation, it is crucial to reduce interference when facing highly conflicting information effectively. In response to these two situations, there have been many attempts to improve the performance, such as distributed weighting [9] and relative reliability evaluation [6, 24].

However, they fail to focus on the measurement of uncertainty between information sources from various agencies, and the methods are somewhat a little bit rough when combining conclusions for the multi-agent information fusion process. If introducing the metric of uncertainty modeling [14, 48], it could characterize the range of the measured value, such as the representative information of the degree to which the measured raw information cannot be determined. The primary process of uncertain modeling is using the original field data, removing the severe errors in the original data, extract and converting the useful information to the fusion layer for selection. For example, Hunter [15] used an adaptive algorithm for merging multiple source uncertain information by assessing the coherence of the information in each subset which helps the fusion process can integrate both conjunctive and disjunctive operators more flexibly.

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In the field of MAIF, evidence theory has good applicability and scalability in dealing with uncertainty information, so it is the potential to be integrated into the MAIF system. However, the D-S evidence theory may fail in the case of high conflicting situations, making it difficult to guarantee the fusion result. Two types of approaches have been identified to improve the performance of MAIF: modifying the fusion rules, and preprocessing the uncertainty information in the system prior to the step of data fusion. The previous studies have put forward the strategy of preprocessing and provided the distance measurement for effectively measuring the uncertainty of MAIF system, but the measurement object is mainly based on multiple sources of data as a whole, lacking measuring a specific distinction between two sources of data. Therefore, the method proposed in this paper will cover the advantages of both ideas and perform better data preprocessing. In this study, we developed a new concept call reconstructed basic probability assignment (BPA) by measuring detailed uncertainties within the framework of evidence theory, and proposed a comprehensive fusion method by combining with evidence distance to show its effectiveness in the application of MAIF systems.

The remaining parts of this paper are arranged as follows. In Section 2, we briefly introduce the basic definition of evidence theory and evidence distance. In Section 3, our proposed method is presented based on reconstructed BPA and the previous concept mentioned. In Section 4, we show the performance of our proposed method based on two numerical examples. Furthermore, the sensitivity test is given in Section 5. Finally, the conclusion of this paper is put forward in Section 6.

2 PRELIMINARIES

In this section, some basic information on MAIF system, Dempster-Shafer evidence theory, Jousselme evidence distance, and its relevant combination rule will be briefly introduced.

2.1 MAIF system

MAIF is short for Multi-Agent Information Fusion System, which presents as an autonomous scheme that solves system problems employing information fusion between agents in a multi-agent system. The concepts of the agent, multi-agent system (MAS), and information fusion will be introduced in detail.

2.1.1 The definition of agent. The following definitions are widely recognized in the field. Wooldridge and Jennings [16] released a simple definition: an agent is a computer system in an environment that has the ability to act autonomously in this environment to achieve its design goals. Then, Minsky [19] believes that each agent possesses its own wisdom to do some simple tasks. When we use a specific method to form these agents into an agent group, the agent intelligence comes along. For the development of agent software tools, it describes agents as the software programs that can perform specific tasks for users, showing a certain degree of intelligence when performing tasks autonomously and interacting with the environment.

2.1.2 Multi-agent system. MAS [34] is an agent society composed of multiple agents and is considered as a distributed autonomous system. It suggests that the multi-agent system contains multiple computing units, referred to as agents, which can interact with each other. The performance of MAS could be achieved through the interaction of agents, by investigating how multiple agents coordinate their knowledge, goals, plans, and strategies to take joint actions or solve problems.

2.1.3 Information fussion. Information fusion [2] is an applied area about combining data from multiple sources to support decision analysis. At present, research methods such as fuzzy theory, neural network, evidence theory and etc. occupy a considerable proportion in this field. Because MAS and information fusion method have some commonality in processing data, agent-based information fusion technology in recent years is expected to provide a new perspective for processing problems in complex engineering application systems and military areas.

2.2 Dempster-Shafer evidence theory

The D-S evidence theory was proposed by Harvard mathematician A.P. Dempster in 1967 [3]who first introduced the concept of upper and lower bound to solve the multivalued mapping problem. After that, his student G. Shafer further refined the evidence theory by proposing belief function, forming a group of evidence combination to illustrate uncertainty reasoning.

D-S evidence theory is an extension [41] of Bayes theory. Different from Bayes theory requiring prior information for probability calculation, evidence theory showed the better performance to deal with uncertain information without prior probability.

2.2.1 Framework of discernment. Suppose there is a problem that needs to be judged. For all the possible solutions (e.g., θ_1) recognized by this problem, they are depicted by an exhaustive and finite set Θ, which is also known as the framework of discernment (FOD). The expression in details is:

$$\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}. \tag{1}$$

Where $\theta_i (i = 1, 2, ..., n)$ is an element or event in the FOD and let 2^Θ indicates the power set of Θ, namely:

$$2^\Theta = \{\emptyset, \theta_1, \cdots, \theta_n, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \cdots, \Theta\}. \tag{2}$$

2.2.2 Basic probability assignment. The D-S evidence theory assigns a probability to each hypothesis in the framework of discernment (FOD), referred as basic probability assignment (BPA) due to good scalability [7, 32]. The corresponding assignment function is called the mass function. Let Θ be the discernment framework, if the set function $m : 2^\Theta \to [0, 1]$ satisfies:

$$\sum_{A \subseteq \Theta} m(A) = 1 \quad m(\emptyset) = 0 \tag{3}$$

where m is the basic probability assignment function or mass function of the FOD 2^Θ and $m(A)$ is the BPA value of proposition A, indicating the degree which evidence trust in A. The value of $m(\emptyset)$ is 0.

2.2.3 D-S evidence combination rule. D-S combination rule is the core of D-S evidence theory, which combines the information generated by multiple subjects [42] (it may be different experts' predictions, data obtained from different sensors or diagnostic results of different electronic devices, etc.). It has the following three advantages: 1) satisfy weaker conditions than Bayes probability

theory; 2) integrate a variety of data or knowledge; 3) express "uncertainty" directly.
Suppose there exist two BPAs m_1 and m_2 on the same FOD. $B_1, ..., B_k$ and $C_1, ..., C_k$ are the focal elements of m_1 and m_2 respectively. According to the D-S combination rule, the belief function $m(A)$ is:

$$\begin{gathered} m_1 \bigoplus m_2(A) = \frac{1}{K} \sum_{B \cap C = A} m_1(B) \cdot m_2(C) \\ \text{where } K = \sum_{B \cap C \neq \phi} m_1(B) \cdot m_2(C) = 1 - \sum_{B \cap C = \phi} m_1(B) \cdot m_2(C) \end{gathered} \tag{4}$$

Where K is a normalization coefficient which represents the conflict degree between two evidences, characterized as an empty intersection of B and C.

2.3 Evidence distance

D-S combination rule can be counterintuitive in the case of highly conflicting evidence occur. To mitigate this problem, a previous study [5] presented the concept of evidence distance to measure the similarity between evidence, which is important in the application of evidence theory. Same as the traditional distance definition, the evidence distance should satisfy the characteristics of non-negative, symmetrical, reflexive, and triangular inequality. The evidence distance is generally divided into two types according to different construction methods. One is indirectly defined by other measures related to the evidence, and the evidence itself directly determines the other. Jousselme proposed the Jousselme distance formula [18] based on evidence theory's geometric interpretation, which is proved to be a rigorous distance definition because it satisfies the four axiomatization conditions mentioned above.

2.3.1 Jousselme evidence distance. Assume that the discernment framework Θ contains N elements, a high-dimensional space can be constructed by identifying the elements in the frame as coordinates. Each evidence is represented as a point or a vector in this space. If m_i, m_j are two independent evidences in the FOD, representing them as vectors in space is $\overrightarrow{m_i}, \overrightarrow{m_j}$. Then, the Jousselme distance between $\overrightarrow{m_i}, \overrightarrow{m_j}$ is defined as:

$$d_{BPA}(m_i, m_j) = \sqrt{\frac{1}{2}(\overrightarrow{m_i} - \overrightarrow{m_j})^T D(\overrightarrow{m_i} - \overrightarrow{m_j})} \tag{5}$$

Where D represents a matrix whose size is $2^N \times 2^N$, and N is the number of elements contained in the FOD. In matrix D, $D(A, B) = \frac{A \cap B}{A \cup B}$ which A, B represent the subset of FOD. The detailed calculation of Jousselme evidence distance is:

$$d_{BPA}(m_i, m_j) = \sqrt{\frac{1}{2}(\|\vec{m}_i\|^2 + \|\vec{m_j}\|^2 - 2 < \overrightarrow{m_i}, \overrightarrow{m_j} >)} \tag{6}$$

$$< \overrightarrow{m_i}, \overrightarrow{m_j} > = \sum_{i=1}^{2^\Theta} \sum_{j=1}^{2^\Theta} m_1(A_s) m_2(B_t) \frac{|A_s \cap B_t|}{|A_s \cup B_t|} \tag{7}$$

$|\cdot|$ is modulo operation, and A_s, B_t are the subsets of FOD. For $d_{BPA} \in [0, 1]$, the greater the value of d_{BPA}, the greater difference between the two evidences.

2.3.2 Combine belief function based on evidence distance. Using evidence distance can optimize the combination of belief function, and several methods [23] have been proposed. Here, we introduce the common one by exploring similarity matrix to generate weights based on Jousselme evidence distance.
The similarity of two bodies of evidence is denoted as Sim_{m_i, m_j}, the definition is:

$$Sim(m_i, m_j) = 1 - d_{BPA}(m_i, m_j) \tag{8}$$

Assume there are k pieces of evidence, the *similarity measure matrix*(SMM) is constructed by calculating all the similarity among these evidences and S_{ij} is short for $Sim(m_i, m_j)$.

$$SMM = \begin{bmatrix} 1 & S_{12} & \cdots & S_{1j} & \cdots & S_{1k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S_{i1} & S_{i2} & \cdots & S_{ij} & \cdots & S_{ik} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S_{k1} & S_{k2} & \cdots & S_{kj} & \cdots & 1 \end{bmatrix} \tag{9}$$

For a evidence $m_i (i = 1, 2, ..., k)$, its support degree is obtained by:

$$Sup(m_i) = \sum_{j=1, j \neq i}^{k} S_{ij} = \sum_{j=1, j \neq i}^{k} Sim(m_i, m_j) \tag{10}$$

The credibility degree Crd_i shows the weight of corresponding evidence after normalization, therefore, it is obvious that $\sum_{i=1}^{k} Crd_i = 1$. The definition of Crd_I is:

$$Crd_i = \frac{Sup(m_i)}{\sum_{i=1}^{k} Sup(m_i)} \tag{11}$$

After obtaining the credibility degree, the modified average of evidence (MAE) is given by assigning the weight [47]:

$$MAE(m) = \sum_{i=1}^{k} (Crd_i \times m_i) \tag{12}$$

3 OUR PROPOSED METHOD FOR UNCERTAINTY MODELING

3.1 Construct uncertainty level of FOD

In the process of information transmits and exchanges, it can often be affected by various factors. For an information detector, the message it receives at the receiving end may be any possible results in its FOD. For example, suppose the FOD of a color detection sensor is $\Theta = \{red, yellow, blue\}$. At a certain moment, an information source emitted a red light signal. The sensor received the signal and gave the recognition result based on its own knowledge which is $m(red) = 0.6$, $m(\{red, yellow\}) = 0.3$, $m(\Theta) = 0.1$. When processing the detected information, the sensor displayed uncertainty [10], mainly in the following two parts. Firstly, the sensor recognized the color with a probability of 0.6, but such recognition accuracy is not accurate enough to be applied in many high-precision systems. Secondly, the three answers given by the sensor all contain the element $\{red\}$, but their weight of saying "the light source is red" is different. Therefore, to solve these two problems, a reconstructed BPA (basic probability assignment) method will be proposed with uncertainty measurement in the context of multi-source information. The following figure shows the construction of the uncertainty level in FOD.

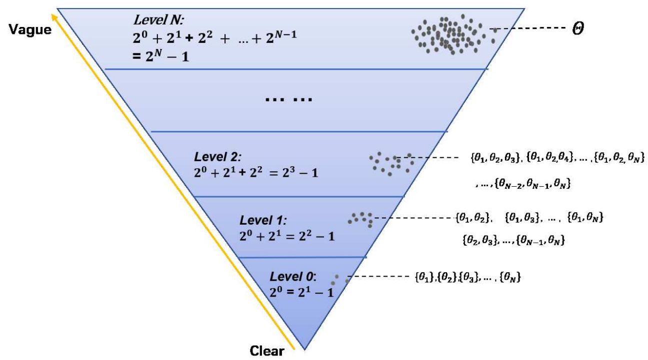

Figure 1: Relationship between uncertainty and identification sets

Each of the cases in the FOD is treated as a recognition set, and there are 2^Θ recognition sets. The uncertainty of the recognition set is related to the number of elements it contains. To construct the uncertainty relationship between the sets of FOD, from bottom to top in Fig.1, it indicates that the information is gradually blurred, and the determined information at level 0 forms the recognition set located in the upper layer after gradual interference. When the information enters the next uncertainty level, an element will be added to the recognition set. Since the newly added element combines with all the recognition sets of the previous layer, the uncertainty is increased by a power series of 2 and the recognition set of a single element at the level 0 is regarded as "1". The purpose of information processing is to reduce the uncertainty to the bottom layer gradually, where the grey dot density indicates the uncertainty degree of recognition set.

3.2 Reconstructed BPA

For a mass function m, the BPA is reconstructed by the uncertainty measurement between recognition sets in the mass function. The definition of reconstructed BPA m_r is as follows. For each $m(A_i) \neq 0$ in m:

$$\begin{cases} m_r(A_i) = \sum_{A_i \subseteq A_j} \dfrac{m(A_j)}{2^k - 1} \quad \forall A_i, A_j \subset \Theta, m(A_i) \neq 0 \\ m_r(\Theta) = \dfrac{m(\Theta)}{2^n - 1} \end{cases} \tag{13}$$

Where A_i, A_j are the subsets in the FOD Θ of the mass function m, and the A_i set can be composed by either a single element or multiple elements. k is the number of elements corresponding to the set A_j, $2^k - 1$ means the potential states in subset A_j in this case.

From the perspective of the reconstruction equation, there are two main advantages when reconstructing BPAs. First, for every statement given by a piece of evidence, the supporting source for it comes not only from its own set, but also from the upper sets which contain it. However, as previously described, the determined information may become vague during the propagation process due to interference. Therefore, the receiving end might get various combinations of interference information and original determined information. For instance, for a recognition set $\{A\}$ which has its own credibility, its upper sets $\{A, B\}$, $\{A, B, C\}$ also contribute a certain degree of support for $\{A\}$ on the premise that this kind of support is different when the elements increase. Secondly, the equation measures the uncertainty factor of mass function, and the relationship of supporting degree between sets is built. When a recognition set contains more elements, it shows more blurred information, which corresponds to higher uncertainty. For , it has no upper set, so there is no need for summation.

To satisfy the format of mass function in evidence theory after the correction, it is necessary to normalize the reconstructed BPA. Use the sum of the calculation results in Eq.(13) as the denominator, and the operation of normalization is:

$$m^{'}(A_i) = \frac{m_r(A_i)}{\sum_i^{2^\Theta} m_r(A_i)} \tag{14}$$

3.3 Comprehensive steps for multi-agent information fusion

For a MAIF system, its task is to enable multiple agents to read the information first correctly, and to combine their information according to the chosen rule to analyze the identification target, which has the highest credibility. In the MAIF system, based on the principle of task decomposition and data classification [26, 40], the system decomposes the tasks to identify and generate multiple agents for performing individually respective tasks. Knowledge sharing [35] between different agents lays a good foundation for completing the information fusion process. In the context of this study, the information fusion method of identifying a certain target is chosen as a task, which is decomposed to ensure that different agents process different types of information. There is a wide variety of agents, such as expert system agents, neural network system agents, or sensoring-based agents, which are collectively referred to as identification agents. To complete an identification task, multiple identification agents are required to cooperate to complete the recognition of the target.

For a multi-agent information fusion system, the assumption is that a group of diagnostic agents $A = \{A_1, A_2, ..., A_m\}$ have the same framework of discernment (FOD), which contains n elements. For a diagnostic agent A_i, the recognition outcome is expressed as the following form.

$$A_i = \begin{Bmatrix} s_1^i & s_2^i & \cdots & s_{2^n}^i \\ c_1^i & c_2^i & \cdots & c_{2^n}^i \end{Bmatrix} \tag{15}$$

The upper line in the equation represents the recognition object given by agent A_i. Since there are a total of n elements in the FOD, there are 2^n recognition objects, and the following line is the credibility corresponding to each recognition object.

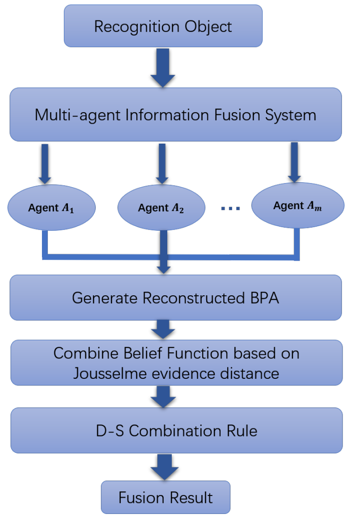

Figure 2: Flow chart shows the process of multi-agent information fusion system

The comprehensive processing flow chart of the multi-agent information fusion system based on evidence theory is shown in Figure 2, and the details is explained as pseudocode form in Algorithm 1:

Step 1. Convert the data of agent A_i to a mass function and the form is (A_i, m_i)= $([\{S_1^i\}, c_1^i], [\{S_2^i\}, c_2^i], \ldots, [\{S_{2^n}^i\}, c_{2^n}^i])$.

Step 2. Generate the reconstructed BPA according to Eq.(13) and execute normalization in Eq.(14).

Step 3. Calculate the Jousselme evidence distance between the BPAs by Eq.(6).

Step 4. By Eq.(8) Eq.(12), combining the belief function based on Jousselme evidence to obtain the MAE.

Step 5. Using the D-S combination rule for fusion multiple times [28] to achieve the result.

Algorithm 1 COMPREHENSIVE FUSION(S_x^i, c_x^i, k)

Input: S_x^i: statement detected by agents; c_x^i: corresponding credibility given by agents; k: the number of agent

Output: $m_c(S_i)$: combined mass function after processed by proposed method

$(A_i, m_i) \leftarrow ([\{S_1^i\}, c_1^i], [\{S_2^i\}, c_2^i], \ldots, [\{S_{2^n}^i\}, c_{2^n}^i])$.
for each subset $S_i \neq 0$ in m_i **do**
  if $S_x \neq \Theta$ **then**
    $m_r^i : m_r(S_x^i) \leftarrow \sum_{S_x \subseteq S_y} \frac{m(S_y)}{2^j - 1}$
    % j: number of element contained in S_y
  else
    $m_r^i : m_r(\Theta^i) \leftarrow \frac{m(\Theta)}{2^n - 1}$
    % n: number of element contained in Θ
  end if
end for
$m_i' \leftarrow m_r^i$ after normalization
for $i = 1 \rightarrow k - 1$ **do**
  for $j = i + 1 \rightarrow k$ **do**
    $d_{BPA}(m_i', m_j') \leftarrow \sqrt{\frac{1}{2}(\|\vec{m_i}\|^2 + \|\vec{m_j}\|^2 - 2 < \vec{m_i}, \vec{m_j} >)}$
    $Sim(m_i', m_j') \leftarrow 1 - d_{BPA}(m_i', m_j')$
    $SMM[i][j] \leftarrow Sim(m_i', m_j')$
    $SMM[j][i] \leftarrow SMM[i][j]$
  end for
end for
for $i = 1 \rightarrow k$ **do**
  for $j = 1 \rightarrow k$ **do**
    if $i \neq j$ **then**
      $Sup(m_i') \leftarrow \sum SMM[i][j]$
    end if
  end for
  $Total\_Sup+ = Sup(m_i')$
end for
$Crd_i \leftarrow \frac{Sup(m_i')}{Total\_Sup}$
$MAE(m) \leftarrow \sum_{i=1}^{k}(Crd_i \times m_i')$
for each subset S_i in $MAE(m_i)$ **do**
  $m_c(S_i) \leftarrow (MAE(m) \bigoplus MAE(m) \bigoplus \cdots MAE(m))(S_i)$
  % $MAE(m)$ fusion by $k - 1$ times
end for
return $m_c(S_i)$

4 NUMERICAL EXAMPLES

In this section, two classic application cases are presented to show the validity of the proposed method in the context of multi-agent information fusion system. One is the situation when system faces highly conflicting evidences [44], and the other one is a typical target recognition task [27] with anti-interfere function.

4.1 Target recognition in a highly conflicting environment

In real-life scenarios [4, 37, 39], agents might be interfered when reading data information, so that they are not able to work properly. One of the most common situations [49] is that the interfered agent makes a high degree of conflict when making a decision [21] or reasoning [46] compared with other agents in the system. Thus, the following example presents how to use the proposed method to effectively avoid such problems in the MAIF system.

In a maritime operation, a group of multi-category sensor agents $A = \{A_1, A_2, A_3, A_4, A_5\}$ are used to identify a object at sea, and the agents are acoustic sensor agent, speed sensor agent, pressure sensitive sensor agent and photosensitive sensor agent respectively. The FOD is $\Theta = \{A, B, C\}$. The data obtained by the corresponding agent is shown as follows (data source from Deng et al.[12]):

$$A_1 = \begin{Bmatrix} A & B & C \\ 0.5 & 0.2 & 0.3 \end{Bmatrix} \quad A_2 = \begin{Bmatrix} A & B & C \\ 0 & 0.9 & 0.1 \end{Bmatrix}$$

$$A_3 = \begin{Bmatrix} A & B & A,C \\ 0.55 & 0.1 & 0.35 \end{Bmatrix} \quad A_4 = \begin{Bmatrix} A & B & A,C \\ 0.55 & 0.1 & 0.35 \end{Bmatrix}$$

$$A_5 = \begin{Bmatrix} A & B & A,C \\ 0.6 & 0.1 & 0.3 \end{Bmatrix}$$

Obviously, the data monitored by agent A_2 is different from that in other agents when the system assigned most of its credibility to

object B while others did A. Therefore, the uncertainty level should be analyzed between the agents. The following shows the main procedure of dealing with the situation.
(1) The form of mass function has been converted as follows.

$$(A_1, m_1) = ([\{A\}, 0.5], [\{B\}, 0.2], [\{C\}, 0.3])$$
$$(A_2, m_2) = ([\{A\}, 0], [\{B\}, 0.9], [\{C\}, 0.1])$$
$$(A_3, m_3) = ([\{A\}, 0.55], [\{B\}, 0.1], [\{A, C\}, 0.35])$$
$$(A_4, m_4) = ([\{A\}, 0.55], [\{B\}, 0.1], [\{A, C\}, 0.35])$$
$$(A_5, m_5) = ([\{A\}, 0.6], [\{B\}, 0.1], [\{A, C\}, 0.3])$$

(2) Reconstruct and normalize the original BPAs based on Eq.(13) and Eq.(14).

	m_1'	m_2'	m_3'	m_4'	m_5'
$\{A\}$	0.5	0	0.7547	0.7547	0.7
$\{B\}$	0.2	0.9	0.1132	0.1132	0.2
$\{C\}$	0.3	0.1	0	0	0
$\{A, C\}$	0	0	0.1321	0.1321	0.1
$\{\Theta\}$	0	0	0	0	0

(3) Get the similarity measure matrix (SMM) by calculating Jousselme evidence distance.

$$\text{SMM}=\begin{bmatrix} 1 & 0.3755 & 0.7052 & 0.7052 & 0.7450 \\ 0.3755 & 1 & 0.1930 & 0.1930 & 0.2720 \\ 0.7052 & 0.1930 & 1 & 1 & 0.9184 \\ 0.7052 & 0.1930 & 1 & 1 & 0.9184 \\ 0.7450 & 0.2720 & 0.9184 & 0.9184 & 1 \end{bmatrix}$$

(4) The support and corresponding credibility degree are obtained by Eq.(10) and Eq.(11)

$$Sup(m_1) = 2.5309 \quad Sup(m_2) = 1.0335 \quad Sup(m_3) = 2.8166$$
$$Sup(m_4) = 2.8166 \quad Sup(m_5) = 2.8538$$
$$Crd_1 = 0.2100 \quad Crd_2 = 0.0858 \quad Crd_3 = 0.2337$$
$$Crd_4 = 0.2337 \quad Crd_5 = 0.2368$$

(5) Assign the credibility weights to mass functions for modified average evidence (MAE).

$$m(\{A\}) = 0.6235 \quad m(\{B\}) = 0.2195$$
$$m(\{C\}) = 0.0716 \quad m(\{A, C\}) = 0.0854$$

(6) By fusing the MAE four times, the result given by multi-agent information fusion system is:

$$m(\{A\}) = 0.9966 \quad m(\{B\}) = 0.0028$$
$$m(\{C\}) = 0.0005 \quad m(\{A, C\}) = 0.00002$$

The table below compares the diagnostic results by using four fusion methods in the context of a multi-agent information fusion system. From the original data, it is clear that the data gave by agent A_2 is highly conflicting. In this case, the traditional D-S combination method did not work well and produced a counter-intuitive answer by treating C as a diagnosis result. The other three methods can correctly diagnose the object A, but there is a divergence in terms of accuracy. Murphy's method is usually mentioned in the test comparison, which achieved the outcome by averaging n pieces of BPAs and fusing them $n-1$ times afterward. In Murphy's method, all evidence weights are equal, but the real situation is often not showing this case. Therefore, although it can solve the problem of highly conflicting scenes to a certain extent, the accuracy is not high enough. Furthermore, Deng et al.'s modified average method is based on the Murphy method and fully considered the differences between the pieces of evidence before the fusion process. The distance between the pieces of evidence can effectively highlight the differences between the agents in the MAIF system. The outcome obtained by using Deng et al.'s modified average combination method was superior to that of Murphy's method. Moving to our proposed method, in the case of reconstructing BPAs to measure the relationship of uncertainty, the deviation was corrected well at the beginning step, and the fusion accuracy performed better than that in the existing two presented methods.

4.2 Identification in a real-time dynamic scene

In some practical engineering applications, objects identified from the system may continuously change over times [13]. In this scenario, the multi-agent information fusion system needs to collect the information in real time and make decision analysis correspondingly [1].
Suppose there is a MAIF system in a military base to identify the type of an target. The system uses three agents for real-time information reading, where A_1 is an expert system agent, A_2 is a neural network agent, and A_3 is a GPS positioning agent. The FOD of the system $\Theta = \{A, H, F\}$ refers to *Airplane*, *Helicopter*, and *Fighter*. To detect the type of dataset correctly, three agents have to read the information in real time. Table 2 shows the details of the multi-agent system reading information at three points in a certain time (data source from Song et al.[30]).
The SMM obtained at three time points is shown as follows.

$$SMM_{t_1} = \begin{bmatrix} 1 & 0.9678 & 0.9678 \\ 0.9678 & 1 & 0.9941 \\ 0.9678 & 0.9941 & 1 \end{bmatrix}$$
$$SMM_{t_2} = \begin{bmatrix} 1 & 0.8869 & 0.7 \\ 0.8869 & 1 & 0.8128 \\ 0.7 & 0.8128 & 1 \end{bmatrix}$$
$$SMM_{t_3} = \begin{bmatrix} 1 & 0.9725 & 0.9579 \\ 0.9725 & 1 & 0.9854 \\ 0.9579 & 0.9854 & 1 \end{bmatrix}$$

Obtain the support degree and credibility degree at three time points.

$$T1 : Sup(m_1) = 1.9356 \quad Sup(m_2) = 1.9619 \quad Sup(m_3) = 1.9619$$
$$Crd_1 = 0.3303 \quad Crd_2 = 0.3348 \quad Crd_3 = 0.3348$$
$$T2 : Sup(m_1) = 1.5869 \quad Sup(m_2) = 1.6997 \quad Sup(m_3) = 1.5128$$
$$Crd_1 = 0.3306 \quad Crd_2 = 0.3541 \quad Crd_3 = 0.3152$$
$$T3 : Sup(m_1) = 1.9304 \quad Sup(m_2) = 1.9579 \quad Sup(m_3) = 1.9433$$
$$Crd_1 = 0.3310 \quad Crd_2 = 0.3357 \quad Crd_3 = 0.3332$$

Three pieces of MAE were calculated as follow:

$$T1 : \quad m(\{A\}) = 0.3953 \quad m(\{H\}) = 0.5206$$
$$m(\{A, H\}) = 0.0777 \quad m(\{\Theta\}) = 0.0064$$
$$T2 : \quad m(\{H\}) = 0.7482 \quad m(\{R\}) = 0.0744$$
$$m(\{A, H\}) = 0.1037 \quad m(\{\Theta\}) = 0.0737$$
$$T3 : \quad m(\{H\}) = 0.9546 \quad m(\{\Theta\}) = 0.0454$$

Table 1: The fusion result obtained by using different methods

	A_1, A_2	A_1, A_2, A_3	A_1, A_2, A_3, A_4	A_1, A_2, A_3, A_4, A_5
Dempster-Shafer's combination rule	$m(A)$=0 $m(B)$=0.8571 $m(C)$=0.1429	$m(A)$=0 $m(B)$=0.6316 $m(C)$=0.3684	$m(A)$=0 $m(B)$ =0.3288 $m(C)$ =0.6712	m(A)=0 m(B)=0.1228 m(C)=0.8772
Murphy's average combination rule [28]	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.3500 $m(B)$=0.5224 $m(C)$=0.1276	$m(A)$=0.6027 $m(B)$ =0.2627 $m(C)$ =0.1346	m(A)=0.7958 m(B)=0.0932 m(C)=0.1110
Deng et al's modified average [12] combination rule	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.4861 $m(B)$=0.3481 $m(C)$=0.1657	$m(A)$=0.7773 $m(B)$ =0.0628 $m(C)$ =0.1600	m(A)=0.8909 m(B)=0.0086 m(C)=0.1005
Proposed method	$m(A)$=0.8718 $m(B)$=0.0848 $m(C)$=0.0306	$m(A)$=0.9610 $m(B)$=0.0286 $m(C)$=0.0088	$m(A)$ =0.9885 $m(B)$ =0.0091 $m(C)$ =0.0022	m(A)=0.9966 m(B)=0.0028 m(C)=0.0005

Table 2: Identification information read by agents

	t_1	t_2	t_3
A_1	$m(\{A\})$=0.3666 $m(\{H\})$=0.4563 $m(\{A,H\})$=0.1185 $m(\{\Theta\})$=0.0586	$m(\{H\})$=0.8176 $m(\{F\})$=0.0003 $m(\{A,H\})$=0.1553 $m(\{\Theta\})$=0.0268	$m(\{H\})$=0.6229 $m(\{\Theta\})$ =0.3771
A_2	$m(\{A\})$=0.2793 $m(\{H\})$=0.4151 $m(\{A,H\})$=0.2652 $m(\{\Theta\})$=0.0404	$m(\{H\})$=0.5658 $m(\{F\})$=0.0009 $m(\{A,H\})$=0.0646 $m(\{\Theta\})$=0.3687	$m(\{H\})$=0.7660 $m(\{\Theta\})$ =0.2340
A_3	$m(\{A\})$=0.2897 $m(\{H\})$=0.4331 $m(\{A,H\})$=0.2470 $m(\{\Theta\})$=0.0302	$m(\{H\})$=0.2403 $m(\{F\})$=0.0004 $m(\{A,H\})$=0.0141 $m(\{\Theta\})$=0.7452	$m(\{H\})$=0.8598 $m(\{\Theta\})$ =0.1402

The fusion result of this multi-agent information fusion system is:

$$T1: \quad m(\{A\}) = 0.3786 \qquad m(\{H\}) = 0.6094$$
$$m(\{A,H\}) = 0.0120$$
$$T2: \quad m(\{H\}) = 0.9451 \qquad m(\{R\}) = 0.00189$$
$$m(\{A,H\}) = 0.0298 \qquad m(\{\Theta\}) = 0.0062$$
$$T3: \quad m(\{H\}) = 0.9979 \qquad m(\{\Theta\}) = 0.0021$$

Using different methods to assess the performance of this application, Table 3 shows the comparison of the fusion results among the five chosen methods. It can be known from the original data that agent A_3 had a problem at time t_2, which was manifested as the inability to determine the information of the identified object. But we can tell from this table that from time node t_1 to t_3, the probability of identifying the target as $\{H\}$ is gradually increasing. Obviously, each method can recognize the flight target *Helicopter* when there is no highly conflicting situation. The fusion result of the proposed method shows high recognition accuracy, and it is consistent with the performance using other methods.

5 SENSITIVITY TEST

In real practical applications, the MAIF system might face a large number of objects to be detected with complex information mixing together, which results in the growth in the scale of the corresponding interference information. Therefore, we will expand the scope of identification and detect the performance of the reconstructed BPA method in a greater demand. The following content will show the sensitivity test of the proposed reconstructed BPA method to proof the validity of it.

Suppose there is a FOD containing 8 elements, which are referred as $\Theta = \{1, 2, ..., 8\}$ with 2^8 subsets in it. Assume that there are two agents $A = \{A_1, A_2\}$ in the MAIF system, and the target has been correctly identified (element *1* was set as the recognition result in this scene). According to the principle of control experiment, the data read by the two agents is the same by default. Afterwards, interference was imposed on one of the agents, which showed that the recognition of element *1* gradually became blurred. The detected information given by the two agents is as follows.

Step 1. Have two originally same detected mass functions (initial value:X=1), whose the recognition result is element 1.

(A_1, m_1):

$m(\{X\}) = 0.8 \;\; m(\{1,2\}) = 0.1 \;\; m(\{3,4,5\}) = 0.05 \;\; m(\{6\}) = 0.05$

(A_2, m_2):

$m(\{1\}) = 0.8 \;\; m(\{1,2\}) = 0.1 \;\; m(\{3,4,5\}) = 0.05 \;\; m(\{6\}) = 0.05$

Step 2. One element is added to the X set of agent A_1 each time.

Step 3.

(1)Use the traditional D-S combination rule, directly combine m_1 and m_2 after each change.

(2)Use reconstructed BPA method, m_1 is firstly reconstructed and normalized, and then combine with m_2 by traditional D-S combination rule.

Table 4: Credibility performance comparison by two methods.

Method	credibility weight of m(X)	
	Reconstructed BPA	D-S combination
$X = \{1\}$	0.9021	0.8000
$X = \{1,2\}$	0.8041	0.9000
$X = \{1,2,3\}$	0.2963	0.8000
$X = \{1,\cdots,4\}$	0.2021	0.8000
$X = \{1,\cdots,5\}$	0.1100	0.8000
$X = \{1,\cdots,6\}$	0.0611	0.8000
$X = \{1,\cdots,7\}$	0.0346	0.8000
$X = \{1,\cdots,8\}$	0.0180	0.8000

Table 3: Fusion results based on different methods

	t_1	t_2	t_3
Dempster-Shafer combination rule	$m(\{A\})$=0.3376 $m(\{H\})$=0.6317 $m(\{A,H\})$=0.0305 $m(\{\Theta\})$=0.0001	$m(\{H\})$=0.9399 $m(\{F\})$=0.0001 $m(\{A,H\})$=0.0526 $m(\{\Theta\})$=0.0074	$m(\{H\})$=0.9876 $m(\{\Theta\})$=0.0124
Song et al's combination method in [30]	$m(\{A\})$=0.3375 $m(\{H\})$=0.6308 $m(\{A,H\})$=0.0315 $m(\{\Theta\})$=0.0002	$m(\{H\})$=0.8998 $m(\{F\})$=0.0002 $m(\{A,H\})$=0.0581 $m(\{\Theta\})$=0.0419	$m(\{H\})$=0.9850 $m(\{\Theta\})$=0.0150
The method in [17]	$m(\{A\})$=0.3384 $m(\{H\})$=0.5904 $m(\{A,H\})$=0.0651 $m(\{\Theta\})$=0.0061	$m(\{H\})$=0.8861 $m(\{F\})$=0.0002 $m(\{A,H\})$=0.0582 $m(\{\Theta\})$=0.0555	$m(\{H\})$=0.9621 $m(\{\Theta\})$=0.0371
The method in [33]	$m(\{A\})$=0.3318 $m(\{H\})$=0.6332 $m(\{A,H\})$=0.0349 $m(\{\Theta\})$=0.0001	$m(\{H\})$=0.8891 $m(\{F\})$=0.0003 $m(\{A,H\})$=0.0427 $m(\{\Theta\})$=0.0679	$m(\{H\})$=0.9784 $m(\{\Theta\})$=0.0216
Proposed method	$m(\{A\})$=0.3786 $m(\{H\})$=0.6094 $m(\{A,H\})$=0.0120	$m(\{H\})$=0.9451 $m(\{F\})$=0.0189 $m(\{A,H\})$=0.0298 $m(\{\Theta\})$=0.0062	$m(\{H\})$=0.9979 $m(\{\Theta\})$=0.0021

Table 4 shows the credibility weights for different $m(X)$ when X gradually increases. Obviously, traditional D-S combination rule faced a counter-intuitive situation. It is manifested that although the interference elements of the set X are gradually increasing, the credibility weight of $m(X)$ is basically unchanged, and the same value is always used throughout the whole fusion process. On the contrary, by reconstructing BPA, the results show that as the number of elements contained in subset X increases, the credibility weight of $m(X)$ decreases at an exponential rate, indicating that more and more interference comes into subset X. Since the increasing interference information will combine with the subsets in X, the number of sets which contain interference increases by a power of 2. Therefore, the uncertainty level of $m(X)$ rises up significantly.

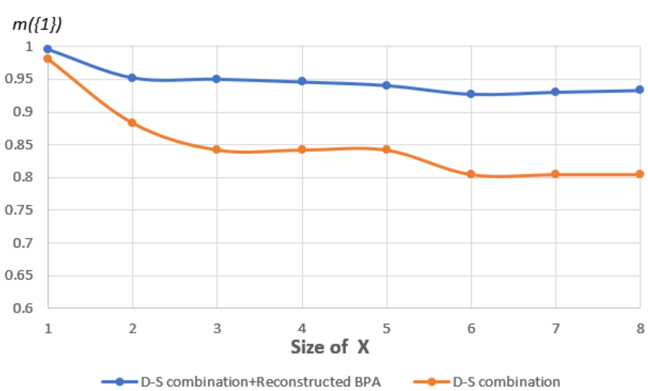

Figure 3: Comparison between reconstructed BPA method and traditional D-S combination rule

Figure 3 compares the recognition rate of the target between the reconstructed BPA method and traditional D-S combination rule. It can be clearly seen that the reconstructed BPA method has a stronger ability of anti-interference and the overall recognition accuracy is better than the traditional method. Moreover, in the traditional method, sometimes the recognition rate is unchanged while the interference increases, which is not so reasonable in real-life scenario. By contrast, the the curve of reconstructed BPA keeps changing within a small range. Therefore, it is suitable to conclude the validity and reliability of proposed method.

6 CONCLUSIONS

As the information of multiple sources is often changeable and easily disturbed, it is critical to improving the performance of the multi-agent system performance by considering the uncertain relationship from the fusion process of various sources. In this study, we addressed the modeling of the uncertainty relationship between the recognition objects in a multi-agent information fusion system. We investigated the improvement of fusion performance during the multi-agent identification process. Here, we developed the uncertainty modeling by reconstructing BPA and combining the factor of evidence distance to leverage the uncertainty relationship in evidence theory. The results showed the validity of the proposed method, especially when multi-agents face a highly adverse situation. In brief, our work provides a valuable view to measure the uncertainty in the context of MAIF system.

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