Hedonic Seat Arrangement Problems*

Extended Abstract

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ABSTRACT

In this paper, we study a variant of hedonic games, called Seat Arrangement. The model is defined by a bijection from agents with preferences to vertices in a graph. The utility of an agent depends on the neighbors in the graph. In this paper, we study the price of stability and fairness in Seat Arrangement, and the computational complexity and the parameterized complexity of finding certain "good" seat arrangements, say Maximum Welfare Arrangement, Maximin Utility Arrangement, Stable Arrangement, and Envy-free Arrangement.

KEYWORDS

hedonic game; stability; envy-freeness; computational complexity; parameterized complexity; local search

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1 INTRODUCTION

Given a set of n agents with preferences for each other and an n-vertex graph, called the *seat graph*, we consider to assign each agent to a vertex in the graph. Each agent has a utility that depends on the agents assigned to neighbors vertices in the graph. Intuitively, if a neighbor is preferable for the agent, his/her utility is high. This models several situations such as seat arrangements in classrooms, offices, restaurants, or vehicles. Here, a vertex corresponds to a seat and an assignment corresponds to a seat arrangement. If we arrange seats in a classroom, the seat graph is a grid. As another example, if we consider a round table in a restaurant, the seat graph is a cycle. We name the model Seat Arrangement.

Seat Arrangement is related to hedonic games [7]. If the seat graph in a Seat Arrangement instance is a disjoint union of cliques, then each clique may be viewed as a potential coalition. Hence an arrangement on that graph naturally corresponds to a coalition forming. In that sense, this model is considered a hedonic game of arrangement on topological structures.

In this paper, we consider the following problems to find four types of *desirable* seat arrangements: Maximum Welfare Arrangement (MWA), Maximin Utility Arrangement (MUA), Stable Arrangement (STA), and Envy-free Arrangement (EFA). MWA is the problem to find a seat arrangement that maximizes the sum of utilities of agents, which is called the *social welfare*.

The concept of MWA is a macroscopic optimality, and hence it may ignore individual utilities. Complementarily, MUA is the problem to find a seat arrangement that maximizes the least utility of an agent. From the viewpoint of economics, the maximin utility of an arrangement can be interpreted as a measure of fairness [3, 10, 18].

Stability is one of the central topics in the field of hedonic games including Stable Matching [2, 7, 14, 19]. Motivated by this, we define a *stable* arrangement as an arrangement with no pair of agents that has an incentive of swapping their seats (i.e., vertices), called a *blocking pair*. This corresponds to the definition of *exchange-stability* proposed by Alcalde in the context of stable matchings [1]. In Seat Arrangement, STA is the problem of deciding whether there is a stable arrangement in a graph.

Finally, we consider the envy-freeness of Seat Arrangement. The envy-freeness is also a natural and well-considered concept in hedonic games.

2 THE MODEL

Let $G = (V, E)$ be a graph where $n = |V|$ and $m = |E|$. We denote by $\mathbf{P}$ the set of agents, and define an *arrangement* as follows.

Definition 2.1 (Arrangement). For a set of agents $\mathbf{P}$ and a graph G, a bijection $\pi : \mathbf{P} \to V(G)$ is called an *arrangement* in G.

We denote by Π the set of all arrangements in G. Note that $|\Pi| = n!$. We call graph G the *seat graph*.

Definition 2.2 ((p, q)-swap arrangement). For a pair of agents $p, q \in \mathbf{P}$, we say that π' is the (p, q)-*swap arrangement* if π' can obtained from swapping the arrangement of p and q.

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Next, we define the *preference* of an agent.

Definition 2.3 (Preference). The *preference* of $p \in \mathbf{P}$ is defined by $f_p : \mathbf{P} \setminus \{p\} \to \mathbb{R}$.

We denote by $\mathcal{F}_\mathbf{P}$ the set of preferences of all agents in $\mathbf{P}$. Here, we say the preferences are *binary* if $f_p : \mathbf{P} \setminus \{p\} \to \{0, 1\}$ for every agent p, are *nonnegative* if $f_p : \mathbf{P} \setminus \{p\} \to \mathbb{R}^+_0$, and are *positive* if $f_p : \mathbf{P} \setminus \{p\} \to \mathbb{R}^+$. Furthermore, we say they are *symmetric* if $f_p(q) = f_q(p)$ holds for any pair of agents $p, q \in \mathbf{P}$ and *strict* if for any $p \in \mathbf{P}$ there is no pair of distinct $q, r \in \mathbf{P}$ such that $f_p(q) = f_p(r)$.

Finally, we define the *utility* of an agent and the *social welfare* of an arrangement π.

Definition 2.4 (Utility and social welfare). Given an arrangement π and the preference of p, the *utility* of p is defined by $U_p(\pi) = \sum_{v \in N(\pi(p))} f_p(\pi^{-1}(v))$. Moreover, the *social welfare* of π for $\mathbf{P}$ is defined by the sum of all utilities of agents and denoted by $\mathrm{sw}(\pi) = \sum_{p \in \mathbf{P}} U_p(\pi)$.

We define four types of Seat Arrangement problems. An arrangement π^* is *maximum* if it satisfies $\mathrm{sw}(\pi^*) \geq \mathrm{sw}(\pi)$ for any arrangement π. Also, an arrangement π^* is a *maximin* arrangement if π^* satisfies $\min_{p \in \mathbf{P}} U_p(\pi^*) \geq \min_{p \in \mathbf{P}} U_p(\pi)$ for any arrangement π. Then, Maximum Welfare Arrangement (MWA) and Maximin Utility Arrangement (MUA) are the problems to find a maximum (maximin) arrangement in G.

Next, we define the *stability* and the *envy-freeness* of Seat Arrangement.

Definition 2.5 (Stablility). Given an arrangement π, a pair of agents p and q is called a *blocking pair* for π if it satisfies that $U_p(\pi') > U_p(\pi)$ and $U_q(\pi') > U_q(\pi)$ where π' is the (p, q)-swap arrangement for π. If there is no blocking pair in π, it is said to be *stable*.

Definition 2.6 (Envy-free). An arrangement π is envy-free if there is no agent p such that there exists $q \in \mathbf{P} \setminus \{p\}$ that satisfies $U_p(\pi') > U_p(\pi)$ where π' is the (p, q)-swap arrangement for π.

Stable Arrangement (STA) and Envy-free Arrangement (EFA) are the problems to decide whether there is a stable (envy-free) arrangement in G.

3 OUR CONTRIBUTION

In this paper, we first investigate the price of stability (PoS) and the price of fairness (PoF) of Seat Arrangement, which are defined as the ratio of the maximum social welfare over the social welfare of a maximum stable solution and a maximin solution, respectively. For the price of stability, we can say the PoS is 1 under symmetric preferences by a result in [17]. For the price of fairness, we show that there is a family of instances such that PoF is unbounded. For the binary case, we show an upper bound of $\tilde{d}(G)$ of PoF, where $\tilde{d}(G)$ is the average degree of the seat graph G. On the other hand, we present an almost tight lower bound $\tilde{d}(G) - 1/4$ of PoF. Furthermore, we give a lower bound $\tilde{d}(G)/2 + 1/12$ for the cases with symmetric preferences.

Next, we give dichotomies of computational complexity of four Seat Arrangement problems from the perspective of the maximum order of connected components in the seat graph. For MWA, MUA, and symmetric EFA, we show that they are solvable in polynomial time if the order of each connected component in the seat graph is at most 2 whereas they are NP-hard even if the order of each connected component of the seat graph is 3. Since a maximum arrangement is always stable under symmetric preferences, symmetric STA can also be solved in polynomial time if the order of each connected component is at most 2. On the other hand, STA is NP-complete even if the order of each connected component in the seat graph is at most 2. Note that if each connected component in the seat graph is of order at most 1, it consists of only isolated vertices, and hence STA is trivially solvable.

For the parameterized complexity, we show that MWA can be solved in time $n^{O(\gamma)}$ whereas it is W[1]-hard with respect to vertex cover number γ of the seat graph and cannot be solved in time $n^{o(n)}$ and $f(\gamma)n^{o(\gamma)}$ under ETH. Moreover, we prove that MUA and symmetric EFA are weakly NP-hard even on seat graphs with $\gamma = 2$.

Finally, we study the parameterized complexity of local search of finding a stable arrangement. We show that determining whether a stable arrangement can be obtained from a given arrangement by k swaps is W[1]-hard when parameterized by $k + \gamma$, whereas it can be solved in time $n^{O(k)}$.

4 RELATED WORK

A *hedonic game* is a non-transferable utility game regarding coalition forming, where each agent's utility depends on the identity of the other agents in the same coalition [6, 11]. It includes the Stable Matching problem [7]. Seat Arrangement can be considered a hedonic game of arrangement on a graph.

Several graph-based variants of hedonic games have been proposed in the literature, see e.g. [2, 8, 9, 12, 16]. However, they typically utilize graphs to define the preferences of agents, and both the preferences and coalitions define the utilities of agents. On the other hand, in Seat Arrangement, the preferences are defined independently of a graph and the utility of an agent is determined by an arrangement in a graph (more precisely, the preferences for the assigned neighbors in the graph).

A major direction of research about hedonic games is the computational complexity of finding desirable solutions such as a solution with maximum social welfare and a stable solution [2, 6]. Peters [20] and Hanaka et al. [15] investigate the parameterized complexity of hedonic games for several graph parameters (e.g., treewidth). For the local search complexity, Gairing and Savani study the PLS-completeness of finding a stable solution [12, 13]. In terms of mechanism design and algorithmic game theory, many researchers study the price of anarchy, the price of stability, and the price of fairness [3, 4, 7, 19].

Very recently, in the context of one-sided markets, Massand and Simon consider the problem of allocating indivisible objects to a set of rational agents where each agent's final utility depends on the intrinsic valuation of the allocated item as well as the allocation within the agent's local neighbourhood [17]. Although the problem is motivated from different contexts, it has a quite similar nature to Seat Arrangement, and they also considered stable and envy-free allocation on the problem.

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