The Price of Anarchy of Self-Selection in Tullock Contests*

Extended Abstract

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ABSTRACT

Crowdsourcing platforms operate by offering their clients the ability to obtain cost-effective solutions for their problems through contests. The top contestants with the best solutions are rewarded, and the submitted solutions are provided to the clients. Within the platforms, the contestants can self-select which contest they compete in. In this paper, we measure crowdsourcing efficiency induces by the strategic behavior of contestants. We first propose a game-theoretic model of self-selection in *Tullock contests* (SSTC). To study the efficiency of SSTC, we establish the existence of a pure-strategy Nash equilibrium (PSNE). We then study the efficiency, via the price of anarchy (PoA), that comes from the worst-case equilibria of SSTC. We develop general efficiency PoA bounds with respect to PSNE, fully mixed NE, and general equilibrium concepts. For the case of identical contestants, we show that the pure and fully mixed PoA are one when the number of contestants is large – implying self-selection is efficient. In simulations, we show that an empirical bound well approximates the pure PoA, and the bound goes to one as the number of contestants becomes large.

KEYWORDS

Crowdsourcing; contest; price of anarchy; effort; Nash equilibrium

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1 INTRODUCTION

Due to the recent successes [9, 11] in using crowdsourcing platforms such as Topcoder and Freelancer to generate high-quality, low-cost solutions for real-world problems, the economy for task completion and problem-solving has now shifted to include crowdsourcing.

Crowdsourcing platforms organize and post a client's problem as an online *contest*, and the online workers or *contestants* sign up to compete by submitting solutions. After the contest, a reward is assigned to the winner, and the submitted solutions are provided to the client. Within a crowdsourcing platform, each contestant can *self-select* which contest s/he competes in, and maybe strategic in this choice to maximize his/her utilities. The platform's goal is to ensure each contest produces quality solutions with sufficient contestants and effort in each contest. However, the contestants' self-selection outcome may be different from an optimal, efficient outcome of the platform. As a result, the platform's solution qualities depend crucially on contestants' strategic behavior.

Contribution. We aim to address the efficiency question of self-selection in crowdsourcing contests from a theoretical perspective. To answer the question, we provide the following contributions. (1) We propose and study a novel model of self-selection in Tullock contests (SSTC) to account for m Tullock contests and each of the n contestants' ability to select a contest to compete in. (2) We show that there is a polynomial time algorithm to compute a pure-strategy Nash equilibrium (PSNE). (3) We define the platform's objective to be the aggregate effort of the contestants in the contests and use the price of anarchy (PoA) to measure the efficiency of self-selection (verse the worst equilibria). We derive PoA bounds with respect to PSNE, fully mixed NE, and general equilibrium concepts (see Table 1). (4) We conduct simulations on randomly generated SSTC and show that our pure PoA bound can be quite large. We show that an empirical bound well approximates the pure PoA, and the bound goes to one as the number of contestants becomes large.

2 SELF-SELECTION IN TULLOCK CONTESTS

To define the game of self-selection in Tullock contests (SSTC), we let $M = \{1, ..., m\}$ be the set of the same-type m Tullock contests and $N = \{1, ..., n\}$ be the set of n contestants. A (pure) strategy of a contestant i is $s_i = (a_i, e_i)$ with a binary selection vector a_i and an effort vector $e_i \in \mathbb{R}_0^{+m}$ of size m where $a_{ij} = 1$ denotes the selection of contest j and e_{ij} denotes the i's effort in contest $j \in M$. Thus, the strategy set of contestant $i \in N$ is $S_i \in A \times \mathbb{R}_0^{+m}$ and $A = \{a_i \in \{0,1\}^m \mid \sum_{j \in M} a_{ij} = 1\}$. We let $S = S_1 \times ... \times S_n$ denote the joint-pure strategy profile of the contestants. We let v_i to be the value of $i \in N$ for winning the contests.

Given a joint pure-strategy profile $\mathbf{s} = (s_i, \mathbf{s}_{-i}) \in S$, the utility of contestant i is $u_i(s_i, \mathbf{s}_{-i}) = \sum_{j=1}^{m} a_{ij}(v_i p_{ij}(\mathbf{s}) - e_{ij})$, where $p_{ij}(\mathbf{s}) = \frac{a_{ij} e_{ij}}{\sum_{k \in N} a_{kj} e_{kj}}$ is i's probability of winning contest j given $\mathbf{s}$; $p_{ij}(\mathbf{s}) = 0$ when $a_{ij} = 0$ for every $i \in N$ and $p_{ij}(\mathbf{s}) = \frac{1}{\sum_{k=1}^{n} a_{kj}}$ when $e_{ij} = 0$ and $a_{ij} = 1$ for each $i \in N$. When there is a single contestant, the contestant wins the prize with probability 1 with zero and negligible effort (i.e., $p_{ij}(\mathbf{s}) = 1$ and $e_{ij} = 0$ if $a_{ij} = 1$ and $\sum_{k \in N} a_{kj} = 1$).

Due to the uniqueness of the PSNE in a single Tullock contest and the two-stage setup of the game, given the contest selection profile $\mathbf{a} = (a_i, \mathbf{a}_{-i}) \in A^n$, the e_{ij} for each contestant $i \in N$ and contest $j \in M$ can be determined deterministically via the equilibrium characterization in [8, 14] (denoted as e_{ij}^*). Thus, a PSNE in SSTC boils down to finding a stable contest selection profile.

Theorem 2.1. *For any game of SSTC, there is a PSNE. Moreover, we can find one in polynomial time.*

*The full version of the paper is available online.

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3 POA ANALYSIS OF SSTC

In the crowdsourcing setting, the contestants' efforts are often translated into solutions of different qualities. While the top solution is often announced as the winner in a contest, the other submitted solutions are not completely discarded (i.e., different machine learning classifiers can be combined to build a more powerful classifier via some ensemble approaches; see also [11]). As a result, the platform seeks many high-quality solutions, which correspond to the overall effort of a contest. The overall effort objective given a profile $\mathbf{a} \in A^n$ is $OE(\mathbf{a}) = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} e^*_{ij}(a_i, a_{-i})$. The platform wants to find an $\mathbf{a}^{opt} \in \arg\max_{\mathbf{a} \in A^n} OE(\mathbf{a})$. The pure PoA [10] of SSTC is $PoA = \max_{\mathbf{a}^* \text{ is a PSNE}} \frac{OE(\mathbf{a}^{opt})}{OE(\mathbf{a}^*)}$, which is the ratio of the overall effort of the optimal profile $OE(\mathbf{a}^{opt})$ and the overall effort of the worst PSNE of SSTC. PoA can be defined with respect to fully mixed NE and other equilibrium concepts. Table 1 summarizes our PoA results. We refer the readers to the full version of the paper for the complete theorems and proofs.

Table 1: PoA Results for Various Equilibrium Concepts

Equilibrium	Heterogeneous	Homogeneous
PSNE	$\leq 2mv_1/v_n$	1 $(n \geq 2m)$
Fully Mixed NE	See Full Paper	1 $(n \to \infty)$
General	See Full Paper	$\leq 2m(m+n) - m$

v_1 (v_n) is the highest (lowest) contestant value for winning
$v_1 = \ldots = v_n$ in the homogenous settings

4 SIMULATIONS: POA AND PSNE

We consider randomly generated SSTC instances where the contestants' values are drawn uniformly from $[0, 1]$. The results are averaged over 100 instances. We denote $\mathbf{a}^*$ to be the PSNE from Theorem 2.1, $Bound1 = mOE_{[n]}(\mathbf{1}_n)$, $Bound2 = mv_1$, and $LB = n_1 \frac{v_n}{2}$ (see the full paper). For small instances of the generated SSTC games, we show (i) the overall effort of our PSNE is close to that of the worst PSNE and (ii) our *Bound*1 can be used to approximate $OE(\mathbf{a}^{opt})$. We then study large numbers of contestants setting.

Comparing PoA Bounds. We show the average *PoA* bounds of the instances in Table 2. Since we use brute force to compute $\mathbf{a}^{opt}$ and the (exact) *PoA*, we are restricted to small instance sizes. Our derived bounds (last two columns) are too loose due to $LB(=v_n/2)$ being too small and v_n is drawn uniformly from $[0, 1]$. We would expect our bounds to be better if the values are distributed closely. Our $\mathbf{a}^*$ provides a good lower bound since *PoA* is very close to $\frac{OE(\mathbf{a}^{opt})}{OE(\mathbf{a}^*)}$, and $\frac{Bound1}{OE(\mathbf{a}^*)}$ is a good approximation to *PoA* when the number of contests/contestants is small/large. We observe that the *PoA* bounds increase as the number of contests increases and decrease as the number of contestants increases.

Table 2: Pure *PoA* Bounds

n, m	PoA	$\frac{OE(\mathbf{a}^{opt})}{OE(\mathbf{a}^*)}$	$\frac{Bound1}{OE(\mathbf{a}^*)}$	$\frac{Bound2}{OE(\mathbf{a}^*)}$	$\frac{Bound1}{LB}$
10,2	1.05	1.05	1.33	2.11	49.51
10,3	1.10	1.10	1.77	2.83	76.45
15,2	1.04	1.03	1.24	1.82	79.55
15,3	1.07	1.07	1.51	2.21	138.68

Large Numbers of Contestants. When the number of contestants is large, we focus on $\frac{Bound1}{OE(\mathbf{a}^*)}$. We consider $n = \{30, 60, 120, 240, 480, 960\}$ and $m = \{10, 20\}$. Figure 1(a) Right shows the values of $\frac{Bound1}{OE(\mathbf{a}^*)}$ as the number of contestants increases when there are 10 and 20 contests. Given that $\frac{Bound1}{OE(\mathbf{a}^*)}$ is an approximation to *PoA*, this suggests that PoA could converge to one as the number of contestants increases. Figure 1(a) Left shows the distributions of contestants over contests and indicates that the contestants are evenly distributed with a large number of contestants ($n > 60$). As the number of contestants grows, the minimum number of contestants in each contest increases while only a small fraction of top contestants (with the highest values) compete in contests (Figure 1(b)). However, more contests lead to more overall participation with fewer contestants in each contest. Figure 1(c) depicts the PSNE generated from Theorem 2.1 for two random instances.

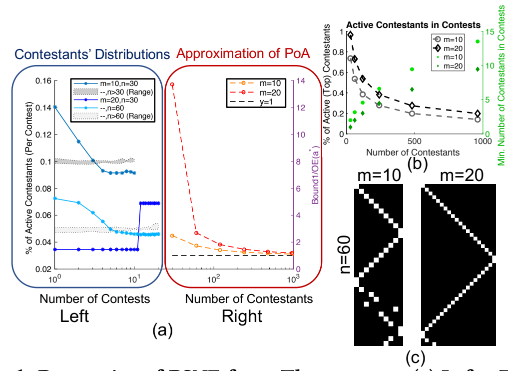

Figure 1: Properties of PSNE from Theorem 2.1. (a) Left - The Distributions of (Active) Contestants in Contests (a) Right - The PoA approximation from $\frac{Bound1}{OE(a^*)}$, (b) The numbers/percentages of active contestants in contests, and (c) $n \times m$-grid representation of $\mathbf{a}^*$ (white = 1, black = 0, $v_1 \geq \ldots . v_n$, only active contestants)

5 RELATED WORK

Multiple Contest Models. The work of [6] considers contestants who select multiple contests to compete in for prizes under the multiple all-pay auctions with incomplete information framework where first each contestant chooses a contest and a bid and then the highest bid winner is announced. The major difference is that we consider multiple Tullock contests with complete information. The work of [1, 2] and [12] considers self-selection of contestants in two contests of various prize structures under different variations of Tullock contests and Lazear and Rosen models, respectively.

The closest work to ours is [4] where all contestants are homogenous but have a different value for each contest. They provide a partial PSNE characterization result when the game satisfies some (integral) conditions. They consider a similar platform's objective and PoA related problem (e.g., difference instead of ratio) for PSNE. Our pure $PoA = 1$ result coincides with theirs in the case of homogenous SSTC. Their result holds when the number of contestants goes to infinity (i.e., $n \to \infty$) while our bound is explicit and tighter.

PoA Analysis in Auctions. Most of the PoA results in auctions are for social welfare functions (platform's objective) defined on the values (in opposed to revenue/effort) of the buyers (e.g., see [5, 13, 15]). The only work we are aware of is [7] where they consider the PoA of revenue, suggesting a PoA analysis for contests when modeled all-pay auctions with incomplete information. Another seemingly related work is that of valid utility games [3, 16]. Valid utility games assume that social welfare is no less than the sum of the utility of the players. This assumption does not hold for us.

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