Mechanism Design for Housing Markets over Social Networks

Takehiro Kawasaki, Ryoji Wada, Taiki Todo, and Makoto Yokoo
Graduate School of Information Science and Electrical Engineering, Kyushu University
Fukuoka, 819-0395 Japan
{kawasaki@agent.,r-wada@agent.,todo@,yokoo@}inf.kyushu-u.ac.jp

ABSTRACT

In this paper we investigate the effect of an underlying social network over agents in a well-known multi-agent resource allocation problem; the *housing market*. We first show that, when a housing market takes place over a social network with more than two agents and these agents have an option to avoid forwarding information about it to their followers, there does not exist an exchange mechanism that simultaneously satisfies strategy-proofness, Pareto efficiency, and individual rationality. It is also impossible to find a strategy-proof exchange mechanism that always chooses an outcome in a weakened core. These results highlight the difficulty of taking into account the agents' incentive of information diffusion in the resource allocation. To overcome these negative results, we consider two different ways of restricting the problem; limiting the domain of preferences and the structure of social networks.

KEYWORDS

Mechanism Design, Social Network, Information Diffusion, Top Trading Cycles Algorithm

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1 INTRODUCTION

The housing market [24] is one of the most investigated model for multi-agent resource allocation. In traditional housing markets, each agent is endowed with a single unit of an indivisible item, usually referred to as a *house*, as well as a strict preference over all the houses in the market. The purpose of housing market problems is to design an algorithm that takes the preferences and finds an appropriate exchange/permutation of houses among agents without monetary compensation. Applications of housing markets include on-campus housing [1] and live-organ exchange [22].

The literature of mechanism design offers many well-known results on strategy-proof mechanisms for several variants and extensions of housing markets. In particular, for traditional housing markets, choosing the unique strict core for cases of strict preferences is individually rational (i.e., truth-telling harms no agent), strategy-proof (i.e., truth-telling is a dominant strategy), and Pareto efficient (i.e., a socially optimal permutation is achieved). Furthermore, Ma [20] showed that such a mechanism is the only one that satisfies all the above three requirements.

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From the perspective of computation, a unique strict core in traditional housing markets can be computed by a polynomial-time algorithm (mechanism) called *top trading cycles (TTC)* [24]. Intuitively, in each step of the procedure, each agent points to the agent who owns the most favorite house remaining in the market. When a finite number of agents exists, at least one directed cycle is constructed. Then, for each cycle, the involved agents exchange houses according to the cycle and are removed from the market with the houses they received. This algorithm obviously terminates in polynomial-time. Indeed, designing such polynomial-time algorithms for variants of housing markets is one of the trends in the field of algorithmic mechanism design [2, 3, 9, 15, 25].

Another recent development in multi-agent resource allocation and algorithmic mechanism design is to design resource allocation mechanisms *over social networks*. In contrast to the traditional auction model where all the agents/buyers can directly access/observe the auction's information, networked models assumed that the information about the auction is only observed when it is forwarded through a social network. Li et al. [19] initiated this new trend by designing a single item auction mechanism that incentivizes agents to forward information about the mechanism to their followers. Based on this seminal work, several extended auction mechanisms have been developed for various complex situations [16, 30, 32, 33].

To the best of our knowledge, there have been virtually no work on housing markets over social networks, which we refer to as *networked housing markets*. One difficulty of extending housing markets to mechanism design over social networks is that monetary compensation is not allowed in such markets, although in the auction model the auctioneer is allowed to collect money from agents/bidders. Indeed, we first show some impossibility results on networked housing markets; the application of TTC does not achieve strategy-proofness, and a very weak notion of the core, which only considers two-agent parent-child blocking coalitions, cannot be achieved by any strategy-proof mechanism.

Therefore, our main interest in this paper is addressing to what extent compatibility persists among strategy-proofness (SP) and other desirable requirements. More specifically, we investigate under which condition (i) TTC satisfies SP and (ii) the above weakened core becomes compatible with SP in a networked housing market. This paper investigates two kinds of conditions: on preference domain and on the structure of social networks. For the former, we show that TTC satisfies SP if and only if a given preference domain satisfies a well-known acyclicity condition. For the latter, we show that TTC satisfies SP if and only if all the agents are connected to a special agent called a moderator, who corresponds to the mechanism itself. We further show that a modified TTC mechanism satisfies SP and always chooses an allocation in the weaker notion of the core if the social network is a directed tree.

2 LITERATURE REVIEW

Shapley and Scarf [24] proposed TTC and showed that it always chooses a unique strict core for traditional housing markets. Ma [20] showed that TTC is the only mechanism that satisfies Pareto efficiency, individual rationality, and strategy-proofness. In the literatur, the model of housing markets is extended in several ways, such as taking into account indifference in preferences [2, 3], multiple houses per agent [9, 25, 28], and asymmetry over houses [29]. Sönmez [26] showed that choosing the strict core is strategy-proof if and only if the strict core is essentially single-valued, such as the cases where each agent has a single house and a strict preference.

Quite a few related works address the approach of weakening the core over social networks, One is so-called *graph-restricted games*, first introduced by Myerson [21], which is one variation of a cooperative game that deals with how agents make a coalition and divide its gains among themselves. In a graph-restricted game, two agents can work in the same coalition only when they share a communication channel. Igarashi and Elkind [13] considered a graph-restricted hedonic game, where an agent has a preference over the coalitions to which she belongs but there is no monetary transfer among agents. They examined solution concepts (which define a stable outcome) and the complexity of finding stable outcomes including the core stability. Both of these works studied the concept of the core on a network among agents, but did not consider resource allocations with ordinal preferences.

Our model of networked housing markets also has another interesting feature; the number of agents participating in a mechanism may vary, according to their actions. In the literature of social choice theory, many works have studied such variable populations in resource allocation without monetary compensation. Specifically, a solidarity property called *population monotonicity* [6, 18, 23, 31], requires that the arrival of a new agent affects all the agents who originally existed in the market in the same direction (i.e., positive or negative). This idea is quite similar to the incentives of agents to incentives to shed some followers. However, all these works only consider such an arrival of a new agent as an external event. To the best of our knowledge, there is virtually no work on resource allocation with variable populations based on agents' actions, and therefore analyzing the incentives of manipulating the population in housing markets is quite new.

Li et al. [19] proposed a new model of auctions, in which buyers are distributed in a social network over which the information on the auction is propagated. Utilizing a social network, a seller can advertise the auction to more potential buyers beyond her followers, as many works studied in network science [5, 7, 14]. Zhao et al. [33] studied a multi-unit unit-demand auction via social networks, where each unit is identical and each buyer requires a unit. Kawasaki et al. [16] proposed another auction mechanism via social networks for a multi-unit unit-demand setting. Takanashi et al. [30] focused on the efficiency of such auctions. However, to the best of our knowledge, there is no work on mechanism design for resource allocations over social networks without monetary compensation. Some works consider resource allocations over social networks without monetary compensation, while they ignore agents' incentives and focused on trades/swaps that are only between neighbors [10, 12].

3 MODEL

We refer to the model defined in this section as a *networked housing market*. A networked housing market has a set of n agents $N = \{1, \dots, i, \dots, n\}$ and a set of indivisible objects, usually referred to as *houses*, $H = \{h_1, \dots, h_i, \dots, h_n\}$. Each agent $i \in N$ is endowed with house h_i. An *allocation* $x = (x_i)_{i \in N}$ is a redistribution of the houses to the agents, where each component $x_i \in H$ is a house assigned to agent i under allocation x. Let X be the set of all possible allocations. Besides the above n agents, there exists a special agent s, a *moderator*. Note that the moderator is neither selfish nor endowed with any house. In practice, it can be considered as a market itself or a trusted third party.

For each agent $i \in N \cup \{s\}$, subset $r_i \subseteq N \setminus \{i\}$ of agents denotes her neighbors. Note that the neighborhood relation can be asymmetric. Each agent $i \in N$ also has strict preference $\succ_i$ over houses H, which is represented as a linear order of all the houses. We write $h \succ_i h'$ if agent i with preference $\succ_i$ prefers house h over h', and $h \succeq_i h'$ if either $h = h'$ or $h \succ_i h'$ holds. Let Π be the set of all $n!$ preferences. In sum, for agent $i \in N$, her type (also known as *private information*) θ_i is given as $(\succ_i, r_i)$.

Now we are ready to describe the mechanism design model considered in this paper. We restrict our attention to direct revelation mechanisms, to which each agent declares her type $\theta'_i = (\succ'_i, r'_i)$. Note that we assume partial verification is possible, i.e., agent i can only declare $\theta'_i = (\succ'_i, r'_i)$ s.t., $r'_i \subseteq r_i$. This partial verification scheme obviously satisfies a well-known *Nested Range Condition* [11], which guarantees that the revelation principle holds. Thus, we can restrict our attention to direct revelation mechanisms without loss of generality. Let $R(\theta_i)$ denote a set of all possible types that agent i with true type θ_i can declare. That is, for any i and any $\theta_i = (\succ_i, r_i)$, $R(\theta_i) := \{(\succ'_i, r'_i) \mid r'_i \subseteq r_i\}$.

Let $\theta' = (\theta'_1, \dots, \theta'_n)$ denote a profile of the agents' declared types. We also use the following standard notations: θ'_{-i} is a profile of the agents' declared types except for agent i, $(\theta'_i, \theta'_{-i})$ is a profile of the agents' declared types where agent i declares θ'_i and other agents declare θ'_{-i}, and $R(\theta_{-i})$ is a set of profiles that agents (except i) can jointly declare when their true type profile is given as θ_{-i}. For notation simplicity, let r denote profile $(r_i)_{i \in N}$ for given θ, which we sometimes refer as a *social network*. A social network can be represented as a directed graph, $G = (V, E)$, where $V := N \cup \{s\}$, and for any pair $i, j \in N \cup \{s\}$, an edge $(i, j) \in E$ if and only if $j \in r_i$. Agent i is *connected* if there is a path $s \to \cdots \to i$ in G defined by reported profile r'. Without loss of generality, we assume that all agents N are connected when they all sincerely report their followers, i.e., r is reported.

A (direct revelation) mechanism f is then defined as a function that takes θ' as input and returns $f(\theta') \in X$. Let $f_i(\theta')$ denote the house assigned to agent i. In this paper, we further restrict our attention to *feasible* mechanisms, which only exchange the houses of connected agents. Formally, allocation $x \in X$ is feasible under reported r' if $x_i = h_i$ for any agent i who is disconnected under r'. A mechanism is feasible if, for any reported θ', $f(\theta')$ is feasible.

As a result, a mechanism design problem for a networked housing market is defined as a tuple (N, Π, r), where N defines the maximum population of the market, and both Π and r define the possible action spaces of the agents.

3.1 Properties

In this section we define four desirable properties for exchange mechanisms: *strategy-proofness*, *strict core*, *individual rationality*, and *Pareto efficiency*. Intuitively, strategy-proofness requires that telling a true type θ_i is a dominant strategy for each agent $i \in N$. Under an allocation in the strict core, no group of agents has an incentive to jointly deviate from the mechanism. Individual rationality requires that telling a true type θ_i guarantees, for each agent i, a weakly better house than her initial endowment, h_i. Finally, for given Pareto efficient allocation $x \in X$, we cannot find another allocation y that is weakly better for all the agents $i \in N$ and strictly better for at least one agent $j \in N$.

Definition 3.1 (Strategy-Proofness). For a networked housing market, a mechanism is said to satisfy *strategy-proofness* (SP) if, for any N, any $i \in N$, any θ_{-i}, any $\theta'_{-i} \in R(\theta_{-i})$, and any $\theta'_i \in R(\theta_i)$, it holds that $f_i(\theta_i, \theta'_{-i}) \succeq_i f_i(\theta'_i, \theta'_{-i})$.

Definition 3.2 (Strict Core). For a networked housing market, allocation $x \in X$ is said to be in the *strict core* (SC) under profile θ if $\nexists S \subseteq N$ such that, under some allocation $y(\neq x) \in X$ satisfying $\bigcup_{i \in S} y_i = \bigcup_{i \in S} h_i$, (a) $y_i \succeq_i x_i$ holds for any $i \in S$, and (b) $y_j \succ_j x_j$ holds for some $j \in S$. Let $\mathrm{SC}(\theta)$ be the set of such allocations for given θ. For a networked housing market, mechanism f is said to satisfy SC if for any θ, $f(\theta) \in \mathrm{SC}(\theta)$.

Set S of agents is usually called a *blocking coalition*, where allocation y weakly dominates allocation x for coalition S. We can analogously define the *weak core* (WC) by replacing conditions (a) and (b) with (c) $y_i \succ_i x_i$ for any $i \in S$. In this case, allocation y *strongly* dominates allocation x for coalition S. Let $\mathrm{WC}(\theta)$ be the weak core for the given θ, and mechanism f is said to satisfy WC if $f(\theta) \in \mathrm{WC}(\theta)$ for any θ.

Definition 3.3 (Individual Rationality). For a networked housing market, feasible allocation $x \in X$ is said to be *individually rational* for given θ if $x_i \succeq_i h_i$ for any $i \in N$. Let $\mathrm{IR}(\theta)$ be a set of such allocations under θ. Mechanism f is said to satisfy *individual rationality* (IR) if $f(\theta) \in \mathrm{IR}(\theta)$ for any θ.

Definition 3.4 (Pareto Efficiency). For a networked housing market, feasible allocation x Pareto dominates another feasible allocation y under θ if $x_i \succeq_i y_i$ for any $i \in N$ and $x_j \succ_j y_j$ for some $j \in N$. Let $\mathrm{PE}(\theta)$ be a set of feasible allocations that is not Pareto dominated by any other feasible allocation. Mechanism f is said to satisfy *Pareto efficiency* (PE) if $f(\theta) \in \mathrm{PE}(\theta)$ for any θ.

It is obvious that requiring a strict core allocation is stronger than requiring both IR and PE. Indeed, by choosing $S = N$, the definition of strict core coincides with Pareto efficiency, and choosing $S = \{i\}$ for each $i \in N$ makes the definition of strict (and weak) core identical to individual rationality. Formally, for any θ, both $\mathrm{SC}(\theta) \subseteq \{\mathrm{PE}(\theta) \cap \mathrm{IR}(\theta)\}$ and $\mathrm{SC}(\theta) \subseteq \mathrm{WC}(\theta) \subseteq \mathrm{IR}(\theta)$ hold.

3.2 Top Trading Cycles

In this paper we apply TTC to our networked housing market model and propose a modified version of TTC in Section 6. Now we formally define TTC and briefly review its characteristics.

Definition 3.5 (Top Trading Cycles (TTC) [24]). The top trading cycles (TTC) mechanism is defined by the following algorithm:

Step t (≥ 1)**.** If no agent remains in the market, then the algorithm terminates; otherwise, construct a directed graph whose vertices correspond to the remaining agents. Each agent points to the agent who has her favorite house remaining in the market. Obviously there is at least one cycle. Assign to each agent in each cycle the house owned by the agent to which she points. Remove all such cycles from the graph and go to ***Step*** $t + 1$.

It is easy to observe that TTC always terminates when there is a finite number of agents. From economic and game theoretic viewpoints, it also has many attractive characteristics, as described in the following two theorems.

Theorem 3.6 (Shapley and Scarf [24]). *For a traditional housing market, the strict core is a singleton and TTC chooses it.*

Theorem 3.7 (Ma [20]). *For a traditional housing market, TTC is a unique mechanism that satisfies SP, IR, and PE.*

4 GENERAL RESULTS

Now we are ready to describe our contributions. We first show that, strategy-proofness in our networked housing market model can be decomposed into the robustness against preference misreports (which is equivalent to strategy-proofness in traditional housing markets) and the robustness against removal of followers.

Theorem 4.1. *Consider mechanism f that satisfies SP in a traditional housing market, and mechanism f^* for a networked housing market that is defined by applying f for the connected agents. Then, f^* also satisfies SP for the networked housing market if*

$$\begin{aligned} &\forall i \in N, \forall \theta_{-i}, \forall \theta_i = (\succ_i, r_i), \forall \theta'_i = (\succ_i, r'_i) \in R(\theta_i), \\ &f^*_i(\theta_i, \theta_{-i}) \succeq_i f^*_i(\theta'_i, \theta_{-i}). \end{aligned} \tag{1}$$

In other words, as long as we know that the current mechanism, like TTC, is strategy-proof in the traditional housing market, it is adequate to check whether no agent can benefit by just hiding some of her followers. All the proofs for strategy-proofness provided in this paper are based on this theorem.

Proof. Since f satisfies SP for the traditional housing market and f^* behaves identically as f for the connected agents,

$$\begin{aligned} &\forall i \in N, \forall \theta_{-i}, \forall \theta_i = (\succ_i, r_i), \forall \theta''_i = (\succ'_i, r_i) \\ &f^*_i(\theta_i, \theta_{-i}) \succeq_i f^*_i(\theta''_i, \theta_{-i}) \end{aligned} \tag{2}$$

holds. Now consider a case where arbitrary agent i with true type $\theta_i = (\succ_i, r_i)$ is misrepresenting its type to $\hat{\theta}_i = (\hat{\succ}_i, \hat{r}_i)$ under f, where $\hat{r}_i \subseteq r_i$. For any θ_{-i}, we have

$$f^*_i(\theta_i, \theta_{-i}) \succeq_i f^*_i((\succ_i, \hat{r}_i), \theta_{-i})$$

from (1), and

$$f^*_i((\succ_i, \hat{r}_i), \theta_{-i}) \succeq_i f^*_i(\hat{\theta}_i, \theta_{-i})$$

from (2). Therefore, $f^*_i(\theta_i, \theta_{-i}) \succeq_i f^*_i(\hat{\theta}_i, \theta_{-i})$ holds. □

For a networked housing market, TTC is applied for the agents connected under reported types r; disconnected agents cannot participate/exchange. The next two theorems show that TTC is strategy-proof in a networked housing market if and only if there are fewer than three agents. They give a complete understanding on the effect of problem restrictions according to N.

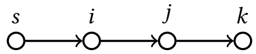

Figure 1: Social network where SP, IR, and PE are incompatible

THEOREM 4.2. *For a networked housing market (N, Π, r) with general r, TTC satisfies SP when $n \leq 2$.*

PROOF. Since TTC satisfies SP for a traditional housing market, it suffices, from Theorem 4.1, to show that no agent can benefit by hiding some of its followers when $n \leq 2$. Obviously, such a manipulation occurs only when the two agents are in a parent-child relation, i.e., $r_s = \{i\}$, $r_i = \{j\}$, and $r_j = \emptyset$, and agent i is hiding its child j. Furthermore, when agent i receives its house h_i as the final assignment, such a manipulation is not beneficial. From the definition of TTC, agent i receives h_j only if $h_j \succ_i h_i$. However, after such a hiding manipulation, agent j becomes disconnected, and thus i receives h_i from the feasibility assumption, which is worse than original assignment h_j under truthfully telling r_i. □

THEOREM 4.3. *For a networked housing market (N, Π, r) with general r, there does not exist a mechanism that satisfies SP IR, and PE when $n \geq 3$.*

PROOF. Since domain Π of strict preferences is rich enough, TTC is the unique mechanism that satisfies SP, IR, and PE (from Theorem 3.7). Therefore, we show that TTC violates SP for $n \geq 3$.

Consider a case with three agents, i, j, and k, where social network r is such that $r_s = \{i\}$, $r_i = \{j\}$, $r_j = \{k\}$, and $r_k = \emptyset$, and their preferences are given as follows:

$$\begin{array}{lcl} \succ_i & : & h_k \succ h_j \succ h_i \\ \succ_j & : & h_k \succ h_i \succ h_j \\ \succ_k & : & h_i \succ h_k \succ h_j \end{array}$$

Figure 1 describes social network r. TTC returns allocation x such that $x_i = h_k$, $x_j = h_j$, and $x_k = h_i$ when all the agents truthfully report their types.

When agent j solely prevents forwarding the information to agent k, i.e., reporting $r'_j = \emptyset$, TTC returns another allocation y such that $y_i = h_j$ and $y_j = h_i$. Since agent j strictly prefers h_i to h_j under her true preference, this manipulation is beneficial, which violates the definition of strategy-proofness. □

Since TTC is the unique mechanism that satisfies both SP and SC, theorem 4.3 implies the incompatibility of SP and SC. We therefore weaken the traditional concept of the core by taking into account the network structure in a networked housing market. The following is one of the weakest variants of the core, which only cares about coalitions by two agents in a parent-child relation, as well as deviations by each single agent.

Definition 4.4 (Strict Core for Neighbors (SC4N)). For a networked housing market, an outcome is said to be in *the strict core for neighbors* (SC4N) under profile θ if the strict core condition holds for any singleton agent $i \in N$ and for any pair i, j of agents such that there is a path $s \to \cdots \to i \to j$ under θ. Let SC4N(θ) be the set of such allocations for given θ. A mechanism is said to satisfy *SC4N* if $f(\theta) \in$ SC4N(θ) for any θ.

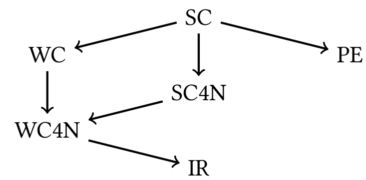

Figure 2: Relations among six properties about quality of allocations. Each edge between two properties denotes that its source property implies its destination property.

We can analogously define the weak core for neighbors (WC4N). For given θ, let WC4N(θ) be the set of allocations in the weak core for neighbors. The relations among SC, WC, SC4N, WC4N, PE, and IR are described in Fig. 2. Note that both SC4N and WC4N have an intersection with PE ∩ IR without being in an inclusion relation, which will be shown in the following two examples. On the other hand, since the preferences are assumed to be strict, there always exists a unique allocation in SC, which implies SC4N ∩{PE∩IR} ≠ ∅ (and therefore WC4N ∩ {PE ∩ IR} ≠ ∅).

Example 4.5 (SC4N and PE). Consider three agents $N = \{i, j, k\}$ who have the same social network shown in Fig. 1, i.e., $r_s = \{i\}$, $r_i = \{j\}$, $r_j = \{k\}$, and $r_k = \emptyset$. Consider the following preferences:

$$\begin{array}{lcl} \succ_i & : & h_k \succ h_j \succ h_i \\ \succ_j & : & h_i \succ h_k \succ h_j \\ \succ_k & : & h_i \succ h_j \succ h_k. \end{array}$$

Allocation x such that $x_i = h_j$, $x_j = h_k$, and $x_k = h_i$ is Pareto efficient and individually rational, but not in SC4N; coalition $\{i, j\}$ weakly blocks x. On the other hand, another allocation y such that $y_i = h_j$, $y_j = h_i$, and $y_k = h_k$ is in SC4N, but not Pareto efficient: allocation z such that $z_i = h_k$, $z_j = h_i$, and $z_k = h_j$ Pareto dominates y.

Example 4.6 (WC4N and PE). Consider four agents, $N = \{i, j, k, \ell\}$, and the social network described in Fig. 3, i.e., $r_s = \{i\}$, $r_i = \{j, k, \ell\}$, $r_j = \{\ell\}$, and $r_k = \{\ell\}$. Their preferences are given as follows:

$$\begin{array}{lcl} \succ_i & : & h_\ell \succ h_k \succ h_i \succ h_j \\ \succ_j & : & h_i \succ h_k \succ h_\ell \succ h_j \\ \succ_k & : & h_\ell \succ h_j \succ h_k \succ h_i \\ \succ_\ell & : & h_i \succ h_j \succ h_\ell \succ h_k. \end{array}$$

Allocation x such that $x_i = h_k$, $x_j = h_i$, $x_k = h_\ell$, and $x_\ell = h_j$ is Pareto efficient and individually rational, but not in WC4N; coalition $\{i, \ell\}$ strongly blocks x. On the other hand, another allocation y such that $y_i = h_k$, $y_j = h_\ell$, $y_k = h_j$, and $y_\ell = h_i$ is in WC4N, but not Pareto efficient: allocation z such that $z_i = h_\ell$, $z_j = h_k$, $z_k = h_j$, and $z_\ell = h_i$ Pareto dominates y.

As mentioned above, WC4N is one of the weakest variants of the core for networked housing markets. However, we obtains another incompatibility result for WC4N and SP, which emphasizes the difficulty of achieving SP for networked housing markets.

THEOREM 4.7. *For a networked housing market (N, Π, r) with general r, there does not exist a mechanism that satisfies SP and WC4N when $n \geq 3$.*

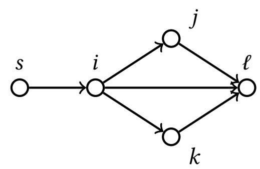

Figure 3: Social network where WC4N and PE∩IR are not in an inclusion relation

Proof. Consider a case with three agents, i, j, and k, where social network r is such that $r_s = \{i\}$, $r_i = \{j, k\}$, $r_j = \{k\}$, and $r_k = \emptyset$, and their preferences are given as follows:

$$\begin{array}{lcl} \succ_i & : & h_k \succ h_i \succ h_j \\ \succ_j & : & h_k \succ h_j \succ h_i \\ \succ_k & : & h_j \succ h_i \succ h_k. \end{array}$$

Figure 4 describes the network.

Note that WC4N implies IR. There are three allocations, x, y, and z, satisfying IR when all the agents truthfully report their types:

$$\begin{array}{l} x = (h_i, h_k, h_j) \\ y = (h_k, h_j, h_i) \\ z = (h_i, h_j, h_k) \end{array}$$

However, for both y and z, there is a strongly blocking coalition, $\{j, k\}$, which violates WC4N.

Thus, if a mechanism f satisfies both SP and WC4N, it returns allocation x for the above input, in which agent i receives her initial endowment house h_i. In this case, however, agent i would have an incentive not to forward the information to agent j; without agent j, the only allocation available, from both feasibility and WC4N, is to swap the houses between agents i and k, which results in agent i getting a better house, h_k. □

5 PREFERENCE RESTRICTIONS

According to Theorem 4.3, we are interested in the structure of preferences under which TTC satisfies SP. Here we focus on *acyclic* preferences, which have been investigated in the literature of two-sided matching to guarantee population monotonicity of TTC [18] and Pareto efficiency of the deferred acceptance mechanism [8]. Indeed, we can show that TTC satisfies SP in a networked housing market if and only if the preference domain is acyclic.

Definition 5.1 (Acyclic Domain). Domain $\Pi' \subseteq \Pi$ of preferences is said to be *acyclic* if for any two preferences, $\succ, \succ' \in \Pi'$, and for any three distinct houses, $h_i, h_j, h_k \in H$, it holds that

$$[h_i \succ h_j \succ h_k] \Rightarrow [h_i \succ' h_k].$$

Intuitively, acyclicity requires that all the involved preferences share a quite similar form, which can be represented by recursively applying the following rule from the top of all the preferences:

(1) the number of houses that are ranked at the top by at least one preference is one or two, and
(2) if there are two such houses, say h_i and h_j, then all the involved preferences rank them at the top and second.

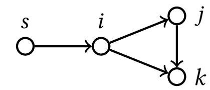

Figure 4: Social network where SP and WC4N are incompatible

For example, domain $\Pi' \subset \Pi$, which consists of the following four preferences over five houses $h_i, h_j, h_k, h_\ell, h_m$, is acyclic.

$$\begin{array}{lcl} \succ & : & h_i \succ h_j \succ h_k \succ h_\ell \succ h_m \\ \succ' & : & h_i \succ h_j \succ h_k \succ h_m \succ h_\ell \\ \succ'' & : & h_j \succ h_i \succ h_k \succ h_\ell \succ h_m \\ \succ''' & : & h_j \succ h_i \succ h_k \succ h_m \succ h_\ell. \end{array}$$

Theorem 5.2. *For a networked housing market (N, Π', r) with $n \geq 3$ and general r, TTC satisfies SP if and only if domain Π' of preferences is acyclic.*

Proof. **If ($\Leftarrow$) Part:** From the definition of acyclicity, it is easy to show that, when domain Π' of preferences is acyclic, there is a hierarchy over all the houses, in each level of which there are at most two houses. For example, in the above domain with four preferences over five houses, the first level consists of two houses, h_i and h_j, the second level consists of one house, h_k, and the third level consists of two houses, h_ℓ and h_m. As long as at least one house of a certain level remains in the market, all the agents in it never prefer the houses in any lower level. An agent therefore can only exchange her house with one specific agent (if any) whose house is in the same level, regardless whether she truthfully forwarded the information. Furthermore, she can exchange her house with that agent if and only if both agree to the trade. Therefore, inviting as many followers as possible is a dominant strategy; for each agent, having more agents whose houses are in any different level does not change her final assignment, and having another agent whose house is in the same level is never disadvantageous. Combining this observation with the fact that TTC satisfies SP in the traditional housing market, according to Theorem 4.1, we can see that TTC satisfies SP in our networked housing market.

Only If ($\Rightarrow$) Part: Assume two preferences, $\succ, \succ' \in \Pi'$, and three distinct houses, $h_i, h_j, h_k \in H$, such that

$$[h_i \succ h_j \succ h_k] \wedge [h_k \succ' h_i].$$

There are two possibilities on $\succ'$: (i) $h_k \succ' h_j$ or (ii) $h_j \succ' h_k$.

For case (i), consider social network r such that $r_s = \{k\}$, $r_k = N \setminus \{i\}$, $r_j = \{i\}$, and $r_\ell = \emptyset$ for any $\ell \in N \setminus \{j, k\}$, and the following preferences:

$$\succ_k = \succ, \succ_i = \succ_j = \succ', \succ_\ell \text{ is arbitrary for any } \ell \in N \setminus \{i, j, k\}.$$

Also, focus on the cases where agent k reports $r'_k = \{j\}$, under which only agents i, j, k are connected when all the other agents truthfully report their types (see case (i) of Fig. 5).

When all the agents (except k) report their types truthfully, allocation x returned by TTC is such that $x_i = h_k$, $x_j = h_j$, and $x_k = h_i$. When agent j solely manipulates by reporting $r'_j = \emptyset$ to remove i from the market, allocation y returned by TTC would be such that $y_j = h_k$ and $y_k = h_j$. Thus, agent j can benefit from this manipulation, which violates the definition of SP.

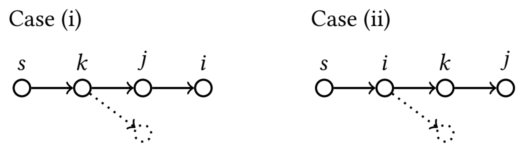

Figure 5: Examples used in proof for only if part of Theorem 5.2. Dotted edges are not reported, and dotted vertices are not connected.

For case (ii), the following holds:

$$[h_j \succ' h_k \succ' h_i] \wedge [h_i \succ h_k]$$

Therefore, by substituting i with j, j with k, and k with i (see case (ii) of Fig. 5), almost the same argument with case (i) holds. □

Note that, when the preference domain is restricted, the uniqueness of TTC may not hold. For example, under the domain of single-peaked preferences for traditional housing markets, the crawler mechanism, proposed by Bade [4], satisfies SP, PE, and IR, although its outcome does not coincide with the one returned by TTC. Therefore, it is possible that some other mechanism also satisfies both SP and SC in the networked housing market.

6 NETWORK RESTRICTION

In this section, we consider another way to overcome the impossibilities presented in Section 4; restricting the possible structure of social networks. While the online social networks in practice could have enormous number of possible structures, other types of social networks might have some typical and specific structures, such as trees (although the mechanism cannot observe the exact structure and each agent can still manipulate the network).

6.1 Pareto Efficiency

We first investigate, under which condition on r, SP is achieved by TTC without restricting the domain of preferences Π. The following theorem gives a complete answer to this question. TTC satisfies SP if and only if r is essentially a star with s at its center.

Theorem 6.1. *For a networked housing market (N, Π, r) with $n \geq 3$ and a fixed r, TTC satisfies SP if and only if $r_s = N$, i.e., all the agents are directly connected to the moderator.*

Note that agents are allowed to be connected with each other. However, since all the agents are directly connected to s, no beneficial hiding strategy exists. Such a star structure is quite common in practical social networks, which often have a powerful leader who directly communicates with all the other members.

Proof. **If (⇐) Part:** Since all the agents are directly connected to the moderator, there is no agent i who can remove any of her followers by misreporting r'_i. This implies that Eq. (1) of Theorem 4.1 holds for TTC, and therefore TTC satisfies SP.

Only If (⇒) Part: Section 4 already showed that TTC satisfies SP for $n \leq 2$. Therefore, to complete the proof for this direction, it suffices to prove that TTC fails to satisfy SP when there is at least one agent who is not directly connected to the moderator. Let j be such an agent, and let i be one of her parents. Without loss of generality, we can assume that i is directly connected to s; otherwise, recursively choose that agent i as j and find her parent who is directly connected to s.

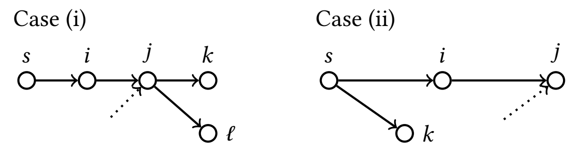

Figure 6: Examples used in proof for only if part of Theorem 6.1. Dotted edges are not reported by agents except for i, j, k.

Consider a situation where all the incoming edges to j (except the one from i) are removed by their source agents. Now only two possible cases exist: if agent i does not forward the information to agent j, i.e., reports $r'_i \not\ni j$, (i) all the agents except i get disconnected, or (ii) at least one agent remains connected.

For case (i), let k denote one of j's direct children; at least one direct child exists since $n \geq 3$ and there only remains agent i if she removes j (see Case (i) of Fig 6). Note that such k is not directly connected to s; otherwise k must still be connected if j is removed, which violates the precondition of this case (i). Now consider the following preferences of agents i, j, k:

$$\begin{aligned} \succ_i &: \quad h_k \succ h_j \succ h_i \succ \cdots \\ \succ_j &: \quad h_i \succ h_j \succ h_k \succ \cdots \\ \succ_k &: \quad h_i \succ h_k \succ h_j \succ \cdots . \end{aligned}$$

All other houses, e.g., h_ℓ, are ranked below these three houses. When agent i forwards the information to agent j, agent j has an incentive not to forward the information to k, which violates SP.

For case (ii), let k denote one such connected agent if i removes j (see Case (ii) of Fig 6). Now consider the following preferences of agents i, j, k:

$$\begin{aligned} \succ_i &: \quad h_k \succ h_i \succ h_j \succ \cdots \\ \succ_j &: \quad h_k \succ h_j \succ h_i \succ \cdots \\ \succ_k &: \quad h_j \succ h_i \succ h_k \succ \cdots . \end{aligned}$$

Agent i has an incentive not to forward the information to agent j, which violates SP. □

In contrast to the preference restriction, we keep preference domain Π rich enough in this network restriction approach. Therefore, TTC is still the only mechanism that satisfies SP, IR, and PE. Hence, the following corollary holds from Theorem 6.1.

Corollary 6.2. *For a networked housing market (N, Π, r) with $n \geq 3$ and a fixed r, there exists a mechanism that satisfies SP, IR, and PE if and only if $r_s = N$.*

6.2 Strict Core for Neighbors

We next investigate whether SP and SC4N can be compatible when network structure r can be controlled. According to Theorem 4.7, the existence of multiple paths to an agent seems critical for incompatibility. Therefore, in this section we restrict our attention to social network r, which is a directed tree rooted at s.

We first introduce a modification to TTC, which restricts the possible actions of agents. Specifically, each agent can only point to

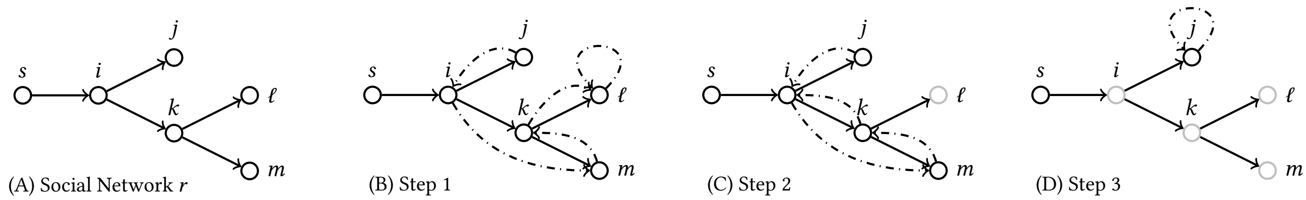

Figure 7: Behavior of Modified TTC. Solid edges represent social network, and dash-dotted edges represent constructed directed graph based on agents' preferences. Agents who left the market are drawn in gray.

herself, her parent, and her descendants. This restriction is static; if an agent cannot point to an existing agent in a certain step, then she cannot point to that agent in any later step.

Definition 6.3 (Modified TTC). The modified TTC mechanism is defined over tree networks as the following algorithm:

Step t (≥ 1)**.** If no agent remains in the market, the algorithm terminates; otherwise, construct a directed graph whose vertices corresponds to the remaining agents. Each agent points to the agent who *is her parent, herself, or one of her descendants* and has her favorite house remaining in the market. There is at least one cycle. Assign to each agent in each cycle the house owned by the agent to which she points. Remove all such cycles from the graph and go to ***Step*** $t+1$.

The following example demonstrates how the modified TTC behaves for a networked housing market.

Example 6.4. Consider the social network with five agents described in Fig. 7-(A). The agents' preferences are given as follows:

$$\begin{array}{rcl} \succ_i & : & h_m \succ h_j \succ h_k \succ h_\ell \succ h_i \\ \succ_j & : & h_m \succ h_i \succ h_j \succ \cdots \\ \succ_k & : & h_\ell \succ h_j \succ h_i \succ h_k \succ h_m \\ \succ_\ell & : & h_m \succ h_\ell \succ \cdots \\ \succ_m & : & h_j \succ h_k \succ h_m \succ \cdots . \end{array}$$

Figure 7-(B) describes the pointing relation among agents at the beginning of Step 1. Notice that although agent j prefers h_m and agent m prefers h_j, they cannot point to each other in the modified TTC; each agent is only allowed to point to herself, her parent, or her descendants. Thus, they point to i and k, respectively. Similarly, although agent ℓ prefers h_m, she is pointing to herself. In this step, agent ℓ is removed from the market with her own house h_ℓ.

Figure 7 (C) describes the pointing relation among agents at the beginning of Step 2. Since agent ℓ left the market at the end of Step 1, agent k must change the agent to whom she is pointing. She next prefers h_j, but is not allowed to point to j. Instead she points to agent i, her parent, who has h_i, which is immediately after h_j in her preference $\succ_k$. In this step, there is a cycle: $i \to m \to k \to i$.

Finally, in Step 3 (Fig. 7-(C)), agent j is removed from the market with h_j. Allocation x, returned by the modified TTC, becomes $x_i = h_m$, $x_j = h_j$, $x_k = h_i$, $x_\ell = h_\ell$, and $x_m = h_k$.

Note that allocation x in Example 6.4 is strongly blocked by coalition $\{j, m\}$, and weakly blocked by a one-parent two-children coalition $\{k, \ell, m\}$. However, these are ignored in both WC4N and SC4N. Indeed, we can show the following positive characteristic of the modified TTC for tree networks.

Theorem 6.5. *The modified TTC satisfies both SP and SC4N for a networked housing market (N, Π, r) when r is a tree network.*

Proof. We first show that the modified TTC satisfies SP. The modified TTC satisfies SP for the traditional housing market, since it is equivalent to the TTC where the preference of each agent i is modified such that she considers the houses owned by agents (except her, her parent, and her descendants) are less preferred than her initial endowment house h_i. Thus, to show that the modified TTC satisfies SP for the networked housing market, it suffices to show that agent i cannot obtain a strictly better house by declaring $\theta'_i = (\succ_i, r'_i)$, where $r'_i \subsetneq r_i$, compared to the case where she declares her true type, $\theta_i = (\succ_i, r_i)$. By way of contradiction, assume agent i obtains a strictly better house when she declares r'_i instead of r_i. Let $\hat{r}$ denote a set of agents in the subtrees, each of which is rooted with each agent in $r_i \setminus r'_i$. In other words, by declaring r'_i instead of r_i, the agents in $\hat{r}$ are disconnected from s and cannot participate in the exchange. Since the modified TTC satisfies IR, the house she obtains when she declares r'_i cannot be her own house, h_i (otherwise, IR is violated when she declares r_i). Assume she obtains h_k, i.e., the house owned by agent k. One of the followings must hold: (i) k is i's descendant, or (ii) k is i's parent.

For case (i), there must exist a pointing sequence starting from k toward i, via one element in r'_i when i declares r'_i. In this situation, if i points to k, a cycle is formed. When i declares r_i, the behaviors of agents in the above pointing sequence do not change, since they cannot point to any agent in $\hat{r}$. In TTC, if there exists a pointing sequence toward i, the sequence remains until i is included in a cycle and removed from the market. This is also true for the modified TTC. Since i prefers h_k over the house she obtained when she declares r_i, i eventually points to k. Also, the above pointing sequence is eventually formed. Thus, a cycle is eventually formed and i obtains h_k. This is a contradiction.

For case (ii), when i declares r'_i, there must exist a pointing cycle $a_1 \to \ldots \to a_m \to a_1$, such that each a_j is a child of a_{j+1}, and $i \to j$ is included in sequence $a_1 \to \ldots \to a_m$. Assume $i = a_\ell$ and $k = a_{\ell+1}$, i.e., the cycle is $a_1 \to \ldots \to a_\ell (= i) \to a_{\ell+1} (= k) \to \ldots \to a_m \to a_1$. Here, agents $a_1, \ldots, a_{\ell-1}$ are i's descendants, and $a_{\ell+1}, \ldots, a_m$ are i's ancestors. When i declares r_i, the behaviors of i's descendants $a_1, \ldots, a_{\ell-1}$ do not change, since they cannot point to any agent in $\hat{r}$. On the other hand, agent $a_j \in \{a_{\ell+1}, \ldots, a_m\}$, who is an ancestor of i, may point to some agent in $\hat{r}$, since they are also descendants of a_j. However, if agent a_j points to some agent in $\hat{r}$ and is included in a cycle, then the cycle must include $i \to k$, since each agent can point only to her parent within her ancestors. Thus,

i obtains h_k. This is a contradiction. Therefore, even if a_j points to some agent in $\hat{r}$, she cannot be included in a cycle. Eventually, she points to a_{j+1} (or a_1 when $a_j = a_m$). As a result, the above cycle is formed and i obtains k. This is a contradiction.

We then show that SC4N also holds. By way of contradiction, assume there is a pair of agents i, j such that i is the parent of j, i obtains h_i, j obtains h_j, and at least one of them obtains a strictly better house than her obtained house in the modified TTC.

Assume agent i is included in a cycle and removed at Step k in the modified TTC, and agent j is included in a cycle and removed at Step k' in the modified TTC. Note that in the modified TTC, i can point to j, and vice versa. Assume $k < k'$ holds. However, at Step k, j remains in the market and i must point to j, since h_j is at least as good as her obtained house in the modified TTC. Since j is not included in a cycle in Step k, i is also not included in a cycle in Step k. This is a contradiction. We obtain a similar contradiction when $k > k'$ holds.

Thus, $k = k'$ must hold. However, at Step k, i must point to j, and j must point to i, since h_j (or h_i) is at least as good as her obtained house in the modified TTC. Then i and j form a cycle; i obtains h_j, and j obtains h_i. This is a contradiction. □

Trees are also quite common structure of social networks. For example, a hierarchy among people/agents, such as the university president, department deans, laboratory PIs, and other staff members, results in a tree structure of information diffusion.

7 DISCUSSION

In network auctions, mechanisms based on a *diffusion critical tree (DCT)* have been successful [16, 19]. Given θ', agent j is a critical parent of agent i, if i becomes disconnected when j does not forward the information. If i remains connected regardless of the manipulation of another agent, we assume s is the only critical parent of i. There can be multiple critical parents of i. We call the critical parent who is closest to i her *least critical parent*. Then, a DCT is constructed such that s is the root, and for each agent, her least critical parent becomes her (direct) parent.

For example, if the original social network is given as Fig. 8-(A), the DCT is given as Fig. 8-(B), assuming all agents truthfully declare their types/followers. Agent ℓ is connected even when either j or k does not forward the information. However, if i does not forward the information, ℓ becomes disconnected. Thus, i is the least critical parent of ℓ; ℓ is directly connected to i.

In network auctions, intuitively, we can construct a strategy-proof mechanism that works with a DCT as follows: for each agent i, (1) having more descendants is advantageous, and (2) the tree structures of other branches do not affect i's outcome. For example, assume agent j does not forward the information to agent ℓ. Then, the DCT changes as described in Fig. 8-(C). Here, compared to Fig. 8-(B), ℓ, who was a child of i, becomes a child of k. If property (2) holds, this manipulation is meaningless for j.

However, in a networked housing market, the tree structure of other branches can crucially affect an agent. Assume we modify the definition of SC4N such that a pair (i, j) of agents must be the parent and the child in the DCT, rather than in the original social network. Furthermore, we modify the definition of the modified TTC such that, the parent and descendants are defined based on the

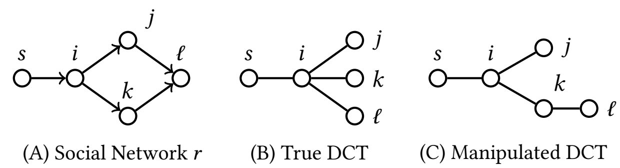

Figure 8: Original social network and two diffusion critical trees; (B) is under the truth declaration by all agents, and (C) is under a manipulation by j.

DCT rather than the original social network. This further modified TTC satisfies the above property (1), but it fails to satisfy (2). As a result, it does not satisfy SP.

More specifically, assume

$$\begin{array}{lcl} \succ_i & : & h_\ell \succ h_j \succ h_i \succ h_k \\ \succ_j & : & h_i \succ h_j \succ \cdots \\ \succ_k & : & h_\ell \succ h_k \succ \cdots \\ \succ_\ell & : & h_i \succ h_k \succ h_\ell \succ h_j. \end{array}$$

When all the agents truthfully declare their followers, the obtained DCT is given as Fig. 8-(B). When we apply the further modified TTC, agent i points to ℓ and vice versa; they exchange houses, and agent j obtains h_j. Assume agent j does not forward the information to ℓ. Then the DCT changes as described in Fig. 8-(C). In this case, agent ℓ cannot point to agent i. So, ℓ points to k and vice versa; they exchange houses. Next, agent i points to j and vice versa; they exchange houses. Thus, agent j obtains h_i, which is better than h_j for j. Intuitively, for agent j, agent ℓ is her rival who competes for h_i. Thus, moving ℓ away from i is beneficial.

We obtain an impossibility result that resembles Theorem 4.7 for WC4N defined on the DCT. Not only the further modified TTC, but also any strategy-proof mechanism defined on the DCT, fails to satisfy WC4N.

8 CONCLUDING REMARKS

In this paper we tackled a new resource allocation problem called the networked housing market. This is the very first work that considers agents' incentives of hiding information in resource allocation without monetary compensation. As well as two impossibility results for the general domain, we provided necessary and sufficient conditions on two problem restrictions under which TTC satisfies SP, and developed a new mechanism that satisfies both SP and SC4N when the social network is a tree. Future works include a further extension of our model, such as considering additional houses freely available from a moderator [27], addressing indifferences and asymmetry of preferences, and allowing multiple houses per agent. Applying mechanism design over social networks to other multi-agent resource allocation problems is also an promissing direction. Furthermore, considering other objectives than Pareto efficiency, such as maximizing the number of exchanges/swaps [17], would also be interesting.

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