Preferences and Constraints in Abstract Argumentation

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Abstract

In recent years there has been an increasing interest in extending Dung's framework to facilitate the knowledge representation and reasoning process. In this paper, we present an extension of Abstract Argumentation Framework (AF) that allows for the representation of preferences over arguments' truth values (3-valued preferences). For instance, we can express a preference stating that extensions where argument a is false (i.e. defeated) are preferred to extensions where argument b is false. Interestingly, such a framework generalizes the well-known Preference-based AF with no additional cost in terms of computational complexity for most of the classical argumentation semantics. Then, we further extend AF by considering both (3-valued) preferences and 3-valued constraints, that is constraints of the form $\varphi \Rightarrow v$ or $v \Rightarrow \varphi$, where φ is a logical formula and v is a 3-valued truth value. After investigating the complexity of the resulting framework, as both constraints and preferences may represent subjective knowledge of agents, we extend our framework by considering multiple agents and study the complexity of deciding acceptance of arguments in this context.

1 Introduction

Reasoning about preferences over a set of alternatives is central to rational decision-making. Preferences have been investigated in many contexts including e.g. decision theory, social choice, knowledge bases, and AI [Rossi *et al.*, 2011; Manlove, 2013; Santhanam *et al.*, 2016; Domshlak *et al.*, 2011; Pigozzi *et al.*, 2016; Conitzer, 2019; Brewka *et al.*, 2003; Staworko *et al.*, 2012]. Preferences are often modeled by means of an irreflexive, asymmetric, and transitive binary relation (expressing the preference of one element w.r.t. another one) and can be represented by an acyclic graph.

Recent years have witnessed intensive formal study, development, and application of Dung's abstract Argumentation Framework (AF) in various directions [Gabbay *et al.*, 2021]. An AF consists of a set $\mathcal{A}$ of arguments and an attack relation $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ that specifies conflicts between arguments (if argument a attacks argument b, then b is acceptable only if a is not). We can think of an AF as a directed graph whose nodes represent arguments and edges represent attacks. The meaning of an AF is given in terms of argumentation semantics, e.g. the well-known *grounded* (gr), *complete* (co), *preferred* (pr), *stable* (st), and *semi-stable* (ss) semantics, which intuitively tells us the sets of arguments (called σ-*extensions*, with $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}\}$) that can collectively be accepted to support a point of view in a dispute. For instance, for AF $\langle \mathcal{A}, \mathcal{R}\rangle = \langle\{\mathtt{a}, \mathtt{b}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a})\}\rangle$ having two arguments, a and b, attacking each other, there are two preferred/stable extensions, $\{\mathtt{a}\}$ and $\{\mathtt{b}\}$; neither a nor b is certainly accepted.

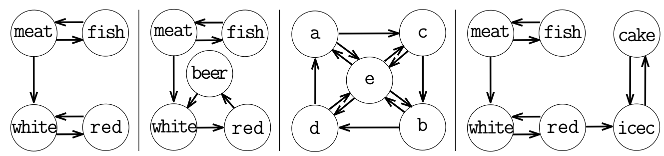

Figure 1: (From left to right) AFs Λ_1,Λ_2, Λ_5, and Λ_9 of Examples 1, 2, 5, and 9, respectively.

Several proposals have been made to extend the Dung's framework with the aim of better modeling the knowledge to be represented. These extensions include Bipolar AF [Nouioua and Risch, 2011], AF with recursive attacks and supports [Cohen *et al.*, 2015; Cayrol *et al.*, 2018], Dialectical framework [Brewka *et al.*, 2013], AF with preferences [Amgoud and Cayrol, 1998; Modgil and Prakken, 2013; Alfano *et al.*, 2022b; 2023a] and AF with constraints [Coste-Marquis *et al.*, 2006; Arieli, 2015; Alfano *et al.*, 2021b], as well as extensions for representing uncertain information [Fazzinga *et al.*, 2020; 2015; Li *et al.*, 2011]. In this paper, we focus on AF with constraints and preferences.

Example 1 Consider AF $\Lambda_1 = \langle\{\mathtt{fish}, \mathtt{meat}, \mathtt{red}, \mathtt{white}\}, \{(\mathtt{fish}, \mathtt{meat}), (\mathtt{meat}, \mathtt{fish}), (\mathtt{meat}, \mathtt{white}), (\mathtt{white}, \mathtt{red}), (\mathtt{red}, \mathtt{white})\}\rangle$, shown in Figure 1(left). Intuitively, Λ_1 describes what a person is going to have for lunch. (S)he will have either fish or meat, and will drink either white wine or red wine. However, if (s)he will have meat, then (s)he will not drink white wine. Λ_1 has three preferred (stable and semi-stable) extensions $E_1 = \{\mathtt{fish}, \mathtt{white}\}$, $E_2 = \{\mathtt{fish}, \mathtt{red}\}$, and $E_3 = \{\mathtt{meat}, \mathtt{red}\}$, which represent alternative menus.

Assume that there is a pescetarian customer and, as a consequence, (s)he wants to discard all menus with meat by putting the constraint $\texttt{meat} \Rightarrow \texttt{false}$, stating that argument meat must be rejected. Thus, feasible preferred extensions are only those where meat is defeated, that is E_1 and E_2.

Assume now that there is another customer which would express the preference on menus having meat instead of fish as main dish; the preference $\texttt{meat} > \texttt{fish}$ can be used to encode such a desideratum. In this case no extension is discarded. Among the three above-mentioned extensions representing the alternative menus, the best one for the considered customer is selected, that is, extension E_3. □

Considering the previous example, one could observe that the (pescetarian) user constraint could be modeled by modifying the AF through the addition of an (unattacked meta-) argument attacking meat. However, such kind of rewriting is not always easy to carry out, e.g. when constraints are defined by complex propositional formulae. In some cases, it is even not possible (e.g. under the complete semantics). In fact, the introduction of constraints and/or preferences is useful not only to separate the objective knowledge represented by the AF from the subjective restrictions and preferences added by users but also because, as it will be clear from our complexity analysis, the rewriting is not always possible.

The extension of AF with constraints, called Constrained AF (CAF), has been studied in [Coste-Marquis *et al.*, 2006; Arieli, 2015; Alfano *et al.*, 2021b], while for AF with preferences different semantics have been proposed in [Amgoud and Cayrol, 1998; 2002; Amgoud and Vesic, 2011; 2014; Cyras, 2016; Silva *et al.*, 2020].

Regarding Preference-based AF (PAF), two main approaches have been proposed in the literature. A first approach defines the PAF semantics in terms of an underlying AF [Amgoud and Cayrol, 2002; Amgoud and Vesic, 2014; Kaci *et al.*, 2018], whereas a second approach uses preferences to select a subset of extensions of the AF, called *best extensions* [Amgoud and Cayrol, 1998; 2002; Amgoud and Vesic, 2011; 2014; Cyras, 2016; Silva *et al.*, 2020]. Considering the first approach, there are cases where the semantics may give counterintuitive results (e.g. the extensions of the rewritten AF are not conflict-free w.r.t. the initial AF). Thus, in the rest of the paper, we focus on the second approach.

A limitation of the forms of preferences proposed in the literature is that, as AF semantics may be 3-valued (arguments can be either *accepted*, *defeated*, or *undecided*) they do not allow expressing preferences referring to the status of arguments. For instance, continuing with our example, classical preferences do not allow us to express a preference for menus (i.e. extensions) containing fish w.r.t. menus not containing fish (i.e. extensions where fish is defeated or undecided) or to express a preference for menus surely not containing fish (i.e. with fish being defeated) w.r.t. menus surely not containing meat (i.e. with meat being defeated).

As most of the AF semantics are 3-valued, in this paper we study AF with *extended preferences*, that is preferences of the form $a^v \succ b^w$, where a and b are arguments and v and w are truth values (*true*, *false*, and *undefined*) denoting the status of associated arguments (accepted, defeated, and undecided, respectively). We also study the combination of extended preferences with 3-valued constraints and propose a framework with multiple agents sharing the same AF while expressing different constraints and preferences.

Contributions. Our main contributions are as follows.

- We introduce the *extended Preference-based AF* (ePAF), an extension of AF where preferences are 3-valued, in the sense that they also refer to the status of arguments (Section 3). We show that ePAF is in general more expressive than PAF (under the so-called KTV interpretation) and that AF semantics can be emulated by ePAF under complete-semantics (cf. Proposition 2). We study the complexity of the verification (Ver_σ) as well as credulous (CA_σ) and skeptical (SA_σ) acceptance problems, and show that in most of the cases (i) it increases by one level in the polynomial hierarchy w.r.t. that of AF, and (ii) is the same as that of PAF under KTV criterion (see Table 1).
- We combine the features of CAF and ePAF to define the *extended Preference-based Constrained AF* (ePCAF), and investigate the complexity of the verification and credulous and skeptical acceptance problems for ePCAF for multi-status semantics $\sigma \in \{\texttt{co}, \texttt{st}, \texttt{pr}, \texttt{ss}\}$. As shown in Table 1, it turns out that ePCAF is more expressive than both CAF and PAF, though the complexity bounds do not increase w.r.t. that of ePAF (Section 4).
- To establish the above-mentioned complexity relationships, *i*) we study the complexity of the verification problem for CAF, showing that it does not increase w.r.t. that of AF; *ii*) we show that CAF is more expressive than AF under preferred semantics by closing the complexity gap (NP-hard, in Σ_2^p) for the credulous acceptance problem (cf. Table 1).
- We further extend our framework by considering a multiple agents scenario. Here the objective knowledge is modeled through an AF, whereas, the agents' subjective knowledge is modeled by means of constraints and (extended) preferences. Also in this context, we study the computational complexity and show that there is no increase w.r.t. ePCAF.

2 Preliminaries

We next review the Dung's framework and its generalizations with constraints (CAF) and preferences (PAF).

2.1 Abstract Argumentation Framework

An abstract *Argumentation Framework* (AF) is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{A}$ is a (finite) set of *arguments* and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a set of *attacks* (also called *defeats*). Different semantics have been defined for AF, leading to the characterization of collectively acceptable sets of arguments, called *extensions* [Dung, 1995].

Given an AF $\Lambda = \langle \mathcal{A}, \mathcal{R} \rangle$ and a set $E \subseteq \mathcal{A}$ of arguments, an argument $a \in \mathcal{A}$ is said to be *i*) *defeated* w.r.t. E iff $\exists b \in E$ such that $(b, a) \in \mathcal{R}$; *ii*) *acceptable* w.r.t. E iff $\forall b \in \mathcal{A}$ with $(b, a) \in \mathcal{R}$, $\exists c \in E$ such that $(c, b) \in \mathcal{R}$. Given an extension E, the sets of defeated ($\mathbf{f}(E)$), acceptable ($\mathbf{t}(E)$), and undecided ($\mathbf{u}(E)$) arguments w.r.t. E are defined as follows (where Λ is understood):

- $\mathbf{f}(E) = \{a \in \mathcal{A} \mid \exists b \in E \,.\, (b, a) \in \mathcal{R}\}$;
- $\mathbf{t}(E) = \{a \in \mathcal{A} \mid \forall b \in \mathcal{A} \,.\, (b, a) \in \mathcal{R} \Rightarrow b \in \mathbf{f}(E)\}$;
- $\mathbf{u}(E) = \mathcal{A} \setminus (\mathbf{t}(E) \cup \mathbf{f}(E))$.

Given an AF $\langle \mathcal{A}, \mathcal{R} \rangle$, a set $E \subseteq \mathcal{A}$ of arguments is said to be:

- *conflict-free* iff $E \cap \mathbf{f}(E) = \emptyset$;
- *admissible* iff it is conflict-free and $E \subseteq \mathbf{t}(E)$.

Given an AF $\langle \mathcal{A}, \mathcal{R} \rangle$, a set $E \subseteq \mathcal{A}$ is an *extension* called:

- *complete* (co) iff it is conflict free and $E = \mathbf{t}(E)$;
- *preferred* (pr) iff it is a $\subseteq$-maximal complete extension;
- *stable* (st) iff it is a total complete extension, i.e. a complete extension s.t. $E \cup \mathbf{f}(E) = \mathcal{A}$ or, equivalently, $\mathbf{u}(E) = \emptyset$;
- *semi-stable* (ss) iff it is a complete extension with a minimal set of undecided arguments, i.e. $\mathbf{f}(E)$ is $\subseteq$-minimal;
- *grounded* (gr) iff it is the $\subseteq$-smallest complete extension.

The set of complete (resp. preferred, stable, semi-stable, grounded) extensions of an AF Λ will be denoted by $\mathtt{co}(\Lambda)$ (resp. $\mathtt{pr}(\Lambda), \mathtt{st}(\Lambda), \mathtt{ss}(\Lambda), \mathtt{gr}(\Lambda)$). It is well-known that the set of complete extensions forms a complete semilattice w.r.t. $\subseteq$, where $\mathtt{gr}(\Lambda)$ is the meet element, whereas the greatest elements are the preferred extensions. All the above-mentioned semantics except the stable admit at least one extension. The grounded semantics, that admits exactly one extension, is said to be a *unique status* semantics, while the others are *multiple status* semantics. With a little abuse of notation, we also use $\mathtt{gr}(\Lambda)$ to denote the grounded extension. For any AF Λ, $\mathtt{st}(\Lambda) \subseteq \mathtt{ss}(\Lambda) \subseteq \mathtt{pr}(\Lambda) \subseteq \mathtt{co}(\Lambda)$, $\mathtt{gr}(\Lambda) \in \mathtt{co}(\Lambda)$ and $\mathtt{st}(\Lambda) \neq \emptyset \Rightarrow \mathtt{st}(\Lambda) = \mathtt{ss}(\Lambda)$.

Example 2 Consider the AF Λ_2 shown in Figure 1 (center-left). Λ_2 has three complete extensions: $E_0 = \emptyset$, $E_1 = \{\mathtt{fish}\}$ (where $\mathbf{f}(E_1) = \{\mathtt{meat}\}$), and $E_2 = \{\mathtt{meat}, \mathtt{red}\}$ (where $\mathbf{f}(E_2) = \{\mathtt{fish}, \mathtt{white}, \mathtt{beer}\}$. E_0 is the grounded extension, whereas E_1 and E_2 are preferred (semi-stable and stable) extensions. □

Given an AF $\Lambda = \langle \mathcal{A}, \mathcal{R} \rangle$ and a semantics $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}, \mathtt{gr}\}$, the *verification* problem, denoted as Ver_σ, is deciding whether a set $S \subseteq \mathcal{A}$ is a σ-extension of Λ. Moreover, for $g \in \mathcal{A}$, the *credulous* (resp. *skeptical*) *acceptance* problem, denoted as CA_σ (resp. SA_σ), is deciding whether g belongs to *any* (resp. *every*) σ-extension of Λ. Clearly, $CA_{\mathtt{gr}}$ and $SA_{\mathtt{gr}}$ are identical problems.

2.2 Constrained AF

We briefly recall the Constrained Argumentation Framework (CAF) extending AF with constraints expressed by means of propositional formulae [Arieli, 2015; Alfano *et al.*, 2021b].

We use $\mathcal{L}_{\mathcal{A}}$ to denote the propositional language defined from a set of arguments $\mathcal{A}$ and the connectives $\wedge, \vee, \Rightarrow, \neg$.

Definition 1 *A* constraint *is a formula of one of the following forms: (i) $\varphi \Rightarrow v$, or (ii) $v \Rightarrow \varphi$, where φ is a propositional formula in $\mathcal{L}_{\mathcal{A}}$ and $v \in \{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$.*

The 3-valued semantics of a formula γ, denoted by $tv(\gamma)$ (truth value of γ), assuming $\neg\mathbf{u} = \mathbf{u}$ and the truth value ordering $\mathbf{f} < \mathbf{u} < \mathbf{t}$, is defined as follows: (i) $tv(v) = v$, for $v \in \{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$, (ii) $tv(\varphi \wedge \psi) = min(tv(\varphi), tv(\psi))$, (iii) $tv(\varphi \vee \psi) = max(tv(\varphi), tv(\psi))$, (iv) $tv(\neg\varphi) = \neg tv(\varphi)$. Regarding the implication operator $\Rightarrow$, different semantics have been defined. For instance, under Kleene's logic $tv(\varphi \Rightarrow \psi) = \neg tv(\varphi) \vee tv(\psi)$, whereas under Lukasiewicz's logic $tv(\varphi \Rightarrow \psi) = (\neg tv(\varphi) \vee tv(\psi)) \vee (tv(\varphi) = tv(\psi)))$. For boolean constraints (that is, with $v \in \{\mathbf{f}, \mathbf{t}\}$) Kleene and Lukasiewicz's logics coincide. A nice property of both Kleene and Lukasiewicz's logics is that literals can be moved from the head to the body (after negating them), and vice versa, analogously to the case of 2-valued semantics. For formulae defining constraints, Lukasiewicz's logic seems to be more appropriate as, for instance, it allows to distinguish $\varphi \Rightarrow \mathbf{f}$ from $\varphi \Rightarrow \mathbf{u}$.

In the following, we refer to the Lukasiewicz's logic and assume that the set of constraints is satisfiable, that is there is an assignment of truth values to arguments which makes all constraints true.

Example 3 The constraint $\mathtt{a} \wedge \mathtt{b} \wedge \mathtt{c} \Rightarrow \mathbf{f}$ states that at least one of the arguments a, b and c must be false, whereas $\mathtt{a} \wedge \mathtt{b} \wedge \mathtt{c} \Rightarrow \mathbf{u}$ states that a, b and c cannot be all true. □

Clearly, constraints of the forms $\mathbf{f} \Rightarrow \varphi$ and $\varphi \Rightarrow \mathbf{t}$ are useless because always satisfied. Regarding the stable semantics, which is 2-valued, only the symbols $\mathbf{f}$ and $\mathbf{t}$ can be used and all interpretations of the implication operator coincide with the classical 2-valued interpretation. Thus, a constraint $\varphi \Rightarrow \mathbf{u}$ is interpreted as $\varphi \Rightarrow \mathbf{f}$, whereas a constraint $\mathbf{u} \Rightarrow \psi$ is interpreted as $\mathbf{t} \Rightarrow \psi$.

Definition 2 *A* Constrained Argumentation Framework (CAF) *is a triple $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ where $\langle \mathcal{A}, \mathcal{R} \rangle$ is an AF and $\mathcal{C}$ is a set of constraints built from $\mathcal{L}_{\mathcal{A}}$.*

Definition 3 *Given a CAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ and a semantics $\sigma \in \{\mathtt{co}, \mathtt{gr}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}\}$, a set $S \subseteq \mathcal{A}$ is a σ-extension for $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ if S is a σ-extension for $\langle \mathcal{A}, \mathcal{R} \rangle$ and $S \models \mathcal{C}$.*

Note that, given a CAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$, if we consider the corresponding AF $\Lambda = \langle \mathcal{A}, \mathcal{R} \rangle$, then the set of complete extensions of Λ that satisfy $\mathcal{C}$ does not always form a complete meet-semilattice. Roughly speaking, the constraints may break the lattice by making unfeasible some extensions. Therefore, even the grounded extension is not guaranteed to exist.

Example 4 Consider the CAF $\langle \mathcal{A}_4 = \{\mathtt{a}\}, \mathcal{R}_4 = \{(\mathtt{a}, \mathtt{a}), \mathcal{C}_4 = \{\mathbf{t} \Rightarrow \mathtt{a}\} \rangle$. AF $\langle \mathcal{A}_4, \mathcal{R}_4 \rangle$ has only one complete extension, $E_1 = \emptyset$, but it does not satisfy the constraint stating that "a must be accepted". Thus, the CAF $\langle \mathcal{A}_4, \mathcal{R}_4, \mathcal{C}_4 \rangle$ has no complete extensions, and thus no grounded extension. □

2.3 Preference-based AF

Several extensions of Dung's framework for handling preferences over arguments have been proposed [Amgoud and Cayrol, 1998; 2002; Amgoud and Vesic, 2011; 2014; Cyras, 2016; Silva *et al.*, 2020].

Definition 4 *A* Preference-based Argumentation Framework (PAF) *is a triple $\langle \mathcal{A}, \mathcal{R}, > \rangle$ such that $\langle \mathcal{A}, \mathcal{R} \rangle$ is an AF and $>$ is a strict partial order (i.e. an irreflexive, asymmetric and transitive relation) over $\mathcal{A}$, called preference relation.*

For arguments a and b, $a > b$ means that a is better than b. To handle preferences, a "best extensions" semantics approach for PAF has been proposed in [Amgoud and Vesic, 2014; Kaci *et al.*, 2018]. Given a PAF $\langle \mathcal{A}, \mathcal{R}, > \rangle$, classical argumentation semantics are used to obtain extensions of the underlying AF $\langle \mathcal{A}, \mathcal{R} \rangle$, and then the preference relation $>$ is used to obtain a preference relation $\sqsupseteq$ over such extensions, so that the *best* extensions w.r.t. $\sqsupseteq$ are eventually selected.

Clearly, $\sqsupseteq$ is not trivial for multiple-status semantics only (for the grounded semantics, its extension is trivially the best one).

There have been different proposals to define the best extensions, corresponding to different criteria to define $\sqsupseteq$.

Definition 5 *Given a PAF $\langle \mathcal{A}, \mathcal{R}, > \rangle$, for $E, F \subseteq A$ with $E \neq F$, we have that under*

- democratic (d) *criterion [Amgoud and Vesic, 2014]: $E \sqsupseteq F$ if $\forall b \in F \setminus E\ \exists a \in E \setminus F$ such that $a > b$;*
- elitist (e) *criterion [Amgoud and Vesic, 2014]: $E \sqsupseteq F$ if $\forall a \in E \setminus F\ \exists b \in F \setminus E$ such that $a > b$;*
- KTV (k) *criterion [Kaci* et al.*, 2018]: $E \sqsupseteq F$ if $\forall a, b \in \mathcal{A}$ the relation $a > b$ with $a \in F \setminus E$ and $b \in E \setminus F$ does not hold.*

Moreover, $E \sqsupset F$, if $E \sqsupseteq F$ and $F \not\sqsupseteq E$.

Definition 6 *Given a PAF $\Delta = \langle \mathcal{A}, \mathcal{R}, > \rangle$, a semantics $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}\}$, and a criterion $* \in \{d, e, k\}$ for $\sqsupseteq$, the best σ-extensions of Δ under criterion $*$ (denoted as $\sigma_*(\Delta)$) are the extensions $E \in \sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ such that there is no $F \in \sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ with $F \sqsupset E$.*

Example 5 Consider the AF $\Lambda_5 = \langle \mathcal{A}_5, \mathcal{R}_5 \rangle$ shown in Figure 1 (center-right) whose preferred extensions are $\{\mathtt{a}, \mathtt{b}\}, \{\mathtt{c}, \mathtt{d}\}$, and $\{\mathtt{e}\}$. For PAF $\Delta_5 = \langle \mathcal{A}_5, \mathcal{R}_5, \{\mathtt{a} > \mathtt{c}, \mathtt{b} > \mathtt{c}, \mathtt{d} > \mathtt{e}\} \rangle$, the best preferred extensions are: $\mathtt{pr}_d(\Delta_5) = \{\{\mathtt{a}, \mathtt{b}\}, \{\mathtt{c}, \mathtt{d}\}\}$, $\mathtt{pr}_e(\Delta_5) = \{\{\mathtt{a}, \mathtt{b}\}, \{\mathtt{e}\}\}$, and $\mathtt{pr}_k(\Delta_5) = \{\{\mathtt{a}, \mathtt{b}\}\}$. □

It is worth noting that, in some situations, the democratic and elitist criteria may lead to counterintuitive solutions. Consider, for instance, the AF of Example 1 with the preference $\mathtt{meat} > \mathtt{fish}$. As discussed in the Introduction, it is expected that the best preferred extension is $E_3 = \{\mathtt{meat}, \mathtt{red}\}$. However, under both democratic and elitist criteria also $E_1 = \{\mathtt{fish}, \mathtt{white}\}$ is a best extension, because white and red are not compared with other arguments. In our opinion, democratic and elitist criteria are too restrictive and, for large AFs, may require a huge number of preferences to be effective. Moreover, for any PAF $\Delta = \langle \mathcal{A}, \mathcal{R}, > \rangle$, (i) $\mathtt{co}_d(\Delta) = \mathtt{pr}_d(\Delta)$, and (ii) $\mathtt{co}_e(\Delta) = \mathtt{gr}(\langle \mathcal{A}, \mathcal{R} \rangle)$ [Alfano *et al.*, 2022b]. This means that under democratic and elitist interpretation, PAF semantics does not extend AF semantics, as for any PAF $\langle \mathcal{A}, \mathcal{R}, \emptyset \rangle$, $\mathtt{co}_*(\Delta)$ may be different from $\mathtt{co}(\langle \mathcal{A}, \mathcal{R} \rangle)$, for $* \in \{d, e\}$.

Thus, in this paper we will introduce and study a preference-based AF which extends PAF with KTV criterion.

Complexity of PAF. The verification, credulous and skeptical acceptance problems for PAF under KTV criterion (denoted as Ver_{σ_k}, CA_{σ_k} and SA_{σ_k}, respectively) naturally extend those for AF by considering the best σ-extensions (instead of all extensions of the underlying AF). As shown in Table 1, the complexity of Ver_{σ_k} increases by one level in the polynomial hierarchy, and also the complexity of CA_{σ_k} and SA_{σ_k} may increase by one level w.r.t. the corresponding problems for AF [Alfano *et al.*, 2022b].

3 Extended Preference-based AF

In this section we introduce a new form of preference for AF and extend the PAF under the KTV criterion.

Definition 7 *Let $\mathcal{A}$ be a set of arguments, an (extended) preference relation, denoted as $\succ$, is a strict partial order (i.e. an irreflexive, asymmetric, and transitive relation) over $\mathcal{A}^V = \{a^v \mid a \in \mathcal{A} \wedge v \in \{\mathbf{f}, \mathbf{u}, \mathbf{t}\}\}$ of the form $a^{v_1} \succ b^{v_2}$.*

Intuitively, we allow defining preference between pairs, where each pair consists of an argument and a truth value in $\{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$, denoting *false*, *undefined*, and *true* truth values, and corresponding to the following statuses of arguments: defeated, undecided, and accepted respectively. [1]

For instance, considering the AF of Example 2, a preference $\mathtt{red}^\mathbf{t} \succ \mathtt{red}^\mathbf{u}$ means that we prefer menus containing red wine w.r.t. menus where red wine is undecided, whereas a preference $\mathtt{fish}^\mathbf{t} \succ \mathtt{red}^\mathbf{f}$ states that we prefer menus containing fish w.r.t. menus where red is false (i.e. defeated).

Definition 8 *An extended PAF (ePAF) is a triple $\langle \mathcal{A}, \mathcal{R}, \succ \rangle$ where $\langle \mathcal{A}, \mathcal{R} \rangle$ is an AF and $\succ$ is an extended preference relation.*

The following definition introduces the semantics of ePAF.

Definition 9 *Given an ePAF $\Delta = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ and two distinct sets of arguments $E, F \subseteq \mathcal{A}$, we have that $E \sqsupseteq F$ under* KTV *(k) criterion if $\nexists\ a^{v_1} \succ b^{v_2}$ such that $a \in v_1(F) \setminus v_1(E)$, $b \in v_2(E) \setminus v_2(F)$ holds (where $v_1, v_2 \in \{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$). Moreover, $E \sqsupset F$, if $E \sqsupseteq F$ and $F \not\sqsupseteq E$.*

Thus best extensions (under *KTV* (k) criterion) are defined as for PAF but using the criterion of Definition 9 to compare extensions. That is, given an ePAF $\Delta = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ and $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}\}$, an extension $E \in \sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ is a best extension for Δ if there is no extension $F \in \sigma(\langle \mathcal{A}, \mathcal{R} \rangle)$ such that $F \sqsupset E$. The set of best σ-extensions for an ePAF Δ under KTV criterion is denoted by $\sigma_k(\Delta)$.

Example 6 Consider the AF of Example 1 under the complete semantics. There are six complete extensions: $E_0 = \emptyset$, $E_1 = \{\mathtt{fish}, \mathtt{white}\}$, $E_2 = \{\mathtt{fish}, \mathtt{red}\}$, $E_3 = \{\mathtt{meat}, \mathtt{red}\}$, $E_4 = \{\mathtt{fish}\}$ (with white and red undecided), and $E_5 = \{\mathtt{red}\}$ (with fish and meat undecided).

Assume that there are the following preferences: $x^\mathbf{t} \succ x^\mathbf{u}$ and $x^\mathbf{t} \succ x^\mathbf{f}$, for every argument x. Then, the best complete extensions are E_1, E_2 and E_3 (which are the preferred ones).

If we also have the preference $\mathtt{fish}^\mathbf{t} \succ \mathtt{meat}^\mathbf{t}$, then the best complete extensions are E_1 and E_2. □

As stated next, ePAF generalizes PAF with KTV criterion.

Proposition 1 *Let $\Delta = \langle \mathcal{A}, \mathcal{R}, \succ \rangle$ be an ePAF and $\Delta' = \langle \mathcal{A}, \mathcal{R}, > \rangle$ be a PAF such that $\succ = \{a^\mathbf{t} \succ b^\mathbf{t} \mid a > b \text{ in } \Delta'\}$ and $> = \{a > b \mid a^\mathbf{t} \succ b^\mathbf{t} \text{ in } \Delta\}$. Then, $\sigma_k(\Delta) = \sigma_k(\Delta')$ for $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{ss}\}$.*

Moreover, AF semantics can be easily expressed in ePAF in terms of best complete extensions.

Proposition 2 *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF, $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{ss}\}$ a semantics, and the ePAF*

$$\Delta_\sigma = \begin{cases} \langle \mathcal{A}, \mathcal{R}, \{a^\mathbf{u} \succ a^\mathbf{t} \mid a \in \mathcal{A}\} \rangle & \text{if } \sigma = \mathtt{gr} \\ \langle \mathcal{A}, \mathcal{R}, \emptyset \rangle & \text{if } \sigma = \mathtt{co} \\ \langle \mathcal{A}, \mathcal{R}, \{a^\mathbf{t} \succ a^\mathbf{u},\ a^\mathbf{t} \succ a^\mathbf{f} \mid a \in \mathcal{A}\} \rangle & \text{if } \sigma = \mathtt{pr} \\ \langle \mathcal{A}, \mathcal{R}, \{a^\mathbf{t} \succ a^\mathbf{u},\ a^\mathbf{f} \succ a^\mathbf{u} \mid a \in \mathcal{A}\} \rangle & \text{if } \sigma = \mathtt{ss}. \end{cases}$$

[1] Instead of notation $a^\mathbf{t}$ (resp. $a^\mathbf{f}$, $a^\mathbf{u}$), we could have used the labelling notation $\mathsf{in}(a)$ (resp. $\mathsf{out}(a)$, $\mathsf{undec}(a)$) [Caminada, 2006].

	AF			CAF			PAF			ePAF / ePCAF / mPCAF		
σ	Ver_σ	CA_σ	SA_σ	Ver_σ	CA_σ	SA_σ	Ver_{σ_k}	CA_{σ_k}	SA_{σ_k}	Ver_{σ_k}	CA_{σ_k}	SA_{σ_k}
co	P	NP-c	P	P	NP-c	coNP-c	coNP-c	Σ_2^p-c	P	coNP-c	Σ_2^p-c	Π_2^p-c
st	P	NP-c	coNP-c	P	NP-c	coNP-c	coNP-c	Σ_2^p-c	Π_2^p-c	coNP-c	Σ_2^p-c	Π_2^p-c
pr	coNP-c	NP-c	Π_2^p-c	coNP-c	Σ_2^p-c	Π_2^p-c	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p
ss	coNP-c	Σ_2^p-c	Π_2^p-c	coNP-c	Σ_2^p-c	Π_2^p-c	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p	Π_2^p-c	Σ_2^p-h, Σ_3^p	Π_2^p-h, Π_3^p

Table 1: Complexity of the verification (Ver) and credulous (CA) and skeptical (SA) acceptance problems under complete (co), stable (st), preferred (pr), and semi-stable (ss) semantics. For any complexity class C, C-c (resp., C-h) means C-complete (resp., C-hard). An interval C-h, C' means C-hard and in C'. For mPCAF, as the complexities of $Ver^{\exists}_{\sigma_k}$ and $Ver^{\forall}_{\sigma_k}$ (resp. $CA^{\exists}_{\sigma_k}$ and $CA^{\forall}_{\sigma_k}$, $SA^{\exists}_{\sigma_k}$ and $SA^{\forall}_{\sigma_k}$) coincide, we use the notation Ver_{σ_k} (resp. CA_{σ_k}, SA_{σ_k}) for both problems. New results are highlighted in grey.

Then, it holds that $\sigma(\langle\mathcal{A},\mathcal{R}\rangle) = \mathtt{co}_k(\Delta_\sigma)$.

An encoding for the stable semantics, that may admit no extensions, is given in Section 4 where we characterize the stable semantics in a simple way by using constraints.

3.1 Complexity of ePAF

The verification, credulous and skeptical acceptance problems for ePAF under KTV criterion (denoted as Ver_{σ_k}, CA_{σ_k} and SA_{σ_k}, respectively) are defined as for PAF but considering the best extensions for ePAF (i.e. according Definition 9). The next theorem states the complexity of these problems.

Theorem 1 *For ePAF, the problem:*

- Ver_{σ_k} *is i) co*NP*-complete for* $\sigma \in \{\mathtt{co},\mathtt{st}\}$*;*
  ii) Π_2^p*-complete for* $\sigma \in \{\mathtt{pr},\mathtt{ss}\}$*;*
- CA_{σ_k} *is i)* Σ_2^p*-complete for* $\sigma \in \{\mathtt{co},\mathtt{st}\}$*;*
  ii) Σ_2^p*-hard and in* Σ_3^p *for* $\sigma \in \{\mathtt{pr},\mathtt{ss}\}$*;*
- SA_{σ_k} *is i)* Π_2^p*-complete for* $\sigma \in \{\mathtt{co},\mathtt{st}\}$*;*
  ii) Π_2^p*-hard and in* Π_3^p *for* $\sigma \in \{\mathtt{pr},\mathtt{ss}\}$.

Thus, the complexity bounds of verification, credulous acceptance and skeptical acceptance for ePAF do not increase w.r.t. those of PAF under KTV semantics, except for skeptical acceptance under complete semantics that becomes Π_2^p-complete (cf. Table 1). Although the form of preference introduced is more flexible than that of PAF, the complexity in most of the cases does not increase.

The following example shows a case where ePAF is used to express preferences not allowed in PAF.

Example 7 Consider the AF Λ_2 of Example 2 shown in Figure 1 (center-left). The PAF preference $\mathtt{red} > \mathtt{white}$ does not allow to restrict the set of extensions and all complete (resp. preferred) extensions are also the best ones. However, the ePAF preference $\mathtt{red}^{\mathbf{t}} \succ \mathtt{red}^{\mathbf{u}}$ allow us to select as best complete (resp. preferred) extension E_2 only. □

Next, we combine extended preferences and constraints and show that the resulting framework, other than offering a compact and easier representation of both preferences and constraints, is also more expressive than both CAF and PAF.

4 Combining Preferences with Constraints

We now extend CAF with (extended) preferences to express several kinds of desiderata among extensions, and propose a novel framework called *extended Preference-based Constrained Argumentation Framework.*

Definition 10 *An* extended Preference-based Constrained Argumentation Framework *(ePCAF) is a tuple* $\Delta = \langle\mathcal{A},\mathcal{R},\mathcal{C},\succ\rangle$*, where* $\langle\mathcal{A},\mathcal{R},\mathcal{C}\rangle$ *is a CAF and* $\succ$ *is an (extended) preference relation (cf. Definition 7).*

The semantics of an ePCAF is given by the best extensions selected among those that satisfy the constraints.

Definition 11 *Given an ePCAF* $\Delta = \langle\mathcal{A},\mathcal{R},\mathcal{C},\succ\rangle$ *and a semantics* $\sigma \in \{\mathtt{co},\mathtt{pr},\mathtt{st},\mathtt{ss}\}$*, a* σ*-extension* E *for* $\langle\mathcal{A},\mathcal{R},\mathcal{C}\rangle$ *is a best* σ*-extension for* Δ *under* KTV *criterion if there is no* σ*-extension* F *for* $\langle\mathcal{A},\mathcal{R},\mathcal{C}\rangle$ *such that* $F \sqsupset E$.

Example 8 Continuing with Example 1, consider the ePCAF $\Delta_8 = \langle\mathcal{A}_8,\mathcal{R}_8,\{\mathtt{white} \Rightarrow \mathbf{f}\},\{\mathtt{meat}^{\mathbf{t}} \succ \mathtt{fish}^{\mathbf{t}}\}\rangle$, where $\langle\mathcal{A}_8,\mathcal{R}_8\rangle$ is the AF in Figure 1 (left). The preferred extensions for AF Λ_8 are $E_1 = \{\mathtt{fish},\mathtt{white}\}$, $E_2 = \{\mathtt{fish},\mathtt{red}\}$ and $E_3 = \{\mathtt{meat},\mathtt{red}\}$. As white must be false, there are only two preferred extensions satisfying the constraint: E_2 and E_3. Then, the only best preferred extension is E_3. □

It is worth noting that, the best extensions would have been different if we had defined the ePCAF $\Delta = \langle\mathcal{A},\mathcal{R},\mathcal{C},\succ\rangle$ as an ePAF $\langle\mathcal{A},\mathcal{R},\succ\rangle$ with a set of constraints $\mathcal{C}$. Indeed, in such a case, the σ-extensions for Δ would have been as the best σ-extensions of $\langle\mathcal{A},\mathcal{R},\succ\rangle$ satisfying constraints $\mathcal{C}$, that is constraints would have been applied after preferences.

We now extend the set of relationships provided in Proposition 2 by showing that the stable semantics of an ePAF can be expressed as the best complete extensions of an ePCAF.

Proposition 3 *For any ePAF* $\langle\mathcal{A},\mathcal{R},\succ\rangle$*, it holds that* $\mathtt{st}_k(\langle\mathcal{A},\mathcal{R},\succ\rangle) = \mathtt{co}_k(\langle\mathcal{A},\mathcal{R},\{a \wedge \neg a \Rightarrow \mathbf{f} \mid a \in \mathcal{A}\},\succ\rangle)$.

Observe that if in the proposition the set of preferences $\succ$ is empty, then the ePAF is an AF and the ePCAF is a CAF.

4.1 Complexity of ePCAF

Before characterizing the complexity of ePCAF, we provide new results concerning the complexity of CAF. Although these results are of independent interest, they are also useful to provide lower bounds on the complexity of ePCAF and to compare the two frameworks from a complexity standpoint.

As observed after Definition 3, the presence of constraints in CAF breaks the meet-semilattice of complete extensions. This entails that the credulous and skeptical acceptance of an argument w.r.t a CAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ may differ from that of the associated AF $\langle \mathcal{A}, \mathcal{R} \rangle$. For instance, the fact that complete extensions may not exist for CAF (cf. Example 4) impacts on the complexity of the skeptical acceptance problem under complete semantics, which cannot be longer decided by simply looking at the grounded extension as for the case of AF (where an argument is skeptically accepted under complete semantics if and only if it is in the grounded extension).

A summary of known results for the complexity of CAF is reported in Table 1 (see cells with white background). The complexity of the verification problem has not been addressed so far. Moreover, an open question is whether credulous acceptance under preferred semantics for CAFs is harder than for AF (where it can be decided by checking credulous acceptance under complete semantics). Indeed, it is known that the complexity of $CA_{\mathtt{pr}}$ is NP-hard and in Σ_2^p [Alfano *et al.*, 2021b]. In all the other cases the complexity of credulous and skeptical reasoning for CAFs and AFs coincides.

As stated next, the complexity of the verification problem for CAF remains the same as that for AF.

Proposition 4 *For CAF, the problem Ver_σ is in P for any semantics $\sigma \in \{\mathtt{co}, \mathtt{st}\}$ and coNP-complete for $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$.*

We now provide a tight characterization of the complexity of $CA_{\mathtt{pr}}$ for CAF, showing that it is harder than for AF. The result follows from the fact we can write some constraints enabling a reduction to our problem from the complement of deciding coherence [Dunne and Bench-Capon, 2002].

Theorem 2 *For CAF, the problem $CA_{\mathtt{pr}}$ is Σ_2^p-complete.*

We are now ready to address the complexity of ePCAF.

Given an ePCAF Δ and a set S of arguments, the *verification* problem under KTV criterion (denoted as Ver_{σ_k}) is deciding whether $S \in \sigma_k(\Delta)$. Moreover, given an argument g, the *credulous* and *skeptical acceptance* problems (denoted as CA_{σ_k} and SA_{σ_k}) are the problems of deciding whether g belongs to any/every σ_k-extension of Δ, respectively.

Theorem 3 *For ePCAF, the problem:*

- *Ver_{σ_k} is i) coNP-complete for $\sigma \in \{\mathtt{co}, \mathtt{st}\}$;*
  ii) Π_2^p-complete for $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$;
- *CA_{σ_k} is i) Σ_2^p-complete for $\sigma \in \{\mathtt{co}, \mathtt{st}\}$;*
  ii) Σ_2^p-hard and in Σ_3^p for $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$;
- *SA_{σ_k} is i) Π_2^p-complete for $\sigma \in \{\mathtt{co}, \mathtt{st}\}$;*
  ii) Π_2^p-hard and in Π_3^p for $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$.

Thus ePCAF is generally more expressive than CAF, particularly if we consider the verification problem whose complexities increase of one level in the polynomial hierarchy for all considered semantics. Also, it turns out that ePCAF has the same complexity bounds as PAF, except for the $SA_{\mathtt{co}_k}$ problem, similarly to what we have observed for ePAF.

5 Dealing with Multiple Agents

Often in knowledge representation and reasoning using argumentation-based frameworks it is assumed that the AF represents the objective knowledge, while constraints and preferences are subjective knowledge of users/agents that are used for collective decision-making. We now extend our framework by considering the case of multiple agents sharing the same AF and having different constraints and preferences (represented by different ePCAFs) that are taken into account to decide for instance the acceptance of a given argument.

Definition 12 *A multi-agent ePCAF (mPCAF) is a set of ePCAFs $\{\langle \mathcal{A}, \mathcal{R}, \mathcal{C}_1, \succ_1 \rangle, \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_2, \succ_2 \rangle, ..., \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_n, \succ_n \rangle\}$, one for each agent $i \in [1, n]$.*

Thus, we assume to have n (distinct) agents and that each agent i has a set of constraints $\mathcal{C}_i$ and a preference relation $\succ_i$. For each agent i we have ePCAF $\Delta_i = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_i, \succ_i \rangle$, and the best σ-extensions for agent i are those in $\sigma(\Delta_i)$.

Definition 13 *Let $\Delta = \{\Delta_1 = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_1, \succ_1 \rangle, ..., \Delta_n = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_n, \succ_n \rangle\}$ be an mPCAF and $\sigma \in \{\mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{ss}\}$ a semantics. Then, a set of arguments $S \subseteq \mathcal{A}$ is said to be a* possible *(resp.* necessary*) best σ-extension of Δ (under* KTV *criterion) iff $S \in \sigma_k(\Delta_i)$ for some (resp. every) $i \in [1, n]$.*

Example 9 Consider the AF $\Lambda_9 = \langle \mathcal{A}_9, \mathcal{R}_9 \rangle$ shown in Figure 1 (right) that extends that of Example 1. It has four preferred extensions: $E_1 = \{\mathtt{fish}, \mathtt{white}, \mathtt{icec}\}$, $E_2 = \{\mathtt{fish}, \mathtt{white}, \mathtt{cake}\}$, $E_3 = \{\mathtt{fish}, \mathtt{red}, \mathtt{cake}\}$, and $E_4 = \{\mathtt{meat}, \mathtt{red}, \mathtt{cake}\}$, where icec denotes ice cream. Assume to have 2 agents α and β whose constraints and preferences are as follows: $(\mathcal{C}_\alpha = \{\mathtt{t} \Rightarrow \mathtt{fish}\}, \succ_\alpha = \{\mathtt{white} \succ \mathtt{red}\})$; $(\mathcal{C}_\beta = \{\mathtt{t} \Rightarrow \mathtt{cake}\}, \succ_\beta = \{\mathtt{fish} \succ \mathtt{meat}\})$. Thus, for agent α the best preferred extensions are E_1 and E_2, whereas for agent β the best preferred extensions are E_2 and E_3. Hence, E_1, E_2 and E_3 are possibly best preferred extensions of the mPCAF $\Delta_9 = \{\Delta_\alpha = \langle \mathcal{A}_9, \mathcal{R}_9, \mathcal{C}_\alpha, \succ_\alpha \rangle, \Delta_\beta = \langle \mathcal{A}_9, \mathcal{R}_9, \mathcal{C}_\beta, \succ_\beta \rangle\}$ while only E_2 is a necessary best preferred extension of mPCAF Δ. □

We have two variants of the verification problem for mPCAF. Given an mPCAF $\Delta = \{\Delta_1 = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_1, \succ_1 \rangle, \dots, \Delta_n = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_n, \succ_n \rangle\}$ and a set $S \subseteq \mathcal{A}$ of arguments, the *possible* (resp. *necessary*) *verification* problem, denoted as $Ver^{\exists}_{\sigma_k}$ (resp. $Ver^{\forall}_{\sigma_k}$), is the problem of deciding whether S is possible (resp. necessary) best σ-extension of Δ.

Analogously, two variants of the credulous and skeptical acceptance problems are defined in what follows.

Definition 14 *Let $\Delta = \{\Delta_1 = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_1, \succ_1 \rangle, \dots, \Delta_n = \langle \mathcal{A}, \mathcal{R}, \mathcal{C}_n, \succ_n \rangle\}$ be an mPCAF, k the KTV criterion, and $\sigma \in \{\mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{ss}\}$ a semantics. Then, $g \in \mathcal{A}$ is said to be:*

- possibly credulously accepted *under σ_k, denoted as $CA^{\exists}_{\sigma_k}(\Delta, g)$, iff $\exists i \in [1, n]$, $\exists E \in \sigma_k(\Delta_i)$ such that $g \in E$;*
- possibly skeptically accepted *under σ_k, denoted as $SA^{\exists}_{\sigma_k}(\Delta, g)$, iff $\exists i \in [1, n]$ s.t. $g \in E$ for all $E \in \sigma_k(\Delta_i)$;*
- necessarily *credulously accepted under σ_k, denoted as $CA^{\forall}_{\sigma_k}(\Delta, g)$, iff $\forall i \in [1, n]$, $\exists E \in \sigma_k(\Delta_i)$ such that $g \in E$;*
- necessarily *skeptically accepted under σ_k, denoted as $SA^{\forall}_{\sigma_k}(\Delta, g)$, iff $\forall i \in [1, n]$, $g \in E$ for all $E \in \sigma_k(\Delta_i)$.*

We now investigate the complexity of the verification and credulous and skeptical acceptance problems in mPCAF. It is worth noting that we are assuming that the number of agents is an arbitrary but fixed number n.

Interestingly, the verification problem in mPCAF is not harder than in PAF, ePAF, and ePCAF. Moreover, an analogous result also holds for the credulous and skeptical acceptance problems, that is, the complexity bounds for mPCAF acceptance problems do not increase w.r.t. those of ePCAF (which coincide with those for ePAF, cf. Table 1).

Theorem 4 *For mPCAF, the problems*

- $Ver^{\exists}_{\sigma_k}$ *and* $Ver^{\forall}_{\sigma_k}$ *are i) co*$\mathbb{NP}$*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*; ii)* Π^p_2*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$*;*
- $CA^{\exists}_{\sigma_k}$ *and* $CA^{\forall}_{\sigma_k}$ *are i)* Σ^p_2*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*; ii)* Σ^p_2*-hard and in* Σ^p_3 *for* $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$*;*
- $SA^{\exists}_{\sigma_k}$ *and* $SA^{\forall}_{\sigma_k}$ *are i)* Π^p_2*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*; ii)* Π^p_2*-hard and in* Π^p_3 *for* $\sigma \in \{\mathtt{pr}, \mathtt{ss}\}$*.*

We conclude this section by observing that reasoning in a context of distributed and conflicting pieces of information underlies many questions related to the field of collective/multi-criteria decisions that are not in the scope of this paper. We have shown that the complexity in the multiple-agent scenarios considered does not increase w.r.t. that of the single-agent one. Our results may be useful for more refined approaches to be explored in future work.

6 Related Work

Different approaches have been proposed to handle preferences in argumentation. A first approach considers AF-based semantics and consists in first defining a defeat relation $\mathcal{R}_i$ that combines original attacks in $\mathcal{R}$ and preference relations, and then apply the usual semantics to the resulting AF $\langle \mathcal{A}, \mathcal{R}_i \rangle$. Here $\mathcal{R}_i$ (with $i \in [1, 4]$) denotes one of the four mappings proposed in the literature [Amgoud and Cayrol, 2002; Amgoud and Vesic, 2014; Kaci *et al.*, 2018]. As mentioned in the Introduction, in some cases these semantics may give counterintuitive results, as for instance the extensions of the rewritten AF are not conflict-free w.r.t. the initial AF. We point out that, in this setting, the complexity of the verification and acceptance problems does not increase as the mapping to AF (i.e. building $\mathcal{R}_i$) is polynomial time. As discussed in Section 2.3, a second approach to handle preferences considers a "best extensions" semantics for PAF [Amgoud and Vesic, 2014; Kaci *et al.*, 2018],

An alternative definition for PAF, based on that defined in [Sakama and Inoue, 2000] for logic programs with preferences, has been proposed in [Wakaki, 2015]. In this context a PAF is a triple $\langle \mathcal{A}, \mathcal{R}, \geq \rangle$, where $\geq$ is a reflexive and transitive relation and $a > b$ if $a \geq b$ and $b \not\geq a$. Moreover, the preference relation $\sqsupseteq$ over extensions is reflexive ($E \sqsupseteq E$), transitive ($E \sqsupseteq F$ and $F \sqsupseteq G$ implies $E \sqsupseteq G$) and satisfies the condition $E \sqsupseteq F$ if $\exists a \in E \setminus F, \exists b \in F \setminus E$ such that $a \geq b$ and $\nexists c \in F \setminus E$ such that $c > a$. In this paper we have only dealt with preferences relation $\sqsupseteq$ that is not transitive as our proposal is intended to extend PAF, where $\sqsupseteq$ is not transitive for all the criteria of Definition 5 (e.g. KTV).

Constrained AF has been firstly proposed in [Coste-Marquis *et al.*, 2006] and then further investigated in [Arieli, 2015; Alfano *et al.*, 2021b]. A drawback of the semantics proposed in [Coste-Marquis *et al.*, 2006] is that, in checking whether an extension satisfies a set of constraints, it does not distinguish between false and undecided arguments. Thus, a constraint of the form $a \wedge \neg a \Rightarrow \mathbf{f}$ is always satisfied, even when a is undecided. The main difference between the approaches in [Arieli, 2015] and [Alfano *et al.*, 2021b] is in the underlying logic. Indeed, while the approach in [Arieli, 2015] is based on Slupecki's logic, interpreting the implication as $tv(\varphi \Rightarrow \psi) = (tv(\varphi) \neq \mathbf{t}) \vee tv(\varphi \wedge \psi)$, the approach in [Alfano *et al.*, 2021b] is based on Lucasiewitcz's logic. A drawback of Arieli's semantics is that it does not distinguish between constraints of the form $\varphi \Rightarrow \mathbf{f}$ and $\varphi \Rightarrow \mathbf{u}$, though it distinguishes between constraints $\mathbf{t} \Rightarrow \varphi$ and $\mathbf{u} \Rightarrow \varphi$.

It is worth mentioning that 3-valued logics [Lukasiewicz, 1920; Kleene, 2009; Berry, 1952] has found application in various research areas, such as in 3-state logical circuits, databases (e.g. well-founded semantics for Datalog [Gelder *et al.*, 1991] and the so-called contension inconsistency measure for databases [Parisi and Grant, 2023]), logic programming (e.g. partial stable models and defeasible logic programming [Alfano *et al.*, 2018; 2021a]), as well as in interpreting null values in SQL [Libkin, 2016].

In *value-based argumentation framework* (VAF) an argument a defeats b only if the value promoted for b is not preferred to that promoted for a (according to a total ordering on values given by an audience) [Bench-Capon *et al.*, 2005]. Moreover, VAF has been extended by incorporating constraints expressed by propositional formulas on arguments' values or arguments, resulting in the *constrained VAF* [Sedki and Yahi, 2016]. Finally, with the aim of generating more skeptically accepted arguments, the idea of comparing extensions and choosing the best ones has been explored in [Konieczny *et al.*, 2015; Bonzon *et al.*, 2018]. Recently, comparing sets of arguments is studied in [Skiba *et al.*, 2021] to identify sets "closed" to become an extension.

To the best of our knowledge, this is the first paper combining both 3-valued constraints and 3-valued preferences (i.e. extended preferences) in AF and providing a thorough investigation of the complexity of the resulting framework.

7 Conclusions and Future Work

Extended preferences and (3-valued) constraints as well as the complexity results for the novel frameworks (ePAF, ePCAF and mPCAF) can carry over to other AF-based frameworks such as (Bipolar) Argumentation Frameworks with necessities (AFN) [Nouioua and Risch, 2011], Argumentation Framework with recursive attacks (AFRA) [Baroni *et al.*, 2011], Recursive Attacks and Supports in Argumentation Framework (ASAF) [Gottifredi *et al.*, 2018], and Recursive Argumentation Framework with Necessity Supports (RAFN) [Cayrol *et al.*, 2018] without collective attacks and supports. Indeed, as these frameworks can be rewritten into AF [Alfano *et al.*, 2020], their extended Preference-based Constrained forms could be rewritten in ePCAF, obtaining upper bounds on their complexity from our results. Lower bounds also follow if those frameworks generalize ePCAF.

As future work, we plan to investigate preferences and constraints in other frameworks extending AF [Baumeister *et al.*, 2021; Alfano *et al.*, 2022a; Fazzinga *et al.*, 2015; 2022; 2023; Alfano *et al.*, 2023c], as well as other forms of constraints such as weak and epistemic constraints [Alfano *et al.*, 2021b; 2023b; Sakama and Son, 2020].

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